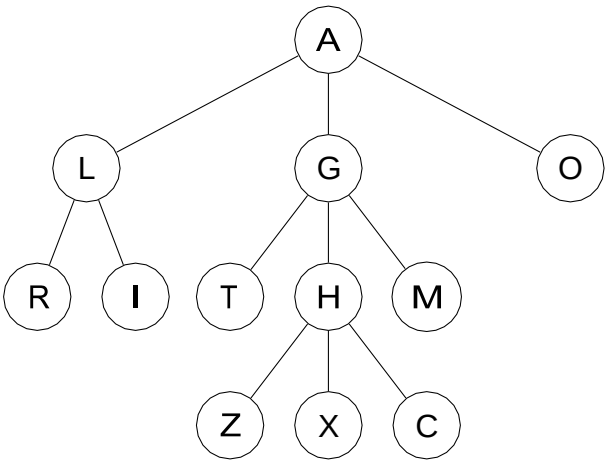
Data Structures and Algorithms

Exercises Week 4 & 5

The following tree is for exercises 2, 4 and 5.



1. Define the following terms.
   (a) Tree
   (b) Root
   (c) Leaf
   (d) Internal node
   (e) Parent
   (f) Child
   (g) Ancestor
   (h) Descendent
   (i) Depth (of a node)
   (j) Height (of a tree)
2. For the given tree (above) list
   (a) the root
   (b) the leaves
   (c) the internal nodes

(d) the children of G
(e) the descendents of G
(f) the parent of R
(g) the ancestors of Z

3. Answer the following:
   (a) How many nodes in a tree have no ancestors?
   (b) What is the name for a node in a tree that has no descendents?
   (c) Given a particular node N in a tree, how many paths connect the root node to node N?
4. For the given tree (above)
   (a) what is the height of the tree?
   (b) what is the depth of H?
5. For the given tree (above) list the nodes according to the three traversals
   (a) pre-order
   (b) post-order
   (c) breadth first
6. Define the following terms.
   (a) Binary tree
   (b) Binary search tree
   (c) Full binary tree
   (d) Proper binary tree
   (e) Depth (of a node)
   (f) Height (of a tree)
7. Use the VisuAlgo package (at https://visualgo.net/en/bst) to show:
   (a) The BST that results from inserting the sequence 6, 3, 11, 9, 8, 5, 4, 2, 12, 7, 1, 10 of keys into an initially empty tree.
   (b) The AVL tree that results from inserting the sequence 6, 3, 11, 9, 8, 5, 4, 2, 12, 7, 1, 10 of keys into an initially empty tree.
8. Give the in-order traversal of the nodes for the tree that you obtained from exercise 7(a).
9. What tree results from deleting key 10 from the tree that you obtained from exercise 7(a)?
10. What tree results from deleting key 3 from the tree that you obtained from exercise 7(a)?
11. Investigate the relationship between the number of nodes in a full binary tree and the height of the tree.
    (a) How many nodes are there in a full tree of height 0, height 1, etc.? Make a table with two columns, headed h for height and n for number of nodes, and complete the entries for h from 0 to 10.

(b) In going from one row to the next, what is the increase in the value of n? Add a third column to your table to show the increments of n.

(c) What is the relation between the values of n and the values of the increments?

(d) Argue that $n = 2^{(h+1)} - 1$ for all values of h.

(e) Argue that $h = O(\log n)$.

12. Download the files AVLNode.java, AVLTree.java, AVLTreeTest.java and read through the code.

(a) Inspect the recursive implementation of methods such as countNodes, search and the three traversals and ensure that you understand how they work. Think how you could implement these without recursion.

(b) Investigate the insert method. Work through some examples trees, perhaps using the VisuAlgo website, to ensure you understand why it is correct.

(c) Read the rotateWithLeftChild and rotateWithRightChild methods and count the number of links changed by their implementation; what is their complexity ($O(\ldots)$)?

13. Investigate the run time performance in AVL trees for

(a) search

(b) insertion

(c) deletion

and explain why they are all $O(\log n)$.