TUMCREATE

CoBMo

Control-oriented Building Model

Technical documentation

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Contents

Notation

Let $\mathbb{R}$ be the domain of real numbers. Non-bold letters x, X denote scalars $\mathbb{R}^{1\times1}$, bold lowercase letters $\boldsymbol{x}$ denote vectors $\mathbb{R}^{n\times1}$ and bold uppercase letters $\boldsymbol{X}$ denote matrices $\mathbb{R}^{n\times m}$. The symbols $\mathbf{0}$ and $\mathbf{1}$ denote vectors or matrices of zeros and ones of appropriate size. The symbol $\boldsymbol{I}$ is the identity matrix of appropriate size. The transpose of a vector or matrix is denoted by $()^{\mathsf{T}}$. Symbols for physical properties are aligned with ISO 80000 and units are based on the international system of units (SI).

1 Introduction

1.1 Demand side flexibility (DSF)

The integration of demand side flexibility (DSF) into the power system is an important step towards dealing with renewable generation and increased peak demand due to electric mobility. In particular, DSF helps to match generation and demand by shifting flexible loads to time periods with high renewable generation [1]. This helps in the avoidance of renewable generation shedding. In a microgrid, this can also help to decrease the need for additional energy storage systems, because a higher share of renewable electricity is instantly consumed [2]. DSF can also ensure that electric grid constraints, i.e., thermal limits and voltage limits, are maintained throughout the operation [3] or they can support the grid stability by offering reserves [4].

Heating, ventilation and air-conditioning (HVAC) systems, are an important candidate for DSF as they account for a large share of the electricity demand in buildings, particularly in tropical cities such as Singapore [5]. DSF from HVAC systems has seen increased attention with the advances in model predictive control (MPC) applications for buildings [6]. With MPC, the control problem of the HVAC system is expressed as an numerical optimization problem aimed at minimizing the operation cost, i.e., the costs for consuming energy, while satisfying the occupant comfort constraints, i.e., the acceptable limits for air temperature and indoor air quality. The building operator benefits from MPC through cost savings which arise from 1) more energy efficient control and 2) the ability to consider dynamic electricity tariffs [7], i.e., the electric demand is shifted to hours with low electricity prices.

1.2 Building modelling for DSF

Appropriate mathematical building models are the key ingredient for MPC and thus for DSF. Specifically, the mathematical equations of the building model are constraints in the numerical optimization problem of the MPC. To this end, convex building model formulations are beneficial for the numerical optimization, because of the high computational efficiency of convex optimization solvers. This will enable solving the MPC efficiently even in low cost computing environments. Additionally, recent advances in DSF have focused on distributed control [8], for which a convex model formulation is essential.

Available convex thermal building modelling approaches feature white-box models, data-driven models and hybrid models. White-box models are formulated based on physical knowledge and technical design information of the building, whereas data-driven models are inferred from experiment data, i.e. measured time-series data during operation of the building. Hybrid models take an initial model setup based on design information and then use experiment data to improve the model parameters.

1.3 Control-oriented building model

The Control-oriented Building Model (CoBMo) is a building modelling framework catering specifically to the formulation of MPC problems for HVAC systems. CoBMo provides a mathematical model which expresses the relationship between the electric load of the HVAC systems and the indoor air climate with consideration for interactions of the building with its environment, its occupants and appliances (fig. 1.1). To this end, CoBMo currently implements models for 1) the thermal comfort of building occupants (chapter 2) as well as 2) the indoor air quality (chapter 3).

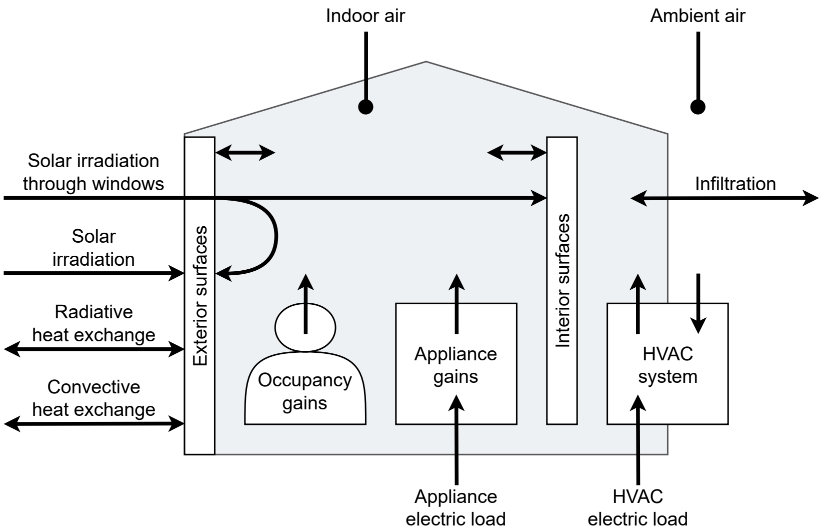

Figure 1.1: Control-oriented building model.

1.3.1 Thermal model

The thermal model focuses on the thermal comfort of the building occupants, which can be expressed as in terms of the indoor air temperature. Therefore, the thermal model caters for the heat exchange between the indoor air and exterior surfaces, interior surfaces, occupants, appliances, the HVAC system and the ambient air through infiltration. The exterior surface model considers convective heat exchange with the ambient air, radiative heat exchange with the ground and the sky, the heat gain due to solar irradiation. Further, both the exterior and the interior surface models consider convective heat transfer with the indoor air as well as incident solar irradiation through windows. Occupancy and appliance gains refer to the radiation and convection heat gains due to human activity, lights and other devices, where the electric demand of these appliances also makes up the fixed base electric load of the building. The HVAC system model translates the thermal load which is provided to the building to maintain thermal comfort constraints into the electric load required to supply conditioned air.

1.3.2 Indoor air quality model

The requirement to provide a fixed amount of outdoor air to ensure sufficient indoor air quality (IAQ), e.g., according to ASHRAE Standard 62.1, is a key limitation for DSF. To this end, demand controlled ventilation (DCV) can be utilized to increase DSF. As cooling and ventilation needs can make up to 60 $\%$ of the energy consumption in commercial buildings in Singapore [5], DCV was first developed as a way to reduce energy consumption by adapting the outdoor airflow rate to the current room conditions, while maintaining IAQ in a comfortable range [9]–[11]. However, as the ventilation requirements are alleviated, some DCV strategies enable ventilation loads to be shifted over time and may thus increase the DSF. Therefore, an IAQ model is derived from a grey clustering model of indoor air and a IAQ-based DCV strategy is formulated along with this.

2 Thermal model

The thermal comfort is expressed in terms of the indoor air temperature. Hence, the thermal building model expresses the relationship between the indoor air temperature, the electric load of the HVAC system, the local weather conditions and the building occupancy. The indoor air temperature, i.e. zone temperature, within each zone is assumed to be uniformly distributed.

As a starting point, the differential equation of the zone temperature T_z of zone z is expressed as:

$$\frac{dT_z}{dt} = \frac{1}{C_z^{thm}} \cdot \left(\left(\sum_{s \in \mathcal{S}_z} \dot{Q}_{s,z}^{cnv,int} \right) + \dot{Q}_z^{inf} + \dot{Q}_z^{occ} + \dot{Q}_z^{hvac} \right) \tag{2.1}$$

Where C_z^{thm} is the heat capacity of zone $z \in \mathcal{Z}_b$, which is obtained according to ISO 13790. The symbol $\mathcal{Z}_b$ is the set of all zones z in building $b \in \mathcal{B}$ and $\mathcal{B}$ is the set of all buildings b. The heat transfer towards zone z is composed of the the convective heat transfer $\dot{Q}_{s,z}^{cnv,int}$ from surfaces $s \in \mathcal{S}_z$ towards zone z, heat transfer towards zone z due to infiltration $\dot{Q}_z^{inf}$, heat transfer towards zone z due to occupancy gains $\dot{Q}_z^{occ}$ and heat transfer towards zone z from the HVAC systems $\dot{Q}_z^{hvac}$, where $\mathcal{S}_z$ is the set of all surfaces adjacent to zone z.

The following assumptions / simplifications are considered in the model formulation:

- Ground heat transfer is omitted, because the considered test case comprises the intermediate storey of an high-rise building, where adiabatic surfaces are assumed for ceiling and floor.
- Emitted radiation on the interior side of surfaces is not modelled, because the difference in interior surface temperatures is assumed to be neglectable.

2.1 Exterior surfaces

Exterior surfaces are modelled as two thermal resistances with a centered heat capacitance between the exterior and zone z. Each surface s is adjacent to exactly one zone z. The heat transfer across exterior surface s is described in the following by (1) the heat balance for

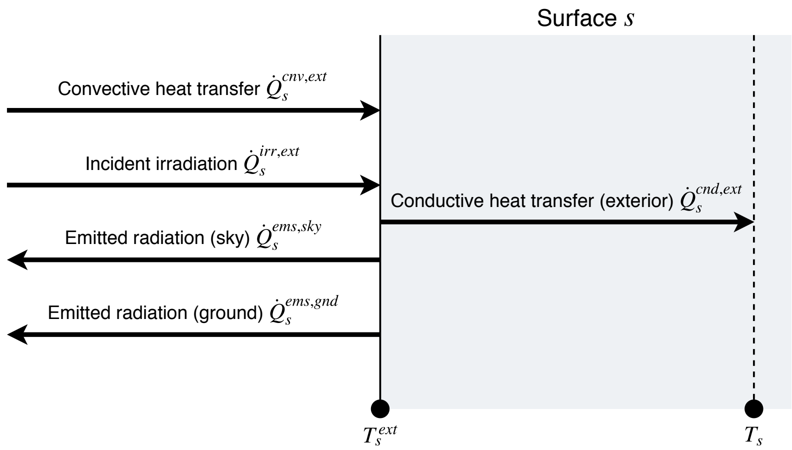

Figure 2.1: Heat balance for the exterior side of exterior surface s.

exterior side (fig. 2.1), (2) the heat balance for the interior side (fig. 2.2) and (3) the heat balance for the core (fig. 2.3). Subsequently, each term of the heat balance equations is defined.

2.1.1 Heat balance

The heat balance for the exterior side of surface s (fig. 2.1) is expressed as:

$$\dot{Q}_s^{cnv,ext} + \dot{Q}_s^{irr,ext} - \dot{Q}_s^{ems,sky} - \dot{Q}_s^{ems,gnd} = \dot{Q}_s^{cnd,ext} \tag{2.2}$$

Where $\dot{Q}_s^{cnv,ext}$ is the convective heat transfer from the exterior towards surface s, $\dot{Q}_s^{irr,ext}$ is the incident irradiation onto surface s, $\dot{Q}_s^{ems,sky}$ and $\dot{Q}_s^{ems,gnd}$ are the emitted radiation from surface s towards the sky and the ground. Note that radiative heat exchange with the sky $\dot{Q}_s^{ems,sky}$ excludes any radiative heat exchange with the sun disc, which is modelled separately as irradiation $\dot{Q}_s^{irr,ext}$. The symbol $\dot{Q}_s^{cnd,ext}$ describes the conductive heat transfer from the exterior towards the core of the surface.

The balance equation for the interior side of surface s (fig. 2.2) is expressed as:

$$\dot{Q}_s^{cnd,int} = \dot{Q}_s^{cnv,int} - \dot{Q}_s^{irr,int} \tag{2.3}$$

On the interior side, $\dot{Q}_s^{cnd,int}$ is the conductive heat transfer from the core towards the interior side of the surface, $\dot{Q}_s^{cnv,int}$ is the convective heat transfer from the interior side of surface s towards zone z and $\dot{Q}_s^{irr,int}$ is the incident irradiation reaching surface s through exterior windows adjacent to the same zone z.

The heat balance for the core of surface s (fig. 2.3) is expressed as:

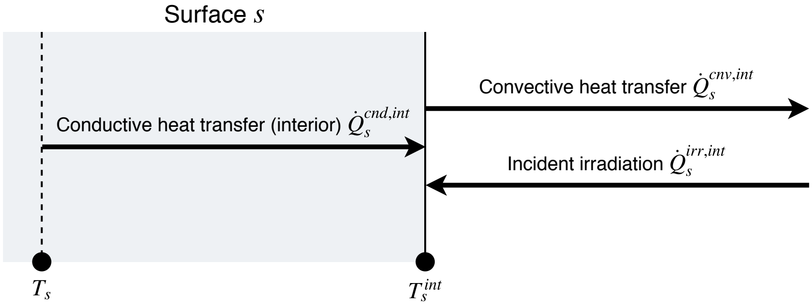

Figure 2.2: Heat balance for the interior side of exterior surface s.

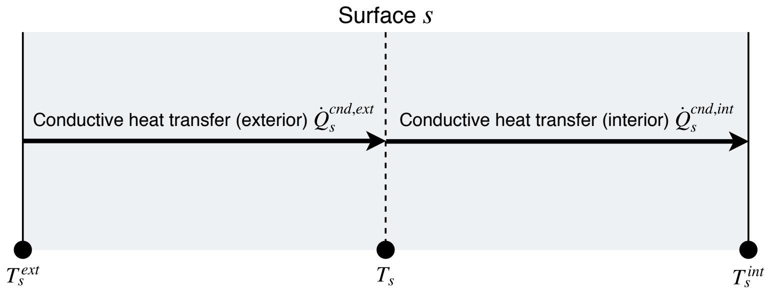

Figure 2.3: Heat balance for the core of exterior surface s.

$$\frac{dT_s}{dt} C_s^{thm} = \dot{Q}_s^{cnd,ext} - \dot{Q}_s^{cnd,int} \tag{2.4}$$

Where C_s^{thm} is the heat capacity of surface s. If the heat capacity of surface s is neglectable $C_s^{thm} = 0$, e.g., for windows, the term simplifies to $\dot{Q}_s^{cnd,ext} = \dot{Q}_s^{cnd,int}$.

2.1.2 Exterior convection

The exterior convective term $\dot{Q}_s^{cnv,ext}$ is expressed as:

$$\dot{Q}_s^{cnv,ext} = A_s h^{cnv,ext} \left(T^{amb} - T_s^{ext} \right) \tag{2.5}$$

Where A_s is the surface area of surface s and $h^{cnv,ext}$ is the exterior convective heat transfer coefficient which is given according to ISO 6946 as $h^{cnv,ext} = \left(0.04\,\mathrm{m^2\,K/W}\right)^{-1}$. The symbol T^{amb} is the ambient temperature and T_s^{ext} is the temperature at the exterior side of surface s.

2.1.3 Exterior irradiation

The exterior irradiation term $\dot{Q}_s^{irr,ext}$ is expressed as:

$$\dot{Q}_s^{irr,ext} = A_s \alpha_s \dot{q}_d^{irr,ext}, \quad d = d(s) \tag{2.6}$$

Where α_s is the absorption coefficient of surface s assuming a uniform absorption across the spectrum of the incident irradiation. The symbol $\dot{q}_d^{irr,ext}$ is the total incident irradiation onto a surface oriented towards direction $d = \{N, E, S, W, H\}$, i.e., vertically facing North N, East E, South S, West W or horizontally facing upwards H, depending on the respective surface's orientation $d = d(s)$.

2.1.4 Exterior emissions

The exterior sky emission term $\dot{Q}_s^{ems,sky}$ describes the radiative heat loss through emission towards the sky. The term is expressed as:

$$\dot{Q}_s^{ems,sky} = A_s h_s^{sky} \left(T_s^{ext} - T^{sky} \right) \tag{2.7}$$

In this linear approximation, the symbol h_s^{sky} is introduced as the sky heat transfer coefficient of surface s, whereas T^{sky} is the sky temperature. The sky heat transfer coefficient h_s^{sky} in turn is defined as:

$$h_s^{sky} = 4\sigma\varepsilon_s F_s^{sky} \left(\frac{T_s^{sur,ext,lin} + T^{sky,lin}}{2} \right)^3 \tag{2.8}$$

Where σ, ε_s and F_s^{sky} are the Stefan-Boltzmann constant, the surface emission coefficient of surface s for long-wave radiations and the view factor of surface s towards the sky. The temperatures $T_s^{sur,ext,lin}$ and $T^{sky,lin}$ are linearization constants of T_s^{ext} and T^{sky}. The term F_s^{sky} is defined as $F_s^{sky} = 0.5$ for $d(s) \in \{N, E, S, W\}$ and $F_s^{sky} = 1$ $d(s) = H$, where $d(s)$ is the direction orientation of the surface.

The exterior ground emission term $\dot{Q}_s^{ems,gnd}$ describes the radiative heat loss through emission towards the ground as well as the built environment. The term is expressed similar to $\dot{Q}_s^{ems,sky}$ as:

$$\dot{Q}_s^{ems,gnd} = A_s h_s^{gnd} \left(T_s^{ext} - T^{amb} \right) \tag{2.9}$$

Where h_s^{gnd} is introduced as the ground heat transfer coefficient of surface s, whereas T^{amb} is the ambient temperature. The ground heat transfer coefficient h_s^{gnd} in turn is defined as:

$$h_s^{gnd} = 4\sigma\varepsilon_s F_s^{gnd} \left(\frac{T_s^{sur,ext,lin} + T^{amb,lin}}{2} \right)^3 \tag{2.10}$$

Where F_s^{gnd} is the view factor of surface s towards the ground. The temperatures $T_s^{sur,ext,lin}$ and $T^{amb,lin}$ are linearization constants of T_s^{ext} and T^{amb}. The term F_s^{gnd} is defined as $F_s^{gnd} = 0.5$ for $d(s) \in \{N, E, S, W\}$ and $F_s^{gnd} = 0$ $d(s) = H$, where $d(s)$ is the direction orientation of the surface.

2.1.5 Interior convection

The interior convective term $\dot{Q}_s^{cnv,int}$ is expressed as:

$$\dot{Q}_s^{cnv,int} = A_s h_s^{cnv,int} \left(T_s^{int} - T_z\right) \tag{2.11}$$

Where $h_s^{cnv,int}$, T_z and T_s^{int} are the interior convective heat transfer coefficient, the zone air temperature and the temperature at the interior side of surface s. The interior heat transfer coefficient $h_s^{cnv,int}$ is defined according to ISO 6946 as $h_s^{cnv,int} = \left(0.13\,\mathrm{m^2\,K/W}\right)^{-1}$ for $d(s) \in \{N, E, S, W\}$ and $h_s^{cnv,int} = \left(0.17\,\mathrm{m^2\,K/W}\right)^{-1}$ for $d(s) = H$, where $d(s)$ is the direction orientation of the surface.

2.1.6 Interior irradiation

The interior irradiation term $\dot{Q}_s^{irr,int}$ is expressed as:

$$\dot{Q}_s^{irr,int} = A_s \alpha_s \dot{q}_z^{irr,int}, \quad d = d(s) \tag{2.12}$$

Where $\dot{q}_z^{irr,int}$ is the interior irradiation incident to all surfaces of zone z. The interior radiation $\dot{q}_z^{irr,int}$ is in fact the irritation which has entered zone z by passing through adjacent windows and is assumed to be uniformly distributed to all surfaces. This term is expressed as:

$$\dot{q}_z^{irr,int} = \frac{\sum_{w \in \mathcal{W}_z} A_w \tau_w \dot{q}_{d(w)}^{irr,ext}}{\sum_{s \in \mathcal{S}_z} A_s} \tag{2.13}$$

Where τ_w is the transmission coefficient of window w. The sets $\mathcal{W}_z$ and $\mathcal{S}_z$ contain all windows w and surfaces s that are adjacent to zone z.

2.1.7 Conduction

Finally, the conductive terms $\dot{Q}_s^{cnd,ext}$ and $\dot{Q}_s^{cnd,int}$ are defined:

$$\begin{aligned} \dot{Q}_s^{cnd,ext} &= A_s 2 h_s^{cnd} \left(T_s^{ext} - T_s\right) \\ \dot{Q}_s^{cnd,int} &= A_s 2 h_s^{cnd} \left(T_s - T_s^{int}\right) \end{aligned} \tag{2.14}$$

Where h_s^{cnd} is the conductive heat transfer coefficient of surface s and T_s is the surface core temperature. For surfaces with a neglectable heat capacity, e.g., windows, the relationship simplifies to:

$$\dot{Q}_s^{ext,int} = A_s h_s^{cnd} \left(T_s^{ext} - T_s^{int} \right) \tag{2.15}$$

2.1.8 Complete model

Equations (2.2) to (2.14) and (2.15) define an overdetermined equation system, such that the temperatures T_s^{ext} and T_s^{int} can be eliminated. The final equations are presented as the complete model for exterior surfaces in the following.

The conductive heat transfer from the exterior towards the core of surface s can be expressed as:

$$\begin{aligned} \dot{Q}_s^{cnd,ext} =& \left(\alpha_s \dot{q}_d^{irr,ext} + h^{cnv,ext} \left(T^{amb} - T_s \right) + h_s^{gnd} \left(T^{amb} - T_s \right) + h_s^{sky} \left(T^{sky} - T_s \right) \right) \\ & \cdot A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{2 h_s^{cnd}} \right)^{-1} \end{aligned} \tag{2.16}$$

To ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$\begin{aligned} \dot{Q}_s^{cnd,ext} =& \dot{q}_d^{irr,ext} \cdot \alpha_s A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{2 h_s^{cnd}} \right)^{-1} \\ & + T^{amb} \cdot \left(h^{cnv,ext} + h_s^{gnd} \right) A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{2 h_s^{cnd}} \right)^{-1} \\ & + T^{sky} \cdot h_s^{sky} A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{2 h_s^{cnd}} \right)^{-1} \\ & + T_s \cdot (-1) A_s \left(\frac{1}{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}} + \frac{1}{2 h_s^{cnd}} \right)^{-1} \end{aligned} \tag{2.17}$$

The conductive heat transfer from the core of surface s towards the interior can be expressed as:

$$\begin{aligned}\dot{Q}_s^{cnd,int} =&\left(-\alpha_s \dot{q}_z^{irr,int} + h_s^{cnv,int}\left(T_s - T_z\right)\right)\\ &\cdot A_s\left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\end{aligned} \tag{2.18}$$

Again, to ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$\begin{aligned}\dot{Q}_s^{cnd,int} =&\dot{q}_z^{irr,int} \cdot (-1)\alpha_s A_s\left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\\ &+ T_s \cdot A_s\left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1}\\ &+ T_z \cdot (-1)A_s\left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1}\end{aligned} \tag{2.19}$$

The convective heat transfer from surface s towards zone z can be expressed as:

$$\begin{aligned}\dot{Q}_s^{cnv,int} =&\alpha_s \dot{q}_z^{irr,int} \cdot A_s\left(1 - \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\right)\\ &+ h_s^{cnv,int}\left(T_s - T_z\right) \cdot A_s\left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\end{aligned} \tag{2.20}$$

Again, to ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$\begin{aligned}\dot{Q}_s^{cnv,int} =&\dot{q}_z^{irr,int} \cdot \alpha_s A_s\left(1 - \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\right)\\ &+ T_s \cdot A_s\left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1}\\ &+ T_z \cdot (-1)A_s\left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1}\end{aligned} \tag{2.21}$$

For surfaces with neglectable heat capacity, the complete heat transfer from the exterior through surface s towards zone z can be expressed as:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int} =& \left(\alpha_s \dot{q}_d^{irr,ext} + h^{cnv,ext}\left(T^{amb} - T_z\right) + h_s^{gnd}\left(T^{amb} - T_z\right) + h_s^{sky}\left(T^{sky} - T_z\right)\right) \\
&\cdot A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnv,int}} + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnd}}\right)^{-1} \\
&+ \alpha_s \dot{q}_z^{irr,int} A_s \left(1 - \left(1 + \frac{h_s^{cnv,int}}{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}} + \frac{h_s^{cnv,int}}{h_s^{cnd}}\right)^{-1}\right)
\end{aligned}
\tag{2.22}
$$

Again, to ease formulating the state space matrix entries (chapter 4), the equations can be separated for each variable:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int} =& \dot{q}_d^{irr,ext} \cdot \alpha_s A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnv,int}} + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnd}}\right)^{-1} \\
&+ T^{amb} \cdot \left(h^{cnv,ext} + h_s^{gnd}\right) \\
&\quad \cdot A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnv,int}} + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnd}}\right)^{-1} \\
&+ T^{sky} \cdot h_s^{sky} A_s \left(1 + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnv,int}} + \frac{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}}{h_s^{cnd}}\right)^{-1} \\
&+ T_z(-1) \cdot A_s \left(\frac{1}{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}} + \frac{1}{h_s^{cnv,int}} + \frac{1}{h_s^{cnd}}\right)^{-1} \\
&+ \dot{q}_z^{irr,int} \cdot \alpha_s A_s \left(1 - \left(1 + \frac{h_s^{cnv,int}}{h^{cnv,ext} + h_s^{gnd} + h_s^{sky}} + \frac{h_s^{cnv,int}}{h_s^{cnd}}\right)^{-1}\right)
\end{aligned}
\tag{2.23}
$$

2.2 Interior surfaces

Interior surfaces are modelled as two thermal resistances with a centered heat capacitance between the zone z_1 and zone z_2. The heat transfer across interior surface s is described in the following by (1) the heat balance for the side which is oriented towards z_1, (2) the heat balance for the side which is oriented towards z_2 and (3) the heat balance for the core. In principle, interior surfaces are modelled equivalently to exterior surfaces, where the heat balance for the interior side (eq. (2.3), fig. 2.2) is applied for both sides of the surface.

2.2.1 Heat balance

The balance equation for the side of surface s which is oriented towards zone z_1 is expressed as:

$$\dot{Q}_s^{cnd,int,1} = \dot{Q}_s^{cnv,int,1} - \dot{Q}_s^{irr,int,1} \tag{2.24}$$

Where $\dot{Q}_s^{cnd,int,1}$ is the conductive heat transfer from the core towards the side of surface s which is oriented towards z_1, $\dot{Q}_s^{cnv,int,1}$ is the convective heat transfer from the side of surface s towards zone z_1 and $\dot{Q}_s^{irr,int,1}$ is the incident irradiation reaching surface s through exterior windows adjacent to zone z_1.

The balance equation for the side of s which is oriented towards z_2 is expressed as:

$$\dot{Q}_s^{cnd,int,2} = \dot{Q}_s^{cnv,int,2} - \dot{Q}_s^{irr,int,2} \tag{2.25}$$

The symbols $\dot{Q}_s^{cnd,int,2}$, $\dot{Q}_s^{cnv,int,2}$ and $\dot{Q}_s^{irr,int,2}$ describe the respective heat transfers on the side of surface s which is oriented towards zone z_2.

The heat balance for the core of surface s (fig. 2.3) is expressed as:

$$\frac{dT_s}{dt} C_s^{thm} = -\dot{Q}_s^{cnd,int,1} - \dot{Q}_s^{cnd,int,2} \tag{2.26}$$

Where C_s^{thm} is the heat capacity of surface s. If the heat capacity of surface s is neglectable $C_s^{thm} = 0$, e.g., for windows, the term simplifies to $\dot{Q}_s^{cnd,int,1} = -\dot{Q}_s^{cnd,int,2}$. Note that the negative signs of $\dot{Q}_s^{cnd,int,1}$ and $\dot{Q}_s^{cnd,int,2}$ are due to applying eq. (2.3) (fig. 2.2) for both sides of the surface.

The definitions of the heat transfers are given in section 2.1 and are omitted here for the sake of brevity.

2.2.2 Complete model

The complete model is derived similar to the complete exterior surface model (section 2.1.8) by eliminating the temperatures $T_s^{int,1}$ and $T_s^{int,2}$.

The conductive heat transfer from the the core of surface s towards zone $z_{1/2}$, i.e. both z_1 and z_2, can be expressed as:

$$\begin{aligned}\dot{Q}_s^{cnd,int,1/2} = &\left(-\alpha_s \dot{q}_{z_{1/2}}^{irr,int} + h_s^{cnv,int,1/2} \left(T_s - T_{z_{1/2}} \right) \right) \\ &\cdot A_s \left(1 + \frac{h_s^{cnv,int,1/2}}{2 h_s^{cnd}} \right)^{-1}\end{aligned} \tag{2.27}$$

To ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$
\begin{aligned}
\dot{Q}_s^{cnd,int,1/2} =& \dot{q}_{z_{1/2}}^{irr,int} \cdot (-1)\alpha_s A_s \left(1 + \frac{h_s^{cnv,int,1/2}}{2h_s^{cnd}}\right)^{-1} \\
&+ T_s \cdot A_s \left(\frac{1}{h_s^{cnv,int,1/2}} + \frac{1}{2h_s^{cnd}}\right)^{-1} \\
&+ T_z \cdot (-1)A_s \left(\frac{1}{h_s^{cnv,int,1/2}} + \frac{1}{2h_s^{cnd}}\right)^{-1}
\end{aligned} \tag{2.28}
$$

The convective heat transfer from surface s towards zone $z_{1/2}$ can be expressed as:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int,1/2} =& \alpha_s \dot{q}_{z_{1/2}}^{irr,int} \cdot A_s \left(1 - \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\right) \\
&+ h_s^{cnv,int} \left(T_s - T_{z_{1/2}}\right) \cdot A_s \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}
\end{aligned} \tag{2.29}
$$

Again, to ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int,1/2} =& \dot{q}_{z_{1/2}}^{irr,int} \cdot \alpha_s A_s \left(1 - \left(1 + \frac{h_s^{cnv,int,1/2}}{2h_s^{cnd}}\right)^{-1}\right) \\
&+ T_s \cdot A_s \left(\frac{1}{h_s^{cnv,int,1/2}} + \frac{1}{2h_s^{cnd}}\right)^{-1} \\
&+ T_{z_{1/2}} \cdot (-1)A_s \left(\frac{1}{h_s^{cnv,int,1/2}} + \frac{1}{2h_s^{cnd}}\right)^{-1}
\end{aligned} \tag{2.30}
$$

For surfaces with neglectable heat capacity, the complete heat transfer from zone z_1 towards zone z_2 can be expressed as:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int,2} =& \left(\alpha_s \dot{q}_{z_1}^{irr,int} + h_s^{cnv,int,1}\left(T_{z_1} - T_{z_2}\right)\right) \\
&\cdot A_s \left(1 + \frac{h_s^{cnv,int,1}}{h_s^{cnv,int,2}} + \frac{h_s^{cnv,int,1}}{h_s^{cnd}}\right)^{-1} \\
&+ \alpha_s \dot{q}_{z_2}^{irr,int} A_s \left(1 - \left(1 + \frac{h_s^{cnv,int,2}}{h_s^{cnv,int,1}} + \frac{h_s^{cnv,int,2}}{h_s^{cnd}}\right)^{-1}\right)
\end{aligned} \tag{2.31}
$$

Again, to ease formulating the state space matrix entries (chapter 4), the equations can be separated for each variable:

$$\begin{aligned}\dot{Q}_s^{cnv,int,2} =&\dot{q}_{z_1}^{irr,int} \cdot \alpha_s A_s \left(1 + \frac{h_s^{cnv,int,1}}{h_s^{cnv,int,2}} + \frac{h_s^{cnv,int,1}}{h_s^{cnd}}\right)^{-1} \\ &+ T_{z_1} \cdot A_s \left(\frac{1}{h_s^{cnv,int,1}} + \frac{1}{h_s^{cnv,int,2}} + \frac{1}{h_s^{cnd}}\right)^{-1} \\ &+ T_{z_2}(-1) \cdot A_s \left(\frac{1}{h_s^{cnv,int,1}} + \frac{1}{h_s^{cnv,int,2}} + \frac{1}{h_s^{cnd}}\right)^{-1} \\ &+ \dot{q}_{z_2}^{irr,int} \cdot \alpha_s A_s \left(1 - \left(1 + \frac{h_s^{cnv,int,2}}{h_s^{cnv,int,1}} + \frac{h_s^{cnv,int,2}}{h_s^{cnd}}\right)^{-1}\right)\end{aligned} \tag{2.32}$$

The complete heat transfer from zone z_2 towards zone z_1 can be obtained by inverting the indices and is omitted for brevity here.

Note that the above equations could be simplified by inserting $h_s^{cnv,int} = h_s^{cnv,int,1} = h_s^{cnv,int,2}$, which holds because to the approximation for $h_s^{cnv,int}$ according to section 2.1.5 depends only on the direction orientation $d(s)$ of surface s. However, the full equation is given here for future reference when a better approximation of $h_s^{cnv,int}$ for each side of surface s might be desired.

2.3 Adiabatic surfaces

Adiabatic surfaces are interior surfaces which neglect heat transfer to one side of the surface, i.e., such a surface is only adjacent to one zone. Adiabatic surfaces are a helpful tool to model well isolated surfaces or surfaces which are positioned entirely within one zone.

Adiabatic surfaces are modelled as a single thermal resistances between a heat capacitance and zone z. The heat transfer between zone z and the adiabatic surface s is described in the following by (1) the heat balance for the side which is oriented towards z and (2) the heat balance for the core. Adiabatic surfaces are modelled equivalently to interior surfaces with only one adjacent zone.

2.3.1 Heat balance

The balance equation for the interior side of surface s (fig. 2.2) is expressed as:

$$\dot{Q}_s^{cnd,int} = \dot{Q}_s^{cnv,int} - \dot{Q}_s^{irr,int} \tag{2.33}$$

On the interior side, $\dot{Q}_s^{cnd,int}$ is the conductive heat transfer from the core towards the interior side of the surface, $\dot{Q}_s^{cnv,int}$ is the convective heat transfer from the interior side of surface

s towards zone z and $\dot{Q}_s^{irr,int}$ is the incident irradiation reaching surface s through exterior windows adjacent to the same zone z.

The heat balance for the core of surface s (fig. 2.3) is expressed as:

$$\frac{dT_s}{dt} C_s^{thm} = -\dot{Q}_s^{cnd,int} \tag{2.34}$$

The definitions of the heat transfers are given in section 2.1 and are omitted here for the sake of brevity.

2.3.2 Complete model

The complete model is derived similar to the complete exterior surface model (section 2.1.8) by eliminating the temperature T_s^{int}.

The conductive heat transfer from the core of surface s towards the interior can be expressed as:

$$\begin{aligned} \dot{Q}_s^{cnd,int} =& \left(-\alpha_s \dot{q}_z^{irr,int} + h_s^{cnv,int} \left(T_s - T_z \right) \right) \\ & \cdot A_s \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}} \right)^{-1} \end{aligned} \tag{2.35}$$

To ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$\begin{aligned} \dot{Q}_s^{cnd,int} =& \dot{q}_z^{irr,int} \cdot (-1) \alpha_s A_s \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}} \right)^{-1} \\ & + T_s \cdot A_s \left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}} \right)^{-1} \\ & + T_z \cdot (-1) A_s \left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}} \right)^{-1} \end{aligned} \tag{2.36}$$

The convective heat transfer from surface s towards zone z can be expressed as:

$$\begin{aligned} \dot{Q}_s^{cnv,int} =& \alpha_s \dot{q}_z^{irr,int} \cdot A_s \left(1 - \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}} \right)^{-1} \right) \\ & + h_s^{cnv,int} \left(T_s - T_z \right) \cdot A_s \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}} \right)^{-1} \end{aligned} \tag{2.37}$$

Again, to ease formulating the state space matrix entries (chapter 4), the equation can be separated for each variable:

$$
\begin{aligned}
\dot{Q}_s^{cnv,int} =& \dot{q}_z^{irr,int} \cdot \alpha_s A_s \left(1 - \left(1 + \frac{h_s^{cnv,int}}{2h_s^{cnd}}\right)^{-1}\right) \\
&+ T_s \cdot A_s \left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1} \\
&+ T_z \cdot (-1) A_s \left(\frac{1}{h_s^{cnv,int}} + \frac{1}{2h_s^{cnd}}\right)^{-1}
\end{aligned}
\tag{2.38}
$$

2.4 Infiltration

The heat transfer towards zone z due to infiltration $\dot{Q}_z^{inf}$ is defined as:

$$
\dot{Q}_z^{inf} = V_z C^{th,air} n_z^{inf} \left(T^{amb} - T_z\right) \tag{2.39}
$$

Where V_z is the volume of zone z, $C^{th,air}$ is the heat capacity of air and n_z^{inf} is the infiltration rate.

2.5 Occupancy gains

Assuming perfect knowledge of the building occupancy schedule, the heat transfer towards zone z due to occupancy gains $\dot{Q}_z^{occ}$, i.e., internal gains, is expressed as:

$$
\dot{Q}_z^{occ} = A_z \dot{q}_z^{occ} \tag{2.40}
$$

Where A_z is the area of zone z and $\dot{q}_z^{occ}$ is the specific thermal gain due to occupancy.

2.6 Heating ventilation and air-conditioning (HVAC) systems

The heating ventilation and air-conditioning (HVAC) systems are distinguished according to fig. 2.4 into 1) generic HVAC system, 2) air handling unit (AHU), 3) terminal units (TUs), 4) heating and chiller plant. Further HVAC systems can in principal be integrated into the modular CoBMo framework, once appropriate linear models are formulated.

The generic HVAC system (section 2.6.1) provides thermal heating / cooling power to each zone z, i.e. it directly adds / removes thermal energy from the zone. This generic HVAC system is an auxiliary system type which helps to model 1) simplified HVAC systems in case that a detailed model is not required or 2) HVAC system types for which a detailed model

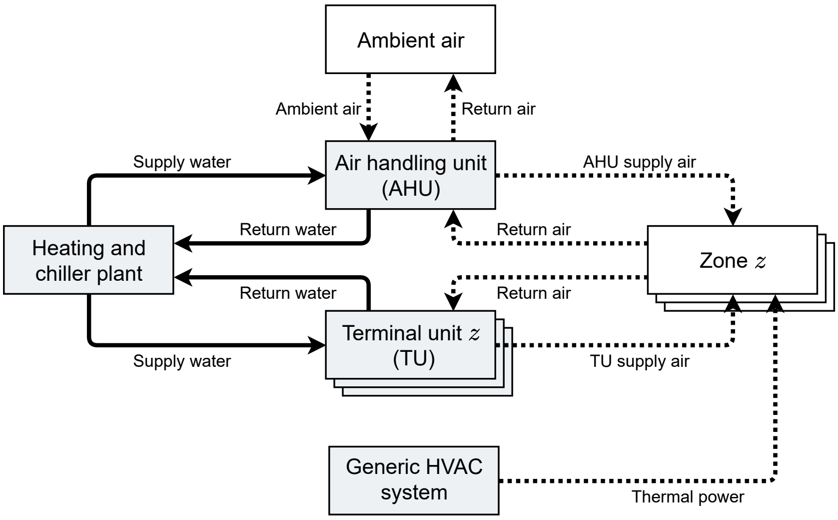

Figure 2.4: HVAC systems overview.

has yet to be implemented. The air handling unit (AHU) (section 2.6.2) serves supply air, i.e., conditioned outdoor air at a fixed temperature and humidity level, to each zone z. A terminal unit (TU) (section 2.6.3) serves supply air, i.e., re-conditioned zone air at a fixed temperature and humidity level, to each zone z. Note that the TU takes in zone air, whereas the AHU draws fresh outdoor air. The heating and cooling demand of the AHU and TUs is provided in form of supply water, i.e., hot and chilled water, by the heating and chiller plant.

The following formulations assume that there is exactly one heating and chiller plant and one AHU per building. However, the AHU supply airflow rate $\dot{V}_z^{ahu}$ can be controlled for each individual zone z, e.g., through variable-air-volume (VAV) boxes. Further, the model assumes that there is exactly one heating and chiller plant per building and one TU per zone, such that the TU supply airflow rate $\dot{V}_z^{tu}$ can be controlled for each individual zone z, e.g., through variable fan rates.

The total heat transfer towards zone z from the HVAC systems $\dot{Q}_z^{hvac}$ is expressed as:

$$\dot{Q}_z^{hvac} = \dot{Q}_z^{gen} + \dot{Q}_z^{ahu} + \dot{Q}_z^{tu} \tag{2.41}$$

Where $\dot{Q}_z^{gen}$, $\dot{Q}_z^{ahu}$ and $\dot{Q}_z^{tu}$ are the heat transfer towards zone z from the generic HVAC system, the AHU and the TU.

2.6.1 Generic HVAC system

The heat transfer towards zone z from a generic HVAC system $\dot{Q}_z^{gen}$ is expressed as:

$$\dot{Q}_z^{gen} = \dot{Q}_z^{gen,heat} + \dot{Q}_z^{gen,cool} \tag{2.42}$$

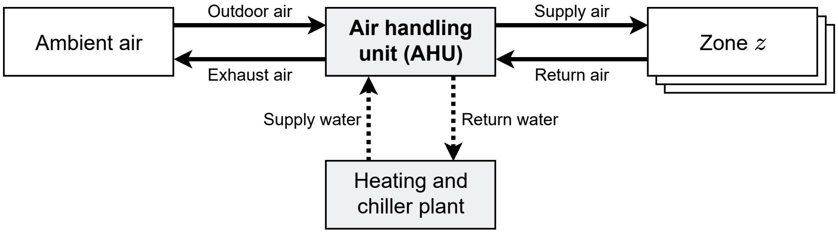

Figure 2.5: Interconnection of the air handling unit (AHU).

Where $\dot{Q}_z^{gen,heat}$ and $\dot{Q}_z^{gen,cool}$ are the thermal heating and cooling power provided to zone z. Note that $\dot{Q}_z^{gen,heat} \geq 0$ and $\dot{Q}_z^{gen,cool} \leq 0$

The electric demand of the generic HVAC system is expressed as:

$$\dot{P}_z^{gen,el} = \eta^{gen,heat}\dot{Q}_z^{gen,heat} + \eta^{gen,cool}\dot{Q}_z^{gen,cool} \tag{2.43}$$

Where $\eta^{gen,heat}$ and $\eta^{gen,cool}$ are the efficiency factor for heating and cooling of the generic HVAC system and $P_z^{gen,el}$ is the electric power consumption of the generic HVAC system associated with thermal demand supplied at zone z. Note that η^{cool} takes a negative value such that $P_z^{gen,el}$ is positive. The part-load behavior of the heating and chiller plant is neglected.

2.6.2 Air handling unit (AHU)

The air handling unit (AHU) serves supply air, i.e., conditioned outdoor air at a fixed temperature and humidity level, to each zone z. The heating and cooling demand of the AHU is provided in form of supply water, i.e., hot and chilled water, by the heating and chiller plant (fig. 2.5). The following formulations assume that there is exactly one heating and chiller plant and one AHU per building. However, the AHU supply airflow rate $\dot{V}_z^{ahu}$ can be controlled for each individual zone z, e.g., through variable-air-volume (VAV) boxes.

As illustrated in fig. 2.6, the AHU model considers a cooling coil, a heating coil, supply air fan, exhaust air fan as well as a heat recovery system. For dehumidification, the air flow is assumed to be cooled below its dew point temperature and humidity is removed from the air by condensation at the cooling coil. A humidifier is not considered in the model.

The heat transfer towards zone z from the AHU $\dot{Q}_z^{ahu}$ is expressed as:

$$\dot{Q}_z^{ahu} = \dot{V}_z^{ahu,heat} C^{th,air}\left(T^{ahu} - T_z^{heat,lin}\right) + \dot{V}_z^{ahu,cool} C^{th,air}\left(T^{ahu} - T_z^{cool,lin}\right) \tag{2.44}$$

Where $\dot{Q}_z^{ahu}$ is the heat transfer from the AHU towards zone z, $\dot{V}_z^{ahu,heat}$ and $\dot{V}_z^{ahu,cool}$ is the air flow rate for heating and cooling from the AHU towards zone z and $C^{th,air}$ is the heat

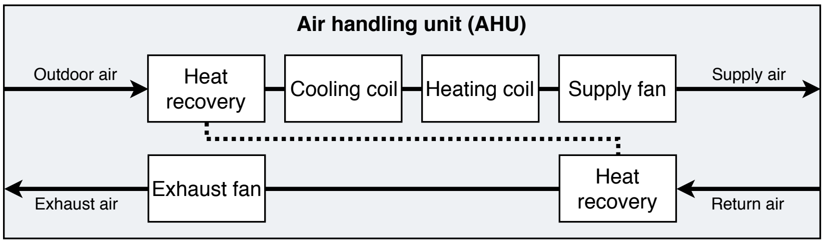

Figure 2.6: Components of the air handling unit (AHU).

capacity of air. The symbols T^{ahu} is the temperature of the conditioned supply air from the AHU and $T_z^{lin,heat}$ as well as $T_z^{lin,cool}$ are the linearization constants of the air temperature at zone z for heating and cooling. Note that the constants $T_z^{lin,heat}$ and $T_z^{lin,cool}$ are taken instead of the variable T_z to avoid a bi-linear term in the model formulation. This linearization approach assumes small changes in the zone temperature $\mathrm{d}T_z \ll T_z$. The linearization constants are separated for heating $T_z^{lin,heat}$ and cooling $T_z^{lin,cool}$ to allow simultaneous heating and cooling, e.g., for intermediate season where both heating and cooling may be desired within one day.

The electric power for ventilation to zone z though the AHU fans $P_z^{ahu,fan,el}$ is expressed as:

$$P_z^{ahu,fan,el} = \dot{V}_z^{ahu} \Delta p^{ahu,fan} \eta^{el,ahu,fan} \tag{2.45}$$

Here, the symbols $\Delta p^{ahu,fan}$ and $\eta^{el,ahu,fan}$ are the total pressure rise across all fans and the average electric efficiency coefficient of the fans.

The AHU heating and cooling demand to supply air for z at set point conditions $\dot{Q}_z^{ahu,heat}$ and $\dot{Q}_z^{ahu,cool}$ are expressed as:

$$\begin{aligned} \dot{Q}_z^{ahu,heat} &= \left(\dot{V}_z^{ahu,heat} + \dot{V}_z^{ahu,cool}\right) \rho^{air} \left(\Delta h^{ahu,heat} - \Delta h^{ahu,heat,rec}\right) \\ \dot{Q}_z^{ahu,cool} &= \left(\dot{V}_z^{ahu,heat} + \dot{V}_z^{ahu,cool}\right) \rho^{air} \left(\Delta h^{ahu,cool} - \Delta h^{ahu,cool,rec}\right) \end{aligned} \tag{2.46}$$

Where $\Delta h^{ahu,heat}$ and $\Delta h^{ahu,cool}$ are the specific heating and cooling power applied to the outdoor air in order to obtain AHU supply air conditions. The symbols $\Delta h^{ahu,heat,rec}$ and $\Delta h^{ahu,cool,rec}$ are the specific heating and cooling power that is recovered from the return air. Heat gains though the supply fan units are neglected.

The specific cooling and heating power are calculated such that the supply air is conditioned to the desired temperature set point T^{ahu} and absolute humidity x^{ahu}. Depending on the ambient air temperature T^{amb} and ambient absolute humidity x^{amb}, $\Delta h^{ahu,cool}$ and $\Delta h^{ahu,heat}$ are defined as:

$$
\begin{aligned}
&x^{amb,lin} \leq x^{ahu} : \\
&\qquad \Delta h^{ahu,cool} &&= \min\left(0, h\left(T^{ahu}, x^{amb,lin}\right) - h\left(T^{amb,lin}, x^{amb,lin}\right)\right) \\
&\qquad \Delta h^{ahu,heat} &&= \max\left(0, h\left(T^{ahu}, x^{amb,lin}\right) - h\left(T^{amb,lin}, x^{amb,lin}\right)\right) \\
&x^{amb,lin} > x^{ahu} : \\
&\qquad \Delta h^{ahu,cool} &&= h\left(x^{ahu}, \varphi = 1\right) - h\left(T^{amb,lin}, x^{amb,lin}\right) \\
&\qquad \Delta h^{ahu,heat} &&= h\left(T^{ahu}, x^{ahu}\right) - h\left(x^{ahu}, \varphi = 1\right)
\end{aligned}
\tag{2.47}
$$

Where T_z^{lin}, $T^{amb,lin}$ and $x^{amb,lin}$ are the linearization constants of the zone air temperature of zone z, the ambient air temperature and the ambient absolute humidity. The linearization constants are taken to remove non-linear terms arising from the functions $h\left(T, x\right)$ and $h\left(x, \varphi\right)$, which give the specific enthalpy of wet air at value pairs of the dry-bulb air temperature T, the absolute humidity x and the relative humidity φ. The scalar x^{ahu} is the absolute humidity at supply air set-point conditions and is defined by $x^{ahu} = x\left(T^{ahu}, \varphi^{ahu}\right)$, where the function $x\left(T, \varphi\right)$ calculates the absolute humidity for a value pair of the dry-bulb temperature T and relative humidity φ. The functions $h()$ and $x()$ are calculated according to [12].

Furthermore, $\Delta h^{ahu,cool,rec}$ and $\Delta h^{ahu,heat,rec}$ are defined as:

$$
\begin{aligned}
&\qquad \Delta h^{ahu,rec} = h\left(T_z^{lin}, x_z^{lin}\right) - h\left(T^{amb,lin}, x_z^{lin}\right) \\
&x^{amb,lin} \leq x^{ahu} : \\
&\qquad \Delta h^{ahu,cool,rec} &&= \max\left(\Delta h^{ahu,cool}, \min\left(0, \eta^{ahu,rec} \Delta h^{ahu,rec}\right)\right) \\
&\qquad \Delta h^{ahu,heat,rec} &&= \min\left(\Delta h^{ahu,heat}, \max\left(0, \eta^{ahu,rec} \Delta h^{ahu,rec}\right)\right) \\
&x^{amb,lin} > x^{ahu} : \\
&\qquad \Delta h^{ahu,cool,rec} &&= \max\left(\Delta h^{ahu,cool}, \min\left(0, \eta^{ahu,rec} \Delta h^{ahu,rec}\right)\right) \\
&\qquad \Delta h^{ahu,heat,rec} &&= 0
\end{aligned}
\tag{2.48}
$$

Where $\eta^{ahu,rec}$ is the efficiency of the heat recovery system and $\Delta h^{ahu,rec}$ is the theoretical maximum recoverable specific heat. With this model, only sensible heat recovery systems, e.g. thermal wheel, plate heat exchanger or runaround coils, can be considered. Furthermore, the heat exchanger is assumed to be positioned before the cooling coil as in fig. 2.6, such that no contribution to re-heating is possible in the case $x^{amb,lin} > x^{ahu}$. Mixing effects in the return air flow due to one AHU serving several zones are neglected.

The electric demand of the heating and chiller plant is expressed as:

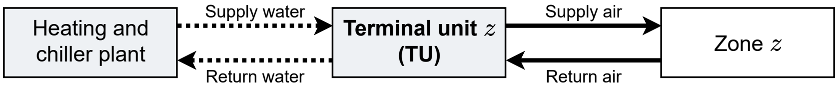

Figure 2.7: Interconnection of the terminal unit (TU).

$$
\begin{aligned}
P_z^{ahu,heat,el} &= \frac{1}{\eta^{heat}} \dot{Q}_z^{ahu,heat} \\
P_z^{ahu,cool,el} &= \frac{1}{\eta^{cool}} \dot{Q}_z^{ahu,cool}
\end{aligned}
\tag{2.49}
$$

Where η^{heat} and η^{cool} are the heating and cooling efficiency, i.e. the COP, of the heating and chiller plant. Note that η^{cool} takes a negative value such that $P_z^{ahu,cool,el}$ is positive. The part-load behavior of the heating and chiller plant is neglected.

Finally, the the electric load due to the air supply to zone z through the AHU $P_z^{ahu,el}$ is expressed as:

$$
P_z^{ahu,el} = P_z^{ahu,heat,el} + P_z^{ahu,cool,el} + P_z^{ahu,fan,el} \tag{2.50}
$$

2.6.3 Terminal unit (TU)

A terminal unit (TU) serves supply air, i.e., re-conditioned zone air at a fixed temperature and humidity level, to each zone z. Note that the TU takes in zone air, whereas the AHU draws fresh outdoor air. The heating and cooling demand of the TU is provided in form of supply water, i.e., hot and chilled water, by the heating and chiller plant (fig. 2.7). The following formulations assume that there is exactly one heating and chiller plant per building and one TU per zone. Further assumption is that the TU supply airflow rate $\dot{V}_z^{tu}$ can be controlled for each individual zone z, e.g., through variable fan rates.

The heat transfer towards zone z from the TU z $\dot{Q}_z^{tu}$ is expressed as:

$$
\begin{aligned}
\dot{Q}_z^{tu} &= \dot{Q}_z^{tu,heat} + \dot{Q}_z^{tu,cool} \\
\dot{Q}_z^{tu,heat} &= \dot{V}_z^{tu,heat} C^{th,air} \left(T^{tu,heat} - T_z^{heat,lin} \right) \\
\dot{Q}_z^{tu,cool} &= \dot{V}_z^{tu,cool} C^{th,air} \left(T^{tu,cool} - T_z^{cool,lin} \right)
\end{aligned}
\tag{2.51}
$$

Where $\dot{Q}_z^{tu,heat}$ and $\dot{Q}_z^{tu,cool}$ are the heat transfer for heating and cooling from the TU towards zone z, $\dot{V}_z^{tu,heat}$ and $\dot{V}_z^{tu,cool}$ are the air flow rate for heating and cooling from the TU towards zone z and $C^{th,air}$ is the heat capacity of air. The symbols $T^{tu,heat}$ and $T^{tu,cool}$ are the temperature of the conditioned supply for heating and cooling air from the TU and $T_z^{heat,lin}$ as well as $T_z^{cool,lin}$ are the the linearization constants of the air temperature at zone z for heating and cooling. Note that the constant T_z^{lin} is taken instead of the variable T_z to avoid a bi-linear term in the model formulation. This linearization approach assumes small changes in the zone temperature $\mathrm{d}T_z \ll T_z$. The linearization constants are separated for heating $T_z^{lin,heat}$ and cooling $T_z^{lin,cool}$ to allow simultaneous heating and cooling, e.g., for intermediate season where both heating and cooling may be desired within one day. Note that the model also assumes the TU supply air temperature to be significantly different from the zone temperature, i.e., $T^{tu,heat} \gg T_z$ and $T^{tu,cool} \ll T_z$.

The electric power for ventilation to zone z though the TU fan $P_z^{tu,fan,el}$ is expressed as:

$$P_z^{tu,fan,el} = \dot{V}_z^{tu} \Delta p^{tu,fan} \eta^{el,tu,fan} \tag{2.52}$$

Here, the symbols $\Delta p^{tu,fan}$ and $\eta^{el,tu,fan}$ are the pressure rise across the fan and the average electric efficiency coefficient of the fan.

The electric demand of the heating and chiller plant is expressed as:

$$\begin{aligned} P_z^{tu,heat,el} &= \eta^{heat} \dot{Q}_z^{tu,heat} \\ P_z^{tu,cool,el} &= \eta^{cool} \dot{Q}_z^{tu,cool} \end{aligned} \tag{2.53}$$

Where η^{heat} and η^{cool} are the heating and cooling efficiency, i.e. the COP, of the heating and chiller plant. Note that η^{cool} takes a negative value such that $P_z^{tu,cool,el}$ is positive. The part-load behavior of the heating and chiller plant is neglected.

Finally, the the electric load due to the air supply to zone z through the TU $P_z^{tu,el}$ is expressed as:

$$P_z^{tu,el} = P_z^{tu,heat,el} + P_z^{tu,cool,el} + P_z^{tu,fan,el} \tag{2.54}$$

3 Indoor air quality model

Ventilation in commercial buildings is meant to ensure good indoor air quality (IAQ), which has significant effects on productivity at work and occupants' health [13], [14]. Particularly, the purpose of outdoor air ventilation is to dilute indoor pollutants, which can be emitted either from (1) occupants and (2) building materials.

IAQ modelling is particularly challenging due to the number of indoor pollutants found in buildings [14]. Studies have quantified the impact on human health only for a few single indoor pollutants, and those impacts may be enhanced when the pollutants are combined. However, a simplified model can be formulated by understanding the correlations within pollutants with similar behaviour, which can then be clustered according to [15] into: (1) pollutants from outdoors (respiratory particles), (2) pollutants from building materials (Formaldehyde, HCHO) and (3) pollutants from human metabolism ($\mathrm{CO_2}$, microorganisms). Pollutants from group (1) are treated with particle filters and other techniques and do not influence the outdoor airflow requirement, whereas pollutants from group (2) and (3) are diluted through outdoor air ventilation.

3.1 Pollutants from building materials

Since pollutants from group (2) are related to building materials, a constant requirement can be defined to remove such pollutants which only depends on the zone area A_z:

$$\dot{V}_z^{min,bld} = \dot{v}^{bld} A_z \tag{3.1}$$

The parameters $\dot{v}^{occ}$, $\dot{v}^{blg}$ are the specific outdoor airflow rate per person and per area respectively. The symbols $n_{z,t}^{occ}$ and A_z are number of occupants in the zone z at time step t and the floor area of zone z. Values for $\dot{v}^{occ}$ and $\dot{v}^{blg}$ are derived according to ASHRAE Standard 62.1. With this strategy, the minimum outdoor airflow rate $\dot{V}_{z,t}^{min,occ}$ varies dynamically with the occupancy.

3.2 Pollutants from human metabolism

Since pollutants within each cluster are correlated, one pollutant can be chosen as the reference for each cluster [15]. The reference pollutant's behaviour would depict the group's behaviour and consequently the current level of IAQ. In our model, $\mathrm{CO_2}$ is chosen as the reference pollutant for group (3). ASHRAE Standard 62.1 defines the maximum $\mathrm{CO_2}$ concentration to ensure good IAQ at 700 ppm above outdoor concentration, i.e. around 1100 ppm when considering an outdoor $\mathrm{CO_2}$ concentration of 400 ppm.

In commercial and residential buildings, the main source for $\mathrm{CO_2}$ is human activity. The $\mathrm{CO_2}$ generation rate per person G depends on the basal metabolic rate, which varies with age, sex and body mass, of the occupants and their level of physical activity. For office work, G ranges between 0.0041 $\mathrm{L/s}$ and 0.0056 $\mathrm{L/s}$ [16]. Assuming a large number of occupants, this paper approximates the mean $\mathrm{CO_2}$ generation rate for office work at 0.0049 $\mathrm{L/s}$. Assuming constant air density, the mass balance for $\mathrm{CO_2}$ at zone z volume yields:

$$V_z \frac{\mathrm{d}c_z^{\mathrm{CO_2}}}{\mathrm{d}t} = G n_z^{occ} + \dot{V}_z c^{\mathrm{CO_2},amb} - \dot{V}_z c_z^{\mathrm{CO_2}} \tag{3.2}$$

Where V_z is the zone volume and $c^{\mathrm{CO_2},amb}$, $c_z^{\mathrm{CO_2}}$ are the concentration of $\mathrm{CO_2}$ in the ambient outdoor air and the zone air respectively. For the sake of brevity, this paper defines $c_z^{\mathrm{CO_2}}$ as the $\mathrm{CO_2}$ concentration in zone air minus the $\mathrm{CO_2}$ concentration in the ambient outdoor air. The number of occupants n_z^{occ} can be determined either with a attendance counter, with an indoor positioning system or even with $\mathrm{CO_2}$ sensors that derive the occupancy from measured concentrations [11], [17].

The $\mathrm{CO_2}$ model (eq. 3.2) is linearized according to:

$$\frac{\mathrm{d}c_z^{\mathrm{CO_2}}}{\mathrm{d}t} = \frac{G n_z^{occ}}{V_z} - \frac{\dot{V}_z^{lin} c_z^{\mathrm{CO_2}}}{V_z} - \frac{\dot{V}_z c_z^{\mathrm{CO_2},lin}}{V_z} + \frac{\dot{V}_z^{lin} c_z^{\mathrm{CO_2},lin}}{V_z} \tag{3.3}$$

Where $\dot{V}_z^{lin}$ and $c_z^{\mathrm{CO_2},lin}$ are the linearization points for the outdoor airflow rate and the $\mathrm{CO_2}$ concentration.

3.3 IAQ constraints

The constraints for maintaining IAQ in a comfortable range are depend on the whether or not a detailed IAQ model is available.

3.3.1 IAQ-based ventilation

Should the a detailed model for the $\mathrm{CO_2}$ concentration be available. The IAQ constraints are expressed as:

$$\begin{aligned} c_z^{\mathrm{CO_2}} &\leq c^{\mathrm{CO_2},max} \\ \dot{V}_z &\geq \dot{V}_z^{min,bld} \end{aligned} \tag{3.4}$$

3.3.2 Occupancy-based ventilation

ASHRAE Standard 62.1 defines the minimum outdoor airflow rate according to the building occupancy and the building size, as a sum to account for both kinds of pollution. This occupancy-based strategy is defined by constraining the ventilation rate at zone z according to:

$$\dot{V}_{z,t} \geq \dot{V}_{z,t}^{min,occ} = \dot{v}^{occ} n_z^{occ} + \dot{v}^{blg} A_z \tag{3.5}$$

The parameters $\dot{v}^{occ}$, $\dot{v}^{blg}$ are the specific outdoor airflow rate per person and per area respectively. The symbols $n_{z,t}^{occ}$ and A_z are number of occupants in the zone z at time step t and the floor area of zone z. Values for $\dot{v}^{occ}$ and $\dot{v}^{blg}$ are derived according to ASHRAE Standard 62.1. With this strategy, the minimum outdoor airflow rate $\dot{V}_{z,t}^{min,occ}$ varies dynamically with the occupancy.

3.3.3 Fixed ventilation

In conservative ventilation strategies, the outdoor airflow rate is fixed based on the designed occupancy rate of the respective zone and does not depend on the actual occupancy rate during operation. Therefore, HVAC systems have to constantly provide a fixed amount of outdoor air. This strategy is defined by constraining the outdoor airflow rate $\dot{V}_z$ in zone z by a fixed minimum requirement $\dot{V}_z^{min}$, which is the minimum outdoor airflow rate in the zone to maintain the desired IAQ:

$$\dot{V}_z \geq \dot{V}_z^{min} \tag{3.6}$$

4 State space formulation

The building model is transformed into state space form, because this allows for a more compact representation which is independent from changes in model configuration or parameters. The state space formulation is obtained by arranging the model variables into vectors and the model parameters into the appropriate matrices:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}^{cnt}\boldsymbol{x} + \boldsymbol{B}^{u,cnt}\boldsymbol{u} + \boldsymbol{B}^{v,cnt}\boldsymbol{v} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}^{u}\boldsymbol{u} + \boldsymbol{D}^{v}\boldsymbol{v} \end{aligned} \tag{4.1}$$

The matrices $\boldsymbol{A}^{cnt}$, $\boldsymbol{C}$ are the state and output matrix, and $\boldsymbol{B}^{u,cnt}$, $\boldsymbol{D}^{u}$, $\boldsymbol{B}^{v,cnt}$, $\boldsymbol{D}^{v}$ are the input and feed-through matrices, on the control and disturbance vectors respectively. The vectors $\boldsymbol{x}$, $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{y}$ are the state, control, disturbance and output vectors. The symbol $()^{cnt}$ denotes the continuous-time instances of the respective matrices. The discrete-time instances are obtained below in eq. (4.3).

Note that the state space model in eq. (4.1) is simply a representation of the differential equations for the zone temperature in eq. (2.1), the surfaces in eqs. (2.4), (2.26) and (2.34) as well as the differential equation for the $\mathrm{CO_2}$ concentration in eq. (3.2), where the model variables are arranged into the vectors as follows:

$$\begin{aligned} \boldsymbol{x} &= \left[[T_z]^\intercal_{z\in\mathcal{Z}}, [T_s]^\intercal_{s\in\mathcal{S}}, \left[c_z^{\mathrm{CO_2}}\right]^\intercal_{z\in\mathcal{Z}} \right]^\intercal \\ \boldsymbol{u} &= \Bigg[\left[\dot{Q}_z^{gen,heat}\right]^\intercal_{z\in\mathcal{Z}}, \left[\dot{Q}_z^{gen,cool}\right]^\intercal_{z\in\mathcal{Z}}, \left[\dot{V}_z^{ahu,heat}\right]^\intercal_{z\in\mathcal{Z}}, \left[\dot{V}_z^{ahu,cool}\right]^\intercal_{z\in\mathcal{Z}}, \\ &\qquad \left[\dot{V}_z^{tu,heat}\right]^\intercal_{z\in\mathcal{Z}}, \left[\dot{V}_z^{tu,cool}\right]^\intercal_{z\in\mathcal{Z}} \Bigg]^\intercal \\ \boldsymbol{v} &= \left[T^{amb}, T^{sky}, \left[\dot{q}_d^{irr}\right]^\intercal_{d\in\{N,E,S,W,H\}}, \left[\dot{q}_z^{occ}\right]^\intercal_{z\in\mathcal{Z}} \right]^\intercal \\ \boldsymbol{y} &= \left[[T_z]^\intercal_{z\in\mathcal{Z}}, \left[\dot{V}_z^{fresh}\right]^\intercal_{z\in\mathcal{Z}}, \left[c_z^{\mathrm{CO_2}}\right]^\intercal_{z\in\mathcal{Z}}, \left[P_z^{gen,el}\right]^\intercal_{z\in\mathcal{Z}}, \left[P_z^{ahu,el}\right]^\intercal_{z\in\mathcal{Z}}, \left[P_z^{tu,el}\right]^\intercal_{z\in\mathcal{Z}} \right]^\intercal \end{aligned} \tag{4.2}$$

The time-discrete form of the thermal building model which is required for the optimization problem is obtained by application of zero-order hold discretization:

$$
\begin{aligned}
\boldsymbol{A} &= e^{\boldsymbol{A}^{cnt}\Delta t} \\
\boldsymbol{B}^{u} &= \left(\boldsymbol{A}^{cnt}\right)^{-1}(\boldsymbol{A}-I)\boldsymbol{B}^{u,cnt} \\
\boldsymbol{B}^{v} &= \left(\boldsymbol{A}^{cnt}\right)^{-1}(\boldsymbol{A}-I)\boldsymbol{B}^{v,cnt}
\end{aligned} \tag{4.3}
$$

Where $\boldsymbol{A}$, $\boldsymbol{B}^u$, $\boldsymbol{B}^v$ are the discrete-time instances of $\boldsymbol{A}^{cnt}$, $\boldsymbol{B}^{u,cnt}$, $\boldsymbol{B}^{v,cnt}$. Note that $\boldsymbol{C}$, $\boldsymbol{D}^u$ and $\boldsymbol{D}^v$ are equivalent in discrete-time as these matrices do not describe a differential equation.

The discrete-time state space model is expressed as:

$$
\begin{aligned}
\boldsymbol{x}_{t+1} &= \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}^u\boldsymbol{u}_t + \boldsymbol{B}^v\boldsymbol{v}_t \\
\boldsymbol{y}_t &= \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{D}^u\boldsymbol{u}_t + \boldsymbol{D}^v\boldsymbol{v}_t
\end{aligned} \tag{4.4}
$$

Where $()_t$ denotes variable instances at time step t.

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