Homework #1

9/2/25

Differentiation Problems

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Graded Problems

(1.) $g(x) = 8 - 4x^3 + 2x^8$

$g'(x) = -12x^3 + 16x$

(2.) $f(z) = z^{10} - 7z^5 + 2z^3 - z^2$

$= 10z^9 - 35z^4 + 6z^2 - 2z$

(3.) $y = 8x^4 - 10x^3 - 9x + 4$

$= 32x^3 - 30x^2 - 9$

(8.) $z = \sqrt{x} - 7\sqrt[4]{x} + 3\sqrt[3]{x}$

$= x^{1/2} - 7x^{1/4} + 3x^{1/3}$

$\boxed{\text{power rule: } \frac{d}{dx}x^n = nx^{n-1}}$

$= x^{1/2 - \frac{2}{2}},\ = \frac{7}{4}x^{1/4 - \frac{4}{4}},\ \frac{3}{3}x^{\frac{1}{3} - \frac{3}{3}}$

$z = \frac{1}{2}x^{-1/2} - \frac{7}{4}x^{-3/4} - x^{-2/3}$

(11.) $g(z) = \frac{4}{z^2} + \frac{1}{7z^5} - \frac{1}{2z}$

$\boxed{\text{Power Rule: } \frac{d}{dx}x^n = nx^{n-1}}$

$= 4z^{-2},\ \frac{1}{7}(g'(x)) \to z^5 \to 5z^4,\ \frac{1}{1}(2z)$

$g'(z) = -8z^{-3} - \frac{5}{7}z^4 - \frac{1}{2}$

(14.) $g(w) = (w-5)(w^2+1)$

$\boxed{\text{Product rule: } = f'(w)h(w) + f(w)h'(w)}$

$f(w) = w - 5,\ f'(w) = 1$

$h(w) = w^2 + 1,\ h'(w) = 2w$

$= (1)(w^2+1) + (w-5)(2w)$

$= (w^2+1) + (2w^2 - 10w)$

$f(w) = 3w^2 - 10w + 1$

(16.) $f(t) = (3 - 2t^3)^2$

$= (3-2t^3)(3-2t^3)$

$g(t) = (3 - 2t^3)\quad g'(t) = -6t^2$

$f(t) = (3 - 2t^3)\quad = -6t^2$

$= (-6t^2)(3-2t^3) + (-6t^2)(3-2t^3)$

$= 2[(-6t^2)(3-2t^3)]$

$= [(-18t^2 - 12t^5)]$

$f'(t) = -36t^2 - 24t^5$

(19.) $y = \frac{t^4 - 2t^2 + 7t}{t^3}$

$= t - 2t^{-1} + 7t^{-2}$

$\boxed{\text{power rule: } \frac{d}{dx}x^n = nx^{n-1}}$

$\boxed{y'(t) = 1 + 2t^{-2} - 14t^{-3}}$

HW#2

U sub Practice Problems

1] $\int (3x^2+2)\sqrt{x^3+2x+5}\,dx$

$u = x^3-2x+5$

$du = (3x^2-2)\,dx$

$\int \sqrt{u}\,du = \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(x^3-2x+5)^{3/2} + C$

2] $\int_1^3 x\cos(x^2)\,dx$

$u = x^2$

$du = 2x\,dx \quad dx = \frac{1}{2}du$

$\int_1^3 \cos(u)\,du$

$\int_1^3 \cos(u)\,\frac{1}{2}\,du$

$\boxed{u=x^2}$

$(3)^2 = 9$

$(1)^2 = 1$

$\int_1^9 \cos(u)\,\frac{1}{2}\,du$

integral ← $\frac{1}{2}\int_1^9 \cos(u)\,du$

cos → sin

$\frac{1}{2}\left[\sin(u)\right]_1^9 =$

$\boxed{= \frac{1}{2}(\sin(9)-\sin(1))}$

3] $\int \frac{e^{5x}}{1+e^{5x}}\,dx$

$u = 1+e^{5x}$

$du = 5e^{5x}\,dx = \frac{1}{5}e^{5x}\,du$

$\int \frac{1}{u}\cdot\frac{1}{5}\,du$

$\frac{1}{5}\ln|u| + C$

$\boxed{1/5\ln|1+e^{5x}|+C}$

4] $\int 4x(x^2+7)^5\,dx$

$u = x^2+7$

$du = 2x\,dx$

$\frac{1}{2}du = dx$

$\int \frac{(4)}{2}(u)^5\,du \Rightarrow 2\int \cancel{2}(u)^5\,du$

$2\cdot\frac{u^6}{6} + C$

$\frac{u^6}{3} + C$

$\frac{(x^2+7)^6}{3} + C$

5] $\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$

$du = e^x\,dx$

$\int_0^{\ln 2} \frac{1}{u}\,du$

$\cancel{\int_0^{\ln 2} \frac{1}{u}\,du}$

bounds

$u = 1+e^0 = \boxed{2}$

$u = 1+e^{\ln 2} =$

$1+[2] = 3$

$\int_2^3 \frac{1}{u}\,du$

$\ln|u|\Big|_2^3 = \ln(3)-\ln(2)$

$= \ln\left(\frac{3}{2}\right)$

6] $\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \cancel{x}\,\frac{\sin(u)}{\cancel{x}}\,du \Rightarrow$

$\int \sin(u)\,du$

integral

sin $\to$ -cos

$= -\cos(\ln x) + C$

7] $\int \frac{5x^4}{(x^5+1)^3}\,dx$

$u = x^5+1$

$du = 5x^4\,dx \quad dx = du$

$\int \frac{1}{u^3}\,du$

$\int u^{-3}\,du$

carry $\to$ negative

$\frac{u^{-2}}{-2} + C$

$\frac{1}{2(x^5+1)^2} + C$

8] $\int_0^{\pi} \cos(2x+1)\,dx$

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

bounds

$2(0)+1 = 1$

$2\pi + 1 = 2\pi+1$

$\int_1^{2\pi+1} \frac{1}{2}\cos(u)\,du$

$\frac{1}{2}\left[\sin(u)\right]_1^{2\pi+1}$

$\frac{1}{2}\left[\sin(2\pi+1) - \sin(1)\right)$

$\frac{1}{2}(0)$

$= 0$

Rest of MIMR HW | $\int v\,dv = uv - \int v\,du$ |

① $I = \int 8te^{7t}\,dt$

$= 8\int te^{7t}\,dt$

$u = t \quad du = dt$
$dv = e^{7t}dt$
$v = \frac{1}{7}e^{7t} \qquad \int e^{7t} = \frac{1}{7}e^{7t}$

$\boxed{\text{IBP}}$ $\int u\,dv = uv - \int v\,du$

$\int te^{7t}dt = t\cdot\frac{1}{7}e^{7t} - \int\frac{1}{7}e^{7t}\,dt$

$= \frac{t}{7}e^{7t} - \frac{1}{49}e^{7t}$

$= 8\left(\frac{1}{7}e^{7t} - \frac{1}{49}e^{7t}\right)$

$= \frac{8}{7}e^{7t} - \frac{8}{49}e^{7t} + C$

② $\int_{\pi}^{2\pi}(1-3x)\sin\left(\frac{x}{2}\right)dx$

$\int_{\pi}^{2\pi}\sin\frac{x}{2}dx - 3\int_{\pi}^{2\pi}x\sin\frac{x}{2}\,dx$

$= -2\cos\frac{x}{2}$

$\int x\sin\frac{x}{2}dx$ $\boxed{\text{IBP}}$ $u = x$
$dv = \sin(x/2)\,dx \Rightarrow du = dx$
$v = -2\cos\frac{x}{2}$

$= \int x\sin\frac{x}{2}\,dx = -2x\cos\frac{x}{2} + 2\int\cos\frac{x}{2}\,dx$

$= -2x\cos\frac{x}{2} + 4\sin\frac{x}{2}$

$= 14 - \frac{12}{\pi}$

③ $\int_{-1}^{2} w^2e^{4w}$

$u = w^2 \quad du = 2w\,dw$
$dv = e^{4w}dw \quad v = \frac{1}{4}e^{4w}$

$\int we^{4w}dw = \frac{w^2}{4}e^{4w} - \frac{1}{2}\int we^{4w}dw$

IBP

$= \frac{w}{4}e^{4w} - \frac{1}{4}\int e^{4w}dw$

$= \frac{w}{4}e^{4w} - \frac{1}{16}e^{4}dw$

$= \frac{25e^8 - 15e^{-4}}{32}$

④ $\int_{-2}^{3}(2-x^2)\ln(4x)\,dx$

$=$

⑤ $\int(6+3z)\cos(1+4z)dz$

$\int 6\cos(1+4z)\,dz + \int 3z\cos(1+4z)dz$

$\int \frac{1}{4}\sin(1+4z) \rightarrow \frac{6}{4}\sin(1+4z)$

$u = z \qquad dz = v = \frac{1}{4}\sin(1+4z)$
$dv = \cos(1+4z) \quad du = dz$

$\int z\cos(1+4z)dz = \frac{z}{4}\sin(1+4z) - \frac{1}{4}\int\sin(1+4z)\,dz$

$= \frac{z}{4}\sin(1+4z) + \frac{1}{16}\cos(1+4z)$

$\boxed{= \frac{3(z+2)}{4}\sin(1+4z) + \frac{3}{16}\cos(1+4z) + C}$

⑥ $\int x^{3/2}\ln(x^{3/2})\,dx$

$\ln(x^{3/2})$ $\qquad u = \ln x$

$\frac{3}{2}\int x^{3/2}\ln x\,dx \qquad du = \frac{1}{x}dx$

$dv = x^{3/2}dx$

$\int x^{3/2}\ln x\,dx = \frac{2}{5}x^{5/2}\ln x - \frac{2}{5}\int x^{3/2}dx \quad v = \frac{2}{5}x^{5/2}$

$= \frac{2}{5}x^{5/2}\ln x - \frac{2}{5}\cdot\frac{2}{5}x^{5/2}$

$= \frac{3x^{5/2}}{25}(5\ln x - 2) + C$

Partial Derivative

① $f(x,y) = 3x^2 + 2y$ $\quad \frac{\partial f}{\partial x}$?

$= 6x$

② $f(x,y) = x^2y + y^3$ $\quad \frac{\partial f}{\partial y}$

$\cancel{= 1 + 3y}$

$= x^2 + 3y^2$

③ $f(x,y) = \sin(xy)$, find $\frac{\partial f}{\partial x}$

$= y\cos(xy)$

④ $f(x,y) = e^{x+y}$ $\quad$ find $\frac{\partial^2 f}{\partial x \partial y}$

$= e^{x+y}$

⑤ $f(x,y,z) = xyz + x^2z^2$

$= xy + x^2 + 2z$

⑥ $f(x,y) = \ln(x^2+y^2)$, $\frac{\partial f}{\partial x}$

$= \frac{2x}{x^2+y^2}$

⑦ $f(x,y) = \frac{x^2y}{x^2+y^2}$, find $\frac{\partial f}{\partial y}$

$= \frac{2xy}{2x+y^2}$

⑧ $f(x,y,z) = x^2\cos(yz)$ $\quad$ Find $\frac{\partial^2 f}{\partial y \partial z}$

$= -x^2 z\cos(yz)$ $\quad \frac{\partial f}{\partial z}$

$= -x^2(\sin(yz) + y\cos(yz)\cdot z)$ $\quad \frac{\partial f}{\partial y}$

$= -x^2\sin(yz) - x^2yz\cos(yz)$

⑨ $f(r,\theta) = r^2\sin\theta + \theta^3\ln r$ $\quad \frac{\partial f}{\partial r}$

$= 2r\cos\theta + \frac{1}{r}(\theta^3)$

$= 2r\cos\theta + \frac{\theta^3}{r}$

$f(x,y) = e^{xy}\sin(x+y)$ $\quad \frac{\partial^2 f}{\partial x^2}$

$=$

Gradient Problems

1) $f(x,y) = 3x^2 + 2y$

$f_x = 6x$

$f_y = 2$

$\nabla f = (6x, 2)$

2) $f(x,y) = x^2y + y^3$

$f_x = 2xy$

$f_y = x^2 + 3y^2$

$\nabla f = (2xy, x^2 + 3y^2)$

3) $f(x,y) = \sin(xy)$

$f_x = \cos(xy) \cdot \partial_x(xy)$

$y\cos(xy)$

$f_y = \cos(xy) \cdot \partial_y(xy)$

$= x\cos(xy)$

$\nabla f = (y\cos(xy), x\cos(xy))$

4) $f(x,y) = e^{x+y}$

$f_x = \partial_x e^{x+y} = e^{x+y}$

$= e^{x+y}$

$\nabla f = (e^{x+y}, e^{x+y})$

5) $f(x,y,z) = xyz + x^2z^2$

$f_x = xz + 2xz^2$

$f_y = xz + \cancel{x^2z^2}$

$f_z = xy + x^2 2z$

$\nabla f = (xz + 2xz^2, xz, xy + x^2 2z)$