Homework 4 (Due October 17)

Grade Distribution (Total=8+8+8+8+12=44).

Please simply answer as much as possible.

1. Suppose that the probability mass function of X is given by

X	-3	-1	0	2	3	5
p(m)	0.20	0.08	0.40	0.10	0.02	0.20

   Find the probability mass function of $Y = X^2$, that is, calculate $P(Y = m)$.

2. Three fair dice (six-sided) are rolled. Let X denote the maximum of the three numbers on the dice and Y the minimum of the three numbers.
   (a) Find the probability mass function of X.
   (b) Find the probability mass function of Y.

3. We choose a number from the set $\{10, 11, 12, \cdots, 99\}$ uniformly at random.
   (a) Let X be the first digit and Y the second digit of the chosen number. Find the probability mass functions of X and Y. Show that for any $1 \le i \le 9$ and $0 \le j \le 9$,
   $$P(X = i, Y = j) = P(X = i) \times P(Y = j).$$
   (b) Let X be the first digit of the chosen number and Z the sum of the two digits. Show that there exist some $1 \le n \le 9$ and $n \le m \le 18$ such that
   $$P(X = n, Z = m) \ne P(X = n) \times P(Z = m).$$

4. Six distinct numbers are randomly distributed to players numbered 1 through 6. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find $P(X = i)$ for $i = 0, 1, 2, 3, 4, 5$.

5. 20 balls are to be distributed among 6 urns, with each ball going into urn i with probability p_i, $\sum_{i=1}^{6} p_i = 1$. Let X_i denote the number of balls that go into urn i. Assume that events corresponding to the locations of different balls are independent.
   (a) For each $1 \le i \le 6$, find the probability mass function of X_i.
   (b) For any $1 \le i < j \le 6$, find the probability mass function of $X_i + X_j$.
   (c) Find $P(X_2 + X_3 + X_4 = 7)$.