Tutorial #4
Numerical Integration in Jupyter

1 Introduction

Earlier this semester you learned how to use Jupyter for data entry and analysis, here you will use Jupyter to learn the fundamentals of numerical integration. Using Jupyter requires the installation of Anaconda. For those who would like to perform the lab on their own personal computer, Anaconda can be downloaded and installed from https://www.anaconda.com/distribution/.

Anaconda is a graphical interface for an installation manager, conda, and comes with many python and python related programs preinstalled. After you open Anaconda Navigator you will see a link to launch the program Jupyter. Jupyter is a program that uses iPython notebooks (.ipynb) and can run several languages incuding Julia, Python, LaTeX, and R. The name itself is an acronym of these languages. Despite being a program, Jupyter has no native graphical interface and will run in your default web browser. Note, ipynb files can't be opened directly from the finder, they must be opened from within Jupyter.

If you have a laptop, please bring it to lab since each student will need a computer.

2 Lab Ticket

Read this section, follow the instructions, and answer the questions before lab.

Bring to lab a pdf printout of a ipynb showing a plot of $\sin(t)$ for one full period.

We'll do the following activities in lab.

3 Tutorial Activities

1. Download, read, and complete the file TUTORIAL - SOLVING ODES NUMERICALLY.IPYNB which is on Canvas. *Do this before moving on to the exercises below.*

2. Create a new ipnyb and complete the following exercises pertaining to a Simple Harmonic Oscillator.

3. If you get stuck there are two files on Canvas that can help you. Both are solutions to the exercises, but with minimal comments or descriptions. The first file, TUTORIAL HELP FILE 1.IPYNB, includes a single cell that runs most of the code. The second file, TUTORIAL HELP FILE 2.IPYNB, is a section by section solution. *Only use these files if you get stuck!*

4 Tutorial Exercises

Originally developed by K. Roos at Bradley University

In this set of exercises you will build a computational model of a hanging mass-spring system that is constrained to move in 1D, using the simple Euler and the Euler-Cromer numerical schemes. The activity will guide you to discover, by using the model to produce graphs of the position, velocity, and energy of the mass as a function of time, that the Euler algorithm does not conserve energy, and that for this simple oscillatory system, a modified algorithm (Euler-Cromer) is necessary to avoid artificial behavior in the model.

4.1 Exercises

Exercise 1: Euler Algorithm Model of a SHO

Build a computational model of a simple hanging harmonic oscillator using the Euler method. Use realistic values for the parameters (i.e., spring constant k and attached mass m, such as would be encountered in a typical introductory mechanics laboratory exercise. Also, assume that the mass of the spring is negligible compared to the attached mass, and that the harmonic oscillator has been stretched vertically downward a distance y_0, relative to its hanging equilibrium position and released from rest. Use the model to produce graphs of the position and velocity of the mass as a function of time, and compare these with the exact functions for the position and velocity,

$$y(t) = y_0 \cos\left(\sqrt{\frac{k}{m}}t\right)$$

and

$$v(t) = -\sqrt{\frac{k}{m}}y_0 \sin\left(\sqrt{\frac{k}{m}}t\right),$$

that result from solving Newton's 2nd Law analytically. Does the angular frequency match that expected for a simple harmonic oscillator of mass m an spring constant k?

Exercise 2: Artificial Behavior with the Euler Algorithm

You may (should!) have noticed that something is not right with the Euler model of your hanging oscillator. Describe in detail the artificial behavior you observe in your model, and explain why it doesn't represent a realistic oscillating mass. Recall that in the Euler method, the accuracy of the solution can be increased by using a smaller value of Δt. Can you get rid of the artificial behavior by making Δt smaller?

Exercise 3: Energy in the Euler Algorithm Model of a SHO

Modify your model to produce a graph of the total energy of the oscillator as a function of time. Describe in detail what happens to the energy, and the artificial behavior observed. Can this artificial behavior in the energy be corrected by making Δt smaller? What can you conclude about using the Euler method to model a simple harmonic oscillator?

Exercise 4: Euler-Cromer Algorithm Model of a SHO

Build a model of the hanging oscillator using the modified Euler, or Euler-Cromer, numerical method. Compare the results you obtain (i.e. position and velocity vs. time) with those obtained from the simple Euler method, and with the exact solution. Comment in detail on your results.

Exercise 5: Energy in the Euler-Cromer Algorithm Model of a SHO

Modify your model to produce a graph of the total energy as a function of time. Is energy conserved for the Euler-Cromer algorithm?

5 Tutorial Assignment

Submit a pdf printout of your results to the SHO exercises.