Factor Maps

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Definition 0.1 (Kra et al. (2022), Definition 2.1)**.** For a system (X, μ, T) we say that the system $(Y, \ u, S)$ is a *measurable factor* of (X, μ, T) if there is a measurable map $\pi : X \to Y$, the *measurable factor map*, such that $\pi(\mu) = \ u$ and $(S \circ \pi)(x) = (\pi \circ T)(x)$ for μ-almost every $x \in X$.

Proposition 0.1 (cf. Host (2019), Proposition 5)**.** *Let (X, Γ) be a topological dynamical system where Γ is an amenable group, $x_0 \in X$, and μ be an ergodic invariant probability measure supported on the closed orbit of x_0 under the action of Γ.*

Let (Z, m_Z, H) be the Kronecker factor of (X, μ, Γ), with factor map $\pi : X \to Z$.

Let $X \times Z$ be endowed with the group action of $\Gamma \times H$. Let $\tilde{\mu}$ be the measure on $X \times Z$ and image of μ under the map $X \to X \times Z$ where $x \mapsto (x, \pi(x))$.

Then there exists a Følner sequence $\tilde{\Phi}$ and a point $z_0 \in Z$ such that (x_0, z_0) is generic for $\tilde{\mu}$ along $\tilde{\Phi}$.

Kra, B., et al. (2022). '*Infinite sumsets in sets with positive density*', Available at: https://arxiv.org/abs/2206.01786.

Host, B. (2019). '*A short proof of a conjecture of erdös proved by moreira, richter and robertson*', Available at: https://arxiv.org/abs/1904.09952.