Assignment

Kevin Guillen

MATH — Class — Quarter

Problem 1 Find the sum of:

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\cdots+\frac{1}{100\cdot 101\cdot 102}$$

Solution. If we look at this as a summation we can describe each term as,

$$\frac{1}{i\cdot(i+1)\cdot(i+2)}$$

for $i \in \{1,\ldots,n\}$. Giving us,

$$\sum_{i=1}^{n}\frac{1}{i\cdot(i+1)\cdot(i+2)}$$

Doing some algebra we can obtain the following,

$$\begin{aligned}\sum_{i=1}^{n}\frac{1}{i\cdot(i+1)\cdot(i+2)} &= \sum_{i=1}^{n}\frac{1}{2}\cdot\frac{2}{i\cdot(i+1)\cdot(i+2)}\\ &= \frac{1}{2}\cdot\sum_{i=1}^{n}\frac{i+2-i}{i\cdot(i+1)\cdot(i+2)}\\ &= \frac{1}{2}\cdot\sum_{i=1}^{n}\frac{(i+2)}{i\cdot(i+1)\cdot(i+2)}-\frac{i}{i\cdot(i+1)\cdot(i+2)}\\ &= \frac{1}{2}\cdot\sum_{i=1}^{n}\frac{1}{i\cdot(i+1)}-\frac{1}{(i+1)\cdot(i+2)}\end{aligned}$$

Finally when we expand the summation out we can see something important,

$$\frac{1}{2}\left(\frac{1}{1\cdot(2)}-\frac{1}{(2)\cdot(3)}+\frac{1}{2\cdot(3)}-\frac{1}{(3)\cdot(4)}+\cdots+\frac{1}{n\cdot(n+1)}-\frac{1}{(n+1)\cdot(n+2)}\right)$$

that the middle terms cancel out giving us,

$$\frac{1}{2}\cdot\left(\frac{1}{1\cdot(1+1)}-\frac{1}{(n+1)\cdot(n+2)}\right)$$

Now plugging in our desired value for the problem, $n=100$, we get,

$$\frac{1}{2}\cdot\left(\frac{1}{2}-\frac{1}{101\cdot 102}\right)=\frac{1}{4}-\frac{1}{20604}$$

$$
\begin{aligned}
&= \frac{5151}{20604} - \frac{1}{20604} \\
&= \frac{5150}{20604}
\end{aligned}
$$

$\square$