Discrete Mathematics for Computer Science

Lecture 1a: Course Content and Information

Dr. Ming Tang

Department of Computer Science and Engineering
Southern University of Science and Technology (SUSTech)
Email: tangm3@sustech.edu.cn

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What is This Course About?

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What is Discrete Mathematics?

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous.

- I.e., the part of mathematics devoted to the study of discrete objects.
- E.g., integers, graphs, statements in logic, ...

Thus, discrete mathematics excludes topics in "continuous mathematics", e.g., real numbers, calculus, ...

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Problems Solved Using Discrete Mathematics

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Problems Solved Using Discrete Mathematics

Logic and Proofs: How to prove a theorem?

	Statements	Reason
1	$\neg p \wedge q$	Premise (given)
2	$\neg p$	Simplification from (1)
3	$r \rightarrow p$	Premise (given)
4	$\neg r$	Modus Tollens from (2) and (3)
5	$\neg r \rightarrow s$	Premise (given)
6	s	Modus Ponens from (4) and (5)
7	$s \rightarrow t$	Premise (given)
8	t	Modus Ponens from (6) and (7)

QED

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Problems Solved Using Discrete Mathematics

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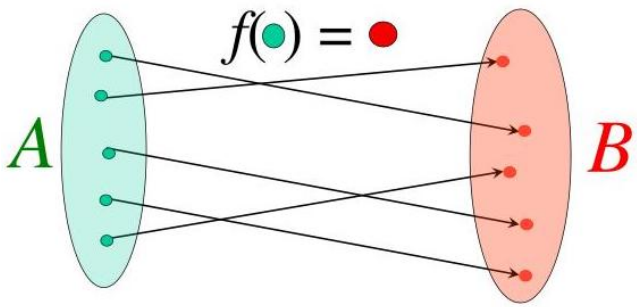
QED		

Set and Function: How to represent discrete objects?



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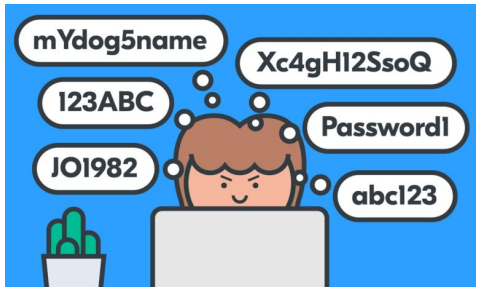
Problems Solved Using Discrete Mathematics

Counting: How many ways are there to choose a valid password on a computer system?



Problems Solved Using Discrete Mathematics

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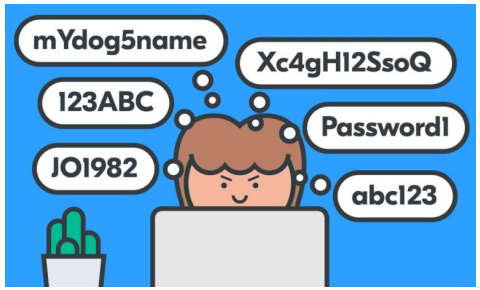

Number Theory and Cryptography: How can I encrypt a message so that no unintended recipient can read it?

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Problems Solved Using Discrete Mathematics

Discrete Probability: What is the probability of winning a lottery?

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Problems Solved Using Discrete Mathematics

Discrete Probability: What is the probability of winning a lottery?

How can I identify spam e-mail messages?

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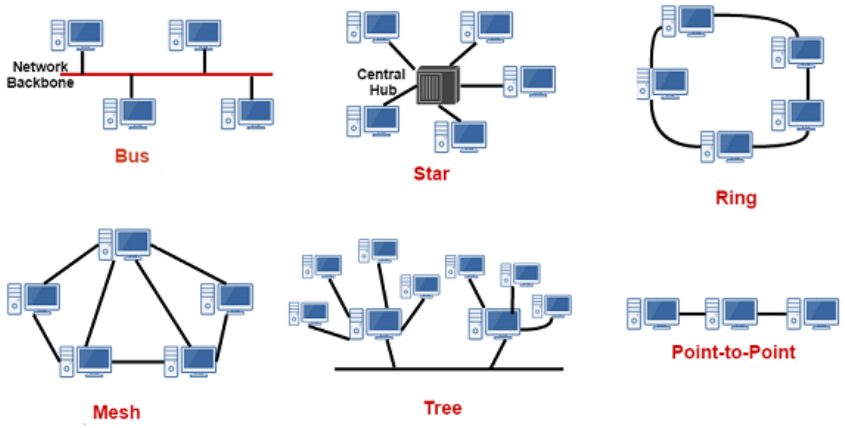
Problems Solved Using Discrete Mathematics

Graph: Is there a link between two computers in a network?



Problems Solved Using Discrete Mathematics

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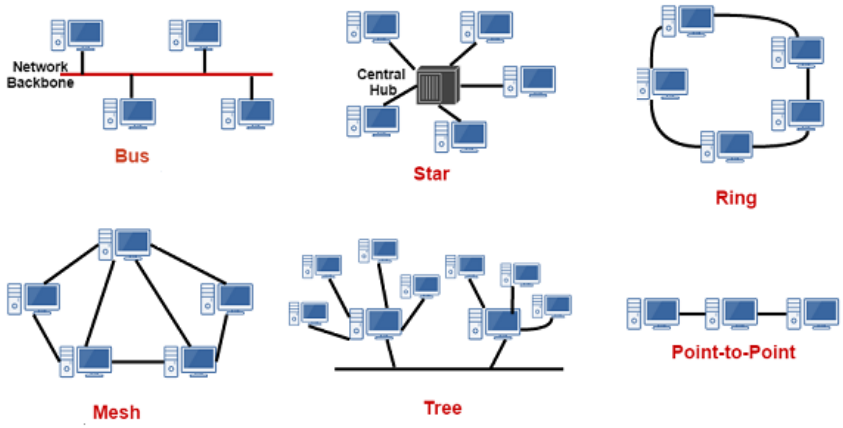
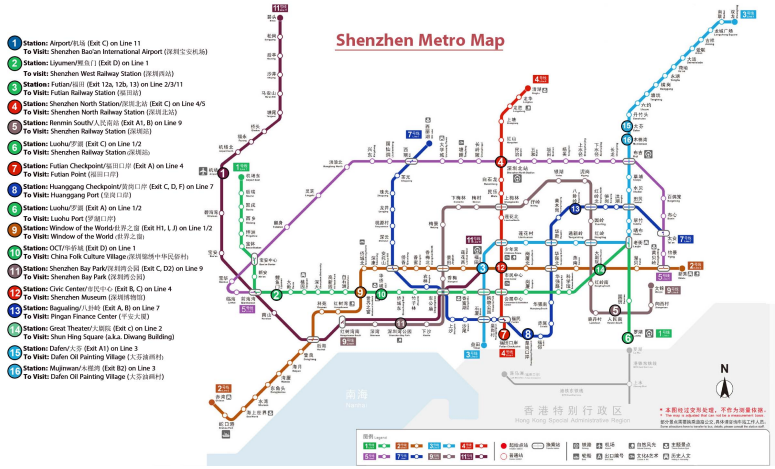


What is the shortest path between two locations?



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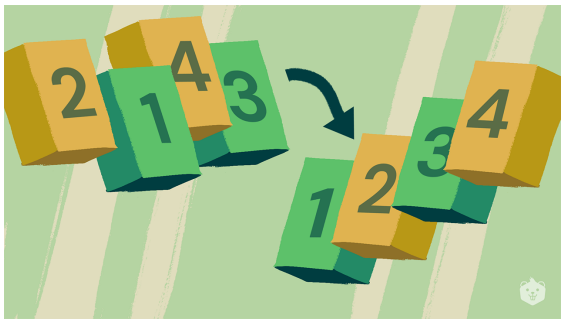
Problems Solved Using Discrete Mathematics

Algorithm and Complexity: How can a list of integers be sorted so that the integers are in increasing order?



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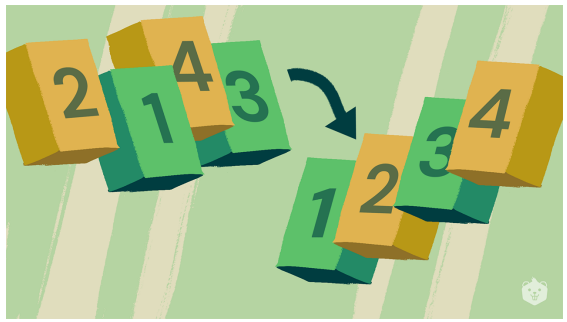

How many steps are required to do such a sorting?

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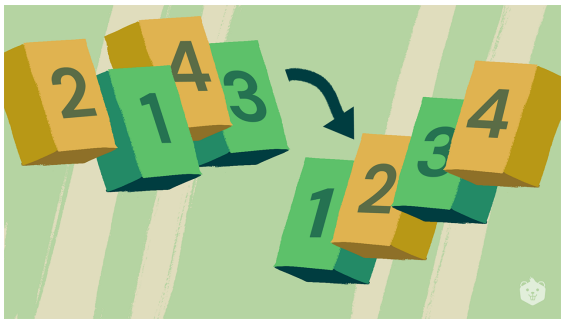

How many steps are required to do such a sorting?

How can it be proved that a sorting algorithm correctly sorts a list?

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Application Areas

- Computer science: algorithms, programming languages, cryptography, artificial intelligence, ...
- Electronic engineering: digital communications, signal processing, information theory, coding theory, ...

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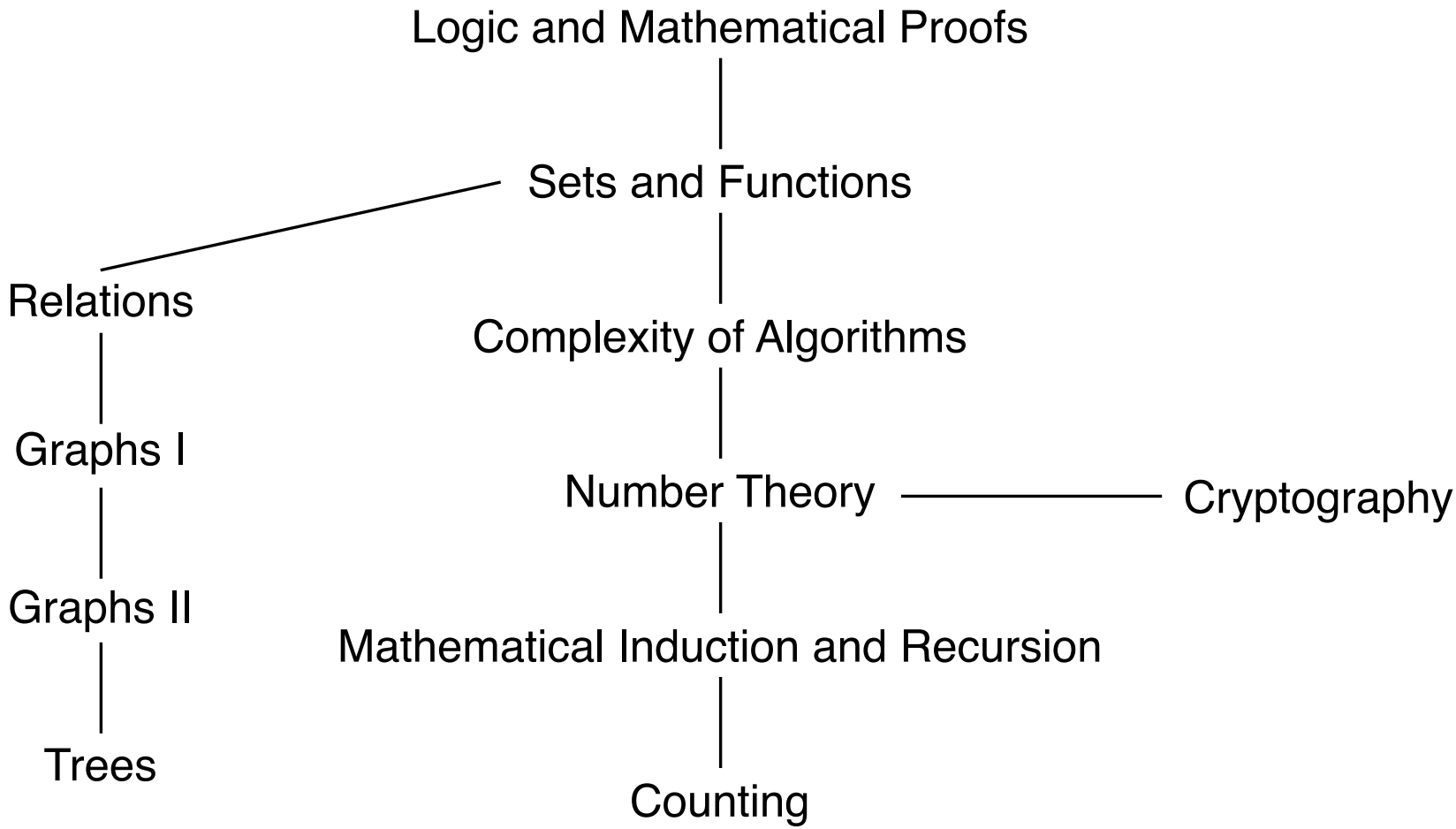
Topics of This Course



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Lecture Schedule (Tentatively)

1. Logic (2 lectures)
2. Mathematical Proofs (1 lecture)
3. Sets and Functions (1 lecture)
4. Complexity of Algorithms (1 lecture)
5. Number Theory (2.5 lecture)
7. Cryptography (0.5 lecture)
8. Midterm Exam
9. Mathematical Induction (1 lecture)
10. Recursion (1 lecture)
11. Counting (2 lectures)
13. Relations (2 lectures)
14. Graph I and II (3 lectures)
15. Trees (1 lecture)
16. Review (1 lecture)

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Learning Objectives

- Find research problems
- Formulate the problem in mathematics

- Read papers and materials

- Solve the problem

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Learning Objectives

- Find research problems
- Formulate the problem in mathematics
  - Understand the formulation of common problems in several areas of discrete mathematics, including counting, graphs, number theory, cryptography, logic and proof, recursions, probability theory, etc.
- Read papers and materials
  - Be able to read, understand, and construct mathematical arguments and proofs
- Solve the problem
  - Learn a number of discrete mathematical tools
  - Apply discrete mathematical tools to solve certain problems in computer science and electronic engineering

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Problem I: Mathematical Proof

Prove that "For an integer n, if $3n + 2$ is odd, then n is odd".

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Note that $3n + 3$ is even, and so is $n + 1$. It then follows that n is odd.

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Note that $3n+3$ is even, and so is $n+1$. It then follows that n is odd.

Proof II (proof by contrapositive):
It is equivalent to prove that "If n is even, then $3n+2$ is even." Why?
W.l.o.g., suppose that $n = 2k$ for some integer k, then $3n+2 = 6k+2$; which is even.

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W.l.o.g., suppose that $n = 2k$ for some integer k, then $3n+2 = 6k+2$; which is even.

Proof III (proof by contradiction): Assume to the contrary that there exists an integer n such that $3n+2$ is odd and n is even. Since n is even, so is $3n$. Then 2 must be odd, leading to a contradiction.

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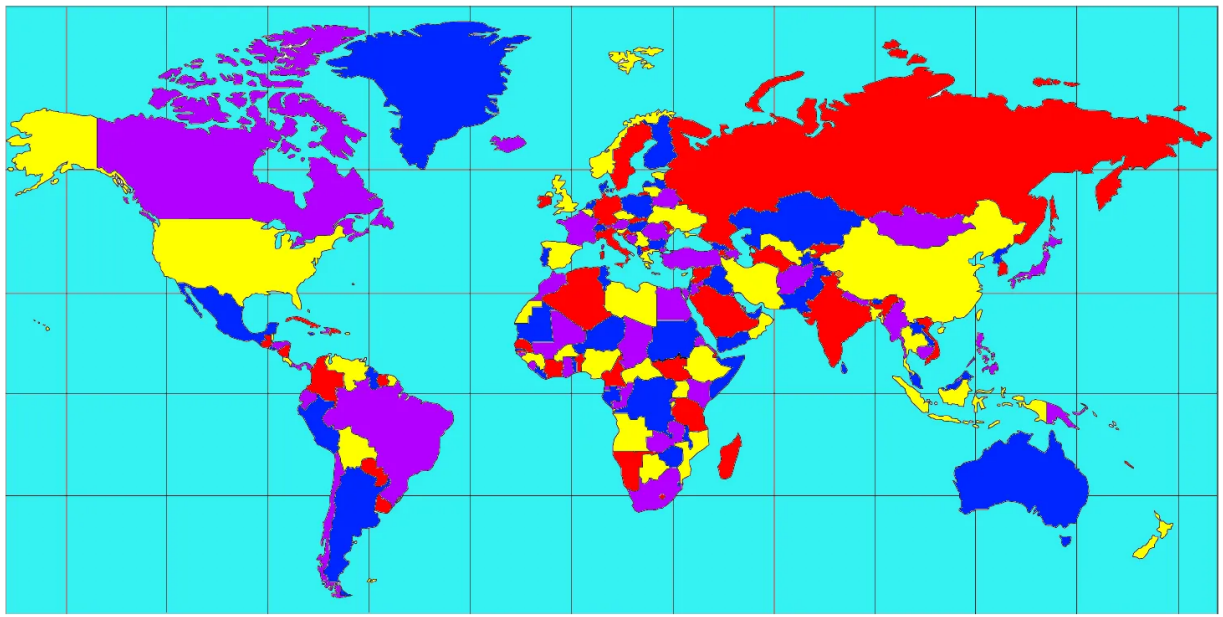
Problem II: Graph Colorings – Four-Color Theorem

Given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.



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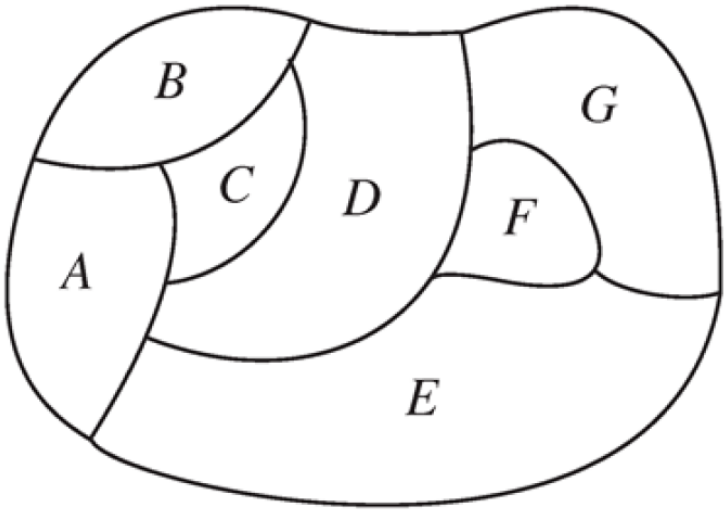

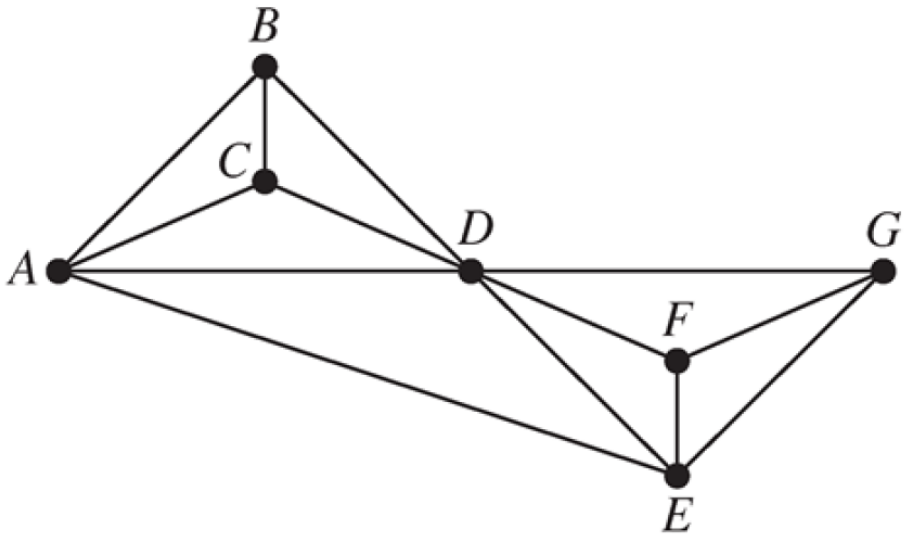

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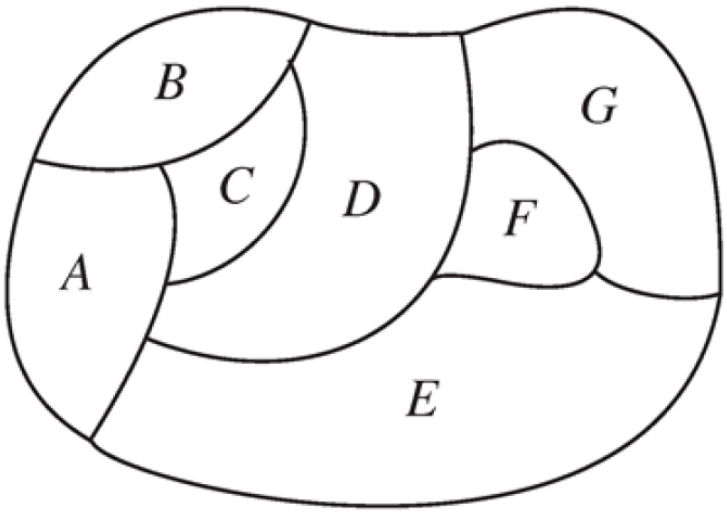

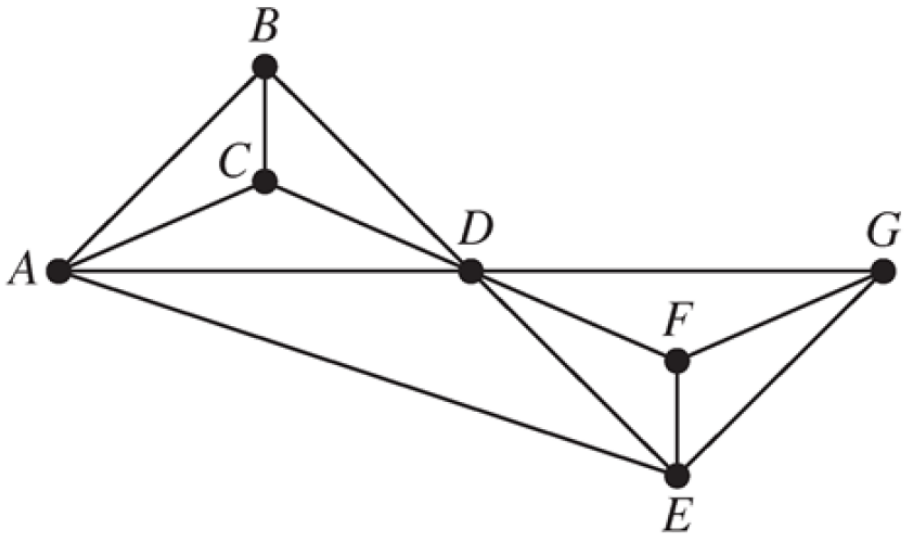

Four-color theorem holds for all planar graph.

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Problem II: Graph Colorings: Scheduling Final Exams

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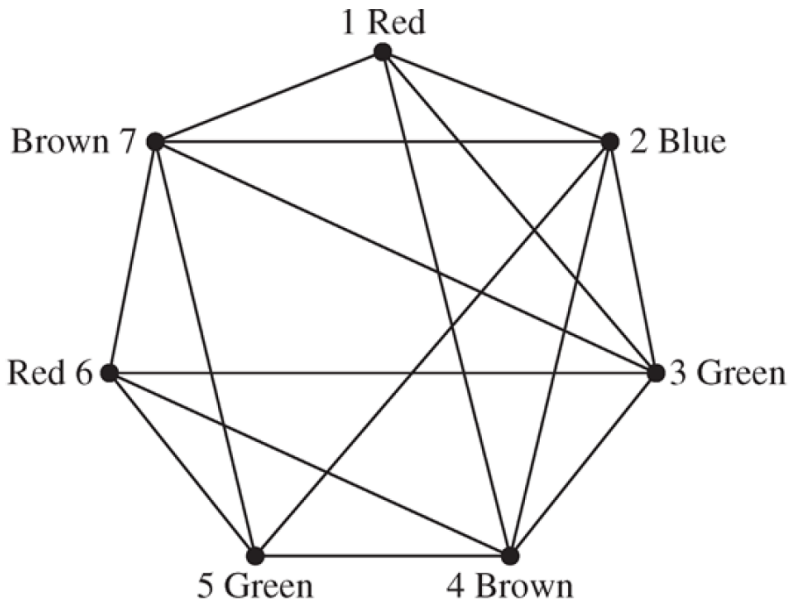
Vertices represent courses. There is an edge between two vertices if there exists at least one student taking both courses.



Time Period	Courses
I	1, 6
II	2
III	3, 5
IV	4, 7

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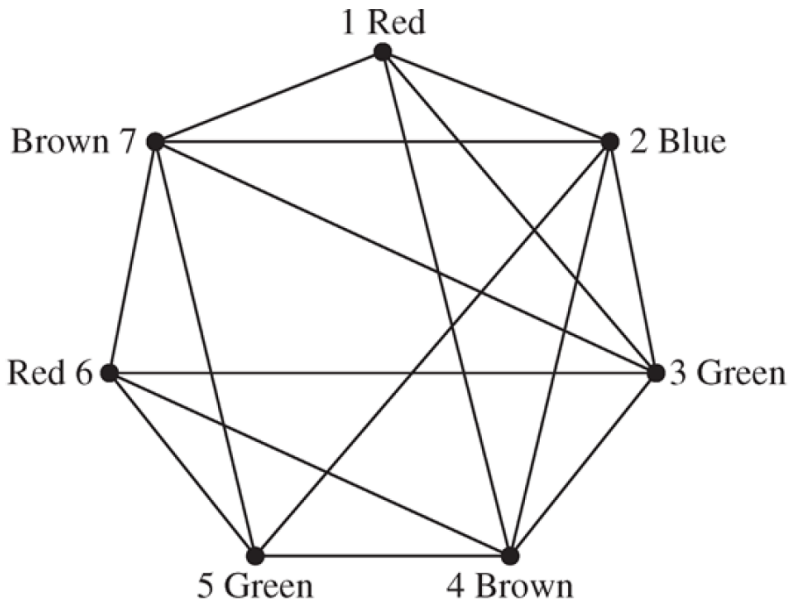

Time Period	Courses
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Graph coloring is computationally hard!!!

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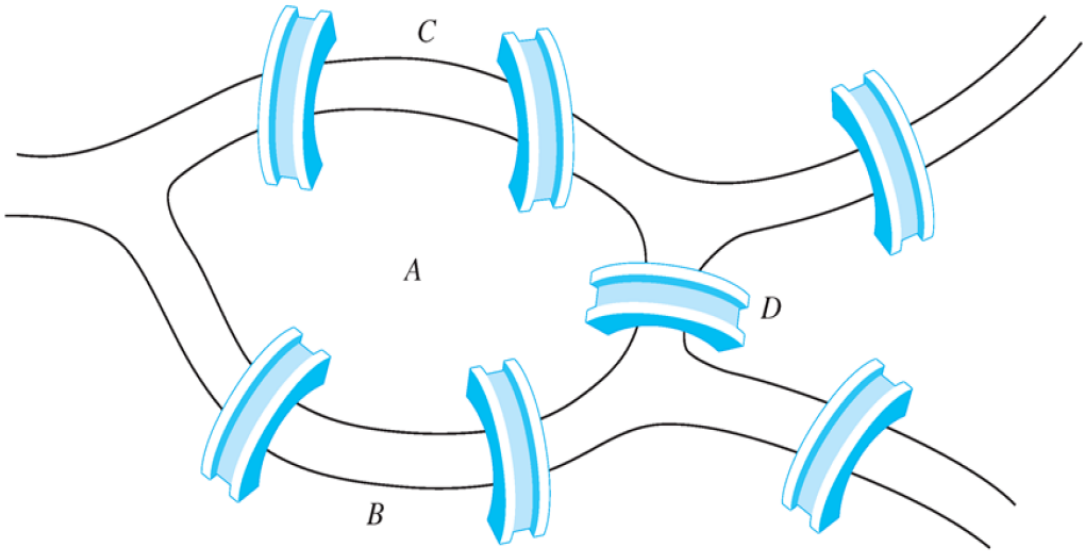
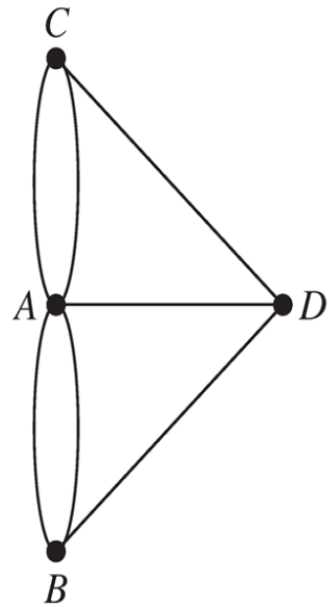
Problem III: Königsberg seven-bridge problem

People wondered whether it was possible to start at some location in the town, travel across all the bridges once without crossing any bridge twice, and return to the starting point.





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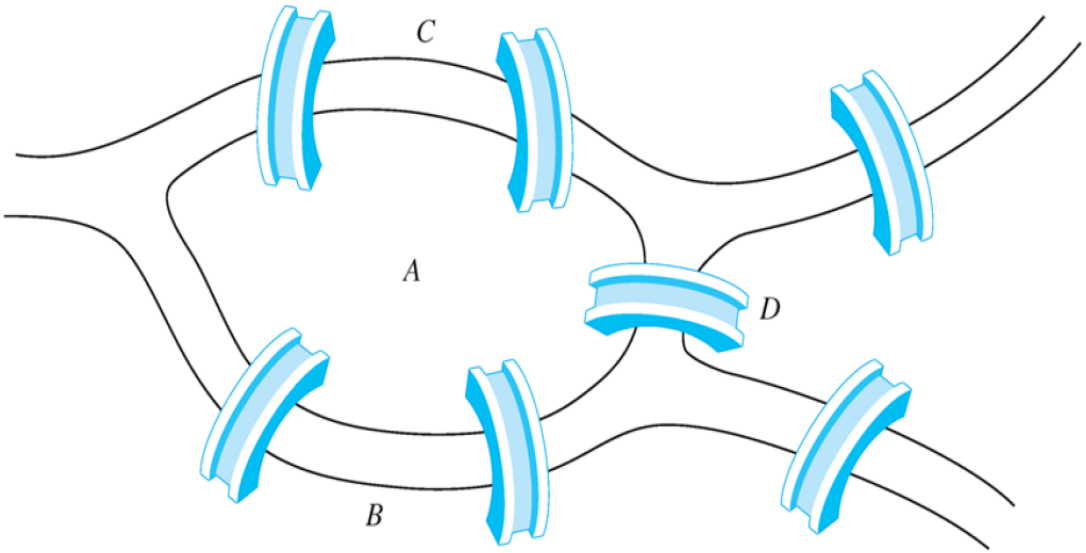

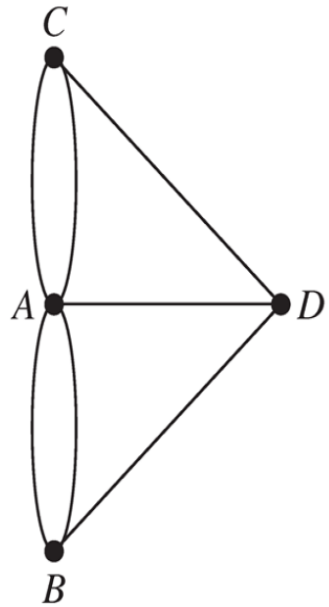

Have such a solution if only if each of its vertices has even degree.

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Problem IV: Fibonacci Number – Population of Rabbits

Reproducing pairs (at least two months old)	Young pairs (less than two months old)	Month	Reproducing pairs	Young pairs	Total pairs
		0	0	1	1
		1	0	1	1
		2	1	1	2
		3	1	2	3
		4	2	3	5

- A young pair of rabbits (one of each sex) is placed on an island.
- A pair of rabbits does not breed until they are 2 months old.
- After they are 2 months old, each pair of rabbits produces another pair each month. Assuming that no rabbits ever die.

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Recursion:

$\Rightarrow F_0 = 1,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$

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Recursion:

$\Rightarrow F_0 = 1,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$

$\Rightarrow$ What is the closed-form expression of F_n?

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Ready to Start?

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Before We Start: Course Information

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About This Course

- Instructor:

  Dr. Ming Tang
  Department of Computer Science and Engineering
  Office: Room 613, South Tower, CoE Building
  Email: tangm3@sustech.edu.cn

- Course Webpage:

  Blackboard: Discrete Mathematics Spring 2025 (not yet created)
  QQ Chat Group: 910716288

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About This Course

- Lectures:
  - Room 106 in Teaching Building 3
  - Monday 19:00–20:50 (every week)
  - Wednesday 19:00–20:50 (every two weeks)
- Office hour:
  - Friday 15:00–17:00 (send an email for an appointment)
  - Or other time periods (send an email for an appointment)
- **Midterm: 2025-04-15 (Tentative)**

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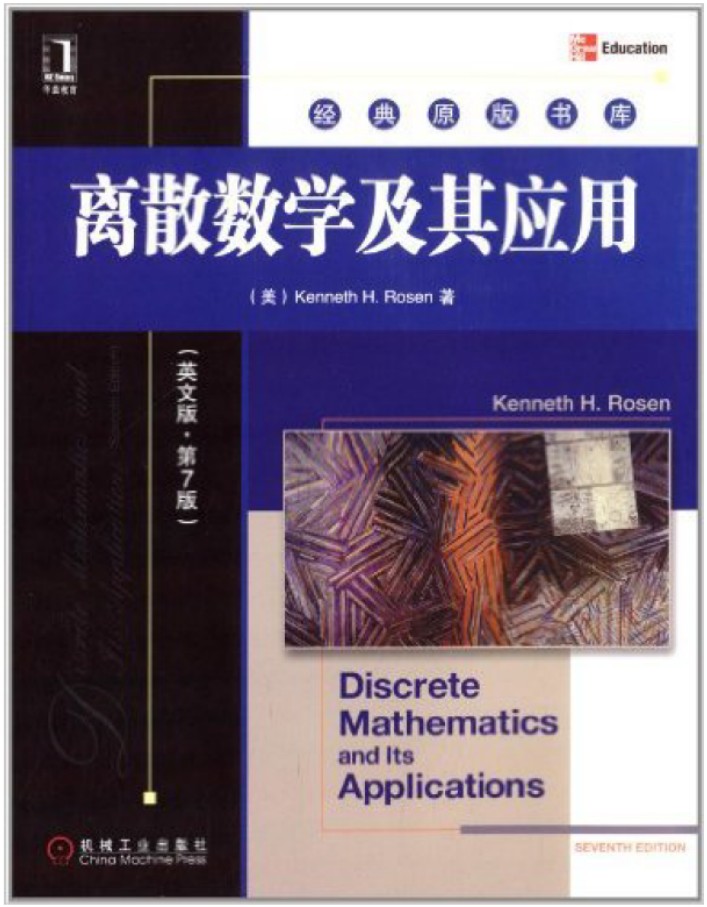
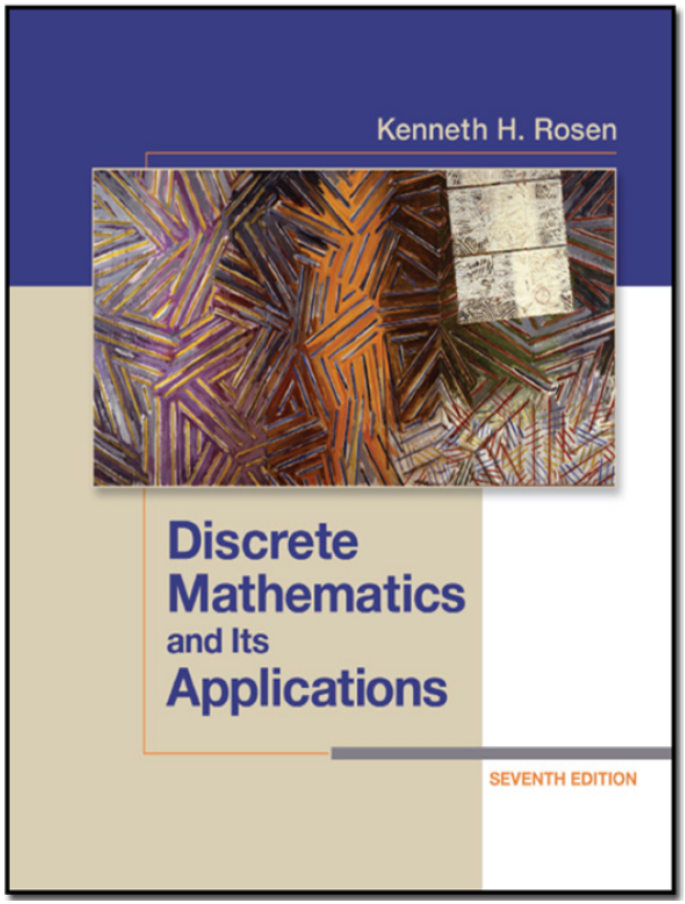
Textbook





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Marking Scheme

- Quiz in class: twice (10%)
- Homework assignments (20%)
  - Six assignments in total
  - Assigned in class and posted on the course webpage
  - Must be submitted online before due date
  - Overdue policy: 80% within 24 hours; 60% within 48 hours; 0 after 48 hours (No Exception)
- Midterm exam (30%)
  - Covers the first half of the course materials
- Final exam (40%)
  - Cover the entire semester's materials
- Project (5%, optional): ranging from -1% to 5%
  - I like the topic of this project, while I do not recommend you to work on this project unless you are very interested in it.
  - You are welcome to discuss this project with me during office hour.

Plagiarism Policy

- Homework assignments should be worked on and written up individually, though group discussions are allowed.
- Any unintellectual behavior and cheating on homework assignments and exams will be dealt with severely.
- If you get the main idea for a solution from someone else or a website, you must acknowledge that source in your submission.

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Plagiarism Policy

Plagiarism in an assignment or a quiz:

- For the first time: the score of the assignment or quiz will be zero
- For the second time: the score of the course will be zero

When two assignments are nearly identical, it may be difficult to tell who actually wrote it. Thus, the policy will apply to BOTH students, unless one confesses having copied without the knowledge of the other.

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What is OK and what is not OK?

- It is OK to discuss an assignment with a friend and share ideas. At the time of actual writing, you should write it alone.
- It is OK to get the main idea for a solution from the web, as long as you acknowledge the source. At the time of actual writing, you should write answers on your own instead of copying from the web.
- It is OK to show your assignment to friends to explain the logic, as long as the friends write their assignment on their own later.
- It is OK to help friends debug their programs. You will probably learn a lot by doing so.

It is **NOT OK** to take the assignment of a friend, make a few cosmetic changes (variable names), and pass it as your work.

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Other Important Messages

- You are encouraged to ask questions in class:
  - If you're having trouble understanding something, then at least 50% of the class is also having trouble: they'll be happy if you ask.
- You are welcome to provide suggestions and comments:
  - If you find mistakes or just think that something's confusing, please email me. Your classmates and future students will thank you.

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Acknowledgements

- Most slides were prepared based on the lecture notes and slides from Prof. Qi Wang in CS Department at SUSTech
- Some slides were prepared based on lecture materials used in the following institutions:

香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

STANFORD UNIVERSITY

Massachusetts Institute of Technology

UNIVERSITY OF PITTSBURGH

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