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Abstract

We follow steps provided by the paper published by (Host, 2019), 'A Short Proof of a Conjecture of Erdős Proved by Moreira, Richter and Robertson', as well as using the results provided by (Kra et al., 2022), to generalise the proof of Erdős's conjecture for amenable groups. The main result that we aim to prove is: 'every positively dense subset of an amenable group contains the group sumset of k infinite sets for every natural number k'.

1 Introduction

Theorem 1.1 (cf. (Host, 2019), Theorem 1)**.** *Let $(G, \cdot)$ be an amenable group. If $A \subseteq G$ has positive density, then there exists infinite subsets B and C of G such that $B \cdot C \subset A$.*

Theorem 1.2 (cf. (Host, 2019), Proposition 2)**.** *There exists a set of positive density not containing any sumset of positive density and an infinite set.*

Theorem 1.3 (cf. (Kra et al., 2022), Theorem 1.1)**.** *Let $(G, \cdot)$ be an amenable group. If $A \subseteq G$ has positive density then, for every $k \in \mathbb{N}$, there are infinite* *subsets* $B_1, ..., B_k \subset G$ *such that* $B_1 \cdot \dots \cdot B_k \subset A$.

2 Preliminaries

We will use $\mathbb{N} = \{1, 2, 3, ...\}$ and e to denote the identity of the group G.

2.1 Amenable Groups and Actions

Throughout this paper, unless otherwise specified, we will let $(G, \cdot)$ be a second countable discrete group. This also means that the Haar measure of G is the counting measure.

Definition 2.1. [Embed definition from folner.qmd]

For simplicity of this paper, we will use the alternative and equivalent definition that a group, Γ, is *amenable* (and second countable) if and only if it has a Følner sequence.

Definition 2.2. [Embed definition from folner.qmd]

Definition 2.3. [Embed definition from density.qmd]

2.2 Topological Dynamics of Group Actions

Definition 2.4. [Embed definition from actions.qmd]

A *topological dynamical system under the action of* G, denoted (X, G), is a compact metric space X that has continuous surjective maps, $(g, x) \mapsto g.x$, for all $g \in G$.

Let $x \in X$, $\Phi = (\Phi_N)_{N\in\mathbb{N}}$ be a Følner sequence in Γ and μ a probability measure on X. Where δ_x is the Dirac mass at x, if

$$\frac{1}{|\Phi_N|} \sum_{g\in\Phi_N} \delta_{g.x} \underset{\text{weakly}*}{\longrightarrow} \mu \text{ as } N \to \infty,$$

then we say x *is generic for* μ *with respect to* Φ and we denote this with $x \in \mathrm{gen}(\mu, \Phi)$.[1]

[1] consider tempered separately as the FCP construction only depends on sequential compactness

2.3 Recurrence Results & Ergodic Theorems

2.4 Erdős Cubes & Cubic Measures

2.5 Factor Maps

2.6 Key Dynamical Results

3 Proof of Theorem (ref)

3.1 Furstenberg's Correspondence Princple

3.2 Kronecker Factor

3.3 Choosing a point x_1

3.4 The joining ν

3.5 Proof Conclusion

4 Proof of Corollary (ref)

5 Proof of Theorem (ref)

Discussion

Host, B. (2019). '*A short proof of a conjecture of erdös proved by moreira, richter and robertson*', Available at: https://arxiv.org/abs/1904.09952.

Kra, B., et al. (2022). '*Infinite sumsets in sets with positive density*', Available at: https://arxiv.org/abs/2206.01786.