

# Group

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## 1 Introduction

**Definition 1.1** (Group). A *group* is a non-empty set  $\Gamma$  together with a binary operation on  $\Gamma$ , denoted “ $\cdot$ ”, that combines any two elements  $\gamma$  and  $\gamma'$  of  $\Gamma$  to form an element of  $\Gamma$ , denoted  $\gamma \cdot \gamma'$ , such that the following three requirements, known as *group axioms*, are satisfied:

- *Associativity*: For all  $g, h, j$  in  $\Gamma$ , one has  $(g \cdot h) \cdot j = g \cdot (h \cdot j)$ .
- *Identity*: There exists an element  $e$  in  $\Gamma$  such that, for every  $g$  in  $\Gamma$ , one has  $e \cdot g = g$  and  $g \cdot e = g$ . Such an element is unique and is called the *identity element*.
- *Unique Inverse*: For each  $g$  in  $\Gamma$ , there exists an element  $h$  in  $\Gamma$  such that  $g \cdot h = e$  and  $h \cdot g = e$ , where  $e$  is the identity element. For each  $g$ , the element  $h$  is unique and is called the *inverse* of  $g$  and is denoted  $g^{-1}$ .

## 2 More Information

You can learn more about controlling the appearance of HTML output here:  
<https://quarto.org/docs/output-formats/html-basics.html>