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07-131: Section A

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September 16, 2020

07-131 Homework 3

Problem 1 Math

LaTeX is awesome for math! Let's look evaluating the value of m if $(m + 2) \times (3 + 4) = 21$ in two ways.

Using the initial equality:

$$(m + 2) \times (3 + 4) = 21 \qquad \text{(start)}$$
$$\implies (m + 2) \times 7 = 21 \qquad \text{(evaluating } 3 + 4 = 7)$$
$$\implies m + 2 = \frac{21}{7} \qquad \text{(dividing by 7 on both sides)}$$
$$\implies m + 2 = 3 \qquad \text{(evaluating } \tfrac{21}{7} = 3)$$
$$\implies m = 3 - 2 \qquad \text{(subtracting 2 on both sides)}$$
$$\implies m = 1 \qquad \text{(evaluating } 3 - 2 = 1)$$

Therefore:

$$m = 1$$

$$*$$

Using the initial equality:

$$(m + 2) \times (3 + 4) = 21 \qquad \text{(start)}$$
$$\implies (m + 2) \times 7 = 21 \qquad \text{(evaluating } 3 + 4 = 7)$$
$$\implies (m \times 7) + (2 \times 7) = 21 \qquad \text{(using distributive property)}$$
$$\implies (m \times 7) + 14 = 21 \qquad \text{(evaluating } 2 \times 7 = 14)$$
$$\implies m \times 7 = 21 - 14 \qquad \text{(subtracting 14 on both sides)}$$
$$\implies m \times 7 = 7 \qquad \text{(evaluating } 21 - 14 = 7)$$
$$\implies m = \frac{7}{7} \qquad \text{(dividing by 7 on both sides)}$$
$$\implies m = 1 \qquad \text{(evaluating } \tfrac{7}{7} = 1)$$

Therefore:

$$m = 1$$

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Problem 2 Induction

Induction becomes so much easier with LaTeX! Have a look!

Define P(n) as $n > n - 1$.

We will now show that P(n) holds for all $n \in \mathbb{N}^+$.

<u>Base Case:</u> $n = 1$

P(1) holds because $1 > 0$.

<u>Induction Hypothesis:</u> Assume that P(k) holds for arbitrary $k \geq 1$.

<u>Induction Step:</u> We want to show that P($k + 1$) holds.

Observe that:

$$
\begin{aligned}
& k \geq k - 1 && \text{(by IH)} \\
\implies\quad & k + 1 \geq k - 1 + 1 && \text{(adding 1 to both sides)} \\
\implies\quad & k + 1 \geq k && \text{(evaluating } 1 - 1 = 0\text{)}
\end{aligned}
$$

Hence, P(k) is true. Since P(k) implies P($k + 1$) and P(1) is true, by the principle of mathematical induction P(n) holds for $n \in \mathbb{N}^+$.

Problem 2 Calculus

Those of you who enjoy calculus will have a blast!

Find $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \lim_{x \to 2} \frac{2x + 1}{2x} \qquad \text{(by L'Hôpital's Rule)}$$
$$= \frac{5}{4}$$

Another one? Ok!

Given $f(x, y) = \frac{xy}{x+y}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

For $(x, y) \neq (0, 0)$,

$$\frac{\partial f}{\partial x}(x, y) = \frac{\partial}{\partial x}\left(\frac{xy}{x+y}\right) \qquad \text{(by substitution)}$$
$$= \frac{y(x+y) - xy(1)}{(x+y)^2} \qquad \text{(by differentiation rules)}$$
$$= \frac{y^2}{(x+y)^2}$$

and

$$\frac{\partial f}{\partial y}(x, y) = \frac{\partial}{\partial y}\left(\frac{xy}{x+y}\right) \qquad \text{(by substitution)}$$
$$= \frac{x(x+y) - xy(1)}{(x+y)^2} \qquad \text{(by differentiation rules)}$$
$$= \frac{x^2}{(x+y)^2}$$

Alright, no more problems. Just two more equations.

$$\iiint_V (\nabla \cdot \mathbf{F}) \, dV = \oiint_{S(V)} \mathbf{F} \cdot \hat{\mathbf{n}} \, dS$$
$$\iiint_V (\nabla \times \mathbf{F}) \, dV = \oiint \hat{\mathbf{n}} \times \mathbf{F} \, dS$$

Problem 3 Programming

You can code in LaTeXtoo! Here's an intro to typesetting programs:

```c
#include<stdio.h>
#include<iostream>

// My first program
int main(void)
{
  printf("Hello World\n");
  return 0;
}
```

You can also change languages really easily, like here:

```sml
(* My second program *)
fun map f xs = let
  fun m ([], acc) = List.rev acc
    | m (x::xs, acc) = m (xs, f x::acc)
  in
    m (xs, [])
  end
```

or here:

```python
# My third program
def UncommonWords(A, B):
  # count will contain all the word counts
  count = {}

  # insert words of string A to hash
  for word in A.split():
    count[word] = count.get(word, 0) + 1

  # insert words of string B to hash
  for word in B.split():
    count[word] = count.get(word, 0) + 1

  # return required list of words
  return [word for word in count if count[word] == 1]
```
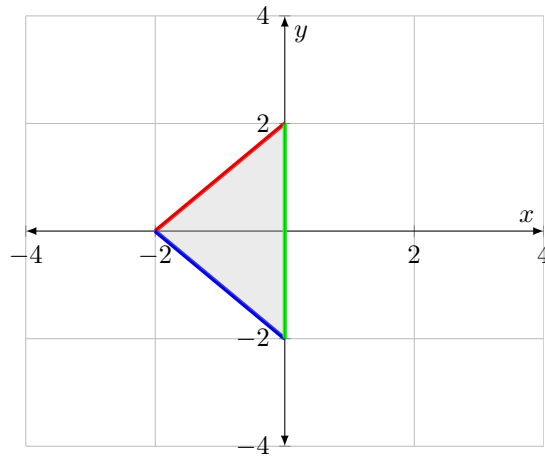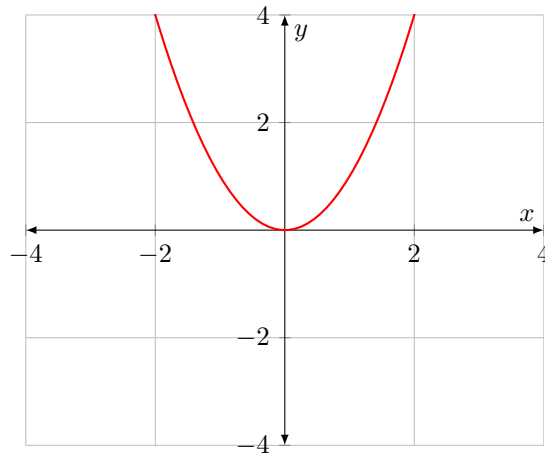
Problem 4 Graphing

Plotting in LATEXis super fun! Here's a mini graph with some shading:

and here's a plot of $y = x^2$:

Problem 4 Images

You can make some really pretty images and figures in LaTeX too!

Figure 1: CMU

(a) Amazon (b) Apple (c) Facebook
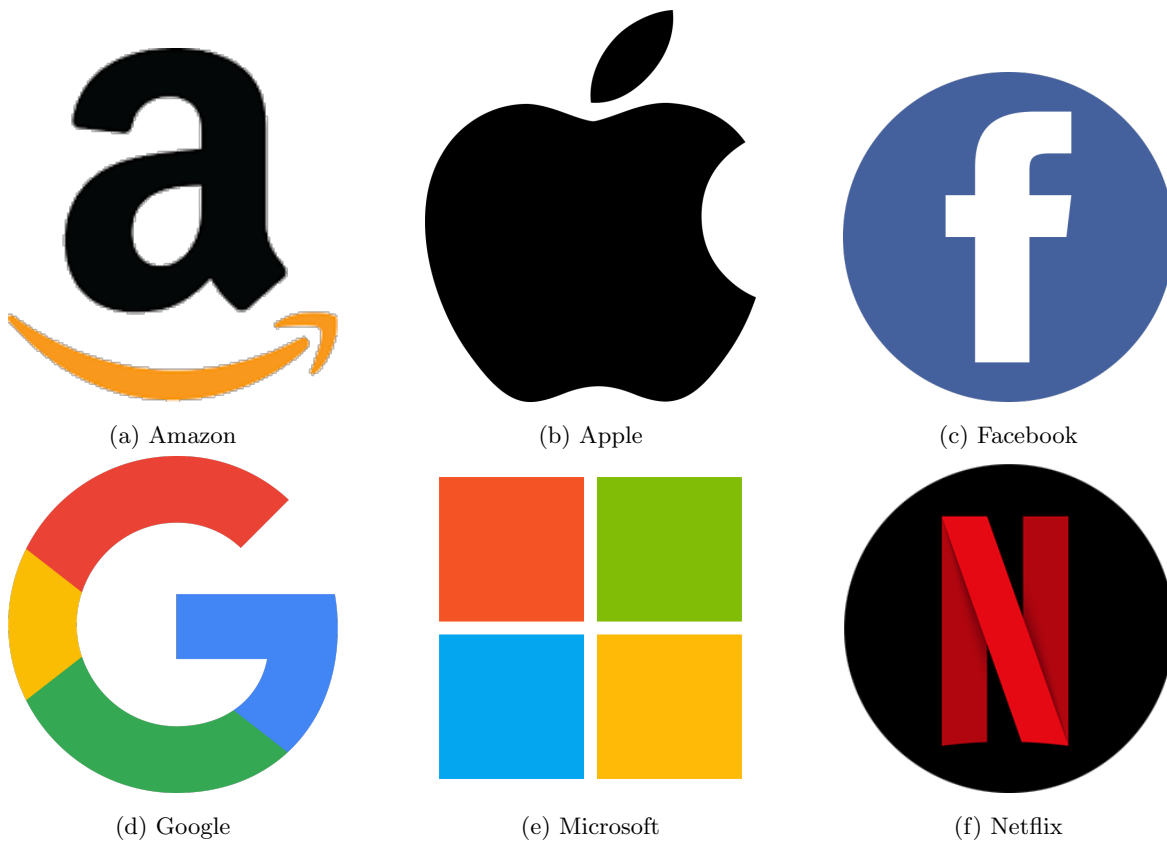
(d) Google (e) Microsoft (f) Netflix

Figure 2: FAANG logos

and can reference back to them! Take me to FAANG logos or take me to CMU.