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1.

a) (1) if we set an edge from the visited node to the highest node, the graph is still directed acyclic and according to the D-Separation algorithm, A and B are still independent.
(2) if we set an edge from B to the lowest node, the graph is still directed acyclic and according to the D-Separation algorithm, A and B are still independent.

b) examining every possible choice for the players, we can see that in both graphs there is a strategy to win for the first player.
In the first graph the first player needs to make the second player to establish a path from B to A to win the game.
In the second graph, since we have a visited node, if due to edge drawing, the visited node parent of A and B has only two choices, it is much easier to maintain independence between A and B, which makes the first player even more convenient to win than in the first graph.

2.

We calculate steps for each path for reaching the required probability.

a. B, E, C, D, H, I
P(E, G, D)⇒P(G, D)⇒P(A|G, H)⇒P(I, F, A)⇒P(A, F|I, H)⇒P(A|G, F)

b. I, H, C, D, E, B
P(C, H)⇒P(C, F, G, D)⇒P(F, G, D)⇒P(F, G, A, B)⇒P(F, G, A, B)⇒P(F, G, A)

In the first path we have 8+4+8+8+16+8=52 cells and in the second path we have 4+16+8+16+16+8=68 cells in CPTs. Accordingly, the first path needs less calculations and is more proper to choose.

3.

a) To feel dizzy on a random day refers to Stationary Distributions. If we assume N as Normal and D as Dizziness, we must calculate value of $P_{\infty}(D)$.
As instructed in the class, for a Stationary Distributions with two variables, P_{∞}s are easily calculated.
So we have equations below for the problem.

$P_{\infty}(D) = P(D|D)P_{\infty}(D) + P(D|N)P_{\infty}(N)$

$P_{\infty}(N) = P(N|N)P_{\infty}(N) + P(N|D)P_{\infty}(D)$

$P_{\infty}(D) + P_{\infty}(N) = 1$

According to the given Bayesian Network we have recent equations as below:

$P_{\infty}(D) = 0.3P_{\infty}(D) + 0.1P_{\infty}(N)$

$P_{\infty}(N) = 0.9P_{\infty}(N) + 0.7P_{\infty}(D)$

Solving these equations gives us $P_{\infty}(D) = 0.125$.

So with 12.5% probability the patient feels dizzy on a random day.

b) We use $P(X|Shortness\ of\ Breath,\ Dizziness)$ model to calculate probability of Heart Muscle Weakness and Sepsis for the patient.

$$P(X|SoB,\ D) = \frac{P(X, SoB, D)}{P(SoB, D)} \propto P(X|SoB,\ D)$$

Now we marginalize P(X, SoB, D) over Y. So we have:

$$P(X|SoB,\ D) = \sum_y P(X, SoB,\ D,\ y) = \sum_y P(X)P(SoB|X)P(y|X)P(D|y)$$

$$= P(X)P(SoB|X)\sum_y P(y|X)P(D|y)$$

Now we can calculate both Sepsis and Heart Muscle Weakness likelihood for the patient.

$P(X = HMW|SoB,\ D) \propto P(HMW)\ P(SoB|HMW)(P(BPD|HMW)P(D|BPD) + P(BC|HMW)P(D|BC))$

$= 0.02 \times 0.5 \times (0.7 \times 0.8 + 0.1 \times 0.7) = 0.0063$

$P(X = S|SoB,\ D) \propto P(S)\ P(SoB|S)(P(BPD|S)P(D|BPD) + P(BC|S)P(D|BC))$

$= 0.003 \times 0.85 \times (0.7 \times 0.8 + 0.5 \times 0.7) = 0.0023205$

Accordingly it is more probable for the patient to have Heart Muscle Weakness than Sepsis.

SoB: Shortness of Breath
HMW: Heart Muscle Weakness
S: Sepsis
BPD: Blood Pressure Drop
BC: Blood Clots