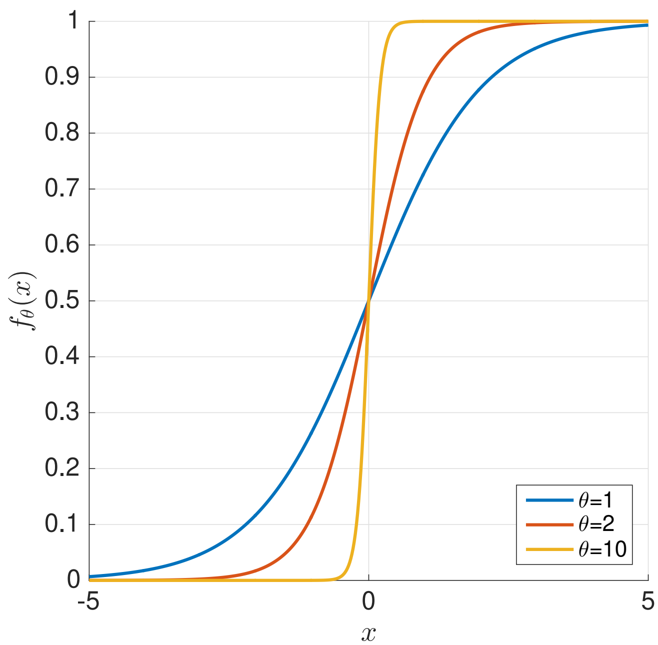
CSCI3656: NUMERICAL COMPUTATION
Homework 7: Due Friday, Oct. 22, 5:00pm

Turn in your own writeup that includes your code. List any resources you used including collaborating with others. Submit a PDF on Canvas by Friday, Oct. 22 at 5pm.

Consider the parameterized family of functions,

$$f_\theta(x) \;=\; \frac{1}{1+\exp(-\theta x)}, \qquad x \in [-5,5].$$

The parameter θ controls how smooth f_θ is near $x = 0$, as shown:



To start this homework, let $\theta = 1$.

1. **Generate training data:** Create a vector with $n = 7$ evenly spaced points in the interval $[-5, 5]$. (Matlab/Numpy: `(np.)linspace`.) For each point x_i in this vector, compute $y_i = f_\theta(x_i)$. You should now have 7 pairs (x_i, y_i). Make a nice table with the seven input/output pairs.

2. **Train the model:** Construct the Vandermonde system and solve for the coefficients of the unique degree-6 interpolating polynomial $p_6(x)$. Make a nice table of the 7 coefficients.

3. **Generate testing data:** Create a new vector with 101 evenly spaced points in $[-5, 5]$. For each point x'_i, compute $y'_i = f_\theta(x'_i)$. Report the mean (`(np.)mean`) and standard deviation (`(np.)std`) from the set of points $y'_1, \ldots, y'_{101}$.

4. **Compute the testing error:** Compute and report the the absolute testing error:

$$\text{error} \;=\; \text{error}_{\theta=1,n=7} \;=\; \underset{i}{\text{maximum}} \;\frac{|\, y'_i - p_6(x'_i)\,|}{|y'_i|}$$

   Note that the error depends on the value of θ and the number of training points / the degree of the polynomial.

5. Repeat steps 1-4 with $\theta = 10$. How does the error change? What does that tell you about the quality of the polynomial approximation for the two functions?

6. EXTRA CREDIT (15pts): Repeat steps 1-4 with both $\theta = 1$ and $\theta = 10$ for $n = 8, 9, \ldots, 15$. Plot error versus n on a semilog scale. Describe the convergence (that is, at what rate the error goes to zero) as n increases.