**CSCI3656: NUMERICAL COMPUTATION**
**Homework 10: Due Friday, Nov. 12, 5:00pm**

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. Submit a PDF on Canvas by Friday, Nov. 12 at 5pm.

1. Implement the following numerical methods for approximating first derivatives: (i) one-sided forward difference, (ii) one-sided backward difference, and (iii) central difference. Consider the function
$$f(x) = \sin(4.8\,\pi\,x).$$
Use the numerical methods to estimate the derivative at $x = 0.2$ using the following values for $h$:
$$h = 2^{-k}, \quad k = 5, 6, \ldots, 30.$$
Compute the derivative by hand to get the *truth*. Plot the relative error versus $h$ on a log-log scale. For each of the three methods, identify the asymptotic regime and estimate the convergence rates from the plot.

2. Consider the finite difference approximation
$$f'(x) \approx \frac{1}{6h}\left[2f(x+h) + 3f(x) - 6f(x-h) + f(x-2h)\right].$$
Using a numerical study similar to Problem 1, identify the rate of convergence for this approximation. Produce a plot that justifies your computed rate. Identify the asymptotic regime in your plot.

BONUS: Construct and implement a 4th-order finite difference approximation of a first derivative.

1. (50 points) Use a Taylor series argument to prove that your method is 4th order.

2. (50 points) Run a numerical experiment to demonstrate that your method is 4th order.