Seven pieces of notation for propositions:
T (a true statement), F (a false one), $\neg$, $\implies$, $\iff$, $\wedge$, $\vee$.

Seven pieces of notation for subsets of a set Ω:
$\subseteq$, $=$, $\emptyset$, Ω, $\cup$, $\cap$, ^{c} (that last one is "complement").

Exercise: match them up! You have one minute now, but we'll come back to this question later. Which one does $\vee$ go with?

I want to match up $\vee$ with $\cup$, and this is because if X and Y are subsets of Ω, then

$$\forall a \in \Omega, a \in X \cup Y \iff (a \in X) \vee (a \in Y).$$

This is a precise relationship between the two concepts. **Make sure you understand it.** I will explain it now. Ask the person next to you if you don't.

Exercise: Write down a precise relationship between $\wedge$ and $\cap$. Show it to someone else and see if they think you got it right.

I want to match up T (a true proposition) with Ω (the biggest subset of Ω), because

$$\forall a \in \Omega, a \in \Omega \iff T.$$

Why don't you try F and X^c (for X a subset of Ω) yourselves now? Write down two precise equations (and show them someone near you if you want to get a second opinion).

I got

$$\forall a \in \Omega, a \in \emptyset \iff F$$

and

$$\forall a \in \Omega, a \in X^c \iff \neg(a \in X).$$

The last two: I want to match up $=$ with $\iff$. Here's why. If X and Y are two subsets of Ω, then

$$X = Y \iff (\forall a \in \Omega, a \in X \iff a \in Y).$$

"Two sets are equal if and only if they have the same elements."

What is the seventh equation?

$$X \subseteq Y \iff (\forall a \in \Omega, a \in X \implies a \in Y).$$

Here's an example of a theorem about propositions. If P and Q are propositions, then $(\neg P) \vee (\neg Q) \iff \neg(P \wedge Q)$. What would be a boring easy way to prove this?

[draw a truth table and check all the cases].
Exercise: what is the corresponding theorem for subsets of Ω?

Homework: here is a theorem about sets.

Theorem: If X and Y are subsets of Ω such that $X \cup Y = \Omega$ and $X \cap Y = \emptyset$, then $Y = X^c$.

Figure out the corresponding theorem about propositions. Prove it, and by applying it to the propositions $a \in X$ and $a \in Y$ for a an arbitrary element of Ω, deduce the theorem for sets.

Great examples of sets: sets of numbers.
Example 1: the *integers* $\mathbb{Z}$.

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

(Z for the German word "Zahlen").

Terrible example 2: the *natural numbers* $\mathbb{N}$.

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}?$$

$$\mathbb{N} = \{1, 2, 3, \ldots\}?$$

A great unsolved problem.

My solution:

$$\mathbb{Z}_{\geq 0} = \{0, 1, 2, 3, \ldots\}$$

$$\mathbb{Z}_{\geq 1} = \{1, 2, 3, \ldots\}$$

In M1F I will never use $\mathbb{N}$ because there is no universal convention on what it means. (at Xena I will use $\mathbb{N} = \{0, 1, 2, \ldots\}$ but at university I was taught $\mathbb{N} = \{1, 2, 3, \ldots\}$).

Other famous sets of numbers:

The *rational numbers* $\mathbb{Q}$ (contains stuff like $-1\frac{3}{7}$)

The *real numbers* $\mathbb{R}$ (contains stuff like $\sqrt{2}$ and π)

The *complex numbers* $\mathbb{C}$ (contains stuff like i with $i^2 = -1$).

These sets of numbers are quite hard to build. We will talk more about these number systems later.

Standard notation for subsets of $\mathbb{R}$.

Let a and b be real numbers.

$[a, b]$ means $\{ x \in \mathbb{R} \mid a \leq x \wedge x \leq b \}$.
Mathematicians often just write $\{ x \in \mathbb{R} \mid a \leq x \leq b \}$.

(a, b) means $\{ x \in \mathbb{R} \mid a < x < b \}$.

$[a, \infty)$ just means $\{ x \in \mathbb{R} \mid a \leq x \}$.

$[a, \infty]$ doesn't mean anything because *infinity is not a number* **and you will lose marks if you write** $x = \infty$. Writing $x < \infty$ is OK (but a waste of ink).

Exercise: guess what $(a, b]$ means. Guess what $(-\infty, b)$ means.

We have talked about the union $X \cup Y$ of *two* things.

But we can talk about the union of *an arbitrary number of things* – including infinitely many things!

Convention alert! What is the union of no things? (the empty set)

What about three things? What is the union of $[1, 2], [2, 3]$ and $[3, 4]$?

If $X_1, X_2, X_3, \ldots$ is a sequence of infinitely many sets, We can take their union! Sets are *better than numbers*. We cannot do $1 + 1 + 1 + 1 + 1 + \ldots$ and get a number as an answer.

But we can write

$$\bigcup_{n=1}^{\infty} X_n$$

for this *countable union*, and get a new set.

Example: if $X_1 = [1, 2]$, $X_2 = [2, 3]$, $\ldots$, $X_n = [n, n + 1]$, $\ldots$, then what is $\bigcup_{n=1}^{\infty} X_n$?

It's $[1, \infty)$. We'll come back to this next time.

Answers to exercises:
$\subseteq$ goes with $\implies$
^{c} goes with $\neg$
$\cup$ goes with $\vee$
$\cap$ goes with $\wedge$ $(\forall a \in \Omega, a \in X \cap Y \iff (a \in X) \wedge (a \in Y))$
T goes with Ω
F goes with $\emptyset$
$=$ goes with $\iff$.
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Truth table for $(\neg P) \vee (\neg Q) \iff \neg(P \wedge Q)$: just observe that in all four cases the values of both sides of the $\iff$ are the same.
The corresponding theorem for subsets is one of De Morgan's laws: $X^c \cup Y^c = (X \cap Y)^c$.

The theorem about propositions in the question marked "homework": if $P \vee Q = T$ and $P \wedge Q = F$ then $Q \iff \neg P$, and if you check all four cases then you can see that indeed the only two that work are either Q false and P true, or Q true and P false. In either case we have $Q \iff \neg P$.
To deduce the theorem about sets, we note that to prove $Y = X^c$ it suffices to prove that $\forall a \in \Omega, a \in Y \iff a \in X^c$ and letting Q be the statement $a \in Y$ and P the statement $a \in X$ we see $Q \vee P = T$ (as $X \cup Y = \Omega$) and $Q \wedge P = F$ (as $X \cap Y = \emptyset$) and hence $Q \iff \neg P$, so $a \in Y \iff \neg(a \in X)$ which is $\iff a \in X^c$.
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$(a, b]$ means $\{\, x \in \mathbb{R} \mid a < x \leq b \,\}$ and $(-\infty, b)$ means $\{x \in \mathbb{R} \mid x < b\,\}$.
$\bigcup_{n=1}^{\infty} X_n = [1, \infty)$, and I guess the proof is that if $r \geq 1$ then $r \in X_n$ for $n = \lfloor r \rfloor$.