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Algebraic structure of gravity in Ashtekar variables

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Abstract. The BRST transformations for gravity in Ashtekar variables are obtained by using the Maurer-Cartan horizontality conditions. The BRST cohomology in Ashtekar variables is calculated with the help of an operator δ introduced by S.P. Sorella [1], which allows to decompose the exterior derivative as a BRST commutator. This BRST cohomology leads to the differential invariants for four-dimensional manifolds.

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1 Introduction

A great amount of work has been done recently in reformulation of general relativity in terms of a new set of variables that replace the spacetime metric [2]. These new variables, called Ashtekar variables, have been intensively used in a large number of problems in gravitational physics. Ashtekar's main task was the quantum gravity issue and these new variables, indeed, have opened a novel line of approach to it.

In any quantum field theory, one of the most important question is the existence and the form of the anomalies. The anomalies, as well as the Schwinger terms and the invariant Lagrangians could be calculated in a purely algebraic way by solving the Wess-Zumino consistency condition [3], which is equivalent to a tower of descent equations, involving the nilpotent BRST operator s, as well as the exterior spacetime derivative d.

The usual procedure for solving the descent equations is based on the Russian formula and the transgression equation [4] (see also [5, 6, 7, 8, 9, 10, 11, 12, 13]). However, for the gravity in Ashtekar variables it is difficult to write down these equations and it is necessary to follow a different scheme.

First, for the Ashtekar variables, the BRST transformations [14] are different from those obtained in the Yang-Mills case. Besides, the Russian formula, in the form given for the Yang-Mills case [5], does not hold and we have to find out a new method for obtaining a generalization of it in this case. On the other hand, for the Ashtekar variables, as well as for the gravity with torsion [15] it is difficult to write down a transgression equation and to obtain the anomalies, the Schwinger terms and the invariant Lagrangians. All these quantities are BRST-invariants modulo d-exact terms and they can be obtained by using an operator δ which allows to express the exterior derivative d as a BRST commutator

$$d = -[s, \delta] \ . \tag{1.1}$$

Once the decomposition (1.1) has been found, successive applications of the operator δ on a polynomial Q which is a nontrivial solution of the equation

$$sQ = 0 \ , \tag{1.2}$$

give an explicit nontrivial member of the BRST cohomology group modulo d-closed terms.

The solving of the equation (1.2) is a problem of local BRST cohomology instead of a modulo-d one. Therefore we see that, due to the operator δ, the study of the cohomology of s modulo d can be reduced to the study of the local cohomology of s which, in turn, could be analyzed by using the spectral sequences method [16].

In this paper we prove the decomposition (1.1) for gravity in Ashtekar variables and we show that the operator δ offers a straightforward way of classifying the BRST cohomology group for these variables. In this way, we can give a cohomological interpretation of the cosmological constant, of the Ashtekar Lagrangian, as well as of the gravitational Chern-Simons terms.

The BRST transformations of the Ashtekar variables will be obtained by making use of the geometrical formalism introduced by L. Baulieu and J. Thierry-Mieg [7, 17, 18, 19]. This formalism allows us to obtain the BRST transformations from Maurer-Cartan horizontality conditions and it turned out [15, 20, 21] (see also [22]) to be very useful in the case of gravity with torsion, as well as the gravity with Ashtekar variables. Moreover, it allows to formulate the diffeomorphism transformations of the Ashtekar variables as *local translations* in the tangent space by means of the introduction of the ghost field $\eta^a = \xi^\mu e^a_\mu$ where ξ^μ are the usual diffeomorphism ghosts and e^a_μ are the tetrads. With these ghosts we can define the linear operator δ from the decomposition (1.1) in a very simple way.

The paper is organized as follows. In section 2, we recall the Ashtekar variables, the Maurer-Cartan horizontality conditions and we derive the BRST transformations and the Bianchi identities. In section 3 we define the linear operator δ and we find out a solution of the descent equations by using this operator and the general method presented in Ref. [23]. Some explicit examples are presented in section 4 and a brief discussion about the commutation properties of the δ-operator can be found in section 5. Finally, the appendices A and B are devoted to some detailed calculations, respectively important commutator relations in the tangent space and the determinant of the tetrad.

2 Ashtekar variables and the Maurer-Cartan horizontality conditions

2.1 The Yang-Mills case

In this case the fields are the Lie-valued 1-form gauge connection $A = A^A_\mu T^A dx^\mu$ and the Lie-valued 0-form ghost field $c = c^A T^A$. T^A are the antihermitian generators of a finite representation of the gauge group obeying the following relations

$$[T^A, T^B] = f^{ABC} T^C \ , \qquad Tr\{T^A T^B\} = \delta^{AB} \ .$$

The BRST transformations

$$\begin{aligned} sA &= dc + Ac + cA = Dc \ , \\ sc &= c^2 \ , \\ s^2 &= 0 \ , \end{aligned} \tag{2.1}$$

could be obtained by reinterpreting (2.1) as a Maurer-Cartan horizontality condition (MCHC). In order to do it, we shall consider the combined gauge-ghost field

$$\tilde{A} = A + c \tag{2.2}$$

which could be considered as an Ehresmann connection on a principal fibre bundle [24], with the differential

$$\tilde{d} = d - s \ , \qquad \tilde{d}^2 = 0 \ . \tag{2.3}$$

The 2-form field strength F is given by

$$F = dA + A^2 \tag{2.4}$$

and

$$dF = [F, A] \tag{2.5}$$

is its Bianchi identity. The field strength $\tilde{F}$ of the connection $\tilde{A}$ is given by

$$\tilde{F} = \tilde{d}\tilde{A} + \tilde{A}^2 \tag{2.6}$$

and it obeys the generalized Bianchi identity

$$\tilde{d}\tilde{F} = [\tilde{F}, \tilde{A}] \ . \tag{2.7}$$

The MCHC reads then

$$\tilde{F} = F \tag{2.8}$$

and it splits into three components (2.1) and (2.4) by expanding $\tilde{F}$ in terms of A and c and collecting the terms with the same form degree and ghost number. Moreover, we have the Bianchi identity

$$\tilde{d}\tilde{F} - [\tilde{F}, \tilde{A}] = dF - [F, A] = 0 \ . \tag{2.9}$$

It is important to emphasize that the BRST transformations (2.1) could be obtained directly from the MCHC (2.8).

2.2 Ashtekar variables

General relativity could be reformulated in term of two fields: a *real* tetrad 1-form

$$e^a = e^a_\mu dx^\mu \tag{2.10}$$

and a *complex self-dual* connection 1-form

$$A^{ab} = A^{ab}_\mu dx^\mu \ . \tag{2.11}$$

Here the indices $(\mu, \nu, \ldots)$ are spacetime indices running from 0 to 3 while the indices $(a, b, c, \ldots)$ are flat tangent space indices running also from 0 to 3. The flat tangent space indices are raised and lowered with the Minkowski metric

$$\eta^{ab} = \eta_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \ . \tag{2.12}$$

By definition, the complex Ashtekar connection 1-form A^{ab} is self-dual, i.e.

$$* A^{ab} = i A^{ab} \ , \tag{2.13}$$

where in the case of $SO(1,3)$ (in four dimensions)

$$* A^{ab} = \frac{1}{2} \varepsilon^{ab}{}_{cd} A^{cd} \tag{2.14}$$

and $\varepsilon^{ab}{}_{cd}$ is the completely antisymmetric tensor, with $\varepsilon^{0123} = 1$ and $\varepsilon_{0123} = -1$.

The tetrad 1-form is related to the Ashtekar torsion 2-form by

$$T^a = de^a + A^a{}_b e^b = De^a \ , \tag{2.15}$$

where $D = d + A$ is the Ashtekar covariant exterior derivative. This Ashtekar torsion 2-form does not vanish even though the spin-connection ω^{ab}, related to A^{ab} by the equation

$$A^{ab} = \frac{1}{2}(\omega^{ab} - i * \omega^{ab}) = \frac{1}{2}(\omega^{ab} - \frac{i}{2}\varepsilon^{ab}{}_{cd}\omega^{cd}) \ , \tag{2.16}$$

is torsion-free. The components of the Ashtekar torsion, $T^a_{\mu\nu}$, are given by

$$T^a_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu + A^a{}_{b\mu} e^b_\nu - A^a{}_{b\nu} e^b_\mu \ . \tag{2.17}$$

The Ashtekar connection 1-form A^{ab} is related to the complex self-dual Ashtekar field strength 2-form by

$$F^{ab} = dA^{ab} + A^a{}_c A^{cb} = \frac{1}{2} F^{ab}_{\mu\nu} dx^\mu dx^\nu \ , \tag{2.18}$$

where F^{ab} is given by

$$F^{ab} = \frac{1}{2}(R^{ab} - i * R^{ab}) \tag{2.19}$$

with the curvature 2-form

$$R^{ab} = d\omega^{ab} + \omega^a{}_c \omega^{cb} \ . \tag{2.20}$$

Applying the covariant exterior derivative to both sides of the equations (2.15) and (2.18) one gets the Bianchi identities

$$\begin{aligned} DT^a &= dT^a + A^a{}_b T^b = F^a{}_b e^b \ , \\ DF^{ab} &= dF^{ab} + A^a{}_c F^{cb} - F^a{}_c A^{cb} = 0 \ . \end{aligned} \tag{2.21}$$

It was proved [25] that the Ashtekar variables (e^a, A^{ab}) are equivalent with the usual metric $g_{\mu\nu}$. This means that if (e^a, A^{ab}) satisfy the equation of motion of the theory with the action

$$S = \int F^{ab} \wedge e_a \wedge e_b \ , \tag{2.22}$$

then the metric

$$g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x) \eta_{ab} \tag{2.23}$$

is the solution of the Einstein equations. Viceversa, every solution of the Einstein equations can be written in terms of the solutions (e^a, A_{ab}) of the equations of motion with the action (2.22), as done in eq.(2.23). In the case of Yang-Mills fields, the MCHC implies the BRST transformations. Similiar conditions can be formulated in the case of gravity [7, 15, 18, 19] and it naturally includes the torsion. Moreover, it allows to formulate the diffeomorphisms as local translations. In this paper we shall write down the MCHC for gravity in Ashtekar variables and we shall obtain BRST transformations for the Ashtekar fields.

2.3 Maurer-Cartan horizontality conditions

The generalized tetrad-ghost field $\tilde{e}^a$ and the extended complex self-dual connection-ghost field $\tilde{A}^{ab}$ are now defined as

$$\tilde{e}^a = e^a + \eta^a \tag{2.24}$$

and

$$\tilde{A}^{ab} = \hat{A}^{ab} + c^{ab} \ , \tag{2.25}$$

where η^a is the ghost field of local translations in the tangent space, c^{ab} is the self-dual Ashtekar ghost and $\hat{A}^{ab}$ is given by

$$\hat{A}^{ab} = A^{ab}{}_c \tilde{e}^c = A^{ab} + A^{ab}{}_c \eta^c \ . \tag{2.26}$$

The 0-form $A^{ab}{}_c$ is defined by the expansion of the 0-form connection A^{ab}_μ in terms of the tetrad fields e^a_μ:

$$A^{ab}{}_\mu = A^{ab}{}_c e^c_\mu \ . \tag{2.27}$$

The ghost field of local translations η^a is related to the usual ghost for the diffeomorphisms ξ^μ by the relations

$$\begin{aligned} \eta^a &= \xi^\mu e^a_\mu \ , \\ \xi^\mu &= E^\mu_a \eta^a \ , \end{aligned} \tag{2.28}$$

where E^μ_a denotes the inverse of the tetrad e^a_μ, i.e.

$$\begin{aligned} e^a_\mu E^\mu_b &= \delta^a_b \ , \\ e^a_\mu E^\nu_a &= \delta^\nu_\mu \ . \end{aligned} \tag{2.29}$$

The generalized Ashtekar torsion 2-form and Ashtekar field strength 2-form are given by

$$\begin{aligned} \tilde{T}^a &= \tilde{d}\tilde{e}^a + \tilde{A}^a{}_b \tilde{e}^b \ , \\ \tilde{F}^{ab} &= \tilde{d}\tilde{A}^{ab} + \tilde{A}^a{}_c \tilde{A}^{cb} \ , \end{aligned} \tag{2.30}$$

and they obey the generalized Bianchi identities

$$\begin{aligned} \tilde{D}\tilde{T}^a &= \tilde{d}\tilde{T}^a + \tilde{A}^a{}_b \tilde{T}^b = \tilde{F}^a{}_b \tilde{e}^b \ , \\ \tilde{D}\tilde{F}^{ab} &= \tilde{d}\tilde{F}^{ab} + \tilde{A}^a{}_c \tilde{F}^{cb} - \tilde{F}^a{}_c \tilde{A}^{cb} = 0 \ , \end{aligned} \tag{2.31}$$

with $\tilde{D}$ the generalized covariant exterior derivative.

Now we are able to formulate the Maurer-Cartan horizontality conditions for the case of gravity in Ashtekar variables. Following [15] we can say that these conditions state that $\tilde{e}^a$ and all its generalized Ashtekar covariant exterior differentials can be expanded over $\tilde{e}^a$ with classical (without tilde) coefficients, i.e.:

$$\tilde{e}^a = \delta^a_b \tilde{e}^b \equiv horizontal \ , \tag{2.32}$$

$$\tilde{T}^a(\tilde{e}, \tilde{A}) = \frac{1}{2} T^a_{bc}(e, A) \tilde{e}^b \tilde{e}^c \equiv horizontal \ , \tag{2.33}$$

$$\tilde{F}^{ab}(\tilde{A}) = \frac{1}{2}F^{ab}{}_{cd}(A)\tilde{e}^c\tilde{e}^d \equiv \mathit{horizontal} \ , \tag{2.34}$$

where the 0-forms T^a_{bc} and $F^{ab}{}_{cd}$ are defined by the tetrad expansion of the Ashtekar 2-form torsion (2.17) and the Ashtekar 2-form field strength (2.18):

$$T^a = \frac{1}{2}T^a_{bc}e^be^c \ , \tag{2.35}$$

$$F^{ab} = \frac{1}{2}F^{ab}{}_{cd}e^ce^d \ . \tag{2.36}$$

It is worthwhile to remind that eq.(2.26) is nothing but the *horizontality condition* for the Ashtekar connection, stating the fact that $\hat{A}^{ab}$ itself can be expanded over $\tilde{e}$. The horizontality conditions (2.32)-(2.34) are equivalent with the statements:

$$\begin{aligned}
\tilde{e}^a &= \exp(i_\xi)e^a = e^a + i_\xi e^a \ , \\
\tilde{T}^a &= \exp(i_\xi)T^a = T^a + i_\xi T^a + \frac{1}{2}i_\xi i_\xi T^a \ , \\
\tilde{F}^{ab} &= \exp(i_\xi)F^{ab} = F^{ab} + i_\xi F^{ab} + \frac{1}{2}i_\xi i_\xi F^{ab} \ ,
\end{aligned} \tag{2.37}$$

since e^a is an 1-form, while T^a and F^{ab} are 2-forms. These conditions reduce to the Russian formula when the diffeomorphism transformation generated by ξ is absent.

Now the MCHC for the case of gravity in Ashtekar variables (2.32)-(2.34) give, when expanded in terms of the elementary fields $(e^a, A^{ab}, \eta^a, c^{ab})$, the nilpotent BRST transformations corresponding to the classical gauge (Lorentz) rotations and to the diffeomorphism transformations. The BRST transformations for the tetrad e^a and for the diffeomorphism ghost η^a could be obtained from (2.33) which yields

$$\begin{aligned}
de^a - se^a + d\eta^a - s\eta^a + \hat{A}^a{}_be^b + \hat{A}^a{}_b\eta^b + c^a{}_be^b + c^a{}_b\eta^b = \\
= \frac{1}{2}T^a_{cd}e^ce^d + T^a_{cd}e^c\eta^d + \frac{1}{2}T^a_{cd}\eta^c\eta^d \ .
\end{aligned} \tag{2.38}$$

Collecting the terms with the same ghost number and form degree, we can easily obtain the BRST transformations for the tetrad 1-form e^a and for the local translation ghost η^a:

$$\begin{aligned}
se^a &= d\eta^a + A^a{}_b\eta^b + A^a{}_{bc}\eta^ce^b + c^a{}_be^b - T^a_{bc}e^b\eta^c \ , \\
s\eta^a &= A^a{}_{bc}\eta^c\eta^b + c^a{}_b\eta^b - \frac{1}{2}T^a_{bc}\eta^b\eta^c \ .
\end{aligned} \tag{2.39}$$

These equations could be rewritten in terms of the diffeomorphism ghost ξ^μ which take the more familiar form [4, 18]:

$$\begin{aligned}
se^a_\mu &= c^a{}_be^b_\mu - \mathcal{L}_\xi e^a_\mu \ , \\
s\xi^\mu &= -\frac{1}{2}\mathcal{L}_\xi\xi^\mu \ ,
\end{aligned} \tag{2.40}$$

where $\mathcal{L}_\xi$ denotes the Lie derivative [24] along the direction ξ^μ, i.e.

$$\mathcal{L}_\xi e^a_\mu = \xi^\lambda\partial_\lambda e^a_\mu + (\partial_\mu\xi^\lambda)e^a_\lambda \ .$$

2.4 BRST transformations and Bianchi identities

In this subsection we are going to give, for the convenience of the reader, the BRST transformations and the Bianchi identities which are contained in the Maurer-Cartan horizontality conditions (2.32)-(2.34) and from eqs.(2.30) and (2.31) for each form sector and ghost number.

- **Form sector two, ghost number zero** $(T^a, F^a_{\ b})$

$$
\begin{aligned}
sT^a &= c^a_{\ b}T^b + A^a_{\ bk}\eta^k T^b - F^a_{\ b}\eta^b \\
&\quad + A^a_{\ b}T^b_{mn}e^m\eta^n - F^a_{\ bmn}e^b e^m\eta^n + (dT^a_{mn})e^m\eta^n \\
&\quad - T^a_{mn}e^m d\eta^n + T^a_{mn}T^m\eta^n - T^a_{kn}A^k_{\ m}e^m\eta^n \ , \\
sF^a_{\ b} &= c^a_{\ c}F^c_{\ b} - c^c_{\ b}F^a_{\ c} + A^a_{\ ck}\eta^k F^c_{\ b} - A^c_{\ bk}\eta^k F^a_{\ c} \\
&\quad + A^a_{\ c}F^c_{\ bmn}e^m\eta^n - A^c_{\ b}F^a_{\ cmn}e^m\eta^n + (dF^a_{\ bmn})e^m\eta^n \\
&\quad + F^a_{\ bmn}T^m\eta^n - F^a_{\ bkn}A^k_{\ m}e^m\eta^n - F^a_{\ bmn}e^m d\eta^n \ .
\end{aligned}
\tag{2.41}
$$

For the Bianchi identities one has

$$
\begin{aligned}
DT^a &= dT^a + A^a_{\ b}T^b = F^a_{\ b}e^b \ , \\
DF^a_{\ b} &= dF^a_{\ b} + A^a_{\ c}F^c_{\ b} - A^c_{\ b}F^a_{\ c} = 0 \ .
\end{aligned}
\tag{2.42}
$$

- **Form sector one, ghost number zero** $(e^a, A^a_{\ b})$

$$
\begin{aligned}
se^a &= d\eta^a + A^a_{\ b}\eta^b + c^a_{\ b}e^b + A^a_{\ bm}\eta^m e^b - T^a_{mn}e^m\eta^n \ , \\
sA^a_{\ b} &= dc^a_{\ b} + c^a_{\ c}A^c_{\ b} + A^a_{\ c}c^c_{\ b} + (dA^a_{\ bm})\eta^m + A^a_{\ bm}d\eta^m \\
&\quad + A^a_{\ c}A^c_{\ bm}\eta^m + A^a_{\ cm}\eta^m A^c_{\ b} - F^a_{\ bmn}e^m\eta^n \ .
\end{aligned}
\tag{2.43}
$$

- **Form sector zero, ghost number zero** $(A^a_{\ bm}, F^a_{\ bmn}, T^a_{mn})$

$$
\begin{aligned}
sA^a_{\ bm} &= -\eta^k\partial_k A^a_{\ bm} - \partial_m c^a_{\ b} + c^a_{\ c}A^c_{\ bm} - c^c_{\ b}A^a_{\ cm} - c^k_{\ m}A^a_{\ bk} \ , \\
sT^a_{mn} &= -\eta^k\partial_k T^a_{mn} + c^a_{\ k}T^k_{mn} - c^k_{\ m}T^a_{kn} - c^k_{\ n}T^a_{mk} \ , \\
sF^a_{\ bmn} &= -\eta^k\partial_k F^a_{\ bmn} + c^a_{\ c}F^c_{\ bmn} - c^c_{\ b}F^a_{\ cmn} \\
&\quad - c^k_{\ m}F^a_{\ bkn} - c^k_{\ n}F^a_{\ bmk} \ .
\end{aligned}
\tag{2.44}
$$

The Bianchi identities (2.42) are projected on the 0-form Ashtekar torsion $T^a_{\ mn}$ and on the 0-form Ashtekar field strength $F^a_{\ bmn}$ to give:

$$
\begin{aligned}
dT^a_{mn} &= (\partial_k T^a_{mn})e^k \\
&= (F^a_{\ kmn} + F^a_{\ mnk} + F^a_{\ nkm} \\
&\quad - A^a_{\ bk}T^b_{mn} - A^a_{\ bm}T^b_{nk} - A^a_{\ bn}T^b_{km} \\
&\quad - T^a_{lk}T^l_{mn} - T^a_{lm}T^l_{nk} - T^a_{ln}T^l_{km}
\end{aligned}
$$

$$
\begin{aligned}
&+ T^a_{lk}A^l{}_{nm} + T^a_{ln}A^l{}_{mk} + T^a_{lm}A^l{}_{kn} \\
&- T^a_{lk}A^l{}_{mn} - T^a_{lm}A^l{}_{nk} - T^a_{ln}A^l{}_{km} \\
&- \partial_m T^a_{nk} - \partial_n T^a_{km})e^k \ ,
\end{aligned}
$$

$$
\begin{aligned}
dF^a{}_{bmn} &= (\partial_k F^a{}_{bmn})e^k \\
&= (-A^a{}_{ck}F^c{}_{bmn} - A^a{}_{cm}F^c{}_{bnk} - A^a{}_{cn}F^c{}_{bkm} \\
&\quad + A^c{}_{bk}F^a{}_{cmn} + A^c{}_{bm}F^a{}_{cnk} + A^c{}_{bn}F^a{}_{ckm} \\
&\quad - F^a{}_{blk}T^l_{mn} - F^a{}_{blm}T^l_{nk} - F^a{}_{bln}T^l_{km} \\
&\quad + F^a{}_{blk}A^l{}_{nm} + F^a{}_{bln}A^l{}_{mk} + F^a{}_{blm}A^l{}_{kn} \\
&\quad - F^a{}_{blk}A^l{}_{mn} - F^a{}_{blm}A^l{}_{nk} - F^a{}_{bln}A^l{}_{km} \\
&\quad - \partial_m F^a{}_{bnk} - \partial_n F^a{}_{bkm})e^k \ .
\end{aligned}
\tag{2.45}
$$

We also have the additional equation

$$
\begin{aligned}
dA^a{}_{bm} &= (\partial_n A^a{}_{bm})e^n \\
&= (-F^a{}_{bmn} + A^a{}_{cm}A^c{}_{bn} - A^a{}_{cn}A^c{}_{bm} \\
&\quad + A^a{}_{bk}T^k_{mn} - A^a{}_{bk}A^k{}_{nm} + A^a{}_{bk}A^k{}_{mn} + \partial_m A^a{}_{bn})e^n \ .
\end{aligned}
\tag{2.46}
$$

- **Form sector zero, ghost number one** $(c^a{}_b, \eta^a)$

$$
\begin{aligned}
s\eta^a &= c^a{}_b\eta^b + A^a{}_{bm}\eta^m\eta^b - \frac{1}{2}T^a_{mn}\eta^m\eta^n \ , \\
sc^a{}_b &= c^a{}_c c^c{}_b - \eta^k\partial_k c^a{}_b \ .
\end{aligned}
\tag{2.47}
$$

- **Algebra between s and d**

  From the above transformations it follows (see also appendix A):

$$
s^2 = 0 \ , \qquad d^2 = 0 \ , \tag{2.48}
$$

  and

$$
\{s, d\} = 0 \ . \tag{2.49}
$$

3 Solution of the descent equations

The question of finding the invariant Lagrangians, the anomalies and the Schwinger terms for the four-dimensional gravity in Ashtekar variables can be solved in a purely algebraic way by solving the BRST consistency condition in the space of the integrated local field polynomials. In order to solve this question, we have to find out the nontrivial solution of the equation

$$
s\Delta = 0 \ , \tag{3.1}
$$

where Δ is an integrated local field polynomial, i.e. $\Delta = \int \mathcal{A}$. The condition (3.1) translates into the local equation

$$
s\mathcal{A} + d\mathcal{Q} = 0 \ , \tag{3.2}
$$

where $\mathcal{Q}$ is some local polynomial and $d = dx^\mu \partial_\mu$ is the nilpotent exterior spacetime derivative which anticommutes with the nilpotent BRST operator s

$$s^2 = d^2 = sd + ds = 0 \ , \tag{3.3}$$

and it is *acyclic* (i.e. its cohomology group vanishes).

The local equation (3.2), due to the algebra (3.3) and the acyclicity of d, generates a tower of descent equations

$$\begin{aligned} s\mathcal{A} + d\mathcal{Q}^1 &= 0 \\ s\mathcal{Q}^1 + d\mathcal{Q}^2 &= 0 \\ &\cdots \\ s\mathcal{Q}^{k-1} + d\mathcal{Q}^k &= 0 \\ s\mathcal{Q}^k &= 0 \end{aligned} \tag{3.4}$$

with $\mathcal{Q}^i$ local polynomials in the fields.

For the Yang-Mills case, these equations can be solved by means of a transgression procedure generated by the Russian formula (2.6) [5]. More recently a new and efficient way of finding nontrivial solutions of the tower (3.4) has been proposed by S.P. Sorella [1] and successfully applied to the study of the Yang-Mills cohomology [23], the gravitational anomalies [26] and the algebraic structure of gravity with torsion [15, 27]. The basic ingredient of the method is an operator δ which allows us to express the exterior derivative d as a BRST commutator, i.e.:

$$d = -[s, \delta] \ . \tag{3.5}$$

Now it is easy to see that, once the decomposition (3.5) has been found, repeated application of the operator δ on the polynomial $\mathcal{Q}$ which is a nontrivial solution of the last equation of (3.4) gives an explicit and nontrivial solution for the other cocycles $\mathcal{Q}^i$ and for $\mathcal{A}$. If $\mathcal{A}$ has ghost number one then it is called an anomaly and if it has ghost number zero then it represents an invariant Lagrangian. In other word using the operator δ we can calculate the solution of the cohomology $H(s \bmod d)$ if we know the solution of the cohomology $H(s)$. Actually, as has been shown in [23], the cocycles obtained by the descent equations (3.4) turn out to be completely equivalent to those one based on the Russian formula.

For the gravity in Ashtekar variables the operator δ introduced in eq.(3.5) can be defined by

$$\begin{aligned} &\delta\eta^a = -e^a \ , \\ &\delta\Phi = 0 \qquad \text{for} \qquad \Phi = (e^a, A^{ab}, T^a, F^{ab}, c^{ab}) \ . \end{aligned} \tag{3.6}$$

Now it is easy to verify that δ is of degree zero[6] and obeys the following algebraic relations

$$d = -[s, \delta] \qquad , \qquad [d, \delta] = 0 \ . \tag{3.7}$$

[6]The degree is given by the sum of the form degree and the ghost number.

In order to solve the tower (3.4) we shall make use of the following identity

$$e^{\delta} s = (s + d) e^{\delta} \ , \tag{3.8}$$

which is a direct consequence of (3.7) (see [23]).

Let us consider now the solution of the descent equations (3.4) with a given ghost number G and form degree N, i.e. a solution of the tower

$$\begin{aligned} s\Omega_4^G + d\Omega_3^{G+1} &= 0 \\ s\Omega_3^{G+1} + d\Omega_2^{G+2} &= 0 \\ s\Omega_2^{G+2} + d\Omega_1^{G+3} &= 0 \\ s\Omega_1^{G+3} + d\Omega_0^{G+4} &= 0 \\ s\Omega_0^{G+4} &= 0 \end{aligned} \tag{3.9}$$

with (Ω_4^G, Ω_3^{G+1}, Ω_2^{G+2}, Ω_1^{G+3}, Ω_0^{G+4}) local polynomials in the variables (e^a, A^{ab}, η^a, c^{ab}) which, without loss of generality, will be always considered as irreducible elements, i.e. they cannot be expressed as the product of several factored terms. In particular Ω_4^0, Ω_3^1 and Ω_2^2 correspond, respectively to an invariant Lagrangian, an anomaly and a Schwinger term.

Due to the identity (3.8) we can obtain the higher cocycles $\Omega_q^{G+4-q} (q = 1, 2, 3, 4)$ once a nontrivial solution for Ω_0^{G+4} is known. Indeed, by applying the identity (3.8) on Ω_0^{G+4} one gets

$$(s + d) \left[e^{\delta} \Omega_0^{G+4} (\eta, c, A, T, F) \right] = 0 \tag{3.10}$$

But as one can see from eq.(3.6), the operator δ acts as a translation on the ghost η^a with an amount $(-e^a)$ and eq.(3.10) can be rewritten as

$$(s + d) \Omega_0^{G+4} (\eta - e, c, A, T, F) = 0 \ . \tag{3.11}$$

Thus the expansion of the 0-form cocycle $\Omega_0^{G+4}(\eta - e, c, A, T, F)$ in power of the 1-form tetrads e^a yields all the cocycles Ω_q^{G+4-q}.

4 Examples

In this section we want to apply the previous algebraic setup to produce some interesting examples. We shall show that all interesting objects which occur in Ashtekar theory as: the cosmological constant, the Ashtekar action, the action for the topological gravity and the Capovilla, Jacobson, Dell action have a cohomological origin, i.e. they are solutions of some descent equations. In the last step we investigate the Chern-Simons terms in five dimensions. The examples are ordered by the power of the Ashtekar field strength.

4.1 The cosmological constant

The simplest local BRST-invariant polynomial 0-form, which can be defined, is given by

$$\Omega_0^4(\eta) = \frac{1}{4!} \varepsilon_{abcd} \eta^a \eta^b \eta^c \eta^d \ . \tag{4.1}$$

Since in four dimensions the product of five ghosts η^a automatically vanishes it is easy to see that Ω^4_0 represents a cohomology class of the BRST operator s, i.e.

$$s\Omega^4_0 = 0 \qquad , \qquad \Omega^4_0 \neq s\widehat{\Omega}^3_0 \ . \tag{4.2}$$

The corresponding 0-ghost term is the invariant Lagrangian

$$\Omega^0_4 = \frac{1}{4!}\delta^4\Omega^4_0 = \frac{1}{4!}\varepsilon_{abcd}e^a e^b e^c e^d = ed^4x \ , \tag{4.3}$$

where e is the determinant of the tetrad e^a_μ given in the appendix B.

4.2 Ashtekar Lagrangian

This time we start with the cocycle

$$\Omega^4_0 = \frac{1}{2i}\frac{1}{2!}\varepsilon_{abcd}F^{ab}{}_{mn}\eta^m\eta^n\eta^c\eta^d \ . \tag{4.4}$$

This cocycle is BRST-closed, $s\Omega^4_0 = 0$, but it is not BRST-exact i.e.

$$\Omega^4_0 \neq s\widehat{\Omega}^3_0 \ .$$

For the case of $SO(1,3)$ the invariant Lagrangian corresponding to (4.4) has the form

$$\begin{aligned} \Omega^0_4 &= \frac{1}{4!}\delta^4\Omega^4_0 = \frac{1}{4i}\varepsilon_{abcd}F^{ab}{}_{mn}e^m e^n e^c e^d = F_{ab}e^a e^b \\ &= iE^\mu_m E^\nu_n F^{mn}_{\mu\nu}ed^4x = iF^{mn}{}_{mn}ed^4x \\ &= \frac{1}{2}e_{a\mu}e_{b\nu}F^{ab}{}_{\tau\sigma}\varepsilon^{\mu\nu\tau\sigma}d^4x \ , \end{aligned} \tag{4.5}$$

where we have used the tetrad 1-forms $e^a = e^a_\mu dx^\mu$, $e_a = \eta_{ab}e^b$ and the selfduality of the Ashtekar field strength. This is just the action introduced by Ashtekar [2] (see also [28, 29]), whose real part is the Palatini Lagrangian.

4.3 The topological action

We also can build an action which is quadratic in the Ashtekar field strength. In this case, using two Ashtekar field strengths one can built up a BRST-invariant local polynomial

$$\Omega^4_0 = -\frac{i}{2}\varepsilon_{abcd}F^{ab}{}_{kl}F^{cd}{}_{mn}\eta^k\eta^l\eta^m\eta^n \ , \tag{4.6}$$

to which it corresponds the invariant Lagrangian

$$\begin{aligned} \Omega^0_4 &= -2i\varepsilon_{abcd}F^{ab}F^{cd} = 4F^{ab}F_{ab} \\ &= F^{ab}{}_{kl}F_{abmn}\varepsilon^{klmn}d^4x = F^{ab}{}_{\mu\nu}F_{ab\tau\sigma}\varepsilon^{\mu\nu\tau\sigma}d^4x \ . \end{aligned} \tag{4.7}$$

Witten has suggested that 4D gravity has a phase described by a topological field theory (TQFT) [30] (see also [31]) and in this phase the observables are global invariants. In particular the Donaldson maps [32] can be identified as BRST-invariants of the corresponding TQFT. In order to extend Witten's analysis for 4D usual gravity (not topological one) with propagating degrees of freedom we have to describe these degrees of freedom using variables that are related naturally to those employed in TQFT. In fact they must be suitable for implementing *both* diffeomorphism and gauge invariance. The Ashtekar connection satisfies these requirements since it replaces the metric and in the reduced phase space, obtained after eliminating the constraints, a restricted sector of the theory is described by the Ashtekar-Renteln ansatz [33]

$$F^{ab} = -\frac{\lambda}{3}[e^a e^b - i * (e^a e^b)] \ . \tag{4.8}$$

With this ansatz the Ashtekar action becomes the topological action, the BRST transformations of the Ashtekar connection coincides with the corresponding one from TQFT [34] if one identifies $i_\xi F$ and $\frac{1}{2} i_\xi i_\xi F$ with the usual ghosts introduced in TQFT.

4.4 An invariant Lagrangian with three Ashtekar fields

In this case, using three Ashtekar field strenghts one can build up the following 0-form s-cocycle

$$\Omega^4_0 = \frac{1}{4} F^{ab}{}_{kl} F^{cd}{}_{mn} F_{ab}{}^{pq} \varepsilon_{cdpq} \eta^k \eta^l \eta^m \eta^n \ . \tag{4.9}$$

Again this cocycle is not trivial, i.e. it cannot be written as a s-coboundary

$$\Omega^4_0 \neq s\widehat{\Omega}^3_0 \ .$$

This term leads to the following invariant Lagrangian

$$\Omega^0_4 = F^{ab} F^{cd} F_{ab}{}^{pq} \varepsilon_{cdpq} \ . \tag{4.10}$$

From this 4-form we can obtain the global invariant proposed by Chang and Soo [34, 35].

4.5 Capovilla, Jacobson, Dell Lagrangian

In this case we shall try to build up a BRST cocycle with four Ashtekar field strengths F^{ab}. Using the 0-form Ashtekar field strength $F^{ab}{}_{cd}$, one gets for the cocycle Ω^4_0:

$$\Omega^4_0 = \frac{1}{4} F^{ab}{}_{kl} F^{cd}{}_{mn} \varepsilon^{klmn} F_{abpq} F_{cdrs} \eta^p \eta^q \eta^r \eta^s \ . \tag{4.11}$$

It can be easy checked that this Ω^4_0 is s-closed, i.e.

$$s\Omega^4_0 = 0 \ . \tag{4.12}$$

Also here it can be identified with a cohomological class of the BRST operator s since

$$\Omega^4_0 \neq s\widehat{\Omega}^3_0 \ .$$

The cocycle (4.11) gives rise to the invariant Lagrangian

$$\Omega_4^0 = F^{ab}{}_{kl} F^{cd}{}_{mn} \varepsilon^{klmn} F_{ab} F_{cd} \ . \tag{4.13}$$

Expression (4.13) is nothing but the Capovilla, Jacobson and Dell Lagrangian [36] for the case of $SO(3, C)$.

It is interesting to remark that this action depends only on the self-dual spin connection, i.e. the Ashtekar variables, and a general scalar-density Lagrange multiplier field, as a coefficient in front of eq.(4.13). The spacetime metric does not appear in this action in any form. On the other hand Capovilla, Jacobson and Dell [36] have shown that the field equations, which follows from this action, reproduce the Einstein equations and that the spacetime metric can be built up entirely from the Ashtekar field strength F. The metric in this case plays no role whatsoever, although it can be reconstructed from the Ashtekar field strength. They also showed that Ashtekar's formulation of general relativity is contained in this new action.

4.6 Chern-Simons term

Let us discuss in details the construction of the five dimensional Chern-Simons term. In this case the descent equations take the form

$$\begin{aligned} &s\Omega_5^0 + d\Omega_4^1 = 0 \\ &s\Omega_4^1 + d\Omega_3^2 = 0 \\ &s\Omega_3^2 + d\Omega_2^3 = 0 \\ &s\Omega_2^3 + d\Omega_1^4 = 0 \\ &s\Omega_1^4 + d\Omega_0^5 = 0 \\ &s\Omega_0^5 = 0 \end{aligned} \tag{4.14}$$

where, using Sorella's method [1], the cocycles can be obtained by

$$\begin{aligned} \Omega_1^4 &= \delta \Omega_0^5 \ , \\ \Omega_2^3 &= \frac{\delta^2}{2!} \Omega_0^5 \ , \\ \Omega_3^2 &= \frac{\delta^3}{3!} \Omega_0^5 \ , \\ \Omega_4^1 &= \frac{\delta^4}{4!} \Omega_0^5 \ , \\ \Omega_5^0 &= \frac{\delta^5}{5!} \Omega_0^5 \ . \end{aligned} \tag{4.15}$$

In order to find a solution for Ω_0^5 we use the redefined Ashtekar ghost

$$\hat{c}^a{}_b = A^a{}_{bm} \eta^m + c^a{}_b \ , \tag{4.16}$$

which, from eq.(3.6), transforms as

$$\delta \hat{c}^a{}_b = -A^a{}_b \ . \tag{4.17}$$

For the 0-form cocycle Ω^5_0 in five dimensions one gets

$$\Omega^5_0 = -\frac{1}{10}\hat{c}^a{}_b\hat{c}^b{}_c\hat{c}^c{}_d\hat{c}^d{}_e\hat{c}^e{}_a + \frac{1}{4}F^a{}_{bmn}\eta^m\eta^n\hat{c}^b{}_c\hat{c}^c{}_d\hat{c}^d{}_a$$
$$- \frac{1}{4}F^a{}_{bmn}\eta^m\eta^n F^b{}_{ckl}\eta^k\eta^l\hat{c}^c{}_a \ , \tag{4.18}$$

which leads to the five dimensional Chern-Simons term in Ashtekar variables

$$\Omega^0_5 = \frac{1}{10}A^a{}_bA^b{}_cA^c{}_dA^d{}_eA^e{}_a - \frac{1}{2}F^a_bA^b{}_cA^c{}_dA^d{}_a + F^a_bF^b_cA^c{}_a \ . \tag{4.19}$$

5 The $\mathcal{G}$-operator

In this section we want to compare the BRST structure of the usual gravity with the gravity in Ashtekar variables. The BRST structure of gravity has been demonstrated by Werneck de Oliveira and Sorella in [26]. They showed that if we work in a local space generated by the spin connection ω, the curvature $R = d\omega + \omega^2$ and their ghosts then the exterior differential d still does have the decomposition (3.5) but d does not commute with δ:

$$2\mathcal{G} = [d, \delta] \neq 0 \ . \tag{5.1}$$

The $\mathcal{G}$-operator does not vanish even if we choose as independent variables the Christoffel connecton Γ, the Riemann tensor $R = d\Gamma + \Gamma^2$ and their ghosts [26].

However, if one uses tetrads e^a_μ together with the spin connection ω, as the independent variables in a first order formalism developed by Palatini, then the operator $\mathcal{G}$ vanishes [15]. Moreover, using tetrads for the Yang-Mills gauge fields in the presence of gravity (with or without torsion) $\mathcal{G}$ vanishes also [15], in spite of the fact that for the pure YM case one has $\mathcal{G} \neq 0$.

So we can say that the diffeomorphisms carry, in some sense, the action of the $\mathcal{G}$-operator through the tetrads.

6 Conclusions

The present paper has shown that the algebraic structure of gravity in the Ashtekar formalism could be entirely obtained from the Maurer-Cartan horizontality conditions and by introducing an operator δ which allows a useful decomposition of the exterior spacetime differential as a BRST commutator. This decomposition offers us a simple possibility to solve the descent equations and to find some elements of the BRST cohomology. In particular we have obtained the actions for the free gravitational field proposed by Ashtekar [2] as well as Capovilla, Jacobson and Dell [36]. The same technique can be applied to study the gravity in Ashtekar variables coupled with Yang-Mills fields as well as to the characterization of the Weyl anomalies in these variables [21].

7 Appendices:

Appendix A is devoted to demonstrate the computation of some commutators involving the tangent space derivative ∂_a. In appendix B one finds some relations concerning the determinant of the tetrad and the ε-tensor.

A Commutator relations

In order to find the commutator of two tangent space derivatives ∂_a, we make use of the fact that the usual spacetime derivatives ∂_μ have a vanishing commutator:

$$[\partial_\mu, \partial_\nu] = 0 \ . \tag{A.1}$$

From

$$\partial_\mu = e^m_\mu \partial_m \tag{A.2}$$

one gets

$$\begin{aligned}
[\partial_\mu, \partial_\nu] = 0 &= [e^m_\mu \partial_m, e^n_\nu \partial_n] \\
&= e^m_\mu e^n_\nu [\partial_m, \partial_n] + e^m_\mu (\partial_m e^n_\nu)\partial_n - e^n_\nu (\partial_n e^m_\mu)\partial_m \\
&= e^m_\mu e^n_\nu [\partial_m, \partial_n] + (\partial_\mu e^k_\nu - \partial_\nu e^k_\mu)\partial_k \\
&= e^m_\mu e^n_\nu [\partial_m, \partial_n] + (T^k_{\mu\nu} - A^k{}_{n\mu} e^n_\nu + A^k{}_{m\nu} e^m_\mu)\partial_k \\
&= e^m_\mu e^n_\nu (T^k_{mn} + A^k{}_{mn} - A^k{}_{nm})\partial_k \\
&\quad + e^m_\mu e^n_\nu [\partial_m, \partial_n] \ ,
\end{aligned} \tag{A.3}$$

so that

$$[\partial_m, \partial_n] = -(T^k_{mn} + A^k{}_{mn} - A^k{}_{nm})\partial_k \ . \tag{A.4}$$

For the commutator of d and ∂_m we get

$$\begin{aligned}
[d, \partial_m] &= [e^n \partial_n, \partial_m] \\
&= -(\partial_m e^k)\partial_k - e^n [\partial_m, \partial_n] \\
&= -(\partial_m e^k)\partial_k + e^n (T^k_{mn} + A^k{}_{mn} - A^k{}_{nm})\partial_k \ ,
\end{aligned} \tag{A.5}$$

and one has therefore

$$[d, \partial_m] = (T^k_{mn} e^n + A^k{}_{mn} e^n - A^k{}_{nm} e^n - (\partial_m e^k))\partial_k \ . \tag{A.6}$$

Analogously, from

$$[s, \partial_\mu] = 0 \tag{A.7}$$

one easily finds

$$\begin{aligned}
[s, \partial_m] &= (\partial_m \eta^k - c^k{}_m)\partial_k + \eta^n [\partial_m, \partial_n] \\
&= (\partial_m \eta^k - c^k{}_m - T^k_{mn}\eta^n - A^k{}_{mn}\eta^n + A^k{}_{nm}\eta^n)\partial_k \ .
\end{aligned} \tag{A.8}$$

B Determinant of the vielbein and the ε-tensor

The definition of the determinant of the vielbein e^a_μ is given by

$$e = det(e^a_\mu) = \frac{1}{4!}\varepsilon_{a_1a_2a_3a_4}\varepsilon^{\mu_1\mu_2\mu_3\mu_4}e^{a_1}_{\mu_1}e^{a_2}_{\mu_2}e^{a_3}_{\mu_3}e^{a_N}_{\mu_N} \ . \tag{B.1}$$

One can easily verify that the BRST transformation of e reads

$$se = -\partial_\lambda(\xi^\lambda e) \ . \tag{B.2}$$

For the case of $SO(1,3)$ one has

$$\begin{aligned} e^0e^1e^2e^3 &= \frac{1}{4!}\epsilon_{a_1a_2a_3a_4}e^{a_1}e^{a_2}e^{a_3}e^{a_4} \\ &= \frac{1}{4!}\epsilon_{a_1a_2a_3a_4}e^{a_1}_{\mu_1}e^{a_2}_{\mu_2}e^{a_3}_{\mu_3}e^{a_4}_{\mu_4}dx^{\mu_1}dx^{\mu_2}dx^{\mu_3}dx^{\mu_4} \\ &= \frac{1}{4!}\epsilon_{a_1a_2a_3a_4}\epsilon^{\mu_1\mu_2\mu_3\mu_4}e^{a_1}_{\mu_1}e^{a_2}_{\mu_2}e^{a_2}_{\mu_2}e^{a_4}_{\mu_4}dx^0dx^1dx^2dx^3 \\ &= ed^4x = \sqrt{-g}d^4x \ , \end{aligned} \tag{B.3}$$

where g denotes the determinant of the metric tensor $g_{\mu\nu}$

$$g = det(g_{\mu\nu}) \ . \tag{B.4}$$

The ε-tensor has the usual norm

$$\varepsilon_{a_1a_2a_3a_4}\varepsilon^{a_1a_2a_3a_4} = -4! \ , \tag{B.5}$$

and obeys the following relation under partial contraction of two indices

$$\varepsilon_{abcd}\varepsilon^{mncd} = -2(\delta^m_a\delta^n_b - \delta^n_a\delta^m_b) \ , \tag{B.6}$$

and in general the contraction of two ε-tensors is given by the determinant of δ-tensors in the following way:

$$\varepsilon_{a_1a_2a_3a_4}\varepsilon^{b_1b_2b_3b_4} = -\begin{vmatrix} \delta^{b_1}_{a_1} & \delta^{b_2}_{a_1} & \delta^{b_3}_{a_1} & \delta^{b_4}_{a_1} \\ \delta^{b_1}_{a_2} & \delta^{b_2}_{a_2} & \delta^{b_3}_{a_2} & \delta^{b_4}_{a_2} \\ \delta^{b_1}_{a_3} & \delta^{b_2}_{a_3} & \delta^{b_3}_{a_3} & \delta^{b_4}_{a_2} \\ \delta^{b_1}_{a_4} & \delta^{b_2}_{a_4} & \delta^{b_3}_{a_4} & \delta^{b_4}_{a_4} \end{vmatrix} \ . \tag{B.7}$$

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