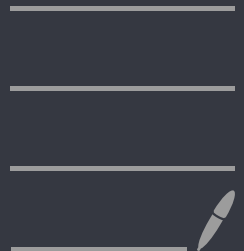


MIMF

HW



MMF HW #8

Problem 1 (Option 1)

$$\frac{3y'}{3} - \frac{1}{2}y = e^x (4x-1)$$

$$-\frac{1}{2}y = \frac{1}{3}e^x (4x-1)$$

$$y' + p(x)y = Q(x)$$

$$P(x) = -\frac{1}{6} \quad Q(x) = \frac{1}{3}e^x (4x-1)$$

$$\mu(x) = e^{\int -\frac{1}{6}dx} = e^{-x/6}$$

$$e^{-x/6}y' - \frac{1}{6}e^{-x/6}y = \frac{1}{3}e^{-x/6}e^x (4x-1)$$

$$\frac{1}{3}e^{5x/6} (4x-1)$$

$$\frac{d}{dx}(e^{5x/6}y) = \frac{1}{3}e^{5x/6} (4x-1)$$

$$du = f dx$$

$$v = \frac{6}{5} \cdot \frac{1}{3}e^{5x/6} = \frac{2}{5}e^{5x/6}$$

$$\int \frac{1}{3}e^{5x/6} (4x-1) dx = \frac{2}{5}(4x-1)e^{5x/6} - \int \frac{2}{5} \cdot 4e^{5x/6} dx$$

$$= \frac{2}{5}(4x-1)e^{5x/6} - \frac{8}{5} \cdot \frac{6}{5}e^{5x/6} = \frac{2}{5}(4x-1)e^{5x/6} - \frac{48}{25}e^{5x/6}$$

$$e^{-x/6}y = \frac{2}{5}(4x-1)e^{5x/6} - \frac{48}{25}e^{5x/6} + C$$

$$y(x) = \left[\frac{2}{5}(4x-1) - \frac{48}{25} \right] e^{x/6} + C e^{x/6}$$

$$y(x) = \left(\frac{8x-2}{5} - \frac{48}{25} \right) e^{x/6} + C e^{x/6}$$

$$= \left(\frac{40x-58}{25} \right) e^{x/6} + C e^{x/6}$$

Problem 1 (Option 2)

$$y' + 7y = 4\cos(2\pi x) - e^{2x} \quad \mu(x) = e^{\int 7dx} = e^{7x}$$

$$e^{7x}y' + 7e^{7x}y = e^{7x}(4\cos(2\pi x) - e^{2x})$$

$$= \int 4e^{7x}\cos(2\pi x) - e^{9x} = 4e^{7x}(7\cos(2\pi x) + 4\pi \sin(2\pi x)) - \frac{1}{9}e^{9x} + C$$

$$e^{7x}y = \frac{4e^{7x}(7\cos(2\pi x) + 2\pi \sin(2\pi x))}{4\pi + 4\pi^2} - \frac{1}{9}e^{9x} + C$$

$$= \frac{4(7\cos(2\pi x) + 2\pi \sin(2\pi x))}{4\pi + 4\pi^2} - \frac{1}{9}e^{9x} + C e^{-7x}$$

$$y(x) = C e^{-7x} - \frac{1}{9}e^{2x} + \frac{28\cos(2\pi x) + 8\pi \sin(2\pi x)}{4\pi + 4\pi^2}$$

#3

3.1

$$y' - y = 3x^2$$

$$y'' - y' + y = 0$$

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} + \mathcal{L}\{y\} = 0$$

$$s^2Y(s) - sy(0) - y'(0) - y'(0) - [sY(s) - y(0)] + Y(s) = 0$$

$$Y(s) = \frac{1}{s^2 - s + 1}$$

$$s^2 - s + 1 = \left(s - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\int \frac{1}{(s-a)^2 + b^2} = \frac{e^{at} \cdot \sin(bt)}{b}$$

$$y(t) = e^{t/2} \left(C_1 \cos\left(\frac{\sqrt{3}}{2}t\right) + C_2 \sin\left(\frac{\sqrt{3}}{2}t\right) \right)$$

3.2

$$4y'' + 2y = e^{5x}$$

$$4y'' + 2y = 0 \Rightarrow y'' + \frac{1}{2}y = 0$$

$$r^2 + \frac{1}{2} = 0 \Rightarrow r = \pm i\sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}i$$

$$y_h(x) = C_1 \cos\left(\frac{\sqrt{2}}{2}x\right) + C_2 \sin\left(\frac{\sqrt{2}}{2}x\right)$$

$$y_p = A e^{5x} \Rightarrow y_p'' = 25A e^{5x}$$

$$4(25A e^{5x}) + 2A e^{5x} = e^{5x} \Rightarrow (100A + 2A) e^{5x} = e^{5x}$$

$$\Rightarrow 102A = 1 \Rightarrow A = \frac{1}{102}$$

$$y(x) = C_1 \cos\left(\frac{\sqrt{2}}{2}x\right) + C_2 \sin\left(\frac{\sqrt{2}}{2}x\right) + \frac{1}{102}e^{5x}$$

Problem #3.3

$$4y'' + 2y = \sin(3x)$$

$$y_p = A \sin(3x) + B \cos(3x) \Rightarrow y_p'' = -9A \sin(3x) - 9B \cos(3x)$$

$$4(-9A \sin(3x) - 9B \cos(3x)) + 2(A \sin(3x) + B \cos(3x)) = \sin(3x)$$

$$(-36A + 2A) \sin(3x) + (-36B + 2B) \cos(3x) = \sin(3x) \Rightarrow -34A \sin(3x) - 34B \cos(3x) = \sin(3x)$$

$$-34A = 1 \Rightarrow A = -\frac{1}{34}$$

$$-34B = 0 \Rightarrow B = 0$$

$$y(x) = C_1 \cos\left(\frac{\sqrt{2}}{2}x\right) + C_2 \sin\left(\frac{\sqrt{2}}{2}x\right) - \frac{1}{34} \sin(3x)$$

Problem 2.1

$$y' - y = 3x^2$$

$$y' - y = 0 \Rightarrow y' = y$$

$$\frac{dy}{y} = dx \Rightarrow \ln|y| = x \Rightarrow y_h = C_1 e^x$$

$$y_p = Ax^2 + Bx + C \Rightarrow y_p' = 2Ax + B$$

$$(2Ax + B) - (Ax^2 + Bx + C) = 3x^2$$

$$-Ax^2 + (2A - B)x + (B - C) = 3x^2$$

$$\cdot -A = 3 \Rightarrow A = -3$$

$$\cdot 2A - B = 0 \Rightarrow -6 - B = 0$$

$$\cdot B - C = 0 \Rightarrow -6 - C = 0 \Rightarrow C = -6$$

$$y(x) = C_1 e^x - 3x^2 - 6x - 6$$

Problem #2.2

$$y'' - y = 5$$

$$y'' - y = 0 \Rightarrow r^2 - 1 = 0 \Rightarrow r = \pm 1$$

$$y_h(x) = C_1 e^x + C_2 e^{-x}$$

$$y_p = A \Rightarrow y_p'' = 0 \Rightarrow 0 - A = 5 \Rightarrow A = -5$$

$$y(x) = C_1 e^x + C_2 e^{-x} - 5$$

Problem 2.3:

$$y'' + y - 3 = e^{-x} + 3$$

$$y'' + y = 0 \Rightarrow r^2 + 1 = 0 \Rightarrow r = \pm i$$

$$y_h(x) = C_1 \cos(x) + C_2 \sin(x)$$

$$y_p = Ae^{-x} + B \Rightarrow y_p' = -Ae^{-x}, y_p'' = Ae^{-x}$$

$$Ae^{-x} + Ae^{-x} + B = e^{-x} + 3 \Rightarrow 2Ae^{-x} + B = e^{-x} + 3$$

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$B = 3$$

$$y(x) = C_1 \cos(x) + C_2 \sin(x) + \frac{1}{2} e^{-x} + 3$$

(4.1)

$$dx/dt = x - 3y$$

$$dy/dt = 2x + 4y$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}, \frac{dX}{dt} = AX$$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 1-\lambda & -3 \\ 2 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda) - (-3)(2) = 0$$

$$(1-\lambda)(4-\lambda) + 6 = \lambda^2 - 5\lambda + 10 = 0$$

$$\lambda = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(10)}}{2} = \frac{5 \pm \sqrt{25 - 40}}{2} = \frac{5 \pm \sqrt{-15}}{2}$$

$$\lambda = \frac{5}{2} \pm \frac{\sqrt{15}}{2}i$$

Problem 4.2

$$dx/dt = 3x - y$$

$$dy/dt = -x - 7y$$

$$A = \begin{bmatrix} 3 & -1 \\ -1 & -7 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & -1 \\ -1 & -7-\lambda \end{vmatrix} = (3-\lambda)(-7-\lambda) - (-1)(-1) = \lambda^2 + 4\lambda - 22$$

$$\lambda = \frac{-4 \pm \sqrt{16 + 88}}{2} = \frac{-4 \pm 2\sqrt{26}}{2} = -2 \pm \sqrt{26}$$