Basic notation.

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1 Propositions

A *proposition* is a true/false statement.

Examples of propositions:

- $2 + 2 = 4$.
- $2 + 2 = 100000000$.
- Fermat's Last Theorem.
- The Riemann hypothesis.

For some propositions, like the Riemann hypothesis, we currently don't know whether they are true or false. But in *classical mathematics*, which is the mathematics of M1F, *every* proposition is either true or false – it's just that we're not yet sure about some of them.

Here are some examples of things which are *not* propositions:

- $2 + 2$.
- $2 = 2 = 4$.

The first thing is a number, not a proposition. It is not "true" or "false" – it is 4.

The second thing doesn't even make sense. It's not even a mathematical object.

2 Notation

2.1 And

If P and Q are propositions, then $P \wedge Q$ is also a proposition, pronounced "P and Q". The idea: $P \wedge Q$ is true exactly when both P and Q are true. Here is the *truth table* for $\wedge$, where we go through all possibilities for P and Q (T means true and F means false).

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Example: $(2 + 2 = 4) \wedge (2 + 2 = 5)$ is false, because $2 + 2 = 5$ is false.

2.2 Or

If P and Q are propositions, then $P \vee Q$ is also a proposition, pronounced "P or Q". We have that $P \vee Q$ is true exactly when *either* P *or* Q *or both* are true. Here is the truth table.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Example: $(2 + 2 = 4) \vee (2 + 2 = 5)$ is true, because $2 + 2 = 4$ is true.

2.3 Not

If P is a proposition, then $\neg P$, pronounced "not P", is the proposition which is "the opposite of P". In other words, if P is true then $\neg P$ is false, and if P is false then $\neg P$ is true. Truth table:

P	$\neg P$
T	F
F	T

Example: if P is the Riemann hypothesis, then $P \vee \neg P$ is true, because in classical mathematics the Riemann hypothesis is either true or false.

2.4 Implies

If P and Q are propositions, then $P \implies Q$ is also a proposition, pronounced "P implies Q". The proposition $P \implies Q$ means: if P is true, then Q is true as well. Here's the truth table.

P	Q	$P \implies Q$
T	T	T
T	F	F
F	T	T
F	F	T

The only time that $P \implies Q$ is false is when P is true and Q is false. For example $(2+2=4) \implies (2+2=5)$ is false, but $(2+2=5) \implies (2+2=4)$ is true.

Note that $Q \impliedby P$ is defined to mean $P \implies Q$.

2.5 Iff

If P and Q are propositions, then so is $P \iff Q$, pronounced "P iff Q" or "P if and only if Q". The proposition $P \iff Q$ is true exactly when P and Q have the same truth value – that is – either they are both true, or both false.

P	Q	$P \iff Q$
T	T	T
T	F	F
F	T	F
F	F	T

For example, if P is any proposition then $P \iff P$ is true. The symbol $\iff$ is the proposition version of $=$ for numbers; if x and y are equal numbers we write $x = y$, but if P and Q are propositions with the same truth value we write $P \iff Q$.

Examples:

- $(P \implies Q) \iff (Q \impliedby P)$ is always true.
- $P \iff (\neg P)$ is always false.