Density

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Definition 0.1. For $A \subseteq G$ and Følner sequence $\Phi = (\Phi_N)_{n=1}^{\infty}$, we write

$$\overline{d}_{\Phi}(A) = \limsup_{N \to \infty} \frac{|\cdot|A \cap \Phi_N}{|\cdot|\Phi_N}$$
$$\underline{d}_{\Phi}(A) = \liminf_{N \to \infty} \frac{|\cdot|A \cap \Phi_N}{|\cdot|\Phi_N}$$

to be the *upper and lower densities of A with respect to* Φ, respectively. If these agree, then we can write

$$d_{\Phi}(A) = \lim_{N \to \infty} \frac{|\cdot|A \cap \Phi_N}{|\cdot|\Phi_N}$$

to be the *density of A with respect to* Φ. We also define the *upper Banach density of A* by

$$\overline{d}(A) = \sup\{d_{\Phi}(A) : \text{for Følner sequences } \Phi \text{ where } d_{\Phi}(A) \text{ exists}\}.$$