that it is possible to apply the parametric t test, the McNemar test, like the binomial test, has power-efficiency of about 95 percent for $A + D = 6$, and the power-efficiency declines as $A + D$ increases to an asymptotic efficiency of about 63 percent.

5.1.5 References

Discussions of this test are presented in McNemar (1969) and Everitt (1977).

5.2 THE SIGN TEST

5.2.1 Function

The *sign test* gets its name from the fact that it is based upon the direction of differences between two measures rather than quantitative measures as its data. It is particularly useful for research in which quantitative measurement is impossible or infeasible, but in which it is possible to determine, for each pair of observations, which is the "greater" (in some sense).

The sign test is applicable to the case of two related samples when the experimenter wishes to establish that two conditions are different. The only assumption underlying this test is that the variable under consideration has a continuous distribution. The test does not make any assumptions about the form of the distribution of differences nor does it assume that all subjects are drawn from the same population. The different pairs may be from different populations with respect to age, sex, intelligence, etc.; the only requirement is that within each pair the experimenter has achieved matching with respect to the relevant extraneous variables. As noted earlier in this chapter, perhaps the best way to accomplish this is to use each subject as its own control.

5.2.2 Method

The null hypothesis tested by the sign test is that

$$P[X_i > Y_i] = P[X_i < Y_i] = \tfrac{1}{2}$$

where X_i is the judgment or score under one condition (or before the treatment) and Y_i is the judgment or score under the other condition (or after the treatment). That is, X_i and Y_i are the two "scores" for a matched pair. Another way of stating H_0 is that the median difference between X and Y is zero.

In applying the sign test, we focus on the direction of the difference between every X_i and Y_i, noting whether the *sign* of the difference is positive or negative (+ or −). When H_0 is true, we would expect the number of pairs which have $X_i > Y_i$ to be equal to the number of pairs which have $X_i < Y_i$. That is, if the null hypothesis were true, we would expect about half of the differences to be negative and half to be positive. H_0 is rejected if too few differences of one sign occur.

5.2.3 Small Samples

The probability associated with the occurrence of a particular number of +'s and −'s can be determined by reference to the binomial distribution with $p = q = \frac{1}{2}$, where N is the number of pairs. If a matched pair shows no difference (i.e., the difference is zero and has no sign), it is dropped from the analysis and N is reduced accordingly. Appendix Table D gives the probabilities associated with the occurrence under H_0 of values as small as x for $N \leq 35$. To use this table, let x be the number of fewer signs.

For example, suppose 20 pairs are observed. Sixteen show differences in one direction (+) and the other four show differences in the other direction (−). In this case, $N = 20$ and $x = 4$. Reference to Appendix Table D reveals that the probability of this few or fewer −'s when H_0 is true (i.e., that $p = \frac{1}{2}$) is .006 (one-tailed).

The sign test may be either one-tailed or two-tailed. In a one-tailed test, the alternative hypothesis states which sign (+ or −) will occur more frequently. In a two-tailed test, the prediction is simply that the frequencies with which the two signs occur will be significantly different. For a two-tailed test, the probability values in Appendix Table D are doubled.

Example 5.2*a* **For small samples.** A researcher was studying husband-wife decision-making processes.[2] A sample of husband-wife pairs was intensively studied to determine the perceived role of each spouse in a major purchase decision—in this case, a home. At one time, each spouse completed a questionnaire concerning the perceived influence that each spouse (in their own marriage) should have in various aspects of the purchase decision. The response to the question was on a scale from husband-dominant to equality to wife-dominant. For each husband-wife pair, the difference between their ratings was determined and was coded as + if the husband judged that the husband should have greater influence than the influence accorded to the husband by the wife. The difference was coded as − if the husband's rating accorded greater influence to the wife than that rated by the wife. The difference was coded as 0 if the couple were in complete agreement on the degree of influence appropriate in the decision.

i. Null hypothesis. H_0: husbands and wives agree on the degree of influence each should have in one aspect of the home purchase decision. H_1: husbands judge that they should have greater influence in the purchase decision than their wives judge that they should.

ii. Statistical test. The rating scale used in this study constitutes at best a partially ordered scale. The information contained in the ratings is preserved if the difference between each couple's two ratings is expressed by a sign (+ or −). Each couple in this study constitutes a matched pair; they are matched in the sense that each responded to the same question concerning spousal influence in the purchase decision and each is a member of the same family. The sign test is appropriate for measures of the sort described and, of course, is appropriate for a case of two related or matched samples.

iii. Significance level. Let $\alpha = .05$ and N is the number of couples in one of the conditions = 17. (N may be reduced if ties occur.)

[2] This example is motivated by Qualls, W. J. (1982). A study of joint decision making between husbands and wives in a housing purchase decision. Unpublished D.B.A. dissertation, Indiana University.

TABLE 5.4
Judged influence in decision making

Couple	Rating of influence: Husband	Rating of influence: Wife	Direction of difference	Sign
A	5	3	$X_H > X_W$	+
B	4	3	$X_H > X_W$	+
C	6	4	$X_H > X_W$	+
D	6	5	$X_H > X_W$	+
E	3	3	$X_H = X_W$	0
F	2	3	$X_H < X_W$	−
G	5	2	$X_H > X_W$	+
H	3	3	$X_H = X_W$	0
I	1	2	$X_H < X_W$	−
J	4	3	$X_H > X_W$	+
K	5	2	$X_H > X_W$	+
L	4	2	$X_H > X_W$	+
M	4	5	$X_H < X_W$	−
N	7	2	$X_H > X_W$	+
O	5	5	$X_H = X_W$	0
P	5	3	$X_H > X_W$	+
Q	5	1	$X_H > X_W$	+

iv. Sampling distribution. The associated probability of occurrence of values as large as x is given by the binomial distribution for $p = q = \frac{1}{2}$. The binomial distribution is tabled for selected values of N in Appendix Table D.

v. Rejection region. Since H_1 predicts the direction of the differences, the rejection region is one-tailed. It consists of all values of x (where x is the number of pluses, since the prediction for H_1 is that the positive differences will predominate) for which the one-tailed probability of occurrence when H_0 is true is equal to or less than $\alpha = .05$.

vi. Decision. The influence judgments of each spouse were rated on a seven-point rating scale. On this scale, a rating of 1 represents a judgment that the wife should have complete authority for the decision, a rating of 7 represents a judgment that the husband should have complete authority for the decision, and intermediate values indicate intermediate degrees of influence. Table 5.4 shows the influence ratings assigned by each husband (H) and wife (W) among the 17 couples. The signs of the differences between each couple's ratings are shown in the final column. Note that three couples showed differences opposite to the predicted difference; these are coded with a minus sign. Three other couples were in complete agreement about the influence and, thus, there was no difference; these are coded with a zero and the sample size is reduced from $N = 17$ to $N = 17 - 3 = 14$. The remaining couples showed differences in the predicted direction.

For the data in Table 5.4, x is the number of positive signs = 11, and N is the number of matched pairs = 14. Appendix Table D shows that for $N = 14$ the probability of observing $x \geq 11$ has a one-tailed probability when H_0 is true of .029. Since this value is in the region of rejection for $\alpha = .05$, our decision is to reject H_0 in favor of H_1. Thus we conclude that husbands believe that they should have greater influence in the home purchase decision than their wives believe that they should.

TIES. For the sign test, a "tie" occurs when it is not possible to discriminate between the values of a matched pair or the two values are equal. In the case of the couples, three ties occurred: the researcher judged that three couples agreed on the degree of influence that each spouse should have in the home purchase decision.

All tied cases are dropped from the analysis for the sign test, and the N is correspondingly reduced. Thus N is the number of matched pairs whose difference score *has a sign*. In the example, 14 of the 17 couples had difference scores with a sign, so for that study $N = 14$.

RELATION TO THE BINOMIAL EXPANSION. In the study just discussed, we should expect that when H_0 is true the frequency of pluses and minuses would be the same as the frequency of heads and tails in a toss of 14 unbiased coins. (More exactly, the analogy is to the toss of 17 unbiased coins, 3 of which rolled out of sight and, thus, could not be included in the analysis.) The probability of getting as extreme an occurrence as 11 heads and 3 tails in a toss of 14 coins is given by the binomial distribution as

$$\sum_{i=x}^{N} \binom{N}{i} p^i q^{N-i}$$

where N = the total number of coins tossed = 14

x = the observed number of heads = 11

and
$$\binom{N}{i} = \frac{N!}{i!(N-i)!}$$

In the case of 11 or more heads when 14 coins are tossed, this is

$$P[x \geq 11] = \frac{\binom{14}{11} + \binom{14}{12} + \binom{14}{13} + \binom{14}{14}}{2^{14}}$$

$$= \frac{364 + 91 + 14 + 1}{16{,}284}$$

$$= .029$$

The probability found by this method is, of course, identical to that found by the method used in the example.

5.2.4 Large Samples

If N is larger than 35, the normal approximation to the binomial distribution can be used. This distribution has

$$\text{Mean} = \mu_x = Np = \frac{N}{2}$$

and
$$\text{Variance} = \sigma_x^2 = Npq = \frac{N}{4}$$

That is, the value of z is given by

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{x - N/2}{.5\sqrt{N}} \tag{5.3}$$

$$= \frac{2x - N}{\sqrt{N}} \tag{5.3a}$$

This expression is approximately normally distributed with zero mean and unit variance. Equation (5.3*a*) is computationally more convenient; however, it does somewhat obscure the form of the test.

The approximation becomes better when a *correction for continuity* is employed. The correction is effected by reducing the difference between the observed number of pluses (or minuses) and the expected number (i.e., the mean) when H_0 is true by .5. (See p. 43 for a more complete discussion of this point.) That is, with the correction for continuity,

$$z = \frac{(x \pm .5) - N/2}{.5\sqrt{N}} \tag{5.4}$$

where $x + .5$ is used when $x < N/2$ and $x - .5$ is used when $x > N/2$. A computationally simpler form of Eq. (5.4) is the following:

$$z = \frac{2x \pm 1 - N}{\sqrt{N}} \tag{5.4a}$$

Here we use $+1$ when $x < N/2$ and -1 when $x > N/2$. The value of z obtained by the application of Eq. (5.4) may be considered to be normally distributed with zero mean and unit variance. Therefore, the significance of an obtained z may be determined by reference to Appendix Table A. That is, Appendix Table A gives the one-tailed probability associated with the occurrence when H_0 is true of values as extreme as an observed x. If a two-tailed test is required, the probability obtained from Table A should be doubled.

Example 5.2*b* For large samples. Suppose an experimenter were interested in determining whether a certain film about juvenile delinquency would change the opinions of the members of a particular community about how severely juvenile delinquents should be punished. He draws a random sample of 100 adults from the community and conducts a "before and after" study, having each subject serve as his or her own control. He asks each subject to take a position on the amount or degree of punitive actions which should be taken against juvenile delinquents. He then shows the film to the 100 adults, after which he repeats the question.

i. Null hypothesis. H_0: the film has no systematic effect on attitudes. That is, of those whose opinions change after seeing the film, just as many decrease as increase the amount of punishment they believe to be appropriate, and any difference observed is of a magnitude which might be expected in a random sample from a population on which the film would have no systematic effect. H_1: the film has a systematic effect on attitudes.

TABLE 5.5
Adult opinions concerning degree of severity of punishment for juvenile delinquents

Judged attitude	Number
Increase in severity	26
Decrease in severity	59
No change	15

ii. Statistical test. The sign test is chosen for this study of two related groups because the study uses ordinal measures within paired replicates, and, therefore, the differences may appropriately be represented by plus and minus signs.

iii. Significance level. Let $\alpha = .01$ and N is the number of adults (out of 100) who show a difference in their attitudes.

iv. Sampling distribution. When H_0 is true, z as computed from Eq. (5.4*a*) [or Eq. (5.4)] is approximately normally distributed for $N > 35$. Appendix Table A gives the probability associated with the occurrence of values as extreme as an obtained z.

v. Rejection region. Since H_1 does not state the direction of the predicted differences, the region of rejection is two-tailed. It consists of all values of z which are so extreme that their associated probability of occurrence when H_0 is true is equal to or less than $\alpha = .01$.

vi. Decision. The results of this study of the effect of a film upon opinion are summarized in Table 5.5. Did the film have any effect? The data show that there were 15 adults who did not change and 85 who did. The analysis is based only on those subjects who did change. If the film had no systematic effect, we would expect about half of those whose attitudes changed after viewing the film to have increased their judgment and about half to have decreased their judgment. That is, of the 85 people whose attitudes changed, we would expect about 42.5 to show one kind of change and 42.5 to show the other change. Now we observe that 59 *decreased* and 26 *increased.* We may determine the probability that, when H_0 is true, a split as extreme or more extreme could occur by chance. Using Eq. (5.4), and noting that $x > N/2$ (that is, $59 > 42.5$), we have

$$z = \frac{2x \pm 1 - N}{\sqrt{N}} \tag{5.4a}$$

$$z = \frac{118 - 1 - 85}{\sqrt{85}}$$

$$= 3.47$$

Reference to Appendix Table A reveals that the probability $|z| \geq 3.47$ when H_0 is true is $2(.0003) = .0006$. (The probability shown in the table is doubled because the tabled values are for a one-tailed test, whereas the region of rejection in this case is two-tailed.) Since .0006 is smaller than $\alpha = .01$, the decision is to reject the null hypothesis in favor of the alternative hypothesis. We conclude from these data that the film had a significant systematic effect on adults' attitudes regarding the severity of punishment desirable for juvenile delinquents.

This example was included not only because it demonstrates a useful application of the sign test but also because data of this sort are often analyzed incorrectly. The data in Table 5.5 are cast in terms of the variables of interest. A fourfold table could be constructed which contained the same information, but would require that we also know the separate frequencies B and C.[3] It is not too uncommon for researchers to analyze such data by using the row and column totals as if they represented independent samples. This is not the case; the row and column totals are separate but not independent representations of the same data.

This example could also have been analyzed by the McNemar test for the significance of changes (Sec. 5.1). With the use of the data in Table 5.5,

$$X^2 = \frac{(|A - D| - 1)^2}{A + D} \qquad \text{with } df = 1 \tag{5.2}$$

$$= \frac{(|59 - 26| - 1)^2}{59 + 26}$$

$$= 12.05$$

Appendix Table C shows that $X^2 \geq 12.05$ with $df = 1$ has a probability of occurrence when H_0 is true of less than .001. This finding is not in conflict with that yielded by the sign test. The slight difference between the two results is due to the limitations of the table of the chi-square distribution used. It should be noted that, if z is computed by using Eq. (5.3) and if X^2 is computed by using Eq. (5.1) (that is, no correction for continuity is made in either case), then z^2 will be identical to X^2 for any set of data. The same is true if the calculations are made by using the correction for continuity [Eqs. (5.2) and (5.4)].

5.2.5 Summary of Procedure

These are the steps in the use of the sign test:

1. Determine the sign of the difference between the two members of each pair.
2. By counting, determine the value of N equal to the number of pairs whose differences show a sign (ties are ignored).
3. The method for determining the probability of occurrence of data as extreme or more extreme when H_0 is true depends on the size of N:
   (*a*) If N is 35 or smaller, Appendix Table D shows the one-tailed probability associated with a value as small as the observed value of x = the number of fewer signs. For a two-tailed test, double the probability value obtained from Appendix Table D.
   (*b*) If N is larger than 35, compute the value of z by using Eq. (5.4*a*). Appendix Table A gives one-tailed probabilities associated with values as extreme as various values of z. For a two-tailed test, double the probability values shown in Appendix Table A.
4. If the probability yielded by the test is less than or equal to α, reject H_0.

[3] The reader is urged to construct the fourfold table as an exercise using $B = 7$ and $C = 8$.

5.2.6 Power-Efficiency

The power-efficiency of the sign test is about 95 percent for $N = 6$, but it declines as the size of the sample increases to an eventual (asymptotic) efficiency of 63 percent. Discussions of the power-efficiency of the sign test for large samples may be found in Lehmann (1975).

5.2.7 References

For other discussions of the sign test, the reader should consult Dixon and Massey (1983), Lehmann (1975), Moses (1952), and Randles and Wolfe (1979).

5.3 THE WILCOXON SIGNED RANKS TEST

The sign test discussed in the previous section utilizes information only about the *direction* of the differences within pairs. If the relative *magnitude* as well as the direction of the differences is considered, a more powerful test can be used. The *Wilcoxon signed ranks test* does just that—it gives more weight to a pair which shows a large difference between the two conditions than to a pair which shows a small difference.

The Wilcoxon signed ranks test is a very useful test for the behavioral scientist. With behavioral data, it is not uncommon that the researcher can (1) tell which member of a pair is "greater than," i.e., tell the sign of the difference between any pair, and (2) rank the differences in order of absolute size. That is, the researcher can make the judgment of "greater than" between any pair's two values as well as between any two difference scores arising from any two pairs. With such information the experimenter may use the Wilcoxon signed ranks test.

5.3.1 Rationale and Method

Let d_i be the difference score for any matched pair, representing the difference between the pair's scores under two treatments X and Y. That is, $d_i = X_i - Y_i$. To use the Wilcoxon signed ranks test, rank all of the d_i's without regard to sign: give the rank of 1 to the smallest $|d_i|$, the rank of 2 to the next smallest, etc. When ranking scores without regard to sign, a d_i of -1 is given a lower rank than a d_i of either $+2$ or -2.

Then to each *rank* affix the sign of the difference. That is, indicate which ranks arose from negative d_i's and which ranks arose from positive d_i's.

TABLE D

Table of probabilities associated with values as small as (or smaller than) observed values of k in the binomial test

Given in the body of the table are one-tailed probabilities under H_0 for the binomial test when $p = q = \frac{1}{2}$.

Entries are $P[Y \leq k]$. Note that entries may also be read as $P[Y \geq N - k]$

N	k = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	062	312	688	938	1.0													
5	031	188	500	812	969	1.0												
6	016	109	344	656	891	984	1.0											
7	008	062	227	500	773	938	992	1.0										
8	004	035	145	363	637	855	965	996	1.0									
9	002	020	090	254	500	746	910	980	998	1.0								
10	001	011	055	172	377	623	828	945	989	999	1.0							
11		006	033	113	274	500	726	887	967	994	999+	1.0						
12		003	019	073	194	387	613	806	927	981	997	999+	1.0					
13		002	011	046	133	291	500	709	867	954	989	998	999+	1.0				
14		001	006	029	090	212	395	605	788	910	971	994	999	999+	1.0			
15			004	018	059	151	304	500	696	849	941	982	996	999+	999+	1.0		
16			002	011	038	105	227	402	598	773	895	962	989	998	999+	999+	1.0	
17			001	006	025	072	166	315	500	685	834	928	975	994	999	999+	999+	1.0
18			001	004	015	048	119	240	407	593	760	881	952	985	996	999	999+	999+
19				002	010	032	084	180	324	500	676	820	916	968	990	998	999+	999+
20				001	006	021	058	132	252	412	588	748	868	942	979	994	999	999+

Note: Decimal points omitted, and values less than .0005 are omitted.

TABLE D (continued)

N	k = 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
21				001	004	013	039	095	192	332	500	668	808	905	961	987	996	999
22					002	008	026	067	143	262	416	584	738	857	933	974	992	998
23					001	005	017	047	105	202	339	500	661	798	895	953	983	995
24					001	003	011	032	076	154	271	419	581	729	846	924	968	989
25						002	007	022	054	115	212	345	500	655	788	885	946	978
26						001	005	014	038	084	163	279	423	577	721	837	916	962
27						001	003	010	026	061	124	221	351	500	649	779	876	939
28							002	006	018	044	092	172	286	425	575	714	828	908
29							001	004	012	031	068	132	229	356	500	644	771	868
30							001	003	008	021	049	100	181	292	428	572	708	819
31								002	005	015	035	075	141	237	360	500	640	763
32								001	004	010	025	055	108	189	298	430	570	702
33								001	002	007	018	040	081	148	243	364	500	636
34									001	005	012	029	061	115	196	304	432	568
35									001	003	008	020	045	088	155	250	368	500

Note: Decimal points omitted, and values less than .0005 are omitted.

TABLE A
Probabilities associated with the upper tail of the normal distribution

The body of the table gives one-tailed probabilities under H_0 of z. The left-hand marginal column gives various values of z to one decimal place. The top row gives various values to the second decimal place. Thus, for example, the one-tailed p of $z \geq .11$ or $z \leq -.11$ is $p = .4562$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007									
3.3	.0005									
3.4	.0003									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

TABLE A (continued)

Selected significance levels for the normal distribution

Two-tailed α	.20	.10	.05	.02	.01	.002	.001	.0001	.00001
One-tailed α	.10	.05	.025	.01	.005	.001	.0005	.00005	.000005
z	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417

TABLE A$_{II}$

Critical z values for $\#c$ multiple comparisons*

Entries in the table for a given $\#c$ and level of significance α is the point on the standard normal distribution such that the upper-tail probability is equal to $\frac{1}{2}\alpha/\#c$. For values of $\#c$ outside the range included in the table, z can be found by using Appendix Table A.

	α					
	Two-Tailed .30	.25	.20	.15	.10	.05
#c	One-tailed .15	.125	.10	.075	.05	.025
1	1.036	1.150	1.282	1.440	1.645	1.960
2	1.440	1.534	1.645	1.780	1.960	2.241
3	1.645	1.732	1.834	1.960	2.128	2.394
4	1.780	1.863	1.960	2.080	2.241	2.498
5	1.881	1.960	2.054	2.170	2.326	2.576
6	1.960	2.037	2.128	2.241	2.394	2.638
7	2.026	2.100	2.189	2.300	2.450	2.690
8	2.080	2.154	2.241	2.350	2.498	2.734
9	2.128	2.200	2.287	2.394	2.539	2.773
10	2.170	2.241	2.326	2.432	2.576	2.807
11	2.208	2.278	2.362	2.467	2.608	2.838
12	2.241	2.301	2.394	2.498	2.638	2.866
15	2.326	2.394	2.475	2.576	2.713	2.935
21	2.450	2.515	2.593	2.690	2.823	3.038
28	2.552	2.615	2.690	2.785	2.913	3.125

* $\#c$ is the number of comparisons.

TABLE A$_{III}$

Critical values $q(\alpha, \#c)$ for $\#c$ dependent multiple comparisons*†‡

Entries in the table for a given $\#c$ and level of significance α are critical values for the maximum absolute values of $\#c$ standard normal random variables with common correlation .5 for the two-tailed test, and critical values for the upper tail of $\#c$ standard normal random variables with common correlation .5 for the one-tailed test.

	Two-Tailed		One-Tailed	
#c α:	.05	.01	.05	.01
1	1.96	2.58	1.65	2.33
2	2.21	2.79	1.92	2.56
3	2.35	2.92	2.06	2.69
4	2.44	3.00	2.16	2.77
5	2.51	3.06	2.24	2.84
6	2.57	3.11	2.29	2.89
7	2.61	3.15	2.34	2.94
8	2.65	3.19	2.38	2.97
9	2.69	3.22	2.42	3.00
10	2.72	3.25	2.45	3.03
11	2.74	3.27	2.48	3.06
12	2.77	3.29	2.50	3.08
15	2.83	3.35	2.57	3.14
20	2.91	3.42	2.64	3.21

* $\#c$ is the number of comparisons.

† Two-tailed entries are adapted from Dunnett, C. W. (1964). New tables for multiple comparisons with a control. *Biometrics*, **20**, 482–491. (With the permission of the author and the editor of *Biometrics*.)

‡ One-tailed entries are adapted from Gupta, S. S. (1963). Probability integrals of multivariate normal and multivariate *t*. *Annals of Mathematical Statistics*, **34**, 792–828. (With the permission of the author and the publisher at *Annals of Mathematical Statistics*.)