CSCI3656: Numerical Computation
Homework 1: Due 5pm on Friday, Sep. 3

Turn in your own writeup that includes your own code. List the names of the people you collaborated with. Submit a PDF via Canvas.

1. Read the following blog post on floating point numbers by Cleve Moler, creator of Matlab: `https://blogs.mathworks.com/cleve/2014/07/07/floating-point-numbers/`. Then answer the following questions: *After reading about floating point numbers, do you trust computers more or less? Why?*

2. Consider the following two polynomials:

$$\begin{aligned} p_1(x) &= (x-2)^9 \\ p_2(x) &= x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512 \end{aligned} \tag{1}$$

   Convince yourself that $p_1(x) = p_2(x)$ in exact arithmetic. (No need to show your work on this in the write-up).

   Given a polynomial expressed as

$$p(x) \;=\; \sum_{i=0}^{n} a_i\, x^i, \tag{2}$$

   Horner's algorithm for evaluating the polynomial at some given x is: (i) initialize $p = a_n$, (ii) for $i = n-1$ down to 0, do $p = p * x + a_i$. Implement Horner's algorithm. (I'm using Matlab.)

   Note that $x = 2$ is a root of $p_1(x)$ and $p_2(x)$. Generate 8000 equally spaced points in the interval $[1.92, 2.08]$. (In Matlab, you can do this with `linspace`.) Evaluate and plot $p_1(x)$ at each point in the interval. In a separate figure, evaluate and plot $p_2(x)$ using Horner's algorithm. In exact arithmetic, these should be the same. What's going on in these plots? (Hint: For a detailed description, see Chapter 0.1 in Sauer's *Numerical Analysis* or Chapter 1.4 and surrounding text in Demmel's *Applied Numerical Linear Algebra*.)

3. Consider the functions

$$f_1(x) = \frac{1-\cos(x)}{\sin^2(x)}, \qquad f_2(x) = \frac{1}{1+\cos(x)}. \tag{3}$$

   Using trig identities, show that $f_1(x) = f_2(x)$. (Please show your work on this one.) Implement f_1 and f_2. Make a table of your implementations evaluated at the points $x_k = 10^{-k}$ for $k = 0, 1, \ldots, 12$. You should see that f_1 loses all accuracy as k increases (that is, as x_k approaches zero), while f_2 retains its accuracy. Explain why. (Hint: For a detailed description, see Chapter 0.4 in Sauer's *Numerical Analysis*.)

4. BONUS 10%: Imagine you need to save the world from killer robots. How might you use your knowledge of floating point systems to defeat the robots? Be creative.