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[2] A bilinear form ω on $V \bigoplus V$ for $\mathbb{F}$-vector space is symmetric if $\omega(x,y) = \omega(y,x)$ for all $x, y \in V$. A quadratic form on V is a function $q : V \to \mathbb{F}$ obtained from a bilinear form ω by writing $q(x) = \omega(x,x)$.

(a) Prove that if $\text{char}(\mathbb{F}) \neq 2$ then every synmetric bilinear form is uniquely determined by the corresponding quadratic form.

Proof. Let ω be a symmetric bilinear form in a vector space V, we will show that $\omega(x) = (x,x)$ is a quadratic form in the same vector space V. We see first that

$$\omega(\alpha x) = (\alpha x, \alpha x) = \alpha^2(x,x) = \alpha^2\omega(x).$$

Now we must show $b_\omega(x,y) = \omega(x+y) - \omega(x) - \omega(y)$ is a symmetric bilinear form. We can see that this is satisfied by the following,

$$\begin{aligned}
b_\omega(x,y) &= \omega(x+y) - \omega(x) - \omega(y) \\
&= (x+y, x+y) - (x,x) - (y,y) \\
&= (x, x+y) + (y, x+y) - (x,x) - (y,y) \\
&= (x,x) + (x,y) + (y,x) + (y,y) - (x,x) - (y,y) \\
&= (x,y) + (y,x) && \omega \text{ is a symmetric bilinear form so,} \\
&= 2(x,y)
\end{aligned}$$

Recall though that ω was defined as a symmetric bilinear form, thus b_ω is a symmetric bilinear form. $\square$