Let $M = \mathbb{Z}^2$ and let N be the $\mathbb{Z}$-submodule generated by the 2 elements

$$\begin{pmatrix} 120 \\ 240 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 360 \\ -300 \end{pmatrix}$$

Find an element $f \in \mathrm{Hom}_{\mathbb{Z}}(M, \mathbb{Z})$ so that $f(N)$ is maximal in $\Sigma_{M,N}$. Where

$$\Sigma_{M,N} = \{(a_f) = \varphi(N) \mid f \in \mathrm{Hom}(M, \mathbb{Z})\}$$

Prove that is is indeed maximal.