Følner sequence

Kai Prince SFHEA

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1 Følner Sequences

Definition 1.1. We define a *right-Følner sequence* in G as a sequence $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ of finite subsets of G satisfying

$$\lim_{N\to\infty} \frac{|\cdot|(\Phi_N \cdot g^{-1}) \cdot \Phi_N}{|\cdot|\Phi_N} = 1$$

for all $g \in G$.

Definition 1.2. Similarly, we define a *left-Følner sequence* in G as a sequence $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ of finite subsets of G satisfying

$$\lim_{N\to\infty} \frac{|\cdot|(g^{-1} \cdot \Phi_N) \cap \Phi_N}{|\cdot|\Phi_N} = 1$$

for all $g \in G$.

Definition 1.3. We call a sequence a *Følner sequence* if it is both a left and right Følner sequence.

2 Tempered Følner Sequences

Definition 2.1 (Lindenstrauss (2001), Definition 1.1)**.** A sequence of sets $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ will be said to be *tempered* if, for some $b > 0$ and all $n \in \mathbb{N}$,

$$|\cdot| \bigcup_{1\le k<N} \Phi_k^{-1}\Phi_N \le b|\cdot|\Phi_N. \tag{1}$$

is referred to as the *Shulman condition.*

Proposition 2.1 (Lindenstrauss (2001), Proposition 1.4)**.**

1. *Every Følner sequence* $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ *has a tempered subsequence.*
2. *Every amenable group has a tempered Følner sequence.*

Lindenstrauss, E. (2001). 'Pointwise theorems for amenable groups', Inventiones mathematicae, 146 (2), pp. 259–295. https://doi.org/10.1007/s002220100162.