HOMEWORK 3

In the next 4 problems, use the Schwarz reflection principle.

1. Find a conformal mapping, transforming the domain $D := \{|z| > 1\} \setminus \big([-2,-1] \cup [i, 2i] \cup [-2i, -i]\big)$ onto Π^+.

2. Find a conformal mapping, transforming the domain $D := \{-2 < \operatorname{Im} z < 2\} \setminus \{\operatorname{Im} z = \pm 1, \operatorname{Re} z \leq 0\}$ onto Π^+.

3. Find a conformal mapping, transforming the domain $D := \overline{\mathbb{C}} \setminus \big([0,1] \cup [0, e^{2\pi i/3}] \cup [0, e^{4\pi i/3}]\big)$ onto Π^+.

4. Detemine the group $\operatorname{Aut}(\Omega)$, where $\Omega = \{1 < |z| < 2\}$ (you can use the Caratheodory Theorem for admissible domains).

5. Find the index w.r.t. 0 of the parameterized curve $f(C)$, where $f(z) = z^3 + 2z$ and C is the standardly parameterized circle $\{z = e^{it},\ 0 \leq t \leq 2\pi\}$.

6. How many roots (with multiplicities) does the equation $z^6 - 6z + 10 = 0$ have in the domain $\{|z| > 2\}$?

7. Show that the equation $z^4 + z^3 - 4z + 1 = 0$ has exactly 3 roots in the annulus $\{1 < |z| < 2\}$.

8. Prove that the equation $\tan z = z$ has only real roots.