Assignment

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MATH — Class — Quarter

Problem 5.1 Writing $z = re^{i\operatorname{Arg} z}$, show that, where $n \in \mathbb{Z}_{>0}$

$$\log(z^{1/n}) = \frac{1}{n}\ln r + i\left(\frac{\operatorname{Arg} z + 2(pn+k)\pi}{n}\right), \quad p \in \mathbb{Z},\ k = 0, \dots, n-1.$$

Now, after writing

$$\frac{1}{n}\log z = \frac{1}{n}\ln r + i\left(\frac{\operatorname{Arg} z + 2q\pi}{n}\right), \quad q \in \mathbb{Z},$$

show that we have equality of sets

$$\log(z^{1/n}) = \frac{1}{n}\log z$$

Proof. First recall from Example 9.9 that $z^{1/n} = e^{\log(z)/n}$. So expanding we have,

$$\begin{aligned}
\log(z^{1/n}) &= \log(e^{\log(z)/n}) && \text{Apply Prop. 9.3(2)} \\
&= \frac{\log z}{n} + 2p\pi i && p \in \mathbb{Z} \\
&= \frac{1}{n}\ln r + \frac{i\operatorname{Arg} z + 2k\pi i}{n} + 2p\pi i && k \in \mathbb{Z} \\
&= \frac{1}{n}\ln r + \frac{i\operatorname{Arg} z + 2k\pi i}{n} + \frac{2p\pi i n}{n} \\
&= \frac{1}{n}\ln r + i\left(\frac{\operatorname{Arg} z + 2(pn+k)\pi}{n}\right) && p \in \mathbb{Z}, k \in \{0, \dots, n-1\}
\end{aligned}$$

giving us the desired expression.

□

Problem 5.2

(a) Find real valued functions u, $v : \mathbb{R}^2 \to \mathbb{R}$ such that $\cos z = u(x, y) + iv(x, y)$.

Solution. $\square$

(b) Show that $e^{\overline{z}} = \overline{e^z}$ and $\cos \overline{z} = \overline{\cos z}$.

Solution. $\square$

(c) Show that $e^{iz} = \cos z + i \sin z$, and prove $\sin \overline{z} = \overline{\sin z}$.

Solution. $\square$

Problem 5.3

(a) Verify that $(z^{\alpha})^{n} = z^{n\alpha}$ for $z \neq 0$ and $n \in \mathbb{Z}$.

Solution. $\square$

(b) Find a counterexample to the statement: $(z^{\alpha})^{\beta} = z^{\alpha\beta}$, where $z \neq 0$ and $\alpha, \beta \in \mathbb{C}$.

Solution. $\square$

Problem 5.4 Determine the points at which the following functions are holomorphic. Find its derivative at those points.

(a) $f(z) = e^{\overline{z}}$.

Solution. □

(b) $g(z) = \cos \overline{z}$.

Solution. □

(c) $a(z) = \dfrac{\operatorname{Log}(2z - i)}{z^2 + 1}$.

Solution. □

Problem 5.5 Find all complex values z satisfying the given equation.

(a) $\cos z = 4$

Solution. $\square$

(b) $\cos z = i \sin z$

Solution. $\square$

Problem 5.6 Let $z \in \mathbb{C}$.

(a) Prove that $|1^z|$ is single-valued if and only if $\operatorname{Im} z = 0$.

Solution. $\square$

(b) Find a necessary and sufficient condition for $\left|i^{iz}\right|$ to be single-valued.

Solution. $\square$

(c) Show by counterexample that the statement is false:

$$1^z \text{ is single-valued if and only if } \operatorname{Im} z = 0.$$

Solution. $\square$

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

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