Math 352 Quiz 3

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The Crank-Nicolson method is convergent if it satisfies both consistency and zero-stability.

- Consistency:

$$j = -1, \; b_{-1} = \frac{1}{2}$$

$$j = 0, \; a_0 = 1, \; b_0 = \frac{1}{2}$$

Consistency is satisfied because

$$\sum_{j=0}^{0} a_j = a_0 = 1$$

and

$$-\sum_{j=0}^{0} j \, a_j + \sum_{j=-1}^{0} b_j = b_{-1} + b_0 = \frac{1}{2} + \frac{1}{2} = 1.$$

Thus, the method is consistent.

It has order at least 2 because

$$\sum_{j=0}^{0} (-j)^2 \, a_j + 2 \sum_{j=-1}^{0} (-j) \, b_j = 2 \, b_{-1} = 1$$

It has order 2 because

$$\sum_{j=0}^{0} (-j)^3 \, a_j + 3 \sum_{j=-1}^{0} (-j) \, b_j = 3 \, b_{-1} = \frac{3}{2} \neq 1.$$

- Zero-Stability:
  The roots of the first characteristic polynomial

$$p_1(\rho) = \rho - 1$$

is

$$\rho = 1.$$

Thus the root condition is satisfied.