Homework 5: Q2

Name: Xinkai Lin, xinkaili

*Don't forget to input your list of collaborators and sources on **AutoLab**.*
Please submit this file as a PDF.

1 Algorithm Idea

I will perform this algorithm as BFS algorithm. I will first select a start node and find all the connected components $\{G_1, G_2, \ldots G_k\}$ of G with BFS algorithm. The select start node will be giving a label as either moth or butterfly. Then, we can run BFS search in each G_i and get a tree T_i with the root r_i. finally, for each G_i, if it has not visited yet, set its value to 0 or 1 as moth or butterfly base on the same or different to its parent. If it has been visited, compare the label with giving definitive judgment, if it is not the same, then it is inconsistent.

2 Algorithm Details

Algorithm: Algorithm1

```
Input:   A set of n caterpillars and m definitive judgments (same, different, ambiguous)
         Construct the graph G = (V, E), s∈V
Consistent = true
CURRLAYER = {s}
Allocate length n array LABEL and set LABEL[i]=-1 for every 0<= I < n
LABEL[s] = 0     // I will assign s as starting node as moth, 0 mean moth, 1 mean butterfly
While CURRLAYER ≠{} do
    LAYER = {}
    For u ∈ CURRLAYER do
        If LABEL[u] == -1 then     //if it hasn't visited
            Add u to LAYER
            If the judgment is same then
                  LABEL[u] = 0   //u is moth
            Else LABEL[u] = 1   //u is butterfly
            End if
        Else    //it has visited
            if the label ≠ definitive judgment
                consistent = false   // inconsistent
        end if
    end for
```

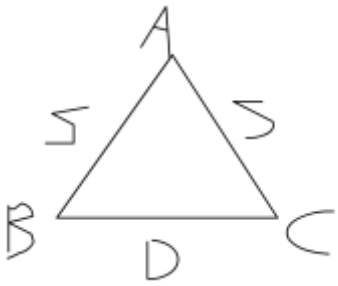
CURRLAYER = LAYER
end while
return consistent

3 Proof of Correctness Idea

To prove the algorithm's correctness, we will show that the judgment is consistent or inconsistent. In this algorithm, it will iterate all the node and mark it as either moth or butterfly depend on their parent node. Once it has its label, we will compare with the definitive judgment, if the label satisfied the definitive judgment, then it is consistent, else it is inconsistent.

4 Proof Details

In this proof, we will know that for any giving input, the algorithm will help us to find out whether or not it is consistent or inconsistent. Consider the following example:
Giving input n = 3, 3 caterpillars and m = 3, with 3 pairs of definitive judgment,



<<A, B>, S>, <<A, C>, S>, <<B, C>, D> S is same, and D is different.

let's set the start node as A. As the algorithm goes, it will first check on all the neighbor node that has not been visited yet, which here are B and C. We found out that B is same A, and C is also same as A. Suppose we know that A is moth. As right now, we can say that B and C is also moth. In other hand, as the algorithm goes, we will find same neighbor node that are already visited. In addition, base on the previous result it will has its label as whether is moth or butterfly. As right now, we know that B and C are same with A which they all moth. Since A have iterated through all its edges. we move to the next layer of the graph. Here we have B and C. Now, we will look at B's neighbor node. Finally, we found that when we compare B with C. the case fail. From previous we set B and C both as moth, but when we look at the definitive judgment it says different. The label didn't match the definitive judgment. So, we concluded that this graph is inconsistent. Similarly, for any giving input n and m. By using this BFS algorithm, we will find out if it is consistent or inconsistent.

5 Runtime Analysis

In this algorithm, first we will use the for loop which set all the caterpillar as unvisited. There is n caterpillars, each caterpillars will take O(1) time. When the for loop finished, it will run O(n) time. Then, we have a for loop inside the while loop, which the for loop will run $O(n_u)$ time, and the while loop will run until the list is empty total run time of $\sum_u O(n_u) = O(m)$. Finally, we can say that our total run times is O(m+n). Also in class, we have proven that the running time of BFS algorithm is O(m+n) time.