

Følner sequence

Kai Prince SFHEA

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1 Introduction

Definition 1.1. We define a *right-Følner sequence* in a group, Γ , as a sequence $\Phi = (\Phi_N)_{N \in \mathbb{N}}$ of finite subsets of Γ satisfying

$$\lim_{N \rightarrow \infty} \frac{|(\Phi_N \cdot \gamma^{-1}) \cap \Phi_N|}{|\Phi_N|} = 1$$

for all $\gamma \in \Gamma$.

Definition 1.2. Similarly, we define a *left-Følner sequence* in a group, Γ , as a sequence $\Phi = (\Phi_N)_{N \in \mathbb{N}}$ of finite subsets of Γ satisfying

$$\lim_{N \rightarrow \infty} \frac{|(\gamma^{-1} \cdot \Phi_N) \cap \Phi_N|}{|\Phi_N|} = 1$$

for all $\gamma \in \Gamma$.

Definition 1.3. We call a sequence a *Følner sequence* if it is both a left and right Følner sequence.

A related definition is the following:

Definition 1.4. We call define *density of a subset $A \subseteq \Gamma$ with respect to a Følner sequence, Φ* , as

$$d_\Phi(A) = \lim_{N \rightarrow \infty} \frac{|\Phi_N \cap A|}{|\Phi_N|},$$

if it exists.

For \mathbb{N} , the *natural density*, d , is defined when the Følner sequence is constructed with $\Phi_N = [1, \dots, N]$.

2 Results

Theorem 2.1 (The Test Theorem). *This is a Theorem.*



Tip

This is a test tip.

3 More Information

You can learn more about controlling the appearance of HTML output here:
<https://quarto.org/docs/output-formats/html-basics.html>