

# Følner sequence

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## 1 Følner Sequences

**Definition 1.1.** We define a *right-Følner sequence* in  $G$  as a sequence  $\Phi = (\Phi_N)_{N \in \mathbb{N}}$  of finite subsets of  $\Gamma$  satisfying

$$\lim_{N \rightarrow \infty} \frac{|(\Phi_N \cdot g^{-1}) \cap \Phi_N|}{|\Phi_N|} = 1$$

for all  $g \in G$ .

**Definition 1.2.** Similarly, we define a *left-Følner sequence* in  $G$  as a sequence  $\Phi = (\Phi_N)_{N \in \mathbb{N}}$  of finite subsets of  $\Gamma$  satisfying

$$\lim_{N \rightarrow \infty} \frac{|(g^{-1} \cdot \Phi_N) \cap \Phi_N|}{|\Phi_N|} = 1$$

for all  $g \in G$ .

**Definition 1.3.** We call a sequence a *Følner sequence* if it is both a left and right Følner sequence.

## 2 Tempered Følner Sequences

**Definition 2.1** (Lindenstrauss (2001), Definition 1.1). A sequence of sets  $\Phi = (\Phi_N)_{N \in \mathbb{N}}$  will be said to be *tempered* if, for some  $b > 0$  and all  $n \in \mathbb{N}$ ,

$$\left| \bigcup_{1 \leq k < n} \Phi_k^{-1} \Phi_n \right| \leq b |\Phi_n|. \quad (1)$$

is referred to as the *Shulman condition*.

**Proposition 2.1** (Lindenstrauss (2001), Proposition 1.4).

1. Every Følner sequence  $\Phi = (\Phi_N)_{N \in \mathbb{N}}$  has a tempered subsequence.
2. Every amenable group has a tempered Følner sequence.

Lindenstrauss, E. (2001). 'Pointwise theorems for amenable groups', *Inventiones mathematicae*, 146 (2), pp. 259–295. <https://doi.org/10.1007/s002220100162>.