Homework 3

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MATH 201 — Algebra II — Winter 2022

Problem 1 Let $M = \mathbb{Z}^2$ and let N be the $\mathbb{Z}$-submodule generated by the 2 elements

$$\begin{bmatrix} 120 \\ 240 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 360 \\ -300 \end{bmatrix}$$

In (a) and (b) we use the notation and terminology adopted in class on the 3rd of February. The terminology in (c) and (d) will be defined on the 8th of February.

(a) Find an element $\nu \in \mathrm{Hom}_{\mathbb{Z}}(M, \mathbb{Z})$ such that $\nu(N)$ is maximal in $\Sigma_{M,N}$: prove that it is indeed maximal.

(b) Find an element $y_1 \in M$ such that $M = \mathbb{Z}y_1 \bigoplus \ker(\nu)$ and $N = \mathbb{Z}a_\nu y_1 \bigoplus (\ker(\nu) \cap N)$.

(c) Find the invariant factors of the quotient $\mathbb{Z}$-module M/N.

(d) Find the elementary divisors of the quotient $\mathbb{Z}$-module M/N

(a) *Proof.* Let $\nu \in \mathrm{Hom}_{\mathbb{Z}}(M, \mathbb{Z})$ be as follows,

$$\nu\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

□

(b)

(c)

(d)