Sayan Chaudhry, Deepayan Patra
07-131: Section A
Professor: Cortina
September 17, 2020

07-131 Homework 3

Problem 1 Math

LaTeX is awesome for math! Let's look evaluating the value of m if $(m+2) \times (3+4) = 21$ in two ways.

Using the initial equality:

$$\begin{aligned}
& (m+2) \times (3+4) = 21 && \text{(start)} \\
\implies \quad & (m+2) \times 7 = 21 && \text{(evaluating } 3+4=7\text{)} \\
\implies \quad & m+2 = \frac{21}{7} && \text{(dividing by 7 on both sides)} \\
\implies \quad & m+2 = 3 && \text{(evaluating } \tfrac{21}{7}=3\text{)} \\
\implies \quad & m = 3-2 && \text{(subtracting 2 on both sides)} \\
\implies \quad & m = 1 && \text{(evaluating } 3-2=1\text{)}
\end{aligned}$$

Therefore:

$$m = 1$$

*

Using the initial equality:

$$\begin{aligned}
& (m+2) \times (3+4) = 21 && \text{(start)} \\
\implies \quad & (m+2) \times 7 = 21 && \text{(evaluating } 3+4=7\text{)} \\
\implies \quad & (m \times 7) + (2 \times 7) = 21 && \text{(using distributive property)} \\
\implies \quad & (m \times 7) + 14 = 21 && \text{(evaluating } 2 \times 7 = 14\text{)} \\
\implies \quad & m \times 7 = 21 - 14 && \text{(subtracting 14 on both sides)} \\
\implies \quad & m \times 7 = 7 && \text{(evaluating } 21-14=7\text{)} \\
\implies \quad & m = \frac{7}{7} && \text{(dividing by 7 on both sides)} \\
\implies \quad & m = 1 && \text{(evaluating } \tfrac{7}{7}=1\text{)}
\end{aligned}$$

Therefore:

$$m = 1$$

Problem 2 Induction

Induction becomes so much easier with LaTeX! Have a look!

Define P(n) as $n > n - 1$.

We will now show that P(n) holds for all $n \in \mathbb{N}^+$.

Base Case: $n = 1$

P(1) holds because $1 > 0$.

Induction Hypothesis: Assume that P(k) holds for arbitrary $k \geq 1$.

Induction Step: We want to show that P($k + 1$) holds.

Observe that:

$$\begin{aligned} & k \geq k - 1 && \text{(by IH)} \\ \implies \quad & k + 1 \geq k - 1 + 1 && \text{(adding 1 to both sides)} \\ \implies \quad & k + 1 \geq k && \text{(evaluating } 1 - 1 = 0\text{)} \end{aligned}$$

Hence, P(k) is true. Since P(k) implies P($k + 1$) and P(1) is true, by the principle of mathematical induction P(n) holds for $n \in \mathbb{N}^+$.

Problem 2 Calculus

Those of you who enjoy calculus will have a blast!

Find $\lim_{x\to 2} \frac{x^2+x-6}{x^2-4}$.

$$\begin{aligned}\lim_{x\to 2} \frac{x^2+x-6}{x^2-4} &= \lim_{x\to 2} \frac{2x+1}{2x} && \text{(by L'Hôpital's Rule)}\\ &= \frac{5}{4}\end{aligned}$$

Another one? Ok!

Given $f(x,y) = \frac{xy}{x+y}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

For $(x,y) \neq (0,0)$,

$$\begin{aligned}\frac{\partial f}{\partial x}(x,y) &= \frac{\partial}{\partial x}\left(\frac{xy}{x+y}\right) && \text{(by substitution)}\\ &= \frac{y(x+y) - xy(1)}{(x+y)^2} && \text{(by differentiation rules)}\\ &= \frac{y^2}{(x+y)^2}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial f}{\partial y}(x,y) &= \frac{\partial}{\partial y}\left(\frac{xy}{x+y}\right) && \text{(by substitution)}\\ &= \frac{x(x+y) - xy(1)}{(x+y)^2} && \text{(by differentiation rules)}\\ &= \frac{x^2}{(x+y)^2}\end{aligned}$$

Alright, no more problems. Just two more equations.

$$\begin{aligned}\iiint_V (\nabla \cdot \mathbf{F})\,\mathrm{d}V &= \oiint_{S(V)} \mathbf{F}\cdot\hat{\mathbf{n}}\,\mathrm{d}S\\ \iiint_V (\nabla \times \mathbf{F})\,\mathrm{d}V &= \oiint \hat{\mathbf{n}}\times\mathbf{F}\,\mathrm{d}S\end{aligned}$$

Problem 3 Programming

You can code in LaTeX too! Here's an intro to typesetting programs:

```c
#include<stdio.h>
#include<iostream>

// My first program
int main(void)
{
  printf("Hello World\n");
  return 0;
}
```

You can also change languages really easily, like here:

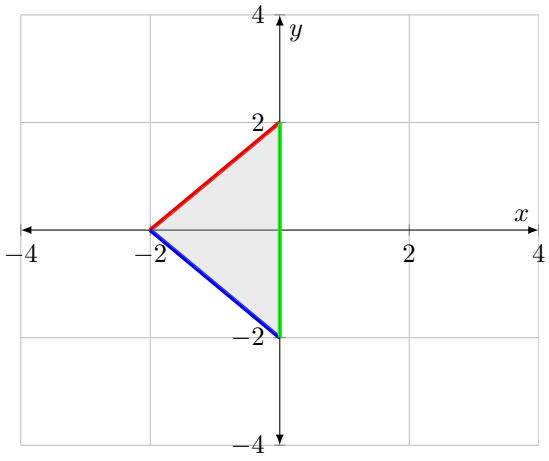
```
(* My second program *)
fun map f xs = let
  fun m ([], acc) = List.rev acc
    | m (x::xs, acc) = m (xs, f x::acc)
  in
    m (xs, [])
  end
```

or here:

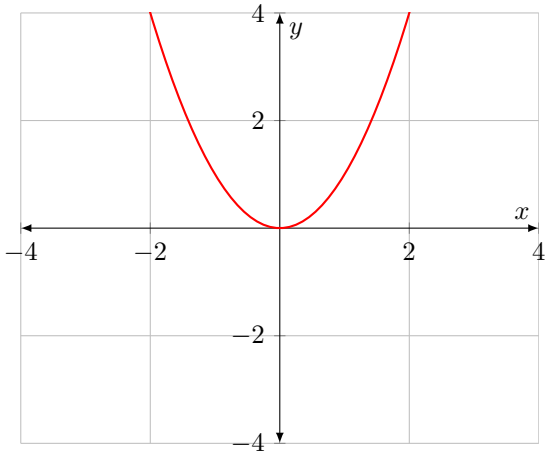
```
# My third program
def UncommonWords(A, B):
  # count will contain all the word counts
  count = {}

  # insert words of string A to hash
  for word in A.split():
    count[word] = count.get(word, 0) + 1

  # insert words of string B to hash
  for word in B.split():
    count[word] = count.get(word, 0) + 1

  # return required list of words
  return [word for word in count if count[word] == 1]
```

Problem 4 Graphing

Plotting in LaTeX is super fun! Here's a mini graph with some shading:



and here's a plot of $y = x^2$:



Problem 4 Images

You can make some really pretty images and figures in LaTeX too!

Carnegie Mellon University

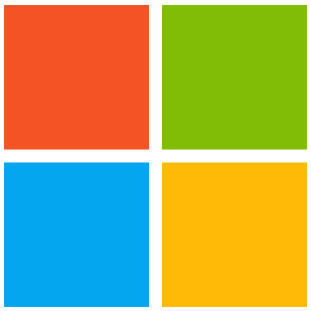
Figure 1: CMU

(a) Amazon

(b) Apple

(c) Facebook

(d) Google



(e) Microsoft

(f) Netflix

Figure 2: FAANG logos

and can reference back to them! Take me to FAANG logos or take me to CMU.