Homework-7

November 1, 2024

1. Prove that if G is a simple group with order 60, then $G \simeq A_5$.

2. (1). Prove there exists no such group G satisfying $G' \simeq S_3$;
(2). Prove there exists no such group G satisfying $G' \simeq S_4$.

3. Prove that all 3-cycles in group $A_n (n \geq 5)$ can be represented by a commutator in A_n. Then prove $A_n' = A_n$.

4. Let p be a prime number, $F = \mathbb{Z}/p\mathbb{Z}$, $G = \mathrm{GL}_n(F)$. Write a specific Sylow p-subgroup of G.

5. Let G be a group, $H \trianglelefteq G$, $K \trianglelefteq G$ and $H \cap K = 1$, prove that $\forall h \in H$ and $k \in K$, $hk = kh$.

6. Let G be a finite group. Prove the minimal normal subgroup of G is a direct product of several (maybe 1) isomorphic simple groups.

7. Prove for any prime p, there exists exact 2 different types of non-abelian groups up to isomorphic.

8. Let G be of order p^3q with $p < q$, p, q are different primes. Prove G is not simple.

9. Let $G = A_4$. Write G into a semidirect product of 2 subgroups.

10. Let $G = S_4$. Write G into a semidirect product of 2 subgroups.