Følner sequence

Kai Prince SFHEA

2025-08-03

1 Introduction

Definition 1.1. We define a *right-Følner sequence* in a group, Γ, as a sequence $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ of finite subsets of Γ satisfying

$$\lim_{N\to\infty} \frac{|(\Phi_N \cdot \gamma^{-1}) \cdot \Phi_N|}{|\Phi_N|} = 1$$

for all $\gamma \in \Gamma$.

Definition 1.2. Similarly, we define a *left-Følner sequence* in a group, Γ, as a sequence $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ of finite subsets of Γ satisfying

$$\lim_{N\to\infty} \frac{|(\gamma^{-1} \cdot \Phi_N) \cap \Phi_N|}{|\Phi_N|} = 1$$

for all $\gamma \in \Gamma$.

Definition 1.3. We call a sequence a *Følner sequence* if it is both a left and right Følner sequence.

A related definition is the following:

Definition 1.4. We call define *density of a subset $A \subseteq \Gamma$ with respect to a Følner sequence, Φ,* as

$$d_\Phi(A) = \lim_{N\to\infty} \frac{|\Phi_N \cap A|}{|\Phi_N|},$$

if it exists.

For $\mathbb{N}$, the *natural density*, d, is defined when the Følner sequence is constructed with $\Phi_N = [1, ..., N]$.

2 Results

Theorem 2.1 (The Test Theorem)**.** *This is a Theorem.*

Tip

This is a test tip.

3 More Information

You can learn more about controlling the appearance of HTML output here: https://quarto.org/docs/output-formats/html-basics.html