Topology - Summer Session 1 - HW3 - 7/16/2021

(5.2) Let $S = \{a, b, c, d\}$ with the discrete topology and let $T = \{a, b, c, d\}$ with the indiscrete topology. Define $f : S \to T$ to be the identitymap. Is f a homeomorphism?

Proof. By defintion the discrete topology on S is simply, $\mathcal{T}_S = \mathcal{P}(S)$. While the indiscrete topology for T is $\mathcal{T}_T = \{\emptyset, T\}$. Meaning the topology $\mathcal{T}_S$ is strictly finer than the topology $\mathcal{T}_T$. This is key since a homeomorphism requires a biijection which is a 1-1 correspondence between the two topologies, and since they are of different size no bijectiction can exist. $\square$

(5.7) Show that the annulus $A = \{(x, y) \in R^2 : 1 \leq x^2 + y^2 \leq 4\}$ is homeomorphic to the cylinder $C = \{(x, y, z) \in R^3 : x^2 + y^2 = 1, 0 \leq z \leq 1\}$