

**CS201: Discrete Math for Computer Science**  
**2025 Spring Semester Written Assignment #2**

Please answer questions in English. Using any other language will lead to a zero point.

**Q. 1.** Suppose that  $A$ ,  $B$  and  $C$  are three finite sets. For each of the following, determine whether or not it is true. Explain your answers.

(a)  $(A - B = A) \rightarrow (B \subset A)$

(b)  $(A \cap B \cap C) \subseteq (A \cup B)$

(c)  $\overline{(A - B)} \cap (B - A) = B$

**Q. 2.** The symmetric difference of  $A$  and  $B$ , denoted by  $A \oplus B$ , is the set containing those elements in either  $A$  or  $B$ , but not in both  $A$  and  $B$ .

(a) Determine whether the symmetric difference is associative; that is, if  $A$ ,  $B$  and  $C$  are sets, does it follow that  $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ ?

(b) Suppose that  $A$ ,  $B$  and  $C$  are sets such that  $A \oplus C = B \oplus C$ . Must it be the case that  $A = B$ ?

**Q. 3.** Prove or disprove that there exists an infinite set  $A$  such that  $|A| < |\mathbb{Z}^+|$ .

**Q. 4.** Suppose that two functions  $g : A \rightarrow B$  and  $f : B \rightarrow C$  and  $f \circ g$  denotes the composition function.

(a) If  $f \circ g$  is one-to-one and  $g$  is one-to-one, must  $f$  be one-to-one? Explain your answer.

(b) If  $f \circ g$  is one-to-one and  $f$  is one-to-one, must  $g$  be one-to-one? Explain your answer.

(c) If  $f \circ g$  is one-to-one, must  $g$  be one-to-one? Explain your answer.

(d) If  $f \circ g$  is onto, must  $f$  be onto? Explain your answer.

(e) If  $f \circ g$  is onto, must  $g$  be onto? Explain your answer.

**Q. 5.** Give an example of two uncountable sets  $A$  and  $B$  such that the difference  $A - B$  is (a) finite, (b) countably infinite, (c) uncountable. Note: one example for each subquestion (a), (b), or (c).

**Q. 6.** If  $A$  is an uncountable set and  $B$  is a countable set, must  $A - B$  be uncountable?

**Q. 7.** Show that the set  $\mathbf{Z}^+ \times \mathbf{Z}^+$  is countable by showing that the polynomial function  $f : \mathbf{Z}^+ \times \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$  with  $f(m, n) = (m + n - 2)(m + n - 1)/2 + m$  is one-to-one and onto.

**Q. 8.** Assume that  $|S|$  denotes the cardinality of the set  $S$ . Show that if  $|A| = |B|$  and  $|B| = |C|$ , then  $|A| = |C|$ .

**Q. 9.** Suppose that  $f(x), g(x)$  and  $h(x)$  are functions such that  $f(x)$  is  $\Theta(g(x))$  and  $g(x)$  is  $\Theta(h(x))$ . Show that  $f(x)$  is  $\Theta(h(x))$ .

**Q. 10.** Suppose that  $f_1(x)$  is  $\Theta(g_1(x))$  and  $f_2(x)$  is  $\Theta(g_2(x))$ . Prove or disprove that  $f_1(x) - f_2(x)$  is  $\Theta(g_1(x) - g_2(x))$ .