Homework 2: Q2

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1 Proof Idea

Begin Proof Idea: I will be using the same example from the walk-through video such that there is a stable matching instance for every n>=1, GS algorithm iterates $\frac{n(n+1)}{2}$ of proposals based on the preference list and the order in which the proposals are made. However, we can rewrite $\frac{n(n+1)}{2}$ as $\frac{n^2+n}{2}$. Eventually, it's just going to be n^2, which prove that while loop in the Gale-Shapley algorithm runs $\Omega(n^2)$ times.

2 Proof Details

Begin Proof Details: Now, we set n=2, which we have
m1: w2>w1
m2: w2>w1
w1: m1>m2
w2: m1>m2
In this case, let w1 be the free woman and propose to her most preferable man m that she has not proposed to yet which here is m1. Since m1 is not engaged, then he accepts w1's proposal, (w1, m1). Now w2 is going to propose to her most preferable man which is also m1. Although, m1 is engaged with w1, m1 prefer w2 over w1. So, w2 and m1 got engaged (w2, m1), and keep the w1 to be free again. Now, we look for w1's most preferable man that she has not proposed to yet which now will be m2. So, when she proposed to m2, they got engaged (w1, m2). This is taking 3 proposal eventually. If we substitute n to the formula $\frac{n(n+1)}{2}$ we got $\frac{2(2+1)}{2}$=3.
In this algorithm, we have loop through all the women to proposed at their most preferable man at once, which takes on n time. There are still some free women out there that are not engaged yet, now we will loop through all the free women by their most preferable man that she has not proposed to yet such that 2nd preferable man at the preference list. There will be n size of preference list (n men). So, the lower bound of the worst case will be n*n, which is n^2. That way proves that after while loop ended in the GS algorithm, it will run $\Omega(n^2)$ times.