END 327E DECISION THEORY

Term Project

WHEN TO STOP LOOKING FOR BETTER

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WHEN TO STOP LOOKING FOR BETTER?

In this project, we investigated when should we stop when there are numerous choices and if we don't have time to examine all of them or if we practically can not. In this situation, we are trying to make the best possible decision but with incomplete information. These problems are often known as “optimal stopping problems”. Our main goal is to determine when to stop examining and make the decision.

In 1949, Merrill M. Flood introduced secretary problems and called it a “fiance” problem in a lecture. (Flood, 1958) These problems have some standard attributes. Let’s say he wants to hire a secretary and examine some elements of decision analysis:

1. Candidates are alternatives and their quality distribution is unknown. We only know there are *n* number of candidates.
2. Decision makers can only make a binary and immediate decision. This means he can only hire or reject the current interviewee at the end of the interview and he cannot turn back the previously rejected.
3. His objective is to hire the best quality candidate. Since we don't know that, it is logical to aim to maximize the probability of hiring the best.

If he randomly chooses from n number of candidates, the probability of choosing the best is 1/n. When n increases even more, when n goes to this probability goes to zero. So this is not a good strategy. Instead, he came up with a better strategy called “Look then Leap”.

In this strategy, “Look” is for gathering information. Spend some portion of the results to get information about candidates. Spend some portion of the resources to get information. Then “Leap” is for exploit and explore. In this stage, you should look for better option than you have seen in the looking phase. So he says, “Reject the first p unconditionally, then accept the next candidate who is better than all.” He proved this strategy dominates any other strategy. It is a good strategy but we need to determine a p value for it. In our examples we examine ways to determine p for when to stop.

Shoe Problem
Finding the best stopping point when buying something without looking back

We can start with an easygoing sample. Let’s imagine that we want to buy a new pair of shoes (it could be buying new apartment or hiring a secretary or anything you want to search with multiple option) and need to decide when to stop look for the better. If we had only 3 shoes stores to search, when should we stop and buy the shoe with or without turning back for finding the best shoes? We only have 3 options and are trying to increase the probability of buying the best shoes.

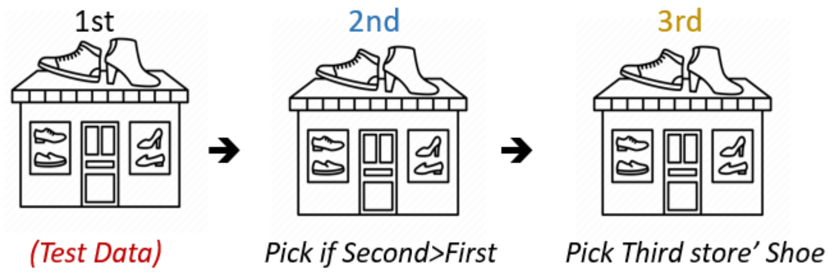
If we had no strategy and just randomly pick a store and buy the shoe in it, then it will be %33 chance of picking the best one. Also in this situation, we consider that we cannot turn back to the one of the previous shoe stores. We can think it like if we pass a store than another customer would buy that shoes that we passed because there are too much demand in that recent time. If we really had 3 options and not capable to turn back, the strategy of “look and leap” could very useful for us instead of randomly selecting it.

Look And Leap Strategy: In this strategy, we always need to set a threshold for stopping point at the beginning of the strategy. In this 3 optioned example, we need to check the first shoe store and pass it no matter what for the gathering the data about shoes for future comparison purposes (look phase). We have 1 out of 3 shoe stores’ information about how good the shoe is. With the data we have, we’ll go and check for the (leap phase) second store if there’s a better one in there. If the shoe in the second store really better than the one in the first store, we stop looking for better one and buy it immedietly without even checking the third store. In any case that the second one is worse than the first, we have no more option but buying the third. With usage of this basic algorithm could provide %50 probability for picking the best shoes instead of randomly picking and getting %33 probability.

1- First store for market info (Look Phase)

2- Pick, if Second better than First

3- If First>Second, pick the third

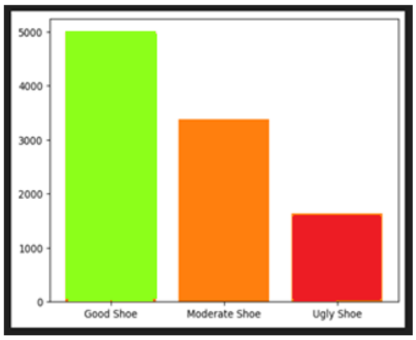


We can mathematically find the probabilities and expected values about this question:

P(2nd Best I 2nd> 1st) + P(3rd Best I 1st > 2nd) =

$$\frac{P(2nd\ is\ Best)\ x\ P(2nd > 1st\ |2nd\ \mathrm{Best})}{P(2nd > 1st)} * \mathrm{P(2nd > 1st)} + \frac{\mathrm{P(3rd\ Best)\ x\ P(1st > 2nd\ l\ 3rd\ is\ Best\ )}}{\mathrm{P(1st > 2nd)}} * \mathrm{P(1st > 2nd)}$$

$$\frac{(1/3\ x\ 1)}{3/6}\ x\ \frac{1}{2} + \frac{(1/3\ x\ 1/2)}{3/6}\ x\ \frac{1}{2} = \frac{1}{3} + \frac{1}{6} = \%50 \textbf{ which is higher than } \%33$$



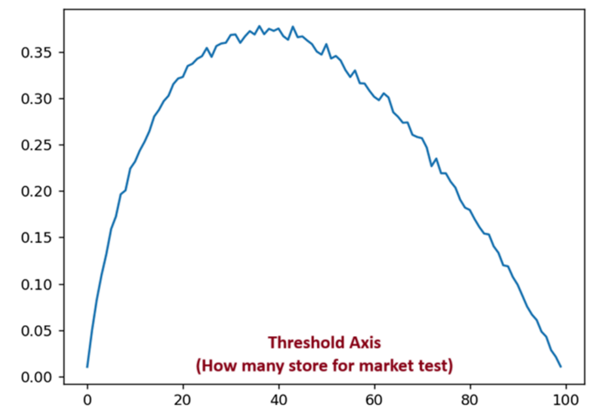
10000 Simulation results in Python

That graph also shows that the highest probability is %50 when we choose the threshold of 1, that means we need to stop searching phase.

What if there are more than 3 shoe stores, for example 100?

We can conduct a calculation of conditional probabilities for all tresholds one by one and find the best expected values or we can simply run a python simulation code for all possible thresholds and its corresponding success rate (getting the best shoes ratio). According to the best success rate we can set our threshold and find the best stopping point for maximizing the benefit. After we found the stopping point, we need to select the first better option in leap phase. For example, if there are 20 shoe stores and we found our threshold to be 5, we should buy the first shoe after the threshold that is greater than all the test data which could be 8th or 9th shoes etc.

The best way to visualize and test what will be the threshold (the stopping point) of our problem is via using python for much more iteration. Basically we will use heuristic techniques and make the approximation according to the results of the multiple iterations:



Best Stopping point at 36th Store with %37.8

Strategy is the same as Look and Leap above:

- Determine threshold for stopping point
- Choose the first that beats best seen so far.

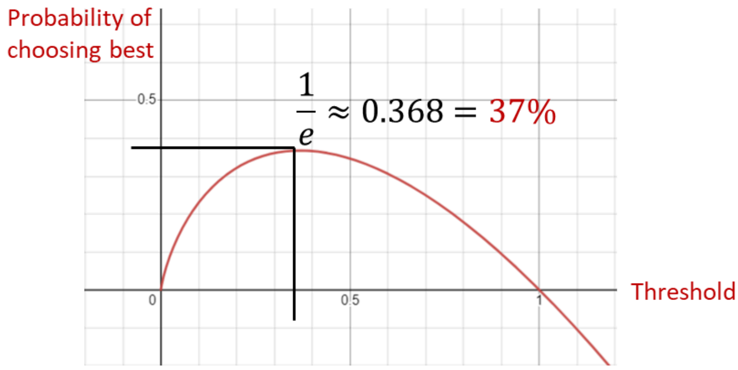
If we keep increase the number of iterations the result will be converges to the threshold = %37 of the total number of shoe stores which it brings us to %37 rule:

(You can find corresponding python simulation codes as .ipynb file in the appendix for trying different parameters such as # of iteration # of markets or probabilities etc.)

%37 Rule

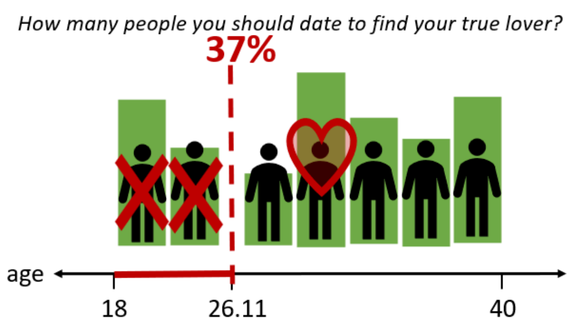
Recall the randomly choosing the best candidate was 1/n. Instead of this function, Mathematicians used the function below for look-then+ leap strategy: $p(x) = x * \int_x^1 \frac{1}{t} dt = -x * \ln(x)$

This is the graph of choosing the best candidate probability according to threshold. (Symonds, 2014)



Derivative of this function gives us the best threshold for the highest probability of choosing the best candidate and it equals to 1/e which is approximately equal to 0,367879 round up to %37. So this suggests the we should look first %37 portion of alternatives.

Michael Trick, who is a professor of operations research, mentions how we applied this %37 rule in his blog. (Trick, 2011) When he was a student, after he learned about secretary problem he wants to apply this to finding “true love”. He decides to use time instead of number of candidates because number of candidates are unknown. He assumed he started dating at 18 years old and byt he time he is 40 years old, he would agreed to just settle down. When he applied %37 rule to this time range, he claims looking phase should be up to 26 years old. But he faces the risk of rejection.

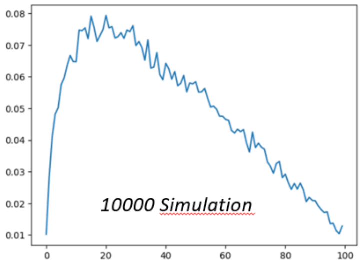


The "37% Rule" is often associated with decision-making and optimal stopping theory. It suggests that to maximize your chances of selecting the best option (whether it's choosing a candidate for shoes, a job, a house to buy, etc.), you should spend the first 37% of your time or resources gathering information and then be prepared to make a decision and commit to the best option you've encountered after that point.

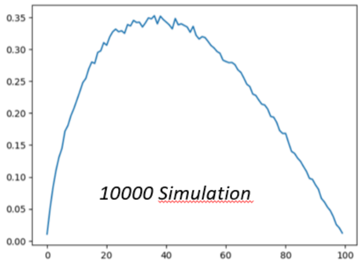
- Gather Information: Spend the first 37% of your time, resources, or options gathering information and understanding the range of possibilities.
- Establish a Threshold: After the initial 37%, set a threshold. This means that you won't make a decision until you've seen an option that is better than the best one you encountered in the first 37%.
- Exploit and Explore: After the initial exploration phase, choose the first option that is better than any you've seen before.

What if the best option you found is not available?

Sometimes even you find the best shoe and decided to buy it, the price could be too expensive to buy it or maybe it simply won’t fit into your feet. There are always some probabilities that the best option you found is not available. In another example, when you try to find the true love of your life, the problem is very similar. If you are an ugly person, your founded best option will reject you.



when p=0.9 is 20th %7.93

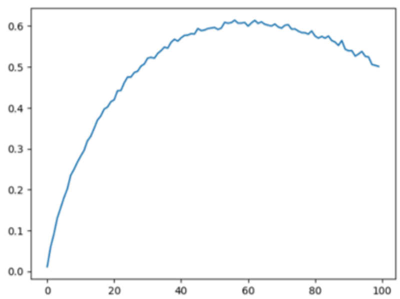


when p=0.1 is 36th %35.26

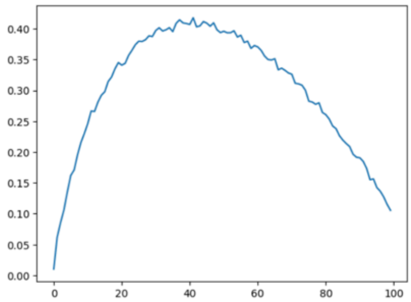
To simulate this we can define the p value that shows that the probability of rejection.

With this rejection feature, we can add some probabilities to the leap phase you can see it in the python codes at the appendix but most of the look and leap algorithm is the same.If If you are rich person, the rejection probability would be low and getting the best shoes and the threshold is higher.

What if we could go back some previous option?



p=0.5 is 56th store with %61.44



p=0.9 is 41th store with %41.77

The most realistic approach is that we are capable of going back to some previous option that we've already searched. For example we searched %37 of the shoe stores as test data and then we want to select the best from the test data (look phase). But there is a chance that the best shoes you've searched had been sold out already and you cannot buy it, in other words you would be rejected.

As you can see, the probability of getting the best shoe is increased to %61 from 1/e = %36,7 when we are capable of turning back. Same as the previous example, when the p gets higher, the best threshold and the probability of getting the best shoe gets increase.

As you can see, when the probability of getting rejected is increased, the threshold and the success rate would be decreased.

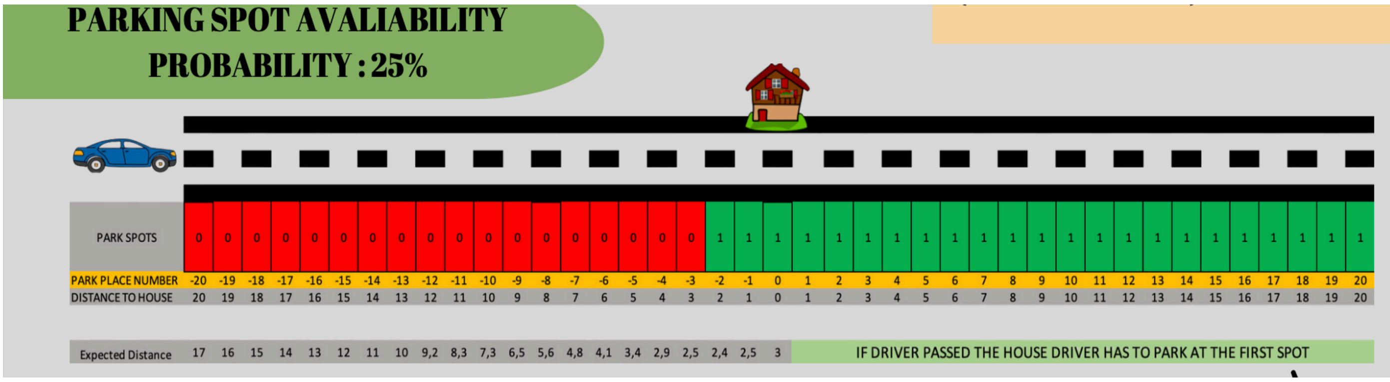
Parking Problem

Finding the best stopping point when parking the car without looking back

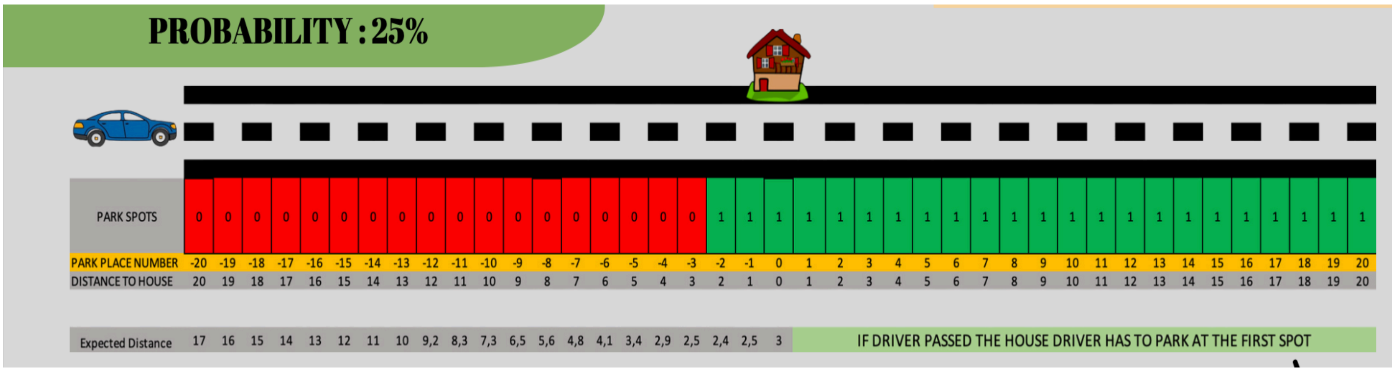
Indecision regarding parking is a common aspect of our daily lives, especially when faced with the dilemma of whether to park in an empty parking space or keep looking for a better parking space. In this scenario, let's imagine that there is an empty parking lot 4 units away from its destination on a one-way straight road. Do you decide to park right away or do you risk continuing to look for a potentially closer or more convenient location?

Threshold for Parking Problems Points: The threshold point in a parking problem represents the critical distance at which a decision must be made to park or continue searching. This is the point at which the expected benefits of finding a better location are balanced against the current distance to available locations. In other words, this is the equilibrium point where the cost of continuing the search is equal to the potential benefit of finding a nearby parking space.

To explain this decision-making process, we introduce the concept of green zone and red zone. Green indicates that the expected distance is greater than the current distance, indicating that continuing the search may result in a better location. On the other hand, red means the expected distance is less than the current distance, indicating that the currently available location may be the best choice. Let's examine two examples with two different possibilities we have studied.



10% Parking Availability: Conversely, our analysis shows that in a 10% availability scenario, the driver would have to keep looking for the next available parking space between -20 and -7 parking spaces.Again, the green zone predominates, indicating that the expected distance is longer than the current distance, encouraging drivers to continue looking for potentially better locations.



25% Parking Availability: In this 25% availability scenario the analysis showed that the driver should keep looking for the next available parking space between -20 and -3 spaces. This area is dominated by green zones, indicating that the expected distance is longer than the current distance and encouraging drivers to continue exploring. Recommended areas are highlighted so the driver can continue searching for the next available space based on the analysis. Understanding green and red zones adds a visual dimension to the decision-making process. It helps drivers decide when to park immediately (red zone) and when to continue searching for a better parking space (green zone).

In conclusion, balancing the current distance with the expected benefits optimizes decision-making in parking scenarios. All studies can be analyzed for all probability values that can be given with this study providing a versatile framework for decision-making across various parking spot availability scenarios.

(You can find corresponding Excel simulation codes as .xlsx file in the appendix for trying different probabilities.)

Honor Code: We pledge our honor that we have not violated the Honor Code during this assignment.

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References

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