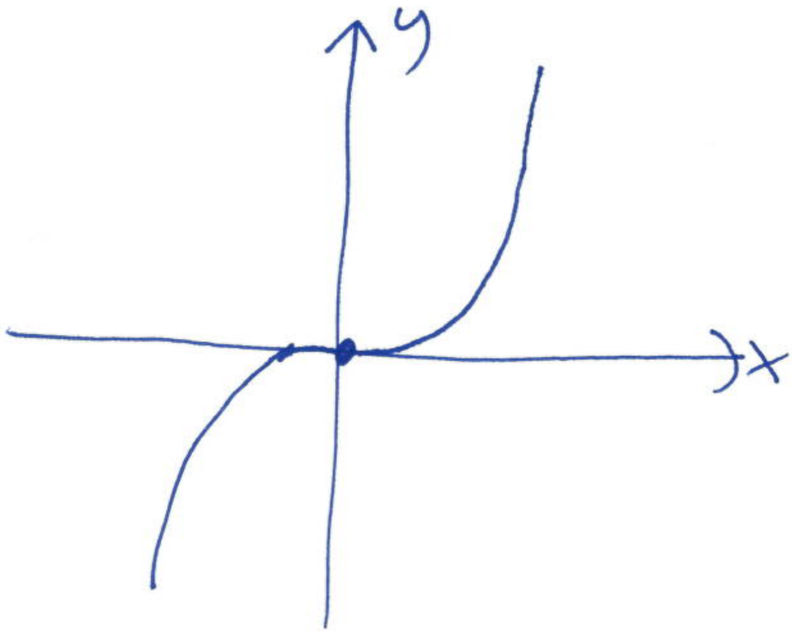
A point of inflection is a point where a curve crossos its own tangent, for examp6 $f(x) = x^3$ graph has a point of inflection at origin $(0,0)$



A point of inflection has property that $y''(x)$ changes sign.

$f(x) = x^3$, $f'(x) = 3x^2$

$f''(x) = 6x$ changes sign at $x = 0$.

A sufficient condition for a to be a point of inflection is $f''(a) = 0$ and $f'''(a) \neq 0$

Example

$f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x = 4x(x^2-1)$

stationary points at $x=0$, $x=\pm 1$

$f''(x) = 12x^2 - 4$

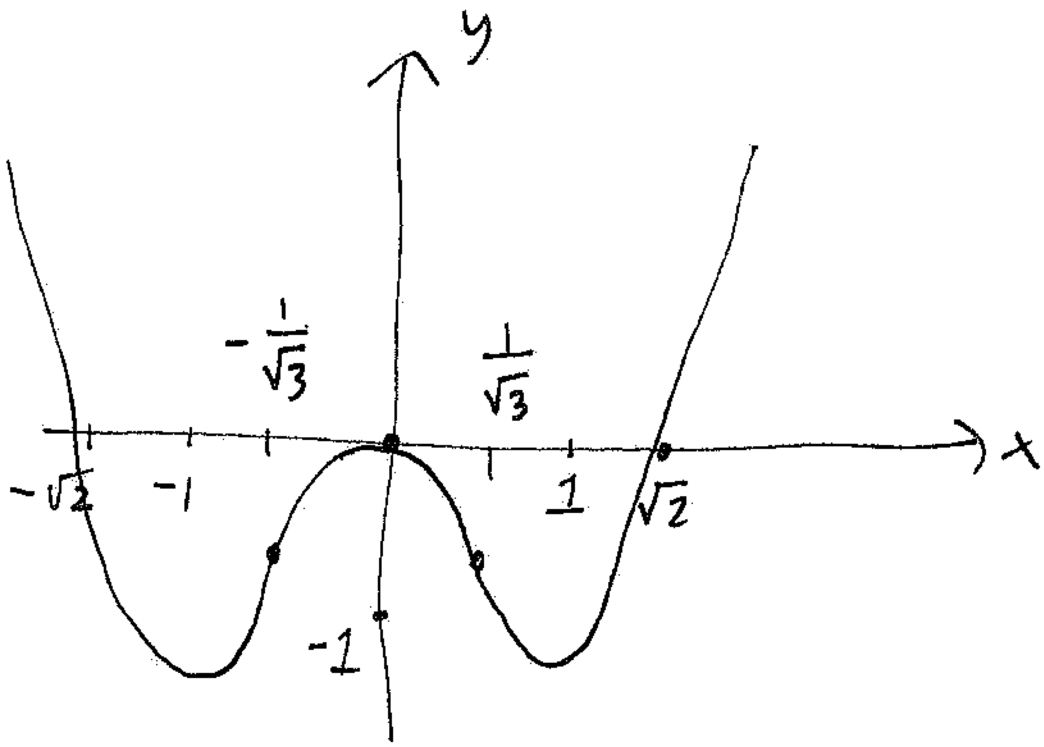
$f''(0) = -4$ $x=0$ is a local maximum

$f''(\pm 1) = 8$ $x=\pm 1$ local minima

$f''(x) = 0$ if $x^2 = \frac{1}{3}$ or $x = \pm \frac{1}{\sqrt{3}}$

$f'''(x) = 24x \neq 0$ if $x = \pm \frac{1}{\sqrt{3}}$

$x = \pm \frac{1}{\sqrt{3}}$ points of inflection



Curve Sketching

No general way to sketch a graph – sometimes free hand ~~skech~~ sketch impossible. The following can be useful:

(i) Any special features, eg odd, even or periodic?

(ii) Intercepts - points where graph intersects x or y axes

(iii) Stationary points or points of inflection

(iv) Linear asymptotes

graph approaches a line as $x \to \infty$ or $x \to -\infty$

or a vertical line as $x \to a$ (a finite)

Linear Asymptotes

Rational functions can have linear asymptotes

(i) $f(x) = \dfrac{x^3}{1-x^2}$ odd

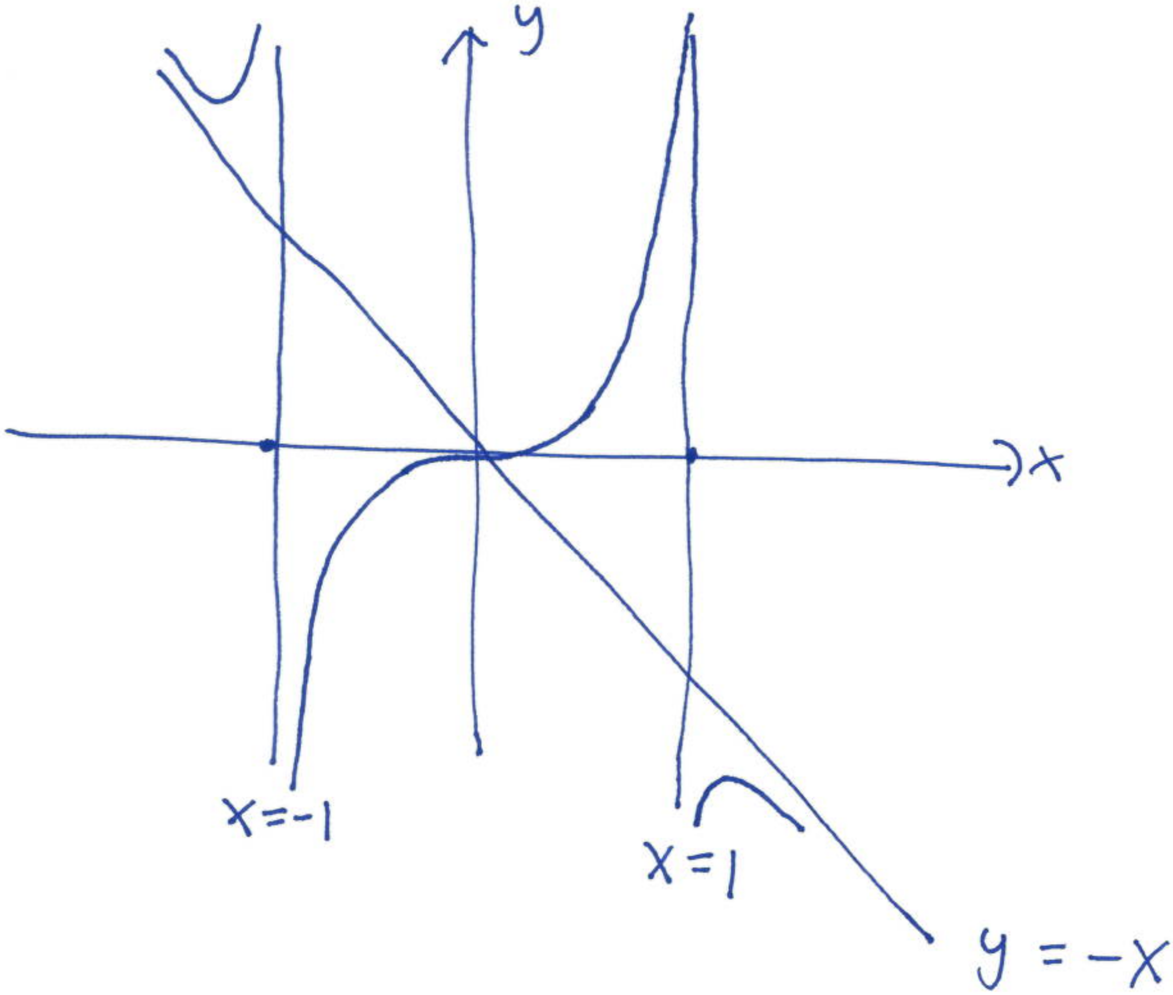
graph has vertical asymptotes at $x = \pm 1$. For large x (positive or negative)

$f(x) \simeq -x$

$$f(x) = \frac{x}{\frac{1}{x^2} - 1} = -x\left(1 - \frac{1}{x^2}\right)^{-1}$$

$$= -x\left(1 + \frac{1}{x^2} + \frac{1}{x^4} + \dots\right)$$

$$= -x - \frac{1}{x} - \frac{1}{x^3}$$



For small x $\quad f(x) \approx x^3$

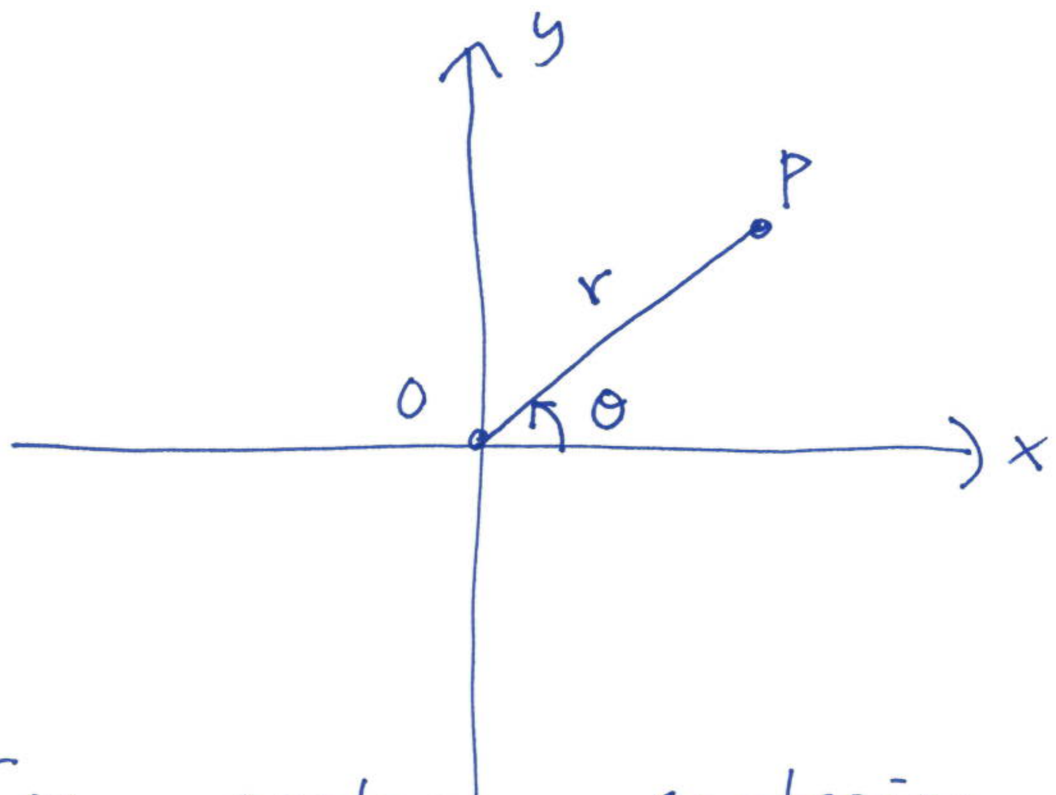
$$f(x) = x^3(1-x^2)^{-1}$$
$$= x^3(1 + x^2 + x^4 + \ldots)$$
$$\approx x^3$$

(ii) What are the linear asymptotes

of $y = \log(\cosh x)$

Polar Graphs

Have described graphs via functions, equations and parametrically. Another way is through polar coordinates



Can replace cartesian coordinates (x, y) with polar coordinates (r, θ)

$r =$ distance from origin

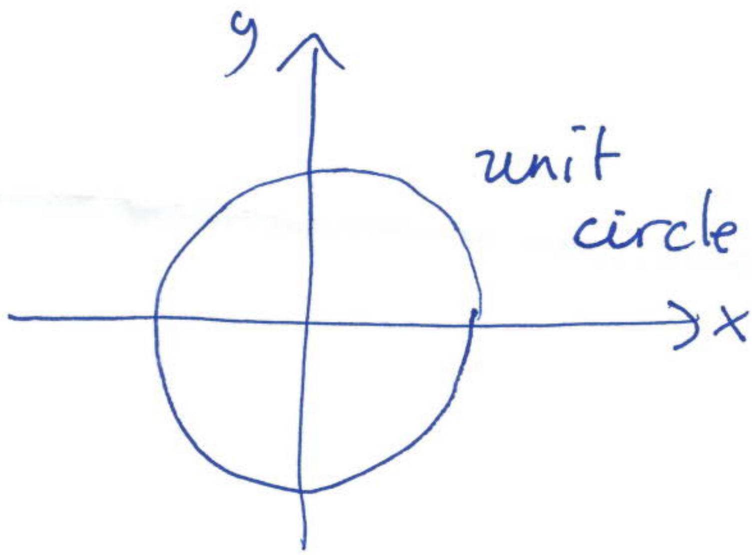
$\theta =$ angle between positive x axis and line segment OP. Have

$$x = r\cos\theta$$
$$y = r\sin\theta$$

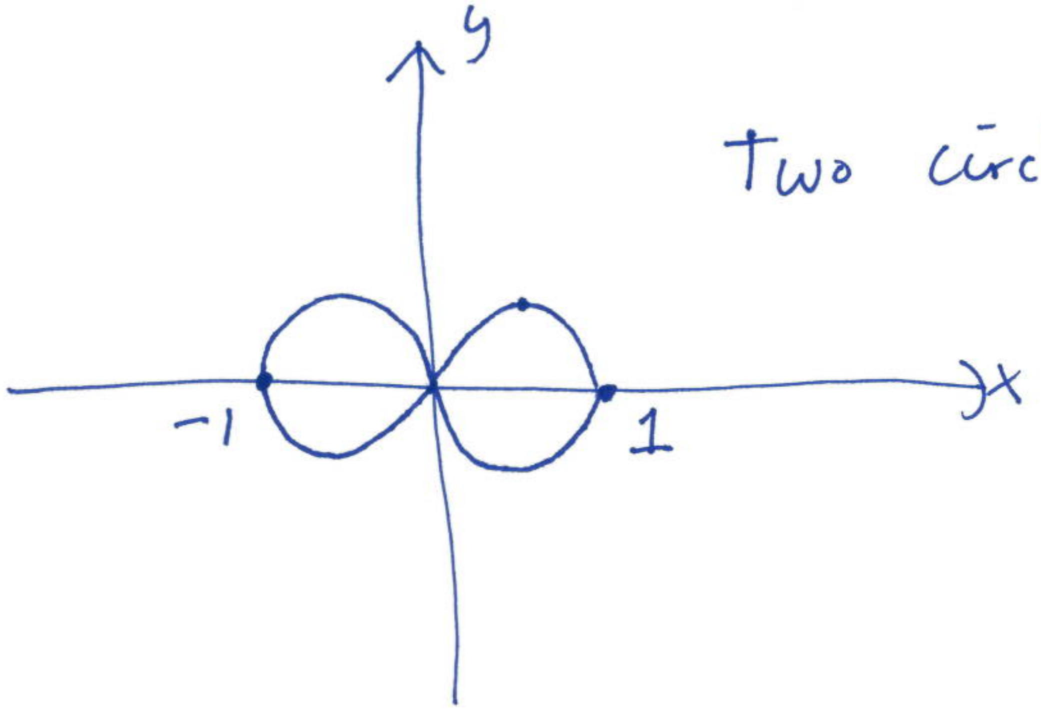
θ is periodic - replacing θ with $\theta + 2\pi$ has no effect.

Examples of polar curves:

(i) $r = 1$



(ii) $r = |\cos\theta|$



Two circles!

(iii)

$$r = \frac{L}{1 + e\cos\theta}$$

L constant

$e \geq 0$ constant

e = eccentricity

This gives a conic section!

Conic Section (in cartesians)

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$$

(A, B, C, D, E, F are constants) defines a conic section

<u>Degenerate cases</u>

nothing
a point
a line
two lines

Non-degenerate case

ellipse

parabola

hyperbola