# Density

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2025-07-23

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**Definition 0.1.** For $A \subseteq G$ and Følner sequence $\Phi = (\Phi_N)_{n=1}^{\infty}$, we write

$$\overline{d}_{\Phi}(A) = \limsup_{N\to\infty} \frac{|A \cap \Phi_N|}{|\Phi_N|}$$
$$\underline{d}_{\Phi}(A) = \liminf_{N\to\infty} \frac{|A \cap \Phi_N|}{|\Phi_N|}$$

to be the *upper and lower densities of $A$ with respect to* $\Phi$, respectively. If these agree, then we can write

$$d_{\Phi}(A) = \lim_{N\to\infty} \frac{|A \cap \Phi_N|}{|\Phi_N|}$$

to be the *density of $A$ with respect to* $\Phi$. We also define the *upper Banach density of $A$* by

$$\overline{d}(A) = \sup\{d_{\Phi}(A) : \text{for Følner sequences } \Phi \text{ where } d_{\Phi}(A) \text{ exists}\}.$$