

Analysis of the predictive power of interest rates for future economic growth - the case of Lithuania

Mikolaj Kahl (50099)

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Disclaimer

The source code for this project and the associated data are available on GitHub. It is highly encouraged to run the Python notebook along with the report as the Python code output provides an even more comprehensive data analysis than the report and greatly supports the report with additional figures, plots and tables. You can access the repository at the following link:

https://github.com/kahlus2001/PredSim_FinalProject

Some of the code has been inspired by the code written by prof. Olga Klinkowska's work that was presented to the author of this paper as part of the course material for the "Predictions and Simulations" course, which is part of Master Big Data Science at Kozminski University, 2024.

1 Introduction

This project aims to analyze the predictive power of interest rates for future economic growth, specifically focusing on Lithuania. The project will evaluate whether the level of interest rates, GDP and several other economic indicators can predict future economic growth values in short-term and long-term horizons.

Interest rates have long been studied for their predictive capabilities regarding economic performance. Early foundational work by Harvey (1988, 1989, 1991) demonstrated that the difference between long and short-term interest rates, known as the term spread, could forecast future recessions and general economic activity. These studies primarily focused on the United States and developed countries, reporting a visible relationship between the term spread and the economic cycle.

More recent research has explored the applicability of these using interest rates as predictors of economic growth over time and across different economies. For instance, Hannikainen (2015, 2017) investigated the predictive power of interest rate spreads in various economic conditions. These studies suggested that the predictive power of interest rates might not be stable and could vary depending on the economic environment and the specific characteristics of the economy under study.

This project aims to analyze the predictive power of interest rates for future economic growth in Lithuania. The analysis will employ advanced econometric techniques, including ARMA modelling, to evaluate the predictive power of interest rates. The primary focus will be on two key interest rate variables: the term spread (the difference between nominal long-term and short-term rates) and the real short-term rate (adjusted for inflation). Additionally, a third variable real gross domestic product (GDP) was selected as an interesting quantity with potential relevance to economic growth.

To ensure the robustness of the analysis, the study will compare the baseline predictive models with a set of alternative models, including nested models with only one exogenous regressor, an autoregressive model with no exogenous regressors, naive forecasts, and an eight-quarter moving average forecast. The evaluation will include both in-sample and out-of-sample analysis.

1.1 Related Academic Studies

The predictive power of interest rates on economic growth has been extensively studied by econometricians in the past. A study by Haubrich and Dombrosky (1996) titled "Predicting Real Growth Using the Yield Curve," discusses the yield curve's usefulness in predicting future economic activity and recessions. The yield curve, which plots bond

yields against their maturities, is a widely watched financial indicator by IR products trades and institutional investors like pension funds and national banks. While the yield curve typically exhibits an upward slope, its flattening or inversion has historically been an indicator of economic downturns.

Haubrich and Dombrosky's (1996) research builds on earlier work by Harvey (1989, 1991, 1993) and extends the analysis by including data up to the mid-1990s. They compare the yield curve's forecasting ability with naive models, traditional leading indicators, and professional forecasts. Utilizing out-of-sample forecasts, their study evaluates the yield curve's predictive power for both the probability of a recession and its severity.

According to the study's results, the 10-year, three-month spread has been shown to have substantial predictive power for forecasting real growth over the 30 years before the publication of the study. It has been one of the best predictors of real growth four quarters into the future. However, in the decade prior to the study's publishing date, its effectiveness has diminished, making it one of the least accurate forecast techniques from the ones under investigation. This decline may be due to a changing relationship between the yield curve and real economic activity. The study also suggests further research could explore methods such as rolling regression models or additional lags to improve its recent performance. The researchers conclude by stating that while not a replacement for professional forecasts, the yield curve still provides valuable information as a check on more complex models.

Another study by Michael Dotsey (1998) titled "The Predictive Content of the Interest Rate Term Spread for Future Economic Growth" examines the predictive power of the yield curve for forecasting real economic activity, particularly real GDP growth and recessions. His article confirms that the yield spread is generally a good indicator for forecasting future GDP growth, although its effectiveness has been declining in recent years. The accuracy of GDP growth predictions using term spread, both in-sample and out-of-sample, has declined compared to earlier periods. However, its reduced predictive power in recent periods may be connected to either some wider economic shift or just a temporary change.

Examining the out-of-sample forecast accuracy for one-year-ahead GDP growth using various models indicates that adding the yield spread does not significantly enhance forecast precision. Forecasts made over the entire period and the 1985 to 1997 period show only a slight improvement in RMSE when the spread is included. Adjusting the regression start date improves forecast accuracy slightly, but the gain remains statistically insignificant. Nonlinear models provide slightly better out-of-sample forecasts

than the spread alone, but the improvement is minimal. Overall, the yield spread offers a 10 per cent increase in forecasting accuracy over models using lagged output growth and short-term interest rates, though this gain is not statistically significant. The study also found that adding a dummy variable for tight monetary policy does not improve out-of-sample forecasting performance.

Specializing study scope on Lithuania, a thesis written by Pilström and Pohl (2009), titled “Forecasting GDP Growth - The case of the Baltic States”, aims to identify a general model for forecasting GDP growth in the Baltic States (Estonia, Latvia and Lithuania) using a vector autoregression (VAR) model. This model uses historical data on GDP, inflation and unemployment to predict future GDP growth. The authors argue that if the model proves reliable for the Baltic States, it should be applicable to other countries. Their forecasts showed promising results, especially for short-term horizons up to eight quarters ahead. Forecasts for 2009 were in line with other established forecasts, although longer-term forecasts up to sixteen quarters showed increased uncertainty, especially during the financial crisis of 2008-2009.

The empirical analysis showed that Lithuania’s GDP was forecast to fall by 12.51% in 2009 and to recover by 4.23% in 2010. The study concluded that the VAR model provided reliable short-term GDP forecasts for the Baltic States and indicated its potential applicability to other countries with different economic profiles. This research underlines the challenges of forecasting in volatile and transition economies and highlights the importance of robust econometric models in economic planning and policy-making.

The earlier studies discussed, including Haubrich and Dombrosky (1996), have laid a solid foundation for understanding the predictive power of the yield curve for future economic growth. However, these studies are over two decades old, and it is essential to revisit and reassess their findings in the context of the current economy. In addition, this study focuses on Lithuania, which is not yet covered by scientific studies to the extent larger economies, like the US, are. By building on these foundational studies and incorporating advanced econometric techniques, this project will investigate the predictive power of interest rates for future economic growth, based on the case study of Lithuania.

2 Methodology

This section outlines the equations for the models included in the study, explains the choice of variables, and describes the evaluation criteria used for in-sample (IS) and out-of-sample (OOS) analysis.

2.1 Baseline Models

The main predictive regressions for short-term and long-term economic growth are defined by the equations below:

Short-term economic growth:

$$Y_{t+1} = \alpha + \sum_{i=0}^{p-1} \beta_i Y_{t-i} + \beta_{TS} TS_t + \beta_{SR} realSR_t + \beta_{realGDP} realGDP_t + \epsilon_{t+1} \quad (1)$$

Long-term economic growth:

$$Z_{t;t+4} = \alpha + \sum_{i=0}^{p-1} \beta_i Z_{t-i} + \beta_{TS} TS_t + \beta_{SR} realSR_t + \beta_{realGDP} realGDP_t + \epsilon_{t+4} \quad (2)$$

where Y_{t+1} represents the short-term economic growth forecast for time $t+1$. $Z_{t;t+4}$ represents the long-term economic growth forecast for the period from t to $t+4$. α is the intercept term. β_i are the coefficients for the lagged terms of the dependent variables. $\sum_{i=0}^{p-1} \beta_i Y_{t-i}$ represents the sum of the lagged terms of short-term economic growth. $\sum_{i=0}^{p-1} \beta_i Z_{t-i}$ represents the sum of the lagged terms of long-term economic growth. β_{TS} is the coefficient for the term spread TS_t . β_{SR} is the coefficient for the short-term real interest rate $realSR_t$. $\beta_{realGDP}$ is the coefficient for the real GDP $realGDP_t$. ϵ_{t+1} and ϵ_{t+4} are the error terms for the short-term and long-term models, respectively.

These two models serve as baseline models for this study. All other models will be compared to the two baseline models.

2.2 Alternative Models

Several alternative models have been developed in order to evaluate the performance of various modelling techniques. This study aims to investigate:

1. Three nested models with only one exogenous regressor each (only realSR, only TS, or only GDP)

2. One autoregressive model with the optimal number of lags and no exogenous regressors (AR model)
3. A naive forecasts
4. One eight-quarter moving average forecast model

2.3 Choice of Variables and Economic Intuition

- **Term Spread (TS):** The term spread, defined as the difference between long-term and short-term interest rates. An upward-sloping yield curve (positive term spread) typically indicates economic growth, while an inverted yield curve (negative term spread) often precedes recessions.
- **Short-term Real Interest Rate (realSR):** The short-term real interest rate influences the cost of borrowing, thus also the ease of borrowing, which might influence borrowing decisions. Lower real short-term rates make borrowing cheaper, encouraging investment and consumption, and leading to economic growth. On the other hand, higher real short-term rates can reduce borrowing and consumption, greatly reducing economic activity.
- **Real GDP (realGDP):** Has been chosen as the additional variable of choice. Including real GDP as an exogenous variable accounts for the current state of economic activity, which can provide additional predictive power. Real GDP captures the aggregate output and income in the economy, making it an important indicator of economic health, as economic growth is positively correlated to GDP growth.

2.4 Evaluation Criteria

In order to evaluate the performance of the models, for both in-sample (IS) and out-of-sample (OOS) analyses, the following metrics are used:

- **Mean Squared Error (MSE):** Measures the average of the squares of the errors, providing a sense of the average magnitude of the errors.
- **Mean Absolute Error (MAE):** Measures the average magnitude of the errors in a set of predictions, without considering their direction.
- **Root Mean Squared Error (RMSE):** The square root of the MSE, which provides an indication of the average magnitude of the error.

3 Data

This section describes the dataset used in the analysis, including the source of the data, the variables included, the time span of the sample, and the transformations applied. The primary data sources are the OECD and Eurostat databases.

Data was collected for various time periods, with some variables available on a monthly basis and others on a quarterly basis. To ensure consistency, all data was transformed into quarterly frequency. Only quarters where data was available for all variables were retained, resulting in a final dataset of 88 rows (quarters) of raw and derived non-null quantities from Q2 2001 to Q1 2023.

3.1 Raw Data

The dataset includes the following raw variables:

- Consumer Price Index (CPI): A measure that examines the weighted average of prices of a basket of consumer goods and services. (Source: OECD (2024), Inflation (CPI) (indicator). doi: 10.1787/eee82e6e-en, Accessed on 10 June 2024)
- Short-term interest rate (str): The interest rate on short-term borrowing. (Source: OECD (2024), Short-term interest rates (indicator). doi: 10.1787/2cc37d77-en, Accessed on 10 June 2024)
- Long-term interest rate (ltr): The interest rate on long-term borrowing. (Source: OECD (2024), Long-term interest rates (indicator). doi: 10.1787/662d712c-en, Accessed on 10 June 2024)
- Gross Domestic Product (GDP): The total value of goods produced and services provided in Lithuania during one year. (Source: OECD (2024), Quarterly GDP (indicator). doi: 10.1787/b86d1fc8-en, Accessed on 17 June 2024)

Summary Statistics were computed for all four raw variables, see table below:

	cpi	ltr	str	gdp
count	88.000000	88.000000	88.000000	8.800000e+01
mean	94.903641	3.671352	1.875061	3.469150e+10
std	18.949942	3.225633	2.327416	1.403695e+10
min	68.990733	0.160000	-0.566377	1.410647e+10
25%	75.909342	0.310000	-0.302637	2.538522e+10
50%	98.821550	3.859333	1.483030	3.316023e+10
75%	105.178742	5.150000	2.999344	4.295819e+10
max	151.858500	14.500000	8.021772	7.095118e+10

Table 1: Summary statistics for raw data

To get a better insight into the raw data at hand, the following time series plots have been produced:

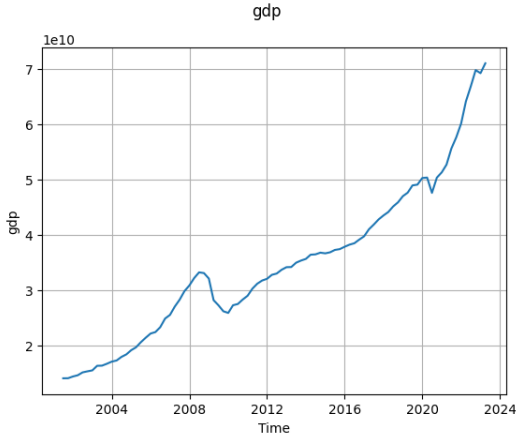


Figure 1: GDP

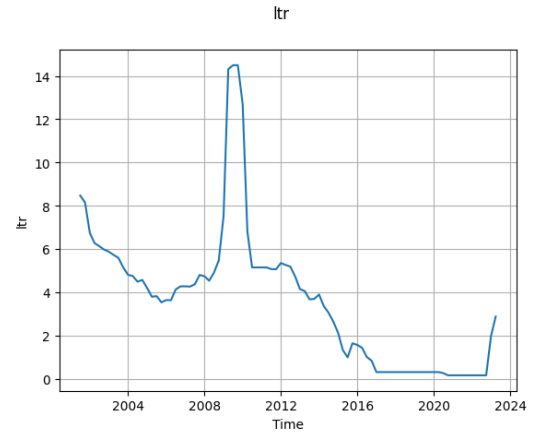


Figure 2: Long Term Interest Rate

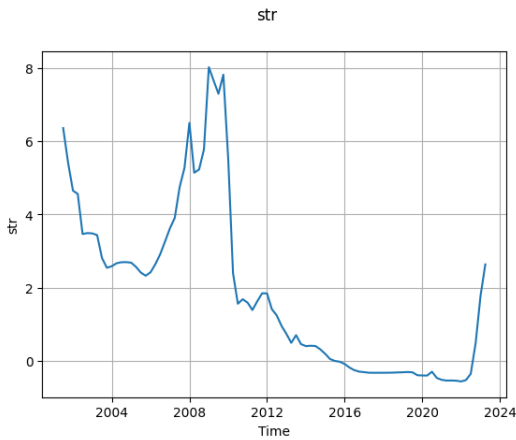


Figure 3: Short Term Interest Rate

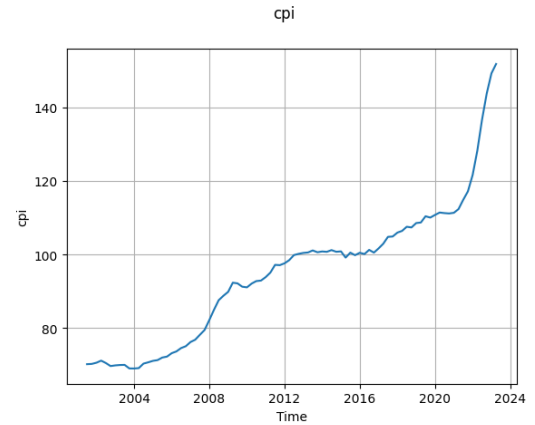


Figure 4: Consumer Price Index (CPI)

Figure 5: Time series plots of raw variables

The GDP plot shows a steady upward trend from 2001 to 2023, indicating consistent economic growth in Lithuania over the period. Notably, there is a sharp increase in GDP around 2010 and a continued rise towards 2023, suggesting significant economic expansion.

The long-term interest rate plot shows visible volatility, particularly around the 2008-2009 financial crisis, where rates spiked sharply. After the crisis, the rates declined steadily and remained low. In recent years (2023-2024) a rapid spike in rates has been observed.

The short-term interest rate plot also shows significant fluctuations, especially during the 2008-2009 financial crisis when rates spiked. Following the crisis, the rates dropped dramatically and stayed low, followed by a spike in interest rates again in 2023. The short-term rate curve has a similar shape to the long-term one.

The CPI plot indicates a steady increase in consumer prices from 2001 to 2023, reflecting rising inflation over the period. The acceleration in CPI growth is particularly noticeable after 2010, suggesting increasing price levels and potential inflationary pressures in the economy.

3.2 Derived Quantities

Having the raw variables, several derived economic quantities were computed, including real GDP, short-term and long-term economic growth rates, real short-term and long-term interest rates, and the term spread.

- Inflation rate (π_t): Calculated as the annualized rate of change in CPI.
- Real short-term rate (real_SR): The short-term interest rate adjusted for inflation.
- Term spread (TS): The difference between the nominal long-term and short-term rates.
- Real GDP (real_gdp): The GDP adjusted for inflation.
- Short-term economic growth (st_growth): The annualized percentage growth in real GDP over the next quarter.
- Long-term economic growth (lt_growth): The annualized percentage growth in real GDP over the next four quarters.
- Real GDP:

$$realGDP_t = \frac{GDP_t}{\left(\frac{CPI_t}{CPI_{base}}\right)} \quad (3)$$

where CPI_{base} is the Consumer Price Index for the base quarter.

- Short-term economic growth (Y_{t+1}):

$$Y_{t+1} = 400 \times \ln \left(\frac{realGDP_{t+1}}{realGDP_t} \right) \quad (4)$$

- Long-term economic growth ($Z_{t;t+4}$):

$$Z_{t;t+4} = 100 \times \ln \left(\frac{realGDP_{t+4}}{realGDP_t} \right) \quad (5)$$

- Term spread (TS):

$$TS_t = LTR_t - STR_t \quad (6)$$

- Real short-term rate (realSR):

$$realSR_t = \left(\frac{1 + STR_t}{1 + \pi_t} \right) - 1 \quad (7)$$

where π_t is the annualized inflation rate for quarter t , defined as:

$$\pi_t = \left(\frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} \right) \times 4 \quad (8)$$

3.3 Transformations

To ensure stationarity in the data, various transformations were applied to variables needed for the model, guided by the Augmented Dickey-Fuller (ADF) test results at a 5% confidence interval. The term spread (TS) and the real short-term rate (realSR) were both differenced, as their first differences (d_TS and d_realSR) were found to be stationary. The short-term economic growth rate (st_growth) was used in its original form since it already exhibited stationarity. For long-term economic growth (lt_growth), the first difference (d_lt.g) was calculated to achieve stationarity. These transformations were essential to meet the assumptions required by the ARMA model.

The following summary statistics can be observed for the transformed variables:

	d_TS	d_real_SR	st_g	d_lt_g
count	87.000000	87.000000	87.000000	87.000000
mean	-0.021479	-0.041653	3.916989	-0.049628
std	0.911163	0.636000	10.811438	3.776400
min	-2.771625	-3.309418	-62.954291	-16.636493
25%	-0.291903	-0.167603	1.269150	-1.304187
50%	-0.011974	-0.026036	4.745078	-0.212919
75%	0.073178	0.122084	8.750791	1.343133
max	7.165042	2.188552	23.289813	19.835918

Table 2: Summary statistics for transformed model variables

Time series plots of the transformed variables were also plotted:

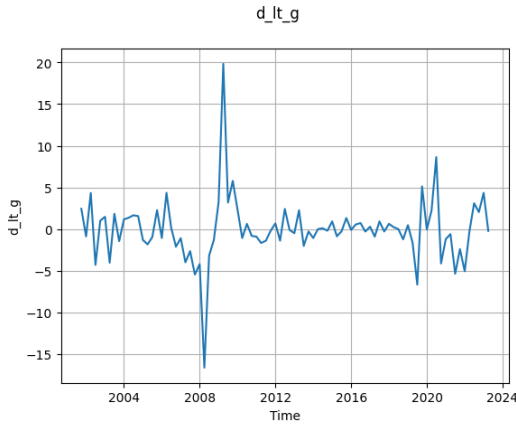


Figure 6: First Difference of Long-term Growth

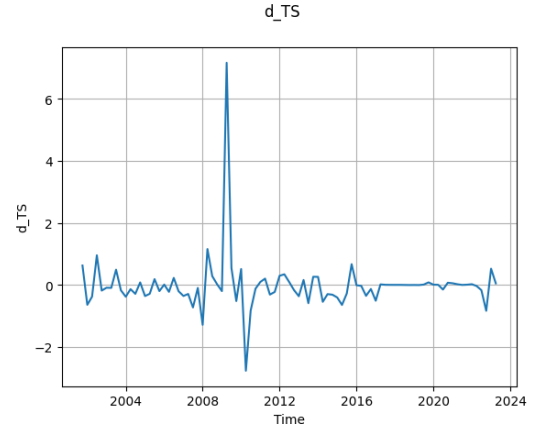


Figure 7: First Difference of Term Spread



Figure 8: First Difference of Short-Term Interest Rate

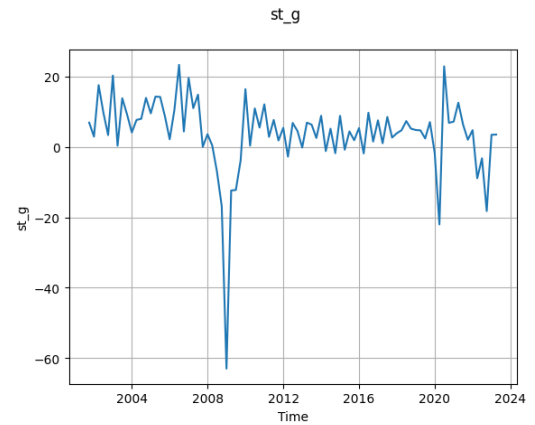


Figure 9: Real Short-Term Growth

Figure 10: Time series plots of transformed model variables

The four graphs illustrate the dynamics and stationarity of key economic variables: long-term growth, term spread, real short-term interest rate, and real short-term growth. Figure 6, showing the first difference of long-term growth (d_lt_g), fluctuates around zero with notable spikes during the 2008-2009 financial crisis and around 2015, reflecting economic instability and recovery. Figure 7, showing the first difference of the term spread (d_TS), reveals significant fluctuations, especially during the financial crisis, indicating shifts in market expectations and investor sentiment. Figure 8, presenting the first difference of the real short-term interest rate (d_real_SR), shows volatility around the 2008-2009 period and other fluctuations, reflecting changes in monetary policy responses to economic conditions. Figure 9, illustrating real short-term growth (st_g), exhibits substantial volatility, particularly during the financial crisis, indicating the responsiveness of short-term economic performance to shocks and policy changes. All graphs present mean-reverting behaviour and fluctuations around a constant mean, suggesting the series are stationary.

3.4 Model Preparation

In this analysis, several models were defined to forecast short-term and long-term growth. The two baseline models are Ordinary Least Squares (OLS) models incorporating autoregressive terms. Based on the Augmented Dickey-Fuller (ADF) and Partial Autocorrelation Function (PACF) analysis, three lags were selected for the short-term growth model, whereas only one lag was chosen for the long-term growth model. The autoregressive terms were accompanied by three exogenous variables, real GDP, the term spread and real short-term rates.

For each of the baseline models, a number of alternative models were defined:

- AR model with only the autoregressive terms of the chosen number of lags as for the baseline model
- three nested models that contained the AR model and one of three exogenous variables (Term Spread, real Short-term interest rate, real GDP)
- a naive prediction - a straightforward one-period shift, i.e. the value of y_t as the direct prediction for y_{t+1} .
- an 8 quarter moving average prediction

In total, 7 models for each of the two quantities under investigation (long-term and short-term economic growth) were created, giving a total of 14 different time-series forecasting models.

4 Study Results

For all of the models, a detailed in-sample and out-of-sample analysis was performed, and the models were compared. Separate comparisons were conducted for short-term and long-term growth models.

4.1 In-Sample Predictions

For short-term growth, various IS forecast evaluation metrics can be seen in the table below:

Model	MSE	RMSE	MAE	MAPE
Baseline	81.494206	9.027414	6.073351	18.549271
AR(3)	95.548546	9.774894	5.965948	26.257733
AR(3) + TS	93.173291	9.652631	6.029796	28.734815
AR(3) + rGDP	95.492523	9.772028	5.948544	25.269149
AR(3) + rSR	83.629474	9.144915	5.874555	18.633401
Naive	158.268131	12.580466	8.682880	47.859152
MA8	106.495057	10.319644	6.212161	36.698801

Table 3: Comparison of Important Metrics for Each Model of Short-term Growth

The Baseline model demonstrates relatively low MSE (81.494206) and RMSE (9.027414) values, indicating reasonable accuracy in predicting short-term growth. The MAE (6.073351) and MAPE (18.549271) values suggest moderate absolute prediction errors. Comparatively, the AR(3) model shows slightly higher MSE (95.548546) and RMSE (9.774894) values, indicating a decrease in prediction accuracy, although its MAE (5.965948) is marginally better than the Baseline model. The AR(3) + TS model's MSE (93.173291) and RMSE (9.652631) values are lower than the AR(3) model but higher than the Baseline model, reflecting moderate prediction accuracy. However, its MAE (6.029796) and MAPE (28.734815) values indicate less reliability in absolute and percentage error measures, compared to the Baseline model.

The AR(3) + rGDP model exhibits high MSE (95.492523) and RMSE (9.772028) values, similar to the AR(3) model, indicating reduced accuracy. Although the MAE (5.948544) is slightly better, the MAPE (25.269149) remains high, showing inconsistencies in prediction accuracy. Conversely, the AR(3) + rSR model has lower MSE (83.629474) and RMSE (9.144915) values compared to other AR models, performing similarly to the Baseline model. Its MAE (5.874555) and MAPE (18.633401) values suggest better consistency in predictions. The Naive model, with the highest MSE (158.268131) and RMSE (12.580466) values, reflects poor prediction accuracy, and its

MAE (8.682880) and MAPE (47.859152) values indicate less reliability. Lastly, the MA8 model, showing high MSE (106.495057) and RMSE (10.319644) values, indicates lower prediction accuracy compared to the Baseline and AR(3) + rSR models. Its MAE (6.212161) is moderate, but the high MAPE (36.698801) reflects inconsistencies in percentage error terms.

For long-term growth, the IS various forecast evaluation metrics can be seen in the table below:

Model	MSE	RMSE	MAE	MAPE
Baseline	6.424274	2.534615	1.487735	6.451892
AR(1)	13.317214	3.649276	2.188883	2.479971
AR(1) + TS	10.449899	3.232630	2.042391	4.730376
AR(1) + rGDP	6.448883	2.539465	1.511991	6.451125
AR(1) + rSR	13.190907	3.631929	2.167943	2.316611
Naive	21.178965	4.602061	3.014436	6.996535
MA8	15.216894	3.900884	2.411565	1.792416

Table 4: Comparison of Important Metrics for Each Model of Long-term Growth

The Baseline model shows low MSE (6.424274) and RMSE (2.534615) values, indicating high accuracy in predicting long-term growth. The MAE (1.487735) and MAPE (6.451892) values suggest a minimal absolute prediction error and a reasonable percentage error, making the Baseline model a robust predictor. The AR(1) model, however, exhibits higher MSE (13.317214) and RMSE (3.649276) values, signifying a decrease in prediction accuracy. Its MAE (2.188883) and MAPE (2.479971) values show a slight increase in absolute and percentage prediction errors, indicating less reliability compared to the Baseline model.

The AR(1) + TS model presents moderate MSE (10.449899) and RMSE (3.232630) values, which are lower than the AR(1) model but still higher than the Baseline model. Its MAE (2.042391) and MAPE (4.730376) values suggest better prediction accuracy than the AR(1) model but less than the Baseline. The AR(1) + rGDP model, with an MSE (6.448883) and RMSE (2.539465) similar to the Baseline model, demonstrates high prediction accuracy. Its MAE (1.511991) and MAPE (6.451125) values are also close to those of the Baseline, indicating strong reliability. Conversely, the AR(1) + rSR model shows higher MSE (13.190907) and RMSE (3.631929) values, reflecting decreased accuracy. Its MAE (2.167943) and MAPE (2.316611) values also indicate increased prediction errors compared to the Baseline and AR(1) + rGDP models.

The Naive model, with the highest MSE (21.178965) and RMSE (4.602061) values, shows poor prediction accuracy. Its MAE (3.014436) and MAPE (6.996535) val-

ues further highlight its unreliability. Finally, the MA8 model, with moderate MSE (15.216894) and RMSE (3.900884) values, indicates lower prediction accuracy than the Baseline and AR(1) + rGDP models but better than the Naive model. Its MAE (2.411565) is moderate, while its MAPE (1.792416) suggests a significant percentage error. In conclusion, the Baseline and AR(1) + rGDP models exhibit the highest prediction accuracy and reliability for long-term growth, while the Naive model shows the poorest performance.

4.2 Out-of-sample Predictions

The OOS predictions were carried out from 2004-01-01, to ensure data availability for all models.

For short-term growth, various OOS forecast evaluation metrics can be seen in the table below:

Model	MSE	RMSE	MAE	MAPE
Baseline	446.134992	21.121908	12.454611	46.330302
AR(3)	313.078230	17.694017	9.309375	30.084663
AR(3) + TS	330.142813	18.169832	10.174535	43.879994
AR(3) + rGDP	273.119131	16.526316	9.842522	30.615063
AR(3) + rSR	303.885695	17.432318	9.435831	33.685793
Naive	205.345975	14.329898	9.690506	43.058673
MA8	83.352252	9.129745	7.334370	32.594780

Table 5: Comparison of Important Metrics for Each Model of Short-term Growth

The Baseline model shows the highest MSE (446.134992) and RMSE (21.121908) values, indicating lower accuracy in predicting short-term growth compared to other models. Its MAE (12.454611) and MAPE (46.330302) values also suggest higher absolute and percentage prediction errors, making it less reliable for out-of-sample predictions.

The AR(3) model demonstrates better performance with lower MSE (313.078230) and RMSE (17.694017) values, reflecting improved accuracy. Its MAE (9.309375) and MAPE (30.084663) values indicate a significant reduction in prediction errors compared to the Baseline model. The AR(3) + TS model shows moderate performance with MSE (330.142813) and RMSE (18.169832) values, slightly higher than the AR(3) model but still much better than the Baseline model.

The AR(3) + rGDP model exhibits the lowest MSE (273.119131) and RMSE (16.526316) values among the AR models, indicating the highest accuracy. Its MAE

(9.842522) and MAPE (30.615063) values also demonstrate significant improvement over the Baseline model. The AR(3) + rSR model, with MSE (303.885695) and RMSE (17.432318) values, shows good accuracy but slightly higher prediction errors (MAE of 9.435831 and MAPE of 33.685793) compared to the AR(3) + rGDP model.

The Naive model shows moderate performance with MSE (205.345975) and RMSE (14.329898) values, reflecting better accuracy than the Baseline model but with higher prediction errors (MAE of 9.690506 and MAPE of 43.058673). Finally, the MA8 model demonstrates the best performance with the lowest MSE (83.352252) and RMSE (9.129745) values, indicating the highest accuracy for short-term growth predictions. Its MAE (7.334370) and MAPE (32.594780) values further confirm its reliability, making it the most accurate model in this comparison.

For long-term growth, various OOS forecast evaluation metrics can be seen in the table below:

Model	MSE	RMSE	MAE	MAPE
Baseline	24.088822	4.908036	2.417257	7.734830
AR(1)	38.173090	6.178438	2.916849	5.908152
AR(1) + TS	9.213908	3.035442	1.831919	7.786987
AR(1) + rGDP	18.741230	4.329114	2.528317	2.970889
AR(1) + rSR	15.968130	3.996014	2.442779	3.147250
Naive	21.178965	4.602061	3.014436	6.996535
MA8	15.216894	3.900884	2.411565	1.792416

Table 6: Comparison of Important Metrics for Each Model of Long-term Growth

The Baseline model exhibits a moderate level of accuracy with an MSE of 24.088822 and RMSE of 4.908036. However, its MAE (2.417257) and MAPE (7.734830) indicate relatively higher prediction errors in terms of absolute and percentage errors.

The AR(1) model shows higher MSE (38.173090) and RMSE (6.178438) values compared to the Baseline model, reflecting lower accuracy. Its MAE (2.916849) and MAPE (5.908152) suggest that while the absolute error is higher, the percentage error is slightly better.

The AR(1) + TS model performs significantly better with the lowest MSE (9.213908) and RMSE (3.035442) values among all models, indicating the highest accuracy. Its MAE (1.831919) is the lowest, although the MAPE (7.786987) is similar to the Baseline model, indicating high accuracy in terms of absolute errors but less improvement in percentage errors.

The AR(1) + rGDP model shows moderate performance with an MSE of 18.741230 and RMSE of 4.329114, suggesting reasonable accuracy. Its MAE (2.528317) is slightly

higher than the Baseline model, but the MAPE (2.970889) indicates a substantial decrease in percentage prediction errors.

The AR(1) + rSR model also demonstrates good performance with an MSE of 15.968130 and RMSE of 3.996014. The MAE (2.442779) is comparable to the Baseline model, and the MAPE (3.147250) shows a significant decrease in percentage prediction errors, indicating better reliability.

The Naive model exhibits higher MSE (21.178965) and RMSE (4.602061) values, reflecting moderate accuracy. However, its MAE (3.014436) and MAPE (6.996535) suggest higher absolute and percentage errors compared to the Baseline model.

The MA8 model performs well with an MSE of 15.216894 and RMSE of 3.900884. Its MAE (2.411565) is comparable to the Baseline model, but the MAPE (1.792416) indicates the lowest percentage of prediction errors among all models.

5 Testing Statistical Significance of Model Predictions

In order to evaluate the predictive accuracy of various models, Diebold-Mariano (DM) and Diebold-Mariano with Harvey, Leybourne, and Newbold (DM-HLN) tests were conducted. These tests compare the forecast errors of different models to determine if there are statistically significant differences in their predictive performance. Additionally, the Unconditional and Conditional Giacomini-White (EPA) tests were also conducted, to further assess the accuracy of these forecasts.

The two baseline models were compared to their alternative AR-based models to evaluate their predictive accuracy. Unfortunately, due to some technical difficulties with the study code, the naive model and moving average model were not compared to the baseline model, and thus ignored for this part of the study.

For the short-term growth models, a summary of the tests can be seen in the table below:

	AR(3) + TS	AR(3) + real GDP	AR(3) + real SR	AR(3)
DM Stat	1.0655	1.1454	1.0412	1.1768
DM p-value	0.2866	0.2520	0.2978	0.2393
DM-HLN Stat	1.0586	1.1379	1.0344	1.1691
DM-HLN p-value	0.2931	0.2587	0.3042	0.2460
Unconditional EPA Stat	1.1189	1.2899	1.0690	1.3603
Unconditional EPA Crit Val	3.8415	3.8415	3.8415	3.8415
Conditional EPA Stat	1.1369	1.3544	1.0809	1.4835
Conditional EPA Crit Val	5.9915	5.9915	5.9915	5.9915

Table 7: Results of DM and DM-HLN tests for short-term growth models

The results of the DM and DM-HLN tests for short-term growth models, summarized in the table, show that none of the alternative models significantly outperform the baseline model. The DM and DM-HLN p-values are all above 0.05, indicating the failure to reject the null hypothesis signifying equal predictive accuracy. Specifically, DM p-values range from 0.2393 to 0.2978, and DM-HLN p-values range from 0.2460 to 0.3042.

Similarly, the EPA test statistics, both unconditional and conditional, do not exceed their critical values. The unconditional EPA statistics range from 1.0690 to 1.3603, and the conditional EPA statistics range from 1.0809 to 1.4835. This indicates no significant evidence that the alternative models offer better forecasting performance than the baseline model.

For the long-term growth models, a summary of the tests can be seen in the table below:

	AR(1) + TS	AR(1) + real GDP	AR(1) + real SR	AR(1)
DM Stat	-1.0950	1.2459	0.8169	1.0425
DM p-value	0.2735	0.2128	0.4140	0.2972
DM-HLN Stat	-1.0879	1.2378	0.8116	1.0357
DM-HLN p-value	0.2801	0.2196	0.4196	0.3036
Unconditional EPA Stat	1.1807	1.5215	0.6616	1.0717
Unconditional EPA Crit Val	3.8415	3.8415	3.8415	3.8415
Conditional EPA Stat	2.4637	3.2250	1.6059	2.3944
Conditional EPA Crit Val	5.9915	5.9915	5.9915	5.9915

Table 8: Results of DM and DM-HLN tests for long-term growth models

The results of the DM and DM-HLN tests for long-term growth models, presented in the table, indicate that none of the alternative models significantly outperform the baseline model. The DM and DM-HLN p-values are all above 0.05, meaning the null

hypothesis of equal predictive accuracy cannot be rejected. Specifically, DM p-values range from 0.2128 to 0.4140, and DM-HLN p-values range from 0.2196 to 0.4196.

Similarly, the EPA test statistics, both unconditional and conditional, do not surpass their critical values. The unconditional EPA statistics range from 0.6616 to 1.5215, and the conditional EPA statistics range from 1.6059 to 3.2250. This suggests no significant evidence that the alternative models provide better forecasting performance than the baseline model.

6 Conclusion

This study investigated the predictive power of interest rates, specifically the term spread, real GDP and real short-term rate, for future economic growth in Lithuania. Employing advanced econometric techniques, including ARMA modelling and nested models, the study aimed to determine the accuracy and reliability of these interest rate variables in forecasting both short-term and long-term economic growth.

For short-term growth, the in-sample (IS) analysis revealed that the Baseline model and the AR(3) + rSR model demonstrated the lowest error metrics, indicating higher prediction accuracy and reliability compared to other models. The Naive and MA8 models showed the poorest performance with significantly higher error metrics. The out-of-sample (OOS) analysis showed that the MA8 model performed the best, followed by the AR(3) + rGDP model, indicating that these models were more accurate in predicting short-term growth outside the sample data.

For long-term growth, the IS analysis highlighted that the Baseline and AR(1) + rGDP models had the lowest error metrics, demonstrating superior prediction accuracy and reliability. The Naive model again showed the poorest performance with the highest error metrics. The OOS analysis indicated that the AR(1) + TS model performed best, followed by the MA8 model, showing that these models were more reliable for long-term growth predictions.

The Diebold-Mariano and Diebold-Mariano with Harvey, Leybourne, and Newbold tests were conducted to statistically compare the predictive accuracy of the alternative models against the baseline models. For short-term growth, the results indicated no significant evidence that the alternative models significantly outperformed the baseline model, as all DM and DM-HLN p-values were above 0.05. Similarly, the Giacomini-White (EPA) tests showed no significant improvement in forecasting performance by the alternative models.

For long-term growth, the DM and DM-HLN tests also indicated that none of the alternative models significantly outperformed the baseline model. The EPA tests further supported these findings, showing no significant evidence that the alternative models provided better forecasting performance than the baseline model.

In conclusion, while incorporating interest rate variables such as the term spread and real short-term rate into AR models showed some improvements in certain contexts, the baseline models generally provided robust and reliable forecasts for both short-term and long-term economic growth. The findings suggest that the additional complexity of incorporating these interest rate variables does not consistently improve predictive

accuracy. Future research could explore the use of other economic indicators and more advanced modelling techniques, to improve economic growth forecasting.

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