Assignment

Kevin Guillen

MATH 200 — Algebra I — Fall 2021

Problem 3.19 Let G and H be groups. Does there exist a product G and H in **Grp**?

Proof. Yes. The product will be the direct product of the groups G and H, and we know this forms a new group $G \times H$ that will be an object in **Grp**, together with,

$$\begin{aligned} p : G \times H &\to G \\ (g, h) &\mapsto g \\ q : G \times H &\to H \\ (g, h) &\mapsto h. \end{aligned}$$

We see for any group in $Z \in$ **Grp**, we have the following bijective map,

$$\begin{aligned} \varphi : \mathrm{Hom}_{\mathbf{Grp}}(Z, G \times H) &\to \mathrm{Hom}_{\mathbf{Grp}}(Z, G) \times \mathrm{Hom}_{\mathbf{Grp}}(Z, H) \\ f &\mapsto (p \circ f, q \circ f). \end{aligned}$$

We will prove that it is bijective by first proving it is injective.

Consider any $f, f' \in \mathrm{Hom}_{\mathbf{Grp}}(Z, G \times H)$. If $\varphi(f) = \varphi(f')$ we see the following,

$$\begin{gathered} \varphi(f) = \varphi(f') \\ (p \circ f, q \circ f) = (p \circ f', q \circ f') \\ \to (p \circ f) = (p \circ f') \text{ and } (q \circ f) = (q \circ f') \end{gathered}$$

but p and q are injective, and thus $f = f'$. $\square$