Suitable Distributions of Returns

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Overview of the study

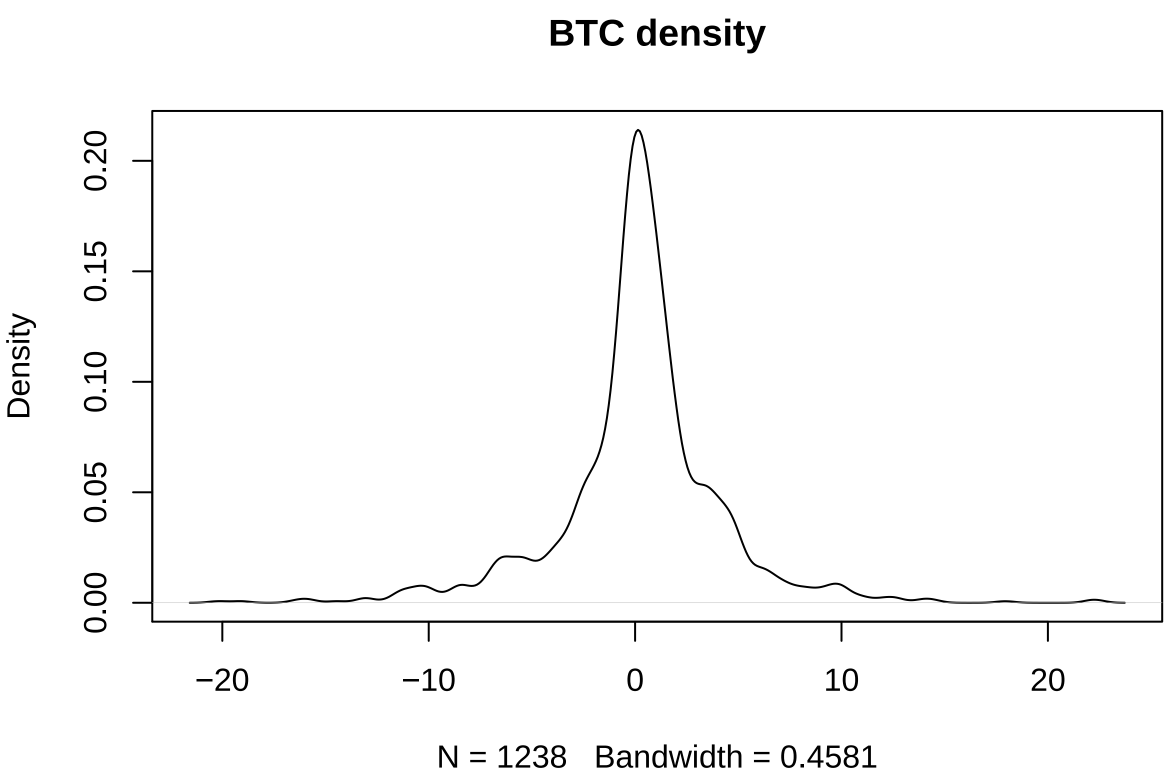
In this section, we will discuss the statistical analysis of financial market data. The classes of the *generalized hyperbolic distribution (GHD)* and its special cases, namely the *hyperbolic (HYP)* and *normal inverse Gaussian (NIG)* distributions and its application to risk modelling.

Goal

The goal is to figure out which of these distributions is best suited for the empirical return's series.

Application

The best suited model will be used to simulate hundreds of thousands of data point in various scenarios with same distribution as that of the empirical return series (real life stock return data) to make accurate predictive models for risk assessment namely the Value at Risk (VaR) and the Expected Shortfall (ES).




```
## Warning in .check.data(data = data, case = "uv", na.rm = na.rm): 1 NA
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## observations removed

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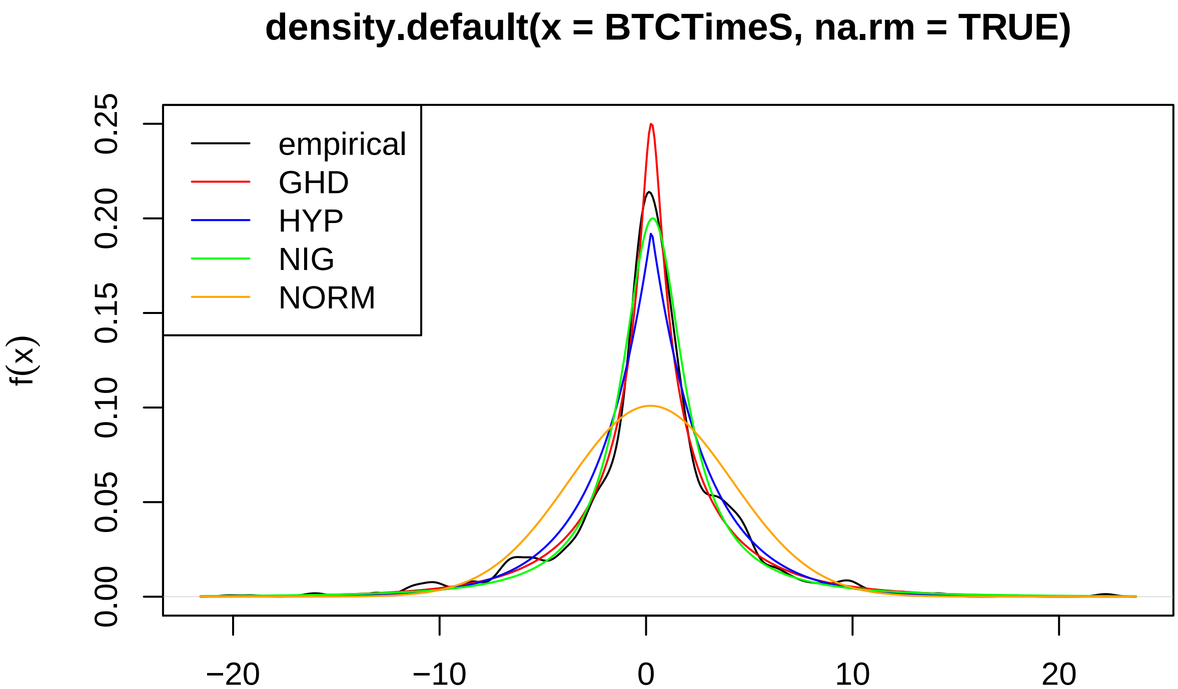
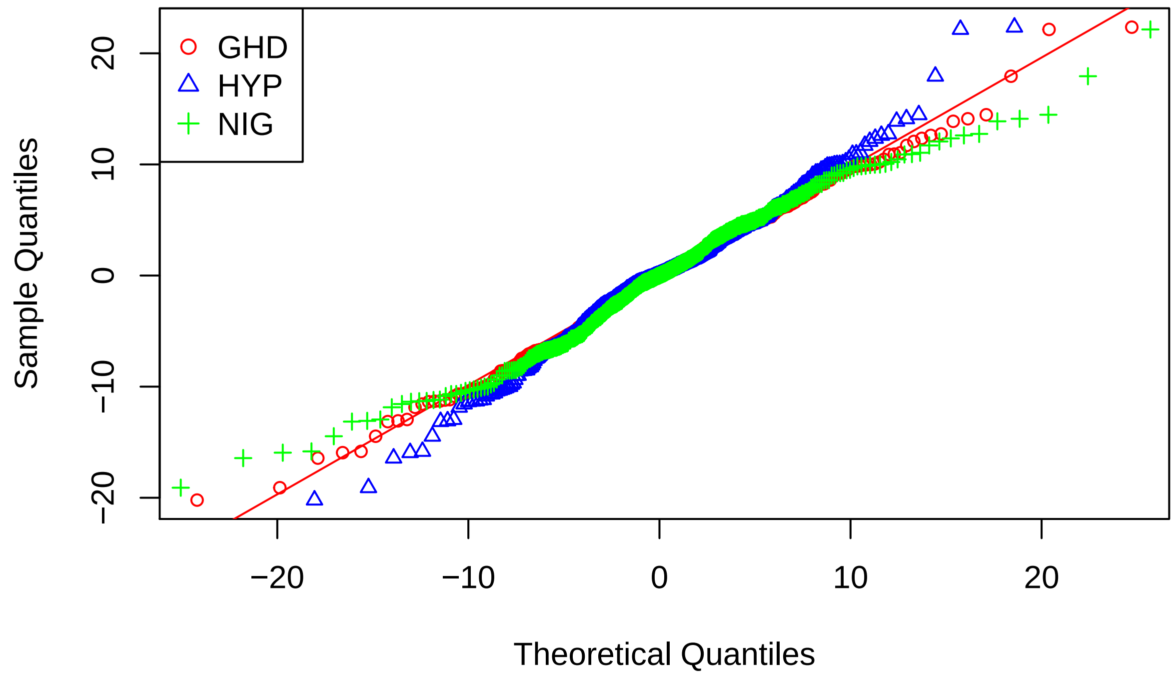


Interpretation

From the density plot of Bitcoin above we see that neither the normal distribution (NORM) nor the hyperbolic (HYP) or normal inverse gaussian (NIG) fit our empirical distribution of Bitcoin. However, the generalized hyperbolic distribution (GHD) seems to suit well the return series of Bitcoin. The quantile plot below will help us



confirm our findings.
From the quantile plot above we can see that only the generalized hyperbolic distribution (red dotted line) follows the straight red line. In conclusion the **generalized hyperbolic distribution (GHD)** is the best suited distribution to be used in risks modeling, assessing, and predicting of **Bitcoin**

Diagnostics to check which model works best

AIC test

Clearly we can now see that GHD is the best suited model for predicting BTC return series

```
##    model symmetric     lambda     alpha.bar        mu      sigma       gamma
## 6   ghyp      TRUE  0.3480425 0.09692303259 0.2614993   4.024953  0.00000000
## 1   ghyp     FALSE  0.3437272 0.09871506604 0.2741003   4.024042 -0.05188980
## 9     VG      TRUE  0.6130611 0.00000000000 0.2007304   3.919506  0.00000000
## 4     VG     FALSE  0.6320125 0.00000000000 0.2007305   3.859283  0.02147761
## 8    NIG      TRUE -0.5000000 0.18838746905 0.3204209   4.244521  0.00000000
## 3    NIG     FALSE -0.5000000 0.19018228628 0.3584791   4.236108 -0.13492695
## 7    hyp      TRUE  1.0000000 0.00166312485 0.2628214   3.637234  0.00000000
## 2    hyp     FALSE  1.0000000 0.00009696075 0.2897956   3.636238 -0.06763449
## 10     t      TRUE -1.0000088 0.00000000000 0.3332417 657.052528  0.00000000
## 5      t     FALSE -1.0000082 0.00000000000 0.3335765 681.096797  2.23683530
## 11 gauss      TRUE         NA           Inf 0.2220988   3.952479  0.00000000
##         aic       llh converged n.iter
## 6  6485.482 -3238.741      TRUE    297
## 1  6487.326 -3238.663      TRUE    422
## 9  6492.248 -3243.124      TRUE    156
## 4  6494.761 -3243.380      TRUE    207
## 8  6521.952 -3257.976      TRUE    154
## 3  6523.031 -3257.516      TRUE    221
## 7  6537.145 -3265.572      TRUE    154
## 2  6538.830 -3265.415      TRUE    319
## 10 6560.900 -3277.450      TRUE    236
## 5  6562.901 -3277.450      TRUE    257
```

```
## 11 6919.165 -3457.583      TRUE      0
```

Likelihood ratio test

The likelihood ratio test also confirms that GHD is the best suited model for predicting BTC return series

```r
LRghdnig <- lik.ratio.test(ghdfit, nigfit)
LRghdnig
```

```
## $statistic
##                  L
## 0.000000004430518
##
## $p.value
## [1] 0.0000000005561613
##
## $df
## [1] 1
##
## $H0
## [1] FALSE
```

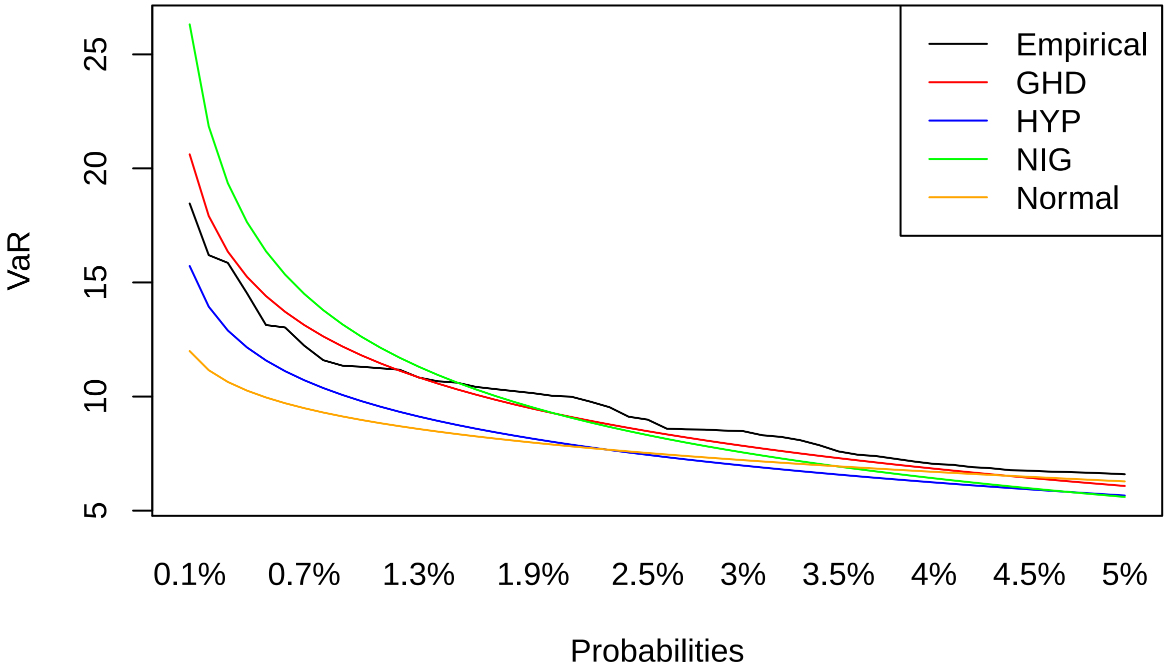
```r
LRghdhyp <- lik.ratio.test(ghdfit, hypfit)
LRghdhyp
```

```
## $statistic
##                     L
## 0.000000000002225205
##
## $p.value
## [1] 0.0000000000002380831
##
## $df
## [1] 1
##
## $H0
## [1] FALSE
```

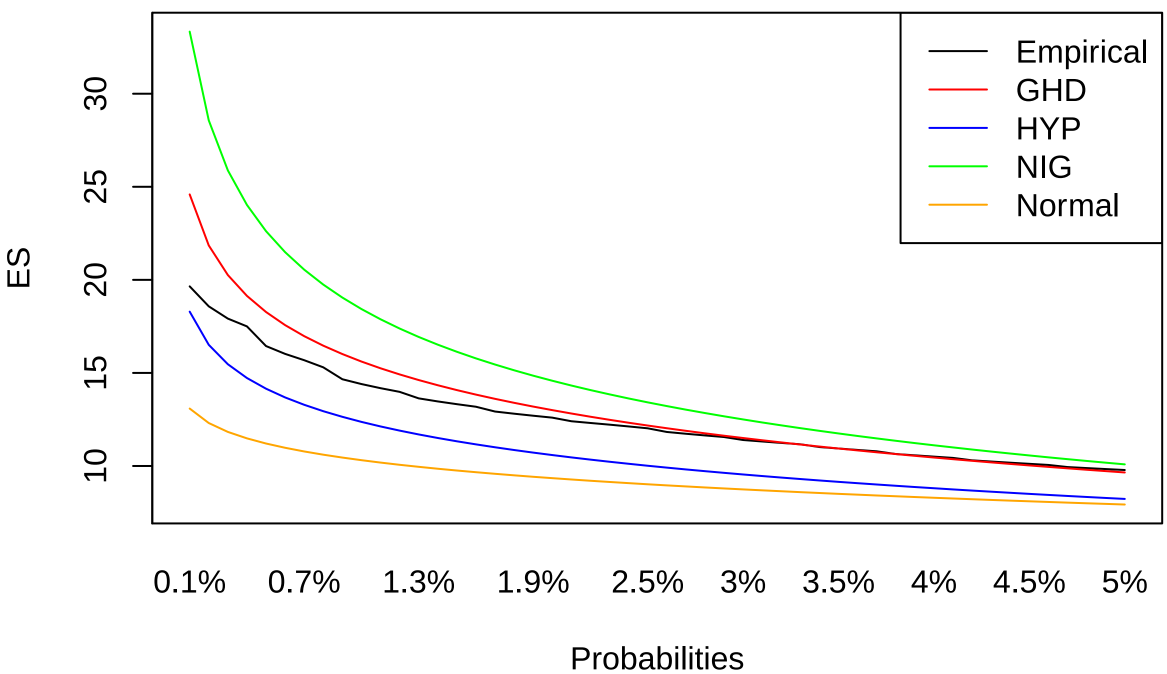
Risk assessment with the GHD

The behavior of the VaR and ES risk measures according to each of the models is investigated. The two risks measures are derived from the fitted GHD, HYP, and NIG distributions for the Bitcoin (BTC) returns from the previous subsection. These measures are calculated over a span from the 95.0% to 99.0% levels. The resulting trajectories of the VaR and ES are then compared to their empirical counterparts.

Probabilities and Value at Risk (VaR)



Probabilities and Expected Shortfall (ES)



From the graphs above and after simulating 100,000 variables, we found that VaR= 6.11 which means the amount that could be lost in the next trading day would be \$6.11 or more and if that happens, on average the lost would be \$9.624748 at 95% confidence level.

At 99% confidnece level, VaR= 11.88 and ES= 15.69 which means that the amount that could be lost in the next trading day would be \$11.88 or more and if that happens, on average the lost would be \$15.69.