PDP assignment

November 10, 2018

Importing Required Libraries

```
In [64]: import numpy as np
         import matplotlib.pyplot as plt
```

Defining Function to get F and Z per iteration based on X.

```
In [53]: def get_F(x):
             f = [0]*5
             f[0] = x[0]+x[1]+x[2]-13.5
             f[1] = 2*x[3]+4*x[0]+2*x[4]-227
             f[2] = x[4]+x[1]+2*x[2]-86.5
             f[3] = x[2]*x[3]/(x[1]*x[4])-0.929
             f[4] = ((12.2/100)**2)*x[1]*(x[4]**3)/(x[0]*x[4])-126.8

             return f
```

```
In [54]: def get_Z(x):
             z = [[1, 1, 1, 0, 0],
                  [4, 0, 0, 2, 2],
                  [0, 1, 2, 0, 1],
                  [0,
                   -(x[2]*x[3]/(x[1]**2*x[4])),
                    x[3]/(x[1]*x[4]),
                    x[2]/(x[1]*x[4]),
                   -(x[2]*x[3]/(x[1]*x[4]**2))],
                  [-(x[1]*x[3]**3/(x[0]*x[4]**2)*0.014884),
                    x[3]**3/(x[0]*x[4])*0.014884,
                    0,
                    3*0.014884*x[1]*x[3]**2/(x[0]*x[4]),
                   -(x[1]*x[3]**3/(x[0]*x[4]**2)*0.014884)]]

             return z
```

Initialize *X_0* as 29, 29, 29, 29, 29 for initial step. Running the loop for 20 iterations based on the system of non-linear equations obtained. Plot shows convergence of each variable over 20 iterations.

```
In [104]: x = [29]*5
          f = get_F(x)
          z = get_Z(x)
          c = []
          iter = []

          for i in range(20):

              z_inv = np.linalg.inv(z)
              iter.append(i)

              print("iter", i+1, "\n======= \n")
              print("X\n", x, "\n\n")
              print("Z\n", z, "\n\n")
              print("Z_inv\n", z_inv, "\n\n")
              print("F\n", f, "\n\n")
              print("=====================================================\n\n")

              x_ = np.matmul(z_inv, ( np.negative(f) + np.matmul(z, x)))
              x = x_
              f = get_F(x_)
              z = get_Z(x_)
              c.append(x_)

          print(len(iter), len(c[0]))
          plt.figure(figsize=(20,10))
          plt.plot(iter, [i[0] for i in c], [i[1] for i in c])
          plt.plot(iter, [i[2] for i in c], [i[3] for i in c])
          plt.plot(iter, [i[4] for i in c])
          plt.show()

iter 1
========

X
 [29, 29, 29, 29, 29]


Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -0.034482758620689655, 0.03448275862068

Z_inv
 [[  0.43589744    0.11538462   -0.25641026    2.23076923   -0.23761712]
 [  0.48717949   -0.07692308   -0.05128205  -11.15384615    0.41582996]
 [  0.07692308   -0.03846154    0.30769231    8.92307692   -0.17821284]
 [ -0.23076923    0.11538462    0.07692308    2.23076923    0.53463852]
 [ -0.64102564    0.15384615    0.43589744   -6.69230769   -0.05940428]]
```

F
  [73.5, 5, 29.5, 0.07099999999999995, -114.282556]

==

iter 2
========

X
  [-23.36515832 43.40377706 -6.53861874 104.05685622 56.17346042]

Z
  [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, 0.006429400640308589, 0.042678780203801

Z_inv
  [[4.69418913e-01 1.03086783e-01 -3.51625750e-01 5.47889572e+00
    1.19760613e-02]
  [5.95978400e-01 -1.26240669e-01 5.38151434e-01 -3.91829678e+01
    -9.21909368e-03]
  [-6.53973136e-02 2.31538857e-02 -1.86525684e-01 3.37040720e+01
    -2.75696763e-03]
  [-4.73654053e-01 2.13893536e-01 -1.31648435e-01 1.72673849e+01
    -3.86851516e-02]
  [-4.65183773e-01 7.99328974e-02 8.34899935e-01 -2.82251763e+01
    1.47330289e-02]]

F
  [-3.552713678800501e-15, 2.842170943040401e-14, -1.4210854715202004e-14, -1.208060272010782, -2

==

iter 3
========

X
  [-14.18290538 -5.90491103 33.58781641 116.63653254 25.22927821]

Z

`[[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -4.453329485283424, -0.7829182485093283`

Z_inv
```
[[ 0.7709753   0.04046133 -0.32067435  0.16556851 -0.00434885]
 [-0.17162702  0.01655853  0.02456656 -0.15645811 -0.00682259]
 [ 0.40065171 -0.05701985  0.29610779 -0.0091104   0.01117144]
 [-0.9122742   0.32159617  0.25813084 -0.50581593  0.02421798]
 [-0.62967641  0.09748118  0.38321786  0.17467891 -0.01552029]]
```

F
`[-2.842170943040401e-14, 8.526512829121202e-14, 0.0, -27.225514393145477, -122.85563164333497]`

==

iter 4
========

X
`[-10.20949788 -11.00275744  34.71225532 105.84074898  28.07824679]`

Z
`[[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -1.0808431424516238, -0.342595282682517`

Z_inv
```
[[ 8.20779094e-01  3.87669564e-02 -3.21491277e-01  5.18969611e-01
  -1.00123720e-03]
 [-3.41212838e-01  3.63904089e-02  4.41748188e-02 -7.38081384e-01
  -8.11048760e-03]
 [ 5.20433744e-01 -7.51573652e-02  2.77316458e-01  2.19111773e-01
   9.11172481e-03]
 [-9.41903537e-01  3.08541766e-01  2.41790288e-01 -1.33779706e+00
   1.21154364e-02]
 [-6.99654650e-01  1.13924322e-01  4.01192265e-01  2.99857838e-01
  -1.01129620e-02]]
```

F
`[1.4210854715202004e-14, 0.0, 1.4210854715202004e-14, -12.82125492460986, -114.15389479225786]`

==

```
iter 5
========

X
 [ -3.66995134 -21.39173076  38.5616821    90.0715361   30.76836657]


Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -0.246687560730117, -0.1368476060700998


Z_inv
 [[ 9.75886636e-01  2.45900561e-02 -3.41644045e-01  2.13813420e+00
    1.10871262e-03]
 [-8.31159085e-01  9.75503577e-02  1.15675365e-01 -4.38303871e+00
  -6.58472709e-03]
 [ 8.55272449e-01 -1.22140414e-01  2.25968680e-01  2.24490452e+00
    5.47601447e-03]
 [-1.07238746e+00  3.04089418e-01  2.50900815e-01 -4.16949807e+00
    2.14987662e-03]
 [-8.79385812e-01  1.46730470e-01  4.32387275e-01 -1.06770320e-01
  -4.36730186e-03]]


F
 [1.4210854715202004e-14, 0.0, 1.4210854715202004e-14, -6.206073880816822, -44.667666544990226]


============================================================


iter 6
========

X
 [  9.64899106 -48.88731723  52.73832617  64.29135298  29.91066489]


Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -0.047430797771553436, -0.0439673502299


Z_inv
 [[ 3.29951793e+00 -2.59266230e-01 -8.91816129e-01  3.44775307e+01
  -5.63606114e-02]
 [-7.82294490e+00  9.71922576e-01  1.74847425e+00 -9.83911104e+01
    1.75684445e-01]
```

```
[ 5.52342698e+00 -7.12656346e-01 -8.56658117e-01  6.39135797e+01
  -1.19323833e-01]
[-3.37512680e+00  5.65142345e-01  8.18790269e-01 -3.95190124e+01
   4.97580008e-02]
[-3.22390905e+00  4.53390116e-01  9.64841989e-01 -2.94360490e+01
   6.29632220e-02]]
```

F
 [0.0, 0.0, 0.0, -3.2477644573455473, -194.2661763049254]

==

iter 7
========

X
 [110.67492942 -334.30892325 237.13399383 -54.39079442 -53.45906441]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -0.002158747891472313, -0.0030433792789

Z_inv
```
[[ 1.43872382e-01  1.42992966e-01 -8.64475817e-03  4.15928657e+01
  -1.12254712e-01]
 [ 1.52260868e+00 -1.69238999e-01 -1.00845009e+00 -1.62415345e+02
   3.34072061e-01]
 [-6.66481062e-01  2.62460335e-02  1.01709485e+00  1.20822480e+02
  -2.21817349e-01]
 [-9.80982083e-02  9.72671364e-02  4.30291211e-02 -3.95611749e+00
   1.14946789e-01]
 [-1.89646556e-01  1.16746932e-01 -2.57396048e-02 -7.92296139e+01
   1.09562636e-01]]
```

F
 [0.0, 1.1368683772161603e-13, 0.0, -1.6506886831606886, -255.28756071079601]

==

iter 8
========
```

X
 [ 150.67457039 -517.12165404  379.94708365  -31.5766275  -156.27251327]


Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -0.0002870918175576559, -0.000390742295


Z_inv
 [[ 2.04620778e-01  1.53104108e-01 -2.15546528e-03  5.12639277e+02
   -2.77816678e+00]
 [ 1.21764936e+00 -1.36917756e-01 -1.01078959e+00 -2.05744254e+03
    1.01731698e+01]
 [-4.22270136e-01 -1.61863513e-02  1.01294505e+00  1.54480326e+03
   -7.39500306e+00]
 [-3.61324711e-02  2.45013255e-02  1.94114502e-02  6.88543150e+00
    9.39497273e-01]
 [-3.73109086e-01  1.69290459e-01 -1.51005197e-02 -1.03216399e+03
    4.61683628e+00]]


F
 [1.1368683772161603e-13, 1.1368683772161603e-13, 2.2737367544323206e-13, -1.0774613955563912, -


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iter 9
========

X
 [-3114.98756128 11246.96491002 -8118.47734874  1266.9853351
  5076.48978746]


Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, 1.601813591060828e-05, 2.21907883426599


Z_inv
 [[ 2.78902126e-01  1.37346716e-01  1.70870847e-04 -1.25837741e+04
    4.04866392e-02]
 [ 9.38341091e-01 -8.00136921e-02 -1.03660578e+00  5.11415124e+04
   -1.45781772e-01]
 [-2.17243217e-01 -5.73330239e-02  1.03643491e+00 -3.85577383e+04
    1.05295133e-01]

7

```
  [-5.39495951e-02  3.06268280e-02  3.59223006e-02 -8.06415967e+02
   -1.61647850e-02]
  [-5.03854657e-01  1.94679740e-01 -3.62640422e-02  2.59739642e+04
   -6.48084934e-02]]
```

F
```
  [0.0, 1.8189894035458565e-12, 0.0, -1.1091554125105658, -1385050.088819058]
```

==

iter 10
========

X
```
  [  39003.67446325 -133944.20572007    94954.03125681   -22016.49213295
   -55877.35679356]
```

Z
```
  [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -2.0853445200970042e-06, -2.94163198444
```

Z_inv
```
  [[ 2.70239553e-01  1.35292755e-01 -2.12877758e-05  9.18527467e+04
   -1.07946013e-03]
  [ 9.98784877e-01 -8.77604504e-02 -1.05080746e+00 -3.74904151e+05
    3.70759292e-03]
  [-2.69024430e-01 -4.75323047e-02  1.05082875e+00  2.83051404e+05
   -2.62813279e-03]
  [-7.97430899e-02  4.65894301e-02  5.08926106e-02  7.49316414e+03
    6.10247589e-04]
  [-4.60736017e-01  1.82825060e-01 -5.08500350e-02 -1.91198658e+05
    1.54867267e-03]]
```

F
```
  [-1.0186340659856796e-10, -1.4551915228366852e-10, 8.731149137020111e-11, -1.2083198153970893,
```

==

iter 11
========
```

X
 [-22281.63786948   4752.49467056   17542.64319892   84428.05680734
 -39751.28106839]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, 0.0016496333108859953, -0.0004469037778

Z_inv
 [[4.08956007e-01 2.05338971e-01 9.40368138e-04 9.19295750e+02
  -1.90486434e-04]
 [3.25381146e-02 -2.03858970e-02 5.21872083e-02 3.06357963e+02
   4.05314627e-05]
 [5.58505878e-01 -1.84953074e-01 -5.31275764e-02 -1.22565371e+03
   1.49954971e-04]
 [3.31637856e-01 -3.00969988e-01 -1.05594868e+00 -3.98354096e+03
   7.21414274e-04]
 [-1.14954987e+00 3.90292045e-01 1.05406794e+00 2.14494946e+03
  -3.40441406e-04]]

F
 [-2.9103830456733704e-11, 2.9103830456733704e-10, 0.0, -8.768873518363174, -5016576.325158303]

==

iter 12
========

X
 [-15176.03944833   7642.23807986   7547.30136847   53115.91971345
 -22650.3408168]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, 0.00030304039972540043, -0.000306852313

Z_inv
 [[4.27631934e-01 2.15241872e-01 1.81833381e-03 1.40545984e+03
  -1.31822349e-04]
 [1.29177946e-01 -6.85186787e-02 1.05473025e-01 1.10842898e+03
   6.61838897e-05]
 [4.43190119e-01 -1.46723194e-01 -1.07291359e-01 -2.51388882e+03
   6.56384594e-05]

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```
[ 1.60294316e-01 -2.92448810e-01 -1.11274636e+00 -6.73026833e+03
   4.61105507e-04]
[-1.01555819e+00  3.61965066e-01  1.10910969e+00  3.91934865e+03
  -1.97460808e-04]]
```

F
```
[-5.45696821063756694e-12, -2.1827872842550278e-11, 5.4569682106375694e-12, -3.244906882518218,
```

==

iter 13
========

X
```
[-11122.36754881   11493.49280912    -357.6252603    33049.97738614
 -10691.74228851]
```

Z
```
[[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -8.36848447213423e-06, -0.0002689494473
```

Z_inv
```
[[ 5.08731488e-01  2.56304484e-01  4.05290094e-03  1.92168935e+03
  -1.09928205e-04]
 [ 5.29000933e-01 -2.70059477e-01 -2.70058131e-02  1.76609140e+03
   1.13487325e-04]
 [-3.77324206e-02  1.37549936e-02  2.29529121e-02 -3.68778076e+03
  -3.55911944e-06]
 [-5.63926884e-01 -2.55158457e-01 -9.89205791e-01 -9.45284882e+03
   3.26225496e-04]
 [-4.53536092e-01  2.42549490e-01  9.81099989e-01  5.60947011e+03
  -1.06369086e-04]]
```

F
```
[7.275957614183426e-12, 2.9103830456733704e-11, 0.0, -0.8328168838963154, -1758339.4941325877]
```

==

iter 14
========

X
 [-9715.24331432 13163.87279328 -3435.12947896 25751.10046402
 -6207.11383537]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -8.22397611923523e-05, -0.0003151536969

Z_inv
 [[6.98362404e-01 3.50636704e-01 6.24129150e-03 2.25555021e+03
  -1.23166867e-04]
 [6.39603710e-01 -5.33476862e-01 -3.70849086e-01 -3.23951341e+02
   1.67178825e-04]
 [-3.37966114e-01 1.82840159e-01 3.64607795e-01 -1.93159887e+03
  -4.40119581e-05]
 [-1.43305333e+00 -3.69069953e-01 -6.54116080e-01 -8.69824950e+03
   3.25488642e-04]
 [3.63285179e-02 1.67796545e-01 6.41633497e-01 4.18714908e+03
  -7.91549086e-05]]

F
 [-2.7284841053187847e-12, -9.094947017729282e-12, 9.094947017729282e-13, 0.15359375488591231, -

==

iter 15
========

X
 [-10157.3997889   13343.55125139 -3172.65146249 27340.04790421
  -6911.74832641]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -7.048405209970633e-05, -0.000296442131

Z_inv
 [[6.54032714e-01 3.28537775e-01 5.67880309e-03 2.24458756e+03
  -1.15751678e-04]
 [6.65102588e-01 -4.84678967e-01 -3.17798807e-01 9.95303012e+01
   1.52267443e-04]
 [-3.19135303e-01 1.56141191e-01 3.12120004e-01 -2.34411786e+03
  -3.65157649e-05]

11

```
[-1.28123345e+00 -3.29472135e-01 -7.04916405e-01 -9.07788054e+03
   3.10739268e-04]
 [-2.68319829e-02  1.72396584e-01  6.93558799e-01  4.58870542e+03
  -7.92359127e-05]]
```

F
```
 [-2.7284841053187847e-12, 3.637978807091713e-12, 1.8189894035458565e-12, 0.011507561598130822,
```

==

iter 16
========

X
```
 [-10291.36559805  13484.65527947  -3179.78968142  27734.80711272
  -7038.57591662]
```

Z
```
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -6.890631878690484e-05, -0.000292213652
```

Z_inv
```
 [[ 6.50808888e-01  3.26915691e-01  5.58273012e-03  2.26537790e+03
  -1.14126610e-04]
 [ 6.66350508e-01 -4.80969925e-01 -3.13429902e-01  1.35143255e+02
   1.49735826e-04]
 [-3.17159396e-01  1.54054234e-01  3.07847172e-01 -2.40052115e+03
  -3.56092152e-05]
 [-1.26958606e+00 -3.26692840e-01 -7.08901018e-01 -9.19665484e+03
   3.06770616e-04]
 [-3.20317159e-02  1.72861457e-01  6.97735558e-01  4.66589905e+03
  -7.85173951e-05]]
```

F
```
 [5.002220859751105e-12, -5.4569682106375694e-12, 0.0, 0.0001779554185555865, -966302.4804960318
```

==

iter 17
========

X
  [-10402.04956101 13629.32132969 -3213.77176868 28032.87691434
  -7115.27779233]

Z
  [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -6.816200859569932e-05, -0.000289069039

Z_inv
  [[6.50715037e-01 3.26852364e-01 5.52186161e-03 2.28927582e+03
  -1.12908771e-04]
 [6.66457193e-01 -4.80869898e-01 -3.13292109e-01 1.37935817e+02
   1.48131955e-04]
 [-3.17172230e-01 1.54017534e-01 3.07770248e-01 -2.42721163e+03
  -3.52231837e-05]
 [-1.26931734e+00 -3.26539560e-01 -7.08795337e-01 -9.29503908e+03
   3.03503129e-04]
 [-3.21127326e-02 1.72834831e-01 6.97751614e-01 4.71648745e+03
  -7.76855873e-05]]

F
  [3.183231456205249e-12, 1.8189894035458565e-12, -1.8189894035458565e-12, 1.9176275652954544e-06

==

iter 18
========

X
  [-10513.54555054 13775.59374554 -3248.54819499 28332.58845664
  -7191.99735555]

Z
  [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -6.743811078655437e-05, -0.000285973906

Z_inv
  [[6.50670136e-01 3.26813596e-01 5.46217898e-03 2.31347853e+03
  -1.11709673e-04]
 [6.66546104e-01 -4.80826443e-01 -3.13221198e-01 1.40235552e+02
   1.46558741e-04]
 [-3.17216239e-01 1.54012847e-01 3.07759019e-01 -2.45371409e+03
  -3.48490680e-05]

13

```
    [-1.26922665e+00 -3.26427941e-01 -7.08627518e-01 -9.39414969e+03
      3.00279951e-04]
    [-3.21136251e-02  1.72800749e-01  6.97703160e-01  4.76719262e+03
     -7.68606050e-05]]


F
  [0.0, 5.4569682106375694e-12, -9.094947017729282e-13, 1.7162152254890373e-08, -1008868.15393263
```

==

```
iter 19
========

X
  [-10626.24592182  13923.45218961  -3283.70626779  28635.53149768
    -7269.53965404]


Z
  [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -6.672195833355478e-05, -0.000282912027


Z_inv
  [[ 6.50626236e-01  3.26775509e-01  5.40315188e-03  2.33794422e+03
    -1.10523252e-04]
   [ 6.66633889e-01 -4.80784065e-01 -3.13151768e-01  1.42554395e+02
     1.45002223e-04]
   [-3.17260124e-01  1.54008556e-01  3.07748616e-01 -2.48049862e+03
    -3.44789710e-05]
   [-1.26913883e+00 -3.26317969e-01 -7.08460840e-01 -9.49433129e+03
     2.97090785e-04]
   [-3.21136398e-02  1.72766952e-01  6.97654536e-01  4.81844284e+03
    -7.60442810e-05]]


F
  [-1.8189894035458565e-12, 1.0913936421275139e-11, 1.8189894035458565e-12, -3.145685378846963e-0
```

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```
iter 20
========
```

X
 [-10740.16792226 14072.91344895 -3319.24552668 28941.75824011
  -7347.92239558]

Z
 [[1, 1, 1, 0, 0], [4, 0, 0, 2, 2], [0, 1, 2, 0, 1], [0, -6.601333832372893e-05, -0.000279882879

Z_inv
 [[6.50582815e-01 3.26737840e-01 5.34476755e-03 2.36267524e+03
  -1.09349314e-04]
 [6.66720736e-01 -4.80742160e-01 -3.13083106e-01 1.44898199e+02
   1.43462083e-04]
 [-3.17303552e-01 1.54004320e-01 3.07738338e-01 -2.50757344e+03
  -3.41127682e-05]
 [-1.26905200e+00 -3.26209200e-01 -7.08295965e-01 -9.59559916e+03
   2.93935175e-04]
 [-3.21136326e-02 1.72733520e-01 6.97606429e-01 4.87024868e+03
  -7.52365462e-05]]

F
 [2.2737367544323206e-12, 1.0913936421275139e-11, -9.094947017729282e-13, -3.2939810878929165e-0

==

20 5

15