

Homework 7 (Due November 14)

Grade Distribution (Total=4+4+4+6=18).

1. (4 points) The joint density of X and Y is

$$f(x, y) = c(x^2 - y^2)e^{-x}, \quad 0 < x < \infty, -x \leq y \leq x.$$

Find the conditional distribution of Y , given $X = x$.

2. (4 points) A television store owner figures that 45 percent of the customers entering his store will purchase an ordinary television set, 15 percent will purchase a plasma television set, and 40 percent will just be browsing. If 5 customers enter his store on a given day, what is the probability that he will sell exactly 2 ordinary sets and 1 plasma set on that day?
3. Suppose that X and Y are independent geometric random variables with the same parameter p .

- (a) (2 points) Without any computations, what do you think is the value of

$$P(X = i | X + Y = n)$$

[Hint: Imagine that you continually flip a coin having probability p of coming up heads. If the second head occurs on the n -th flip, what is the probability mass function of the time of the first head?]

- (b) (2 points) Verify your conjecture in part (a).

4. A model proposed for NBA basketball supposes that when two teams with roughly the same record play each other, the number of points scored in a quarter by the home team minus the number scored by the visiting team is approximately a normal random variable with mean 1.5 and variance 0.6. In addition, the model supposes that the point differentials for the four quarters are independent. Assume that this model is correct. You may express the following answers in terms of some integrals regarding $\varphi(x)$ with $\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$.

- (a) (2 points) What is the probability that the home team wins?

- (b) (2 points) What is the conditional probability that the home team wins, given that it is behind by 5 points at halftime?

- (c) (2 points) What is the conditional probability that the home team wins, given that it is ahead by 5 points at the end of the first quarter?