

About this course

Textbooks

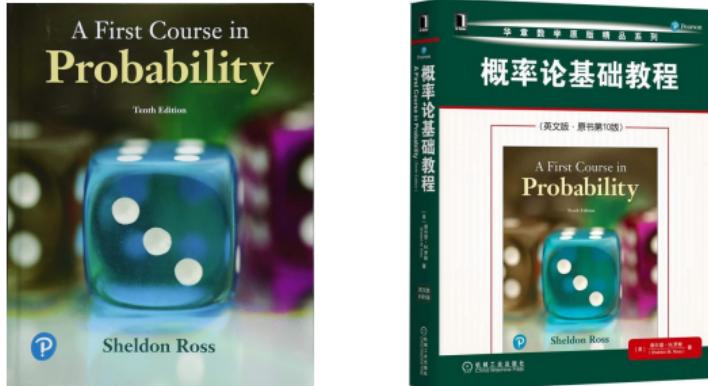


Figure: Textbook: Sheldon Ross, A first course in probability.

Further reading



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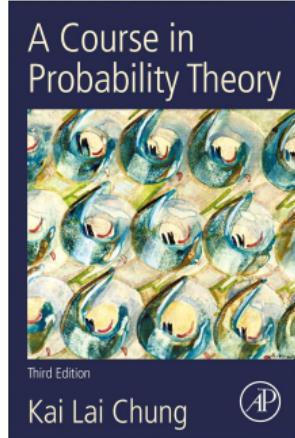
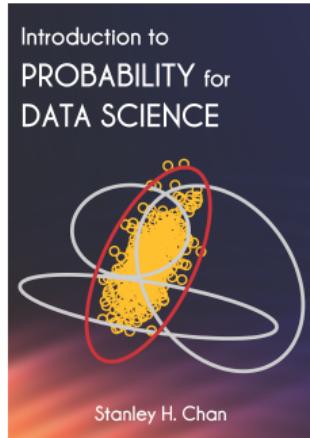
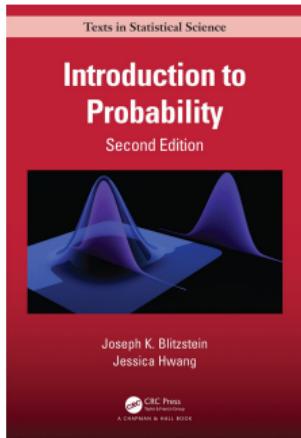


Figure: Further materials

Softwares



In this course, we might use several types of free software:





Lecture 1: Data and Probability

Foundation of Probability Theory/STA 203

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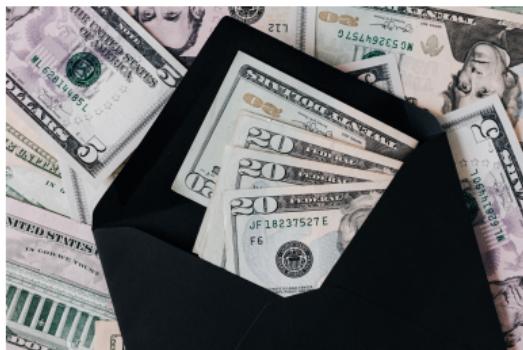
Fall, 2023

Randomness

Data in our real world



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Data in our real world



- We may think **Data** as any collection of numbers, characters, images, or other items that provide information about something.
- Statisticians are working with data.
- An example of data set.

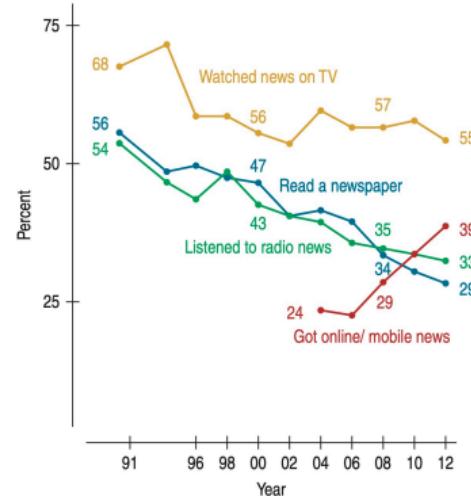
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3	Johnson	Tom	23/08/1972	France	02/11/2016	83	
4	Lewis	Peter	18/10/1979	Germany	22/11/2016	27	
5	Koenig	Edward	13/05/1983	Argentina	26/03/2015	43	
6	Preston	Jack	16/06/1991	United States	06/11/2016	77	
7	Smith	David	11/03/1965	Canada	15/11/2016	23	
8	Brown	Luis	03/09/1997	Australia	03/07/2015	74	
9	Miller	Thomas	07/01/1980	Germany	07/11/2016	13	
10	Williams	Bill	26/07/1966	United States	20/11/2015	80	
11	Gemini	Alexia	12/09/1995	Canada	11/03/2017	35	
12	Bond	James	25/02/1975	United Kingdom	12/08/2017	40	
13	Burgle	Patricia	01/12/1990	United States	18/01/2015	55	
14	Reding	Michelle	07/04/1985	Canada	23/02/2017	28	
15	Harvey	Billy	14/07/1971	United Kingdom	12/01/2016	41	
16							
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18							

Sample and Population

- **Target of a statistician:** know about an entire group of individuals.

However, the entire group is usually impossible to get access.

A timeplot showing the responses of 18–29 year olds to Pew polls asking them where they get most of their news about national and international news. ➔



■ Ideas of sampling:

- a. Examine a part of the whole:

Select a sample from the population for examining.

- b. Randomizing:

On average, the sample looks like the rest of the population.

- c. Large sample size:

Sample size is the number of individuals in the sample.

■ Simple random sampling.

The principle of simple random sampling is that every set of items has the same chance of being chosen.



Randomness



- Statistics and Data Science is based on the theory of “randomness”.
- This world is also filled with randomness.



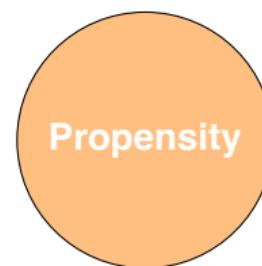
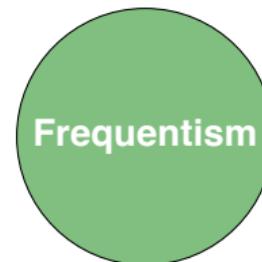
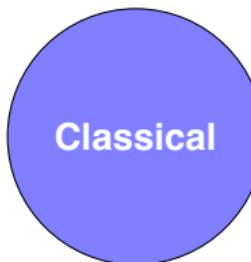
- How to understand “randomness”?
- The surprising fact is that in the long run, many truly random phenomena settle down in a way that's **consistent** and **predictable**.
- **Random phenomena**: we know what outcomes can possibly occur, but we don't know which particular outcome will happen.
- **Trials**: each occasion upon which we observe a random phenomenon is called a trial.
- **Events**: Any Combination of outcomes is called an event.
- **Sample Space**: The collection of all possible outcomes is called the sample space.

Explanations of probabilities

How likely does an event happen?



We use **probability** to measure how likely does an event happen.
There are several explanations of the term “probability”:



- Classical Probability: Pierre-Simon Laplace (拉普拉斯, 1749–1827).



Developed from studies of gambling it states that probability is shared equally between all the possible outcomes, provided these outcomes can be deemed equally likely.

- If a random experiment can result in N mutually exclusive and equally likely outcomes and if N_A of these outcomes result in the occurrence of the event A , the probability of A is defined by

$$P(A) = \frac{N_A}{N}.$$

- Examples:

Cards

The probability of drawing a face card (JQK) from a deck of playing cards (without Jokers) is

$$P(\text{face card}) = \frac{\# \text{ of face cards}}{\# \text{ of total cards}} = \frac{12}{52} = \frac{3}{13}.$$

Classical Probability

■ Limitations:

- (a) Only applicable to situations where there is only a “finite” number of possible outcomes.
- (b) Require a priori determination that all possible outcomes are equally likely.



Frequency probability



Frequentists posit that the probability of an event is its **relative frequency** over time (**the long-run probability**).

John Venn (韦恩, 1834–1923) provided a thorough exposition of frequentist probability in his book, *The Logic of Chance* (1866):



Probabilities can be found (in principle) by a repeatable objective process (and are thus ideally devoid of opinion).

Frequency probability



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Traffic lights

Day	1	2	3	4	5	6	...	100
Light	Green	Red	Green	Green	Red	Red	...	Red
% Green	100	50	66.7	75	60	50	...	33.35

Coin Tossing

In the case of tossing a fair coin, frequentists say that the probability of getting a heads is $1/2$, not because there are two equally likely outcomes but because repeated series of large numbers of trials demonstrate that the empirical frequency converges to the limit $1/2$ as the number of trials goes to infinity.



Definition 1 (Frequency probability)

Denote by n_A the number of occurrences of an event A in n trials, then if

$$\lim_{n \rightarrow \infty} \frac{n_A}{n} = p$$

we say that $P(A) = p$.

■ Limitations:

- (a) It is of course impossible to actually perform an infinity of repetitions of a random experiment to determine the probability of an event.
- (b) If only a finite number of repetitions, different values of probabilities for different trials.

- Bayesian probability is an interpretation of the concept of probability, in which probability is interpreted as reasonable expectation representing a state of knowledge or as quantification of a personal belief.
- The propensity probability is a probability interpretation in which the probability is thought of as a physical propensity, disposition, or tendency of a given type of situation to yield an outcome of a certain kind, or to yield a long-run relative frequency of such an outcome.
- Will be discussed later.