Numerical Computing - CSCI 3656--001
Homework 1 - 9/3/21
Kai Handelman

1. After reading about floating-point numbers, I didn't feel like my trust has changed when it comes to approximation. As the post said, there is a finite-ness of the range of numbers able to be expressed, however, the range is basically irrelevant in everyday use. The one thing that does concern me is if a person using floating numbers forgets its limitations and the consequence result in something fatal.

2. Figure 1 is $p_1(x)$ plotted and figure 2 is $p_2(x)$ plotted while incorporating Horner's Algorithm. The reason to why figure 2 looks like it's more scattered is because of floating-point approximation errors. Since Horner's algorithm deals with multiple compounded calculations for any given point, and because of the outputs are extremely small (less than one) the rounding errors become very evident.

3. Figure 4 shows that as $k_{0\to12}$, $f_1(x)$ starts to lose accuracy while $f_2(x)$ retains it, despite $f_1(x)$ and $f_2(x)$ being equal (seen in Figure 3). This difference can be explained by the loss of significance through subtraction (Chapter 0.4 of Sauer). $f_2(x)$ contains a substraction and as $k_{0\to12}$ we can observe the two sides of the substraction becoming closer and closer.

Figure 1:

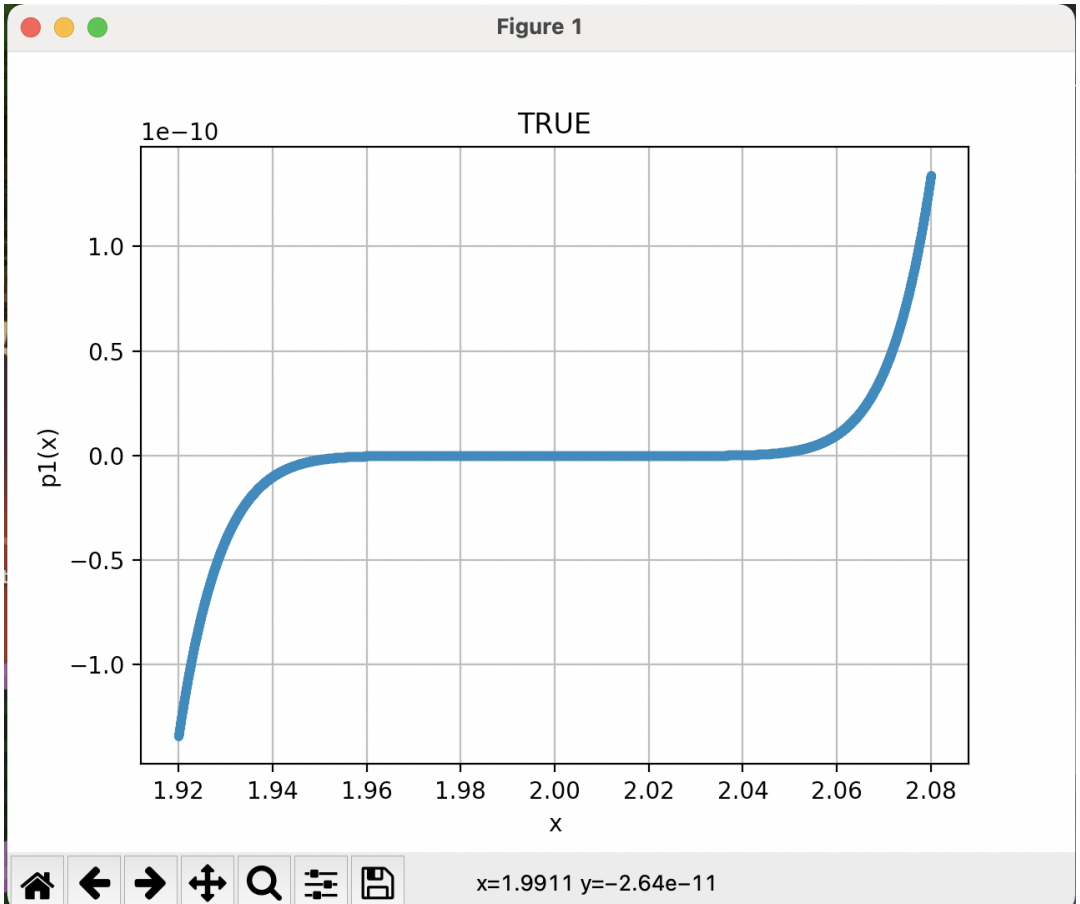

Figure 2:

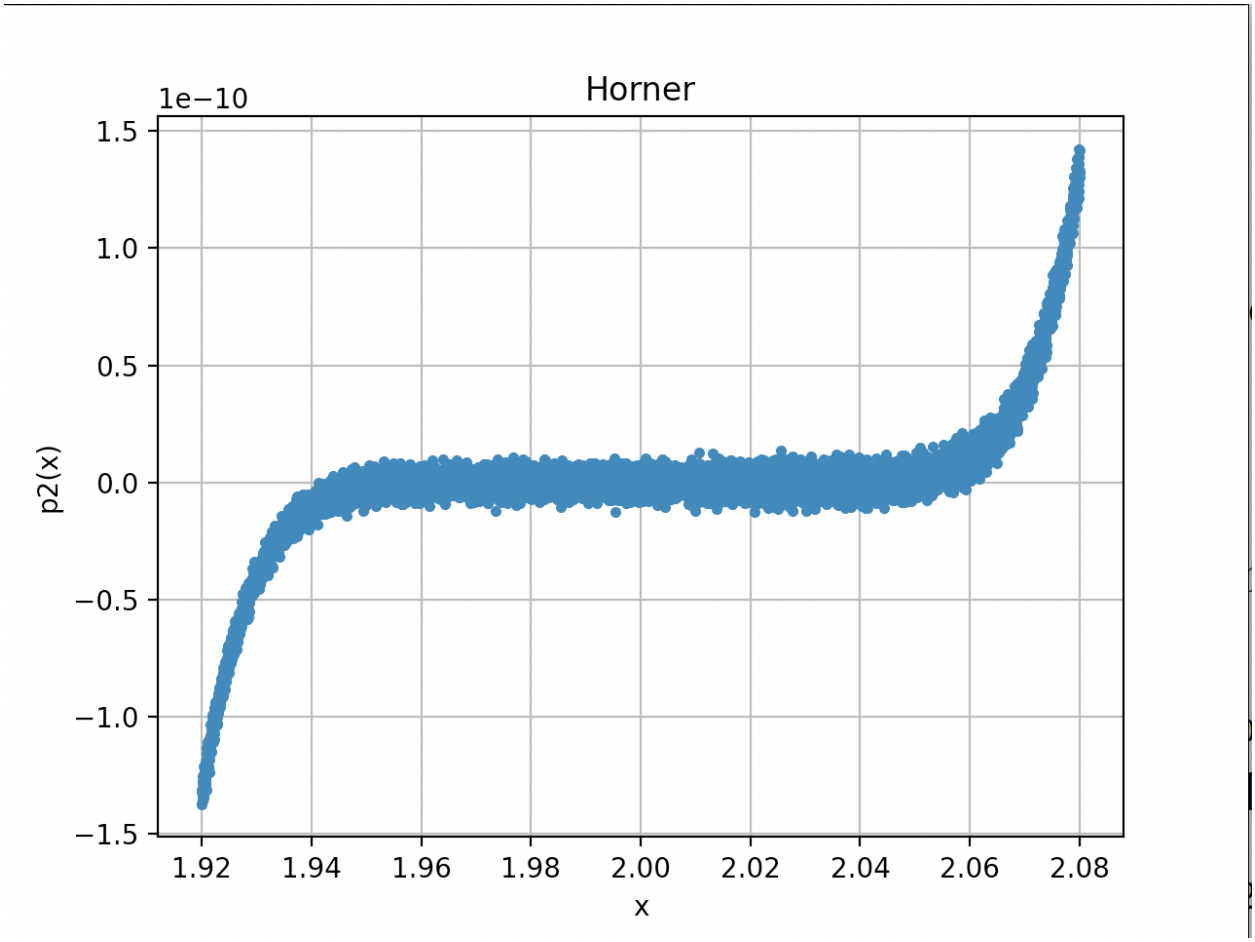

Figure 3:

$$f_1(x) = \frac{1-\cos(x)}{\sin^2(x)}$$

$$= \frac{1-\cos(x)}{(1-\cos^2(x))} \quad (\text{using: } 1 = \sin^2(x) + \cos^2(x))$$

$$= \frac{1-\cos(x)}{(1-\cos(x))(1+\cos(x))} = \frac{\cancel{(1-\cos(x))}}{\cancel{(1-\cos(x))}(1+\cos(x))}$$

$$= \frac{1}{1+\cos(x)}$$

$$= f_2(x)$$

Figure 4:

```
(base) kai@Kais-MacBook-Pro-2 HW1 % python3.8 hwScript.py
+--------+----------+----------+
|    x_k |       f1 |       f2 |
+========+==========+==========+
| 1      | 0.649223 | 0.649223 |
+--------+----------+----------+
| 0.1    | 0.501252 | 0.501252 |
+--------+----------+----------+
| 0.01   | 0.500013 | 0.500013 |
+--------+----------+----------+
| 0.001  | 0.5      | 0.5      |
+--------+----------+----------+
| 0.0001 | 0.5      | 0.5      |
+--------+----------+----------+
| 1e-05  | 0.5      | 0.5      |
+--------+----------+----------+
| 1e-06  | 0.500044 | 0.5      |
+--------+----------+----------+
| 1e-07  | 0.4996   | 0.5      |
+--------+----------+----------+
| 1e-08  | 0        | 0.5      |
+--------+----------+----------+
| 1e-09  | 0        | 0.5      |
+--------+----------+----------+
| 1e-10  | 0        | 0.5      |
+--------+----------+----------+
| 1e-11  | 0        | 0.5      |
+--------+----------+----------+
| 1e-12  | 0        | 0.5      |
+--------+----------+----------+
```