CS 70 — Discrete Mathematics and Probability Theory — **DIS 0B**

Fall 2024 — Rao, Hug

1 Propositional Practice

Note 1

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational. $(\exists x \in R)(x \notin Q)$

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

$(\forall x \in N)((6|x) \Rightarrow ((2|x) \vee (3|x)))$

(d) $(\forall x \in \mathbb{Z})\ (x \in \mathbb{Q})$

(e) $(\forall x \in \mathbb{Z})\ (((2 \mid x) \vee (3 \mid x)) \implies (6 \mid x))$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$ (T)

(a) $\exists x, x \notin Q$ (T)

(b) $(\forall x \in Z)\ (x \in N \vee x < 0) \wedge (\neg(x \in N \wedge x < 0))$ (T)

(c) $x \in N$, $\frac{x}{6} \in Z \Rightarrow \frac{x}{2} \in Z \vee \frac{x}{3} \in Z$ (T)

(d) All integers are rational (T)

(e) All integers that are divisible by 2 or 3 it is divisible by 6 (F)

(f)

2 Truth Tables

Note 1

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

P	Q	$P \wedge (Q \vee P)$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

R	P	Q	$(P \vee Q) \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

$(P \wedge Q) \vee R$		$(P \vee R) \wedge (Q \vee R)$
T		T
T		T
T		T
T		T
T		T
T	F	F
T	F	F
F		F

3 Implication

Note 0
Note 1

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

$\forall x$ $\exists y$

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

$\exists x$ $\forall y$