352 Quiz 8

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1.

$$u_{ex} = \frac{e^{\frac{\beta}{\mu}x} - 1}{e^{\frac{\beta}{\mu}} - 1}$$

2.

$$-\mu \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2} + \beta \frac{u_{i+1} - u_{i-1}}{2\Delta x} = 0$$

$$A = \begin{bmatrix} \text{a} & \text{b} & 0 & 0 & ... & 0 \\ \text{c} & \text{a} & \text{b} & 0 & ... & 0 \\ 0 & \text{c} & \text{a} & \text{b} & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & ... & ... & \text{c} & \text{a} \end{bmatrix}$$

$$a = \frac{2\mu}{(\Delta x)^2},\ b = -\frac{\mu}{(\Delta x)^2} + \frac{\beta}{2\Delta x},\ c = -\frac{\mu}{(\Delta x)^2} - \frac{\beta}{2\Delta x}$$

$$u = \begin{bmatrix} \text{u}_1 \\ \text{u}_2 \\ ... \\ ... \\ \text{u}_{n-1} \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ ... \\ ... \\ b_{n-1} \end{bmatrix}, \text{ where n} = \lfloor \frac{1}{\Delta x} \rfloor$$

Notice that when $i = n - 1$,

$$-\mu \frac{u_n - 2u_{n-1} + u_{n-2}}{(\Delta x)^2} + \beta \frac{u_n - u_{n-2}}{2\Delta x} = 0.$$

Since $u_n = 1$, we have

$$\frac{2\mu}{(\Delta x)^2} u_{n-1} - \frac{\mu}{(\Delta x)^2} u_{n-2} - \frac{\beta}{2\Delta x} u_{n-2} = \frac{\mu}{(\Delta x)^2} - \frac{\beta}{2\Delta x}.$$

Thus, $b_{n-1} = \frac{\mu}{(\Delta x)^2} - \frac{\beta}{2\Delta x}$.