Final SAS Optimization Report

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Introduction:

Building T currently receives their water through two sources: the *Water Company* and their own *XYZ Water Storage Tank.* The company is currently deciding which Water Company contract to sign.

- **Contract 1:** provides water at $0.15 per gallon with a minimum of 25,000 gallons purchased per week.
- **Contract 2*:*** provides water at $0.12 per gallon with a minimum of 35,000 gallons purchased per week.

Problem Statement:

Which of the two proposed contracts provides XYZ Corporation with the lowest total water cost over the next four weeks?

Recommendations:

We recommend that XYZ Corporation use Contract 2 as it provides Building T with the lowest total water cost over the next four weeks. The following provides the constraints of the problem along with a week-by-week breakdown of the recommended gallons purchased through the contract and supplied by the water tank.

<u>Constraints and Assumptions:</u>

Contract 1		
Weekly Price Per Gallon	=	0.15
Weekly Gallon Purchase Quantity	>=	25,000
Contract 2		
Weekly Price Per Gallon	=	0.12
Weekly Gallon Purchase Quantity	>=	35,000
Water Tank		
Weekly End Balance of Gallons in Tank	>=	30,000
Percentage of Total Gallons Supplied By Tank	>=	25%
Week 1 & 2 Treatment Cost Per Gallon	=	0.18
Week 3 & 4 Treatment Cost Per Gallon	=	0.10

Expected Precipitation		
Week 1	=	12,000
Week 2	=	18,000
Week 3	=	20,000
Week 4	=	22,000
Building Use		
Week 1	=	55,940
Week 2	=	56,264
Week 3	=	65,031
Week 4	=	67,348

Weekly Breakdown:

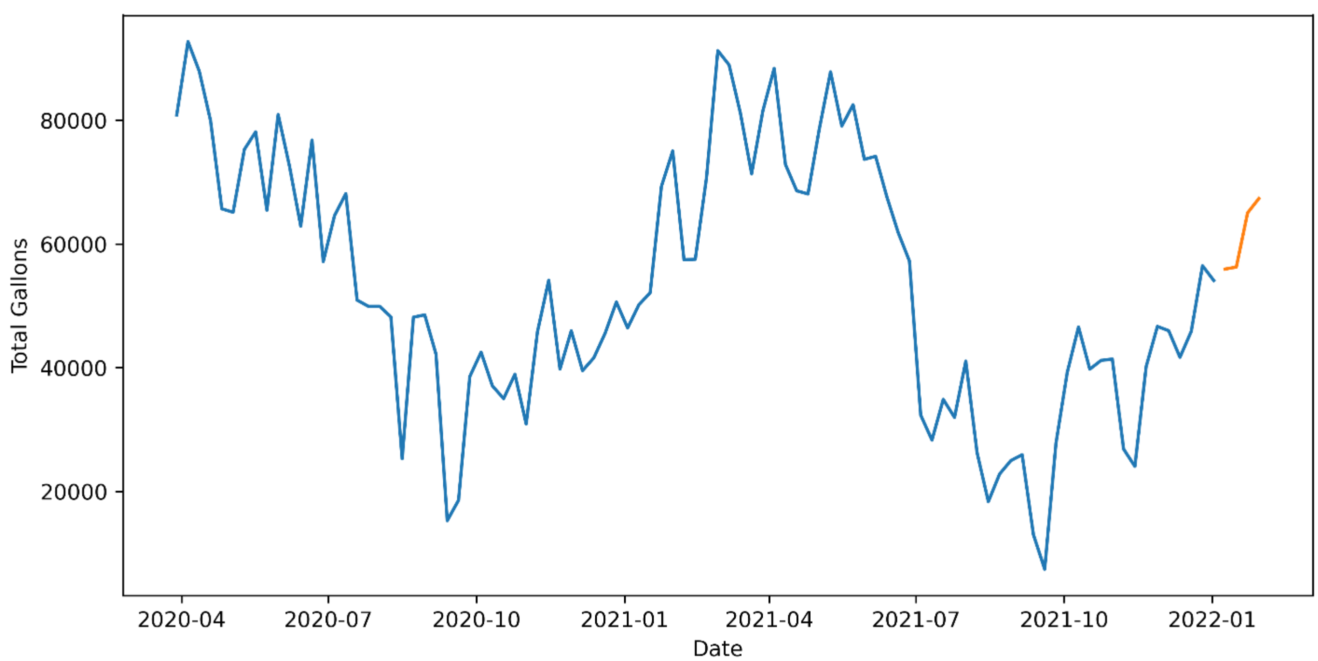
	Contract 1	Contract 2	Difference
Week 1			
Predicted Use	55,940	55,940	0
Gallons from Storage Tank	13,985	13,985	0
Cost from Storage Tank	$2,517	$2,517	0
Gallons from Contract	41,955	41,955	0
Cost from Contract	$6,294	$5,035	$1,259
Total Cost	**$8,811**	**$7,552**	**$1,259**
Week 2			
Predicted Use	56,264	56,264	0
Gallons from Storage Tank	14,066	14,066	0
Cost from Storage Tank	$2,547	$2,532	0
Gallons from Contract	42,197	42,198	0
Cost from Contract	$6,315	$5,064	$1,266
Total Cost	**$8,862**	**$7,596**	**$1,266**
Week 3			
Predicted Use	65,031	65,031	0
Gallons from Storage Tank	34,101	30,031	4,071
Cost from Storage Tank	$3,410	$3,003	$407
Gallons from Contract	30,930	35,000	-4,071
Cost from Contract	$4,639	$4,200	$439
Total Cost	**$8,049**	**$7,203**	**$846**
Week 4			
Predicted Use	67,348	67,348	0
Gallons from Storage Tank	42,348	32,348	10,000
Cost from Storage Tank	$4,234	$3,235	$1,000
Gallons from Contract	25,000	35,000	-10,000
Cost from Contract	$3,750	$4,200	$(450)
Total Cost	**$7,984**	**$7,435**	**$549**
TOTAL COST	**$33,706**	**$29,786**	**$3,920**

Methodology:

Demand Forecasting

Predicted Use	
Week 1	55,940
Week 2	56,264
Week 3	65,031
Week 4	67,348

We utilized Python's autots library to forecast water demand for the following four weeks. We set the number of generations to 15 in the AutoTS. Each generation experiments with new models, which adds time but improves accuracy. Additionally, we set ensemble to all, which retains additional information about each model. Finally, all models are evaluated using the most recent data set. This is because the most recent data are typically the most indicative of the anticipated future. We limited the number of validations to 2 and used backward cross validation as the validation method because it is the safest and works backwards from the most recent data. This makes it more suitable for smaller or dynamic datasets such as ours. By dividing the data into 2 equal parts, the final validation findings would then be composed of three components: the results from the three cross validation samples, as well as the original validation sample results. *The full python code file is attached.*



Blue= Historical, Orange = Forecast

Optimization

We constructed 3 SAS proc optmodel to determine which contract XYZ Corporation should adopt. In the first model, we used only the parameters specified in contract 1, in the second model, we used the parameters provided in contract 2, and in the third model, we combined the parameters specified in both contracts to create a nonlinear model. In model, we created binary decision variables called $Contract_i$. The decision variables will determine the optimal contract for XYZ. The objective function, parameters, decision variables, and mathematical formulation constraints that were used in the third model are listed below. SAS optimization software analyzed the given constraints, costs, and predicted demand to determine the most cost-effective water supply distribution. The included code is largely dynamic, which enables the user to easily update it as new monthly demand data becomes available. The total cost of the first model is $33,707, the second model is $29,786, and the third model is also $29,786, confirming our conclusion. *The full SAS code files are attached.*

Parameter	Parameter Description
$useQ_i$	The predicted amount of water usage for week *i*
$rain_i$	The amount of precipitation for week *i*
$trmt_i$	The treatment costs for week *i*
$price_i$	The price of per gallon of water in the contract *i*
min_i	The minimum order quantity for each contract *i*
WaterTankAvailable	The amount of water in tank that can be used

Decision Variable	Variable Description
x_i	The water purchased from Water Co. in week *i (contract1)*
z_i	The water purchased from Water Co. in week *i (contract2)*
y_i	The water used from Water Tank in week *i*
$Contract_i$	If we choose contract *i*, it should be 1 otherwise 0

Mathematical Formulation

Objective Function

Min $(Contract_1 \times \sum_{i=1}^{4} x_i \times price_1) + (Contract_2 \times \sum_{i=1}^{4} z_i \times price_2) + (\sum_{i=1}^{4} y_i \times trmt_i)$

Subject to:

$\sum_{i=1}^{4} x_i * Contract1 + z_i * Contract2 + y_i \geq useQ_i$ (1)
$\sum_{i=1}^{4} y_i \geq 0.25 \times useQ_i$ (2)
$\sum_{i=1}^{4} x_i \geq min_1$ (3)
$\sum_{i=1}^{4} z_i \geq min_2$ (4)
$\sum_{i=1}^{2} Contract_i = 1$ (5)
$\sum_{i=1}^{4} y_i \leq WaterTankAvailable_i + rain_i - y_{i-1}$ (6)
$Contract_i \in \{0, 1\}$

Constraints Description
(1) The combined water usage from the Water Company and the water tank must be greater than or equal to the weekly demand forecast. (2) The weekly water usage from the water tank must be greater than or equal to 25% of the weekly demand forecast. (3) In contract 1, the weekly water consumption from Water Co. must be greater than or equal to 25,000 gallons. (4) In contract 2, the weekly water consumption from Water Co. must be greater than or equal to 35,000 gallons. (5) Only one contract may be selected. (6) The amount of weekly water consumption from the water tank must not exceed that of the remaining water in the tank.

Conclusion:

Contract 2 will have a lower cost based on our demand forecasting. At the end of the next four weeks, the total cost of water will be $29,786. By selecting the recommended contract over the alternative contract, XYZ will save $3,921. XYZ will consume 14,069 *fewer* gallons of water from the water tank, in other words, XYZ will purchase 14,069 *more* gallons from the Water Co., in comparison to Contract 1.

Gallon Supply Distribution in Contract 2			
	Buy from Contract 2	**Use from Water Tank**	**Water Left in Tank**
Week 1	41,955	13,985	60,515
Week 2	42,198	14,066	64,449
Week 3	35,000	30,031	54,418
Week 4	35,000	32,348	44,070

	Contract 1	Contract 2	Difference
Total Cost	$33,707	$29,786	$3,921
Treatment Cost	$12,694	$11,287	$1,407
Purchase Cost	$21,013	$18,499	$2,514
Total Purchased	140,083 Gallons	154,152 Gallons	14,069 Gallons
End Balance in Tank	30,001 Gallons	44,070 Gallons	14,069 Gallons