Financial Econometrics and Statistical Arbitrage

Master of Science Program in Mathematical Finance
New York University

Homework 1

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Problem 1. A stationary process $\{ Y_t \}$ is given, show that:

a) $Z_t = (Y_t - Y_{t-1})$ is a stationary process.

b) $Z'_t = (Y_t - Y_{t-d})$ is a stationary process

Problem 2. Write the autocovariance function for the MA(1) process and show that MA(1) is a stationary process.

Problem 3. Load the file "Data_For_HW1.xls" file from the course website.

a) Plot the time series.
b) Obtain the equation for its linear trend, de-trend the original time series, and plot it.
c) Plot the sample autocorrelation function for the de-trended time series. What process do you recommend to model the de-trended time series?
d) Plot the first difference of the original time series.
e) Plot the sample autocorrelation function for the differenced time series. What process model do you recommend for the differenced time series?

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Problem 4. Let $\{Y_t\}$ be a stationary time series with mean 0 and covariance function γ_Y. If $\sum_{j=-\infty}^{\infty} |\psi_j| < \infty$ then show that the time series

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Y_{t-j}$$

is stationary with mean 0 and autocovariance function

$$\gamma_X(h) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \psi_j \psi_k \gamma_Y(h+k-j)$$

Problem 5. In Problem 4, when $\{X_t\}$ is a linear process (i.e. $\{Y_t\} = \{Z_t\}$), show that:

$$\gamma_X(h) = \sum_{j=-\infty}^{\infty} \psi_j \psi_{j+h} \sigma^2$$