GINav User Manual

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1. Introduction

GINav is an open-source software, which focuses on the data processing and analysis of GNSS/INS integrated Navigation system, and can also process multi-constellation and multi-frequency GNSS data. GINav is suitable for in-vehicle scenarios and is aimed at providing a useful tool for carrying out GNSS/INS-related research. It is a convenient platform for testing new algorithms and experimental functionalities. GINav is developed in MATLAB environment. It provides the user-friendly graphical user interface (GUI) to facilitate users to quickly use it, and a visualization tool, GINavPlot, is also provided for solution presentation and error analysis.

The main features of the software are:

- Support GNSS/INS loosely coupled (LC) modes, include standard single positioning (SPP)/INS LC, post-processing differenced (PPD)/INS LC, post-processing kinematic (PPK)/INS LC and precise point positioning (PPP)/INS LC
- Support GNSS/INS tightly coupled (TC) modes, include SPP/INS TC, PPD/INS TC, PPK/INS TC and PPP/INS TC
- INS-aided cycle slip detection and robust estimation for GNSS/INS integration
- Support multi-constellation and multi-frequency GNSS data processing
- Support GNSS absolute positioning modes, include SPP and PPP
- Support GNSS relative positioning modes, include PPD, PPK and post-processing static (PPS)
- Convenient visualization

2. Requirements

GINav is developed and tested in MATLAB version 2016a. For this reason, MATLAB version 2016a or newer is required for running GINav. GINav can be used on Windows or Unix/Linux platform, but Windows is preferred because it is developed on Windows platform. Furthermore, GINav uses LAMBDA v3.0 toolbox to resolve ambiguity. If you use PPK, PPS or PPK/INS mode to process data, please download and install the lambda-3.0.zip file from http://gnss.curtin.edu.au/research/lambda.cfm.

3. Installation

The installation steps of GINav are as follows:

(1). Unzip GINav.zip to the GINav folder

(2). Unzip lambda-3.0.zip and place the lambda-3.0 folder into the **GINav\3rd** folder

(3). Open the MATLAB application and set the current folder to the unzipped GINav folder

4. Quick Start

(1). Run the script file, **GINav\GINavExe.m**, to configure the input file. The GUI that used to configure the input file is shown in **Figure 1**.

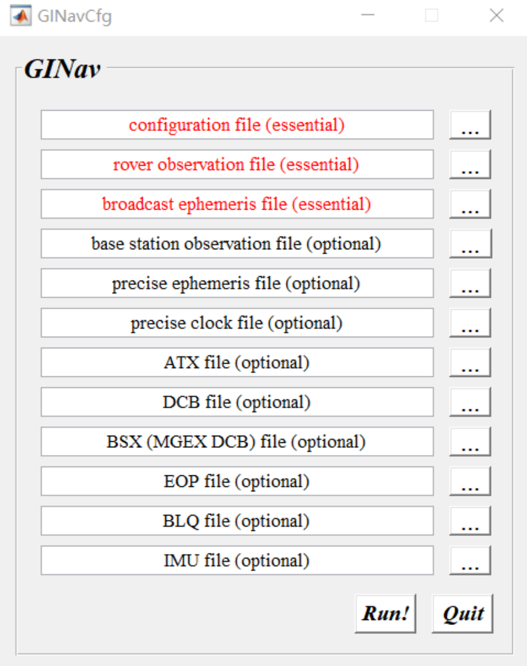


Figure 1 The GUI for configuration of input file

(2). Input the configuration file path in the text field "configuration file (essential)". You can fill in the file path manually or select a file using the corresponding selection dialog by pushing the ... button. After entering the configuration file path, the program decodes the configuration file and automatically matches the required files based on the options in the configuration file. The **Figure 2** shows the matched files. Automatic file matching function is convenient and useful, but the configuration file needs to be set according to the requirements of GINav. The configuration file setting will be described in detail in the **Section 5**. In addition, the following points should be emphasized:

- The automatic file matching function may fail to match the required file or match the wrong one due to some reasons (such as incorrect settings in the configuration file). In this case, you can manually select the required file by pushing the corresponding ... button.
- If you choose to use the absolute positioning mode, the base station observation file will not be automatically matched.
- If you choose to use the broadcast ephemeris, the precise ephemeris and clock file will not be automatically matched.
- If you choose SPP or GNSS/INS SPP processing mode, the ATX will not be automatically matched.
- BLQ file will be automatically matched only when you choose PPP mode.
- IMU file will be automatically matched only when you choose GNSS/INS integration mode.

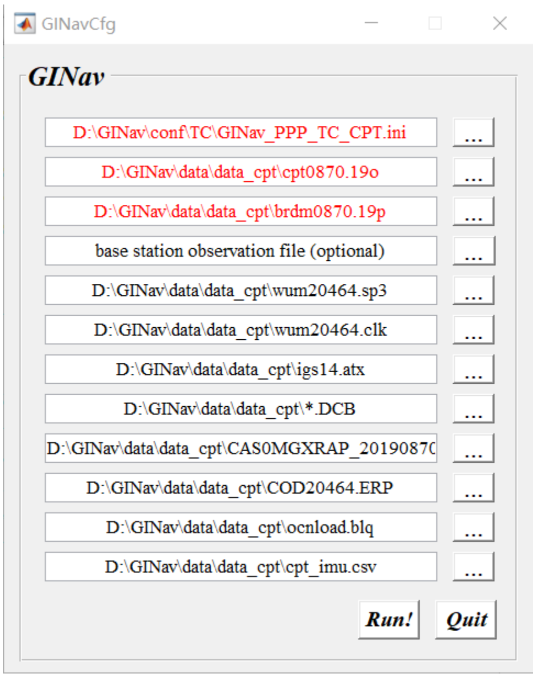


Figure 2 The input file matched by the GINav

(3). Push Run! button to start the processing. GINav first reads the input file, and the progress of data reading will be displayed in the MATLAB command window, as shown in **Figure 3**. When the data preparation is complete, GINav will pop up the progress bar and plot the dynamic trajectory, as shown in **Figures 4** and **Figure 5**.

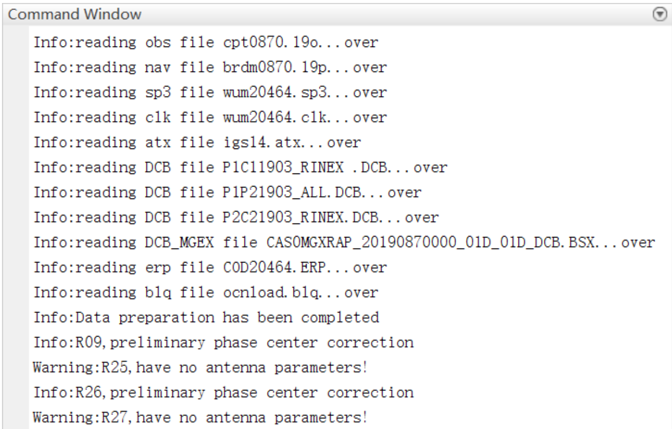


Figure 3 Input file reading progress

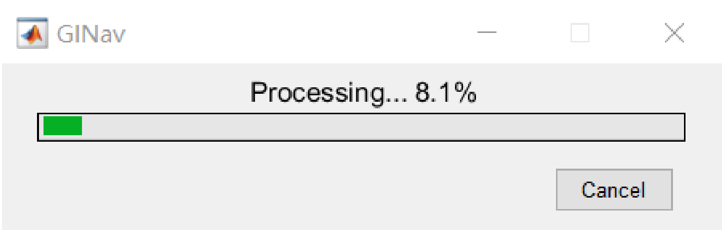


Figure 4 Progress bar

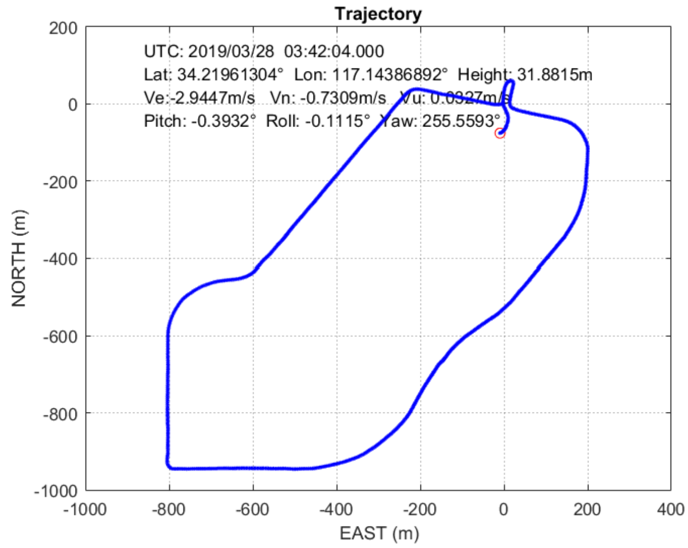


Figure 5 Dynamic trajectory during the processing

(4). After completing the processing, you can obtain a solution file, namely pos file that saved in **GINav\result**. A pos file example are shown in **Figure 6**.

```
% program    : GINav v0.1.0
% obs start : 2019/03/28 03:28:10.0
% obs end   : 2019/03/28 03:59:01.0
% mode      : PPK/INS TC
% system    : GPS
% nfreqs    : 1
% elev mask : 10.0 deg
% ephemeris : broadcast
% ionos opt : broadcast model
% trops opt : Saastamoinen model
%
%  GPST              x-ecef(m)      y-ecef(m)      z-ecef(m)   Q  ns  sdx(m)  sdy(m)  sdz(m)  sdxy(m)  sdyz(m) sdzx(m) age(s) ratio  vx(m/s)   vy(m/s)   vz(m/s)     sdvx
2046 358097.000 -2408691.0763 4698108.5138 3566698.3480 3  0 25.9956 29.0088 29.8845 -10.7365 1.3054 -0.9347 0.00   0.0 1.46918 0.91718 -0.16143 10.00000 10.
2046 358098.000 -2408689.3208 4698109.4442 3566698.0148 1  8  0.0074  0.0122  0.0099  -0.0073 0.0084 -0.0049 0.00   3.3 1.61537 0.97224 -0.27464  9.32036 9.
2046 358099.000 -2408687.6556 4698110.6293 3566697.5997 1  8  0.0074  0.0122  0.0099  -0.0073 0.0084 -0.0049 0.00   3.4 1.67596 1.12481 -0.38600  0.34181 0.
2046 358100.000 -2408685.7925 4698111.8309 3566697.2318 1  8  0.0074  0.0122  0.0099  -0.0073 0.0084 -0.0049 0.00   3.9 1.96597 1.27996 -0.38989  0.20136 0.
2046 358101.000 -2408683.7442 4698113.1974 3566696.8814 1  8  0.0074  0.0122  0.0099  -0.0073 0.0084 -0.0049 0.00   4.6 2.12041 1.26139 -0.20417  0.15347 0.
2046 358102.000 -2408681.5953 4698114.5226 3566696.6636 1  8  0.0074  0.0122  0.0099  -0.0073 0.0084 -0.0049 0.00   4.7 2.24030 1.18382 -0.03646  0.13223 0.
2046 358103.000 -2408679.2843 4698115.7798 3566696.6439 1  8  0.0074  0.0122  0.0099  -0.0073 0.0083 -0.0049 0.00 540.9 2.39688 1.21006  0.06662  0.02099 0.
2046 358104.000 -2408676.7782 4698116.9932 3566696.8079 1  8  0.0072  0.0119  0.0096  -0.0071 0.0080 -0.0047 0.00 985.7 2.60979 1.22133  0.25547  0.01618 0.
2046 358105.000 -2408674.1017 4698118.1985 3566697.1127 1  9  0.0071  0.0118  0.0095  -0.0070 0.0080 -0.0047 0.00 999.9 2.74377 1.19629  0.37761  0.01136 0.
2046 358106.000 -2408671.2532 4698119.3463 3566697.5496 1  8  0.0070  0.0115  0.0093  -0.0068 0.0078 -0.0046 0.00 999.9 2.94318 1.14897  0.42427  0.00870 0.
2046 358107.000 -2408668.2829 4698120.5100 3566698.0840 1  8  0.0068  0.0111  0.0090  -0.0066 0.0075 -0.0044 0.00 999.9 2.99243 1.11902  0.56249  0.00722 0.
2046 358108.000 -2408665.2830 4698121.6508 3566698.6791 1  8  0.0066  0.0107  0.0087  -0.0063 0.0072 -0.0042 0.00 999.9 2.98811 1.14580  0.58784  0.00627 0.
2046 358109.000 -2408662.2887 4698122.8170 3566699.2770 1  8  0.0064  0.0104  0.0085  -0.0061 0.0070 -0.0040 0.00 999.9 2.99629 1.16988  0.61206  0.00545 0.
2046 358110.000 -2408659.2661 4698123.9256 3566699.9011 1  7  0.0065  0.0127  0.0083  -0.0069 0.0074 -0.0041 0.00 999.9 3.03686 1.07840  0.65393  0.00493 0.
2046 358111.000 -2408656.2319 4698125.0417 3566700.5349 1  8  0.0064  0.0135  0.0082  -0.0072 0.0076 -0.0041 0.00 999.9 3.06200 1.14738  0.57125  0.00459 0.
2046 358112.000 -2408653.1464 4698126.2092 3566701.1376 1  8  0.0063  0.0129  0.0080  -0.0069 0.0074 -0.0040 0.00 999.9 3.10497 1.17135  0.62704  0.00418 0.
2046 358113.000 -2408650.0420 4698127.3918 3566701.7307 1  7  0.0062  0.0130  0.0079  -0.0069 0.0073 -0.0040 0.00 999.9 3.11562 1.21010  0.55533  0.00409 0.
2046 358114.000 -2408646.9444 4698128.6106 3566702.2813 1  6  0.0068  0.0132  0.0081  -0.0073 0.0075 -0.0046 0.00 999.9 3.09662 1.20559  0.57503  0.00420 0.
2046 358115.000 -2408643.8563 4698129.7940 3566702.8760 1  6  0.0070  0.0131  0.0081  -0.0074 0.0075 -0.0048 0.00 999.9 3.10533 1.13361  0.63620  0.00416 0.
2046 358116.000 -2408640.7360 4698130.9256 3566703.5450 1  6  0.0070  0.0129  0.0081  -0.0074 0.0075 -0.0048 0.00 999.9 3.16475 1.11069  0.71471  0.00416 0.
2046 358117.000 -2408637.5637 4698132.0771 3566704.2411 1  8  0.0070  0.0128  0.0080  -0.0073 0.0073 -0.0047 0.00 999.9 3.18685 1.13697  0.67515  0.00399 0.
2046 358118.000 -2408634.3763 4698133.2625 3566704.8933 1  6  0.0069  0.0127  0.0079  -0.0072 0.0072 -0.0046 0.00 999.9 3.18419 1.16966  0.68309  0.00392 0.
2046 358119.000 -2408631.1869 4698134.4297 3566705.5544 1  7  0.0069  0.0125  0.0079  -0.0071 0.0071 -0.0046 0.00 999.9 3.18526 1.15164  0.66800  0.00389 0.
2046 358120.000 -2408628.0130 4698135.5812 3566706.2535 1  7  0.0067  0.0124  0.0078  -0.0069 0.0069 -0.0044 0.00 999.9 3.17278 1.11963  0.73700  0.00386 0.
2046 358121.000 -2408624.8297 4698136.7296 3566706.9573 1  7  0.0067  0.0123  0.0077  -0.0069 0.0068 -0.0044 0.00 999.9 3.21101 1.13517  0.66319  0.00383 0.
2046 358122.000 -2408621.6333 4698137.8802 3566707.6645 1  7  0.0066  0.0122  0.0077  -0.0068 0.0067 -0.0043 0.00 999.9 3.19946 1.14420  0.75804  0.00379 0.
2046 358123.000 -2408618.3873 4698139.0647 3566708.3602 1  7  0.0065  0.0121  0.0076  -0.0067 0.0066 -0.0041 0.00 999.9 3.24540 1.20123  0.67801  0.00378 0.
2046 358124.000 -2408615.1267 4698140.2922 3566709.0097 1  7  0.0064  0.0120  0.0075  -0.0066 0.0065 -0.0040 0.00 999.9 3.25627 1.26410  0.64683  0.00390 0.
2046 358125.000 -2408611.8642 4698141.5687 3566709.5956 1  7  0.0063  0.0119  0.0075  -0.0065 0.0064 -0.0039 0.00 999.9 3.26222 1.26537  0.57896  0.00405 0.
2046 358126.000 -2408608.6510 4698142.8400 3566710.1483 1  7  0.0059  0.0115  0.0073  -0.0060 0.0060 -0.0033 0.00 314.8 3.20305 1.24474  0.53848  0.00399 0.
2046 358127.000 -2408605.4763 4698144.0817 3566710.7221 1  7  0.0059  0.0115  0.0073  -0.0059 0.0059 -0.0032 0.00 604.7 3.19563 1.21039  0.63921  0.00383 0.
```

Figure 6 A pos file example obtained from GINav.

(5). Run the script file, **GINav\ Plot_Analysis.m**, to plot and analyze the result. **Figure 7** show the dialog of GINavPlot. More details and usages about GINavPlot can be found in **Section 6**.

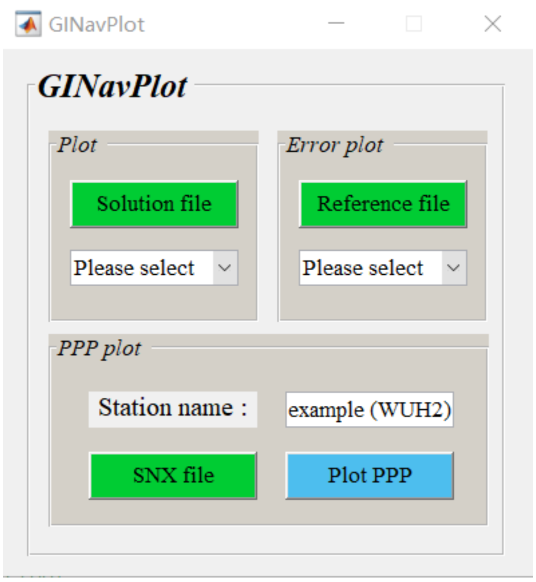


Figure 7 The dialog of GINavPlot

5. Configuration file

Properly configuring the options in the configuration file is a crucial step to running GINav successfully and getting good results. The following describe each option in more detail:

Options	Descriptions	Notes
data_dir	The path of raw data file	All files that need to be automatically matched should be placed under this path. The name of the base station observation file should include the keyword "base", and the name of the IMU data file should include the keyword "imu". GINav does not support IMU file formats of any manufacturer, and you need to save IMU data in the format GINav requires. A detailed description of the IMU data format can be found in the **Appendix**.
site_name	Specified site name	The file name of both rover and base station should include this keyword, otherwise the file automatic matching cannot be completed.
start_time	The start time of processing 0: the start time will be set at the first epoch of the specific observation file, 1: the start time is set in the	

	configuration file	
end_time	The end time of processing 0:the end time will be set at the last epoch of the specific observation file, 1:the end time is set in the configuration file.	
t_interval	The time interval of processing 0:the time interval will be set according to observation file, non-zero: specified interval	
gnss_mode	GNSS positioning mode 1:SPP 2:PPD 3:PPK 4:PPS 5:PPP KINE 6:PPP STATIC	
navsys	Specified navigation system G:GPS R:GLONASS E:GALILEO C:BDS J:QZSS	
nfreq	The number of used frequencies	Max frequency is 3
elmin	Elevation mask angle (deg)	
sateph	Satellite ephemeris/clock 1:broadcast ephemeris 2:precise ephemeris	
ionoopt	Ionosphere correction 0:off 1:broadcast model 2:iono-free LC 3:estimation	SPP mode only support 0,1
tropopt	Troposphere correction 0:off 1:Saastamoinen model 2:ZTD estimation 3:ZTD+grid	SPP mode only support 0,1
dynamics	Dynamics model 0:off 1:on	SPP, PPS and PPP STATIC modes do not support dynamics model
tidecorr	Earth tide correction 0:off 1:on	SPP do not support earth tide correction
armode	Ambiguity resolution (AR) mode 0:off 1:continuous 2:instantaneous 3:fix and hold	only PPK or PPS modes support AR, PPP mode does not support AR
gloar	GLONASS AR mode 0:off 1:on 2:auto calibration:receiver inter-channel bias terms are estimated	
bdsar	BDS AR mode 0:off 1:on	
elmaskar	Elevation mask of AR (deg)	
elmaskhold	Elevation mask to hold ambiguity(deg)	
LAMBDAtype	LAMBDA algorithm	

	1:all AR, 2:part AR (PAR)	
thresar	AR threshold [1] AR ratio test [2] success rate threshold of PAR	
bd2frq	specified the used frequency order for BDS-2	For example, 1,3,2 or 1,2,3. [1]B1 [2]B2 [3]B3
bd3frq	Specified the used frequency order for BDS-3	For example, 1,3,4 or 1,3,5 [1]B1 [3]B3 [4]B1C [5]B2a [6]B2b
gloicb	GLONASS inter-frequency code bias 0: off 1: linear 2: quadratic	
gnsproac	GNSS precise product AC 1: wum 2: gbm 3: com 4: grm	only the DCB correction of BDS3 need this option
posopt	Positioning options 0: off 1: on [1] satellite PCV [2] receiver PCV [3] phase wind up [4] reject GPS Block IIA [5] RAIM FDE [6] handle day-boundary clock jump [7] gravitational delay correct	
maxout	Obs outage count to reset ambiguity	
minlock	Min lock count to fix ambiguity	
minfix	Min fix count to hold ambiguity	
niter	Number of filter iteration	
maxinno	Reject threshold of innovation(m)	
maxgdop	Reject threshold of gdop	
csthres	Reject threshold of cycle slip detection [1] GF(m) [2] MW(m) [3] Doppler Integration(cycle)	
prn	Process-noise standard deviation [1] bias [2] iono [3] trop [4] acch [5] accv [6] pos	
std	Initial-state standard deviation [1] bias [2] iono [3] trop	
err	Measurement error factor [1] reserved [2-4] error factor a/b/c of phase (m) [5] doppler frequency (Hz)	
sclkstab	Satellite clock stability (sec/sec)	
eratio	code/phase error ratio	
antdelsrc	The source of antenna delta 0: from obs 1: from options	
antdel	Antenna delta	(ENU frame) (unit: m)

	[1-3]rover [4-6]base station	
basepostype	Reference position type of base station 1:pos in options 2:average of SPP 3:rinex header	
basepos	Base station reference position(WGS84-XYZ)	
ins_mode	GNSS/INS mode 0:off 1:LC 2:TC	GNSS/INS mode use local navigation system (i.e., ENU-frame)
ins_aid	INS aid GNSS (0:off 1:on) [1]INS-aid cycle slip detection [2]INS-aid robust estimation	INS-aid cycle slip detection: PPK/INS TC and PPP/INS TC modes support this option INS-aid robust estimation: SPP/INS LC, SPP/INS TC, PPD/INS TC, PPK/INS TC and PPP/INS TC support this option
data_format	IMU data format 1:rate 2:increment	
sample_rate	IMU sample rate(Hz)	
lever	Lever arm from INS to GNSS under body frame(i.e., RFU-frame)	
init_att_unc	Initial uncertainty of attitude(deg) [1] pitch [2] roll [3] yaw	
init_vel_unc	Initial uncertainty of velocity(m/s) [1] east [2] north [3] up	
init_pos_unc	Initial uncertainty of position(m) [1] latitude [2] longitude [3] height	The unit will be converted in the program
init_bg_unc	Initial uncertainty of the gyro bias(rad/s)	
init_ba_unc	Initial uncertainty of the accelerometer bias(m/s^2)	
psd_gyro	Gyro noise PSD (rad^2/s)	
psd_acc	Accelerometer noise PSD (m^2/s^3)	
psd_bg	Gyro bias random walk PSD (rad^2/s^3)	
psd_ba	Accelerometer bias random walk PSD (m^2/s^5)	
timef	Time format (1:GPST 2:UTC)	
posf	Position format 1:ECEF-XYZ 2:LLH(latitude longitude height)	If choose ECEF-XYZ, the output velocity use ECEF-XYZ format. If choose LLH, the output velocity use ENU format.

outvel	Output velocity 0:off 1:on	
outatt	Output attitude 0:off 1:on	only for GNSS/INS integration mode

6. GINavPlot

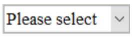
GINavPlot is a visualization tool for solution presentation and error analysis, it provides three modules, namely, Plot, Error Plot and PPP Plot modules. You can run **GINav\Plot_Analysis.m** to start GINAvPlot, the dialog of GINavPlot is shown in **Figure 7**.

(1). Plot module

Plot module can plot trajectory, position, velocity, the number of satellite and ambiguity resolution (AR) ratio factor. The steps are as follows:

a. Select solution file by pushing 

button.

b. Select plot type by pulling down 

menu.

Figure 8 to **Figure 12** show the trajectory, position, velocity, the number of satellite and AR ratio factor of a vehicle during the test.

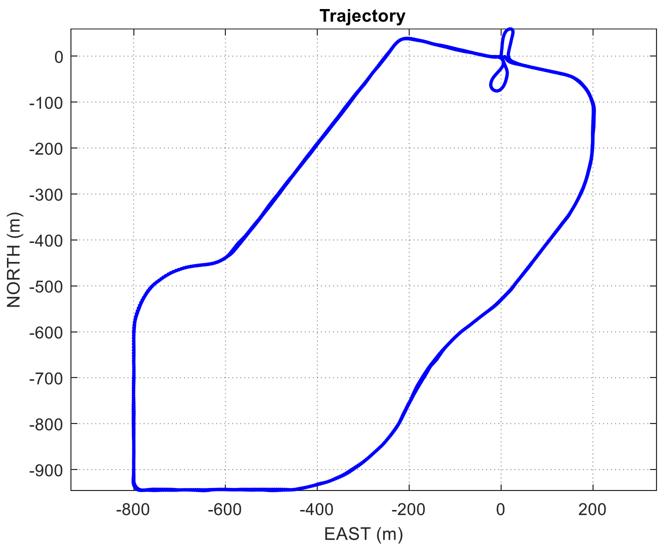


Figure 8 Trajectory of a vehicle

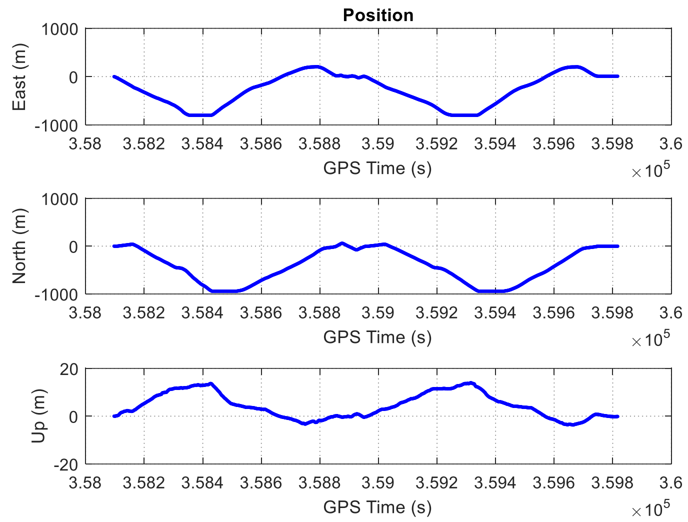


Figure 9 E/N/U components of vehicle position

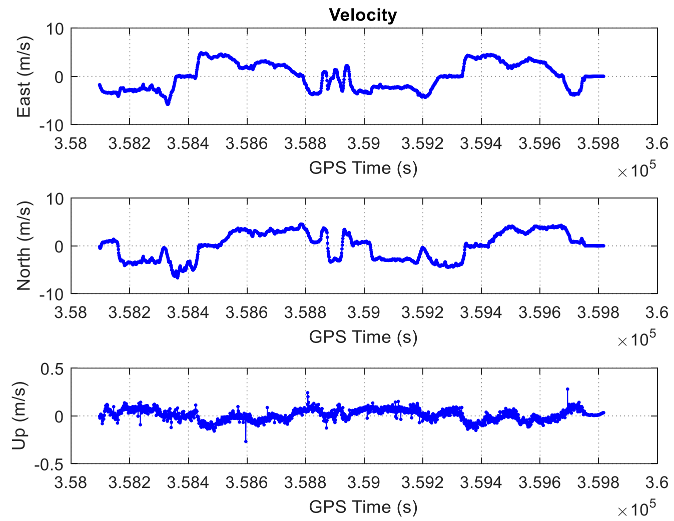


Figure 10 E/N/U components of vehicle velocity

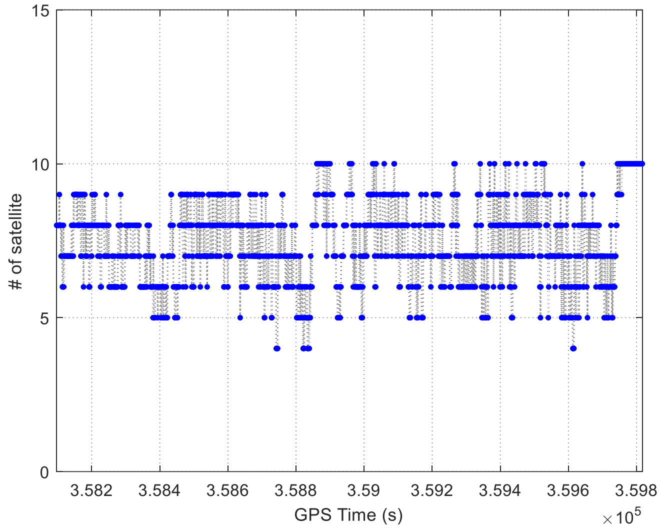


Figure 11 Number of satellites during test

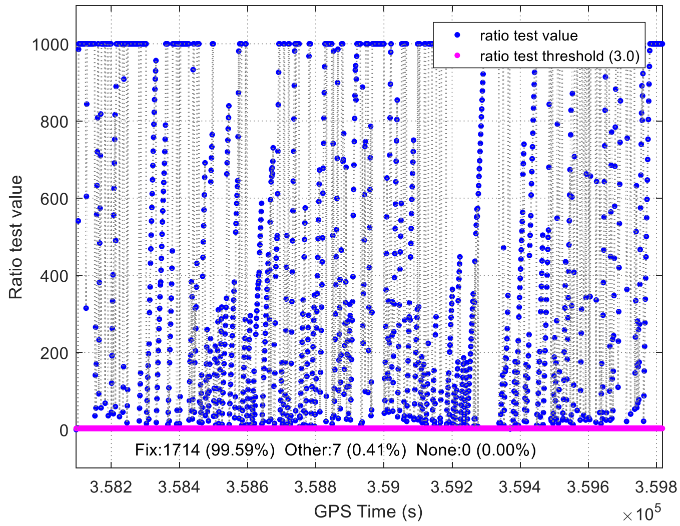


Figure 12 Ratio test value for ambiguity resolution

(2). Error Plot module

Error Plot module can plot position, velocity and attitude error when the reference truth is known. The steps are as follows:

a. Select solution file by pushing Solution file button.

b. Select reference truth file by pushing Reference file button. Note that the reference truth file should be kept in a format required by GINav, and more details can be found in **Appendix**.

c. Select plot type by pulling down Please select menu.

Figure 13 to **Figure 15** show the shows the position, velocity and attitude error of a vehicle in PPK/INS tightly coupled mode.

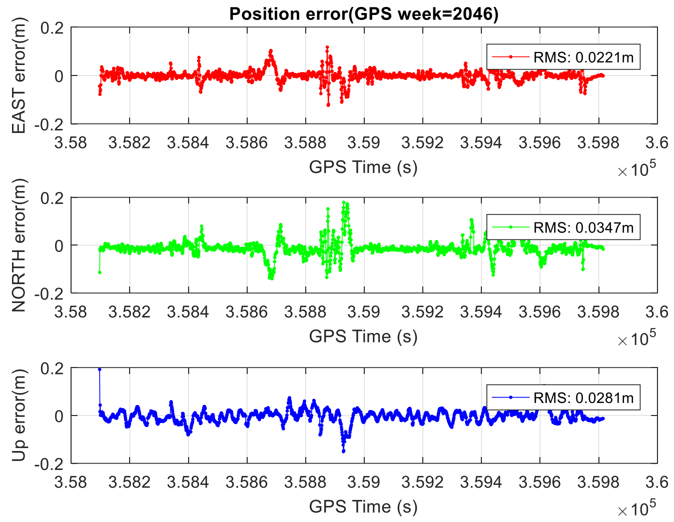


Figure 13 Position error of a vehicle in PPK/INS tightly coupled mode

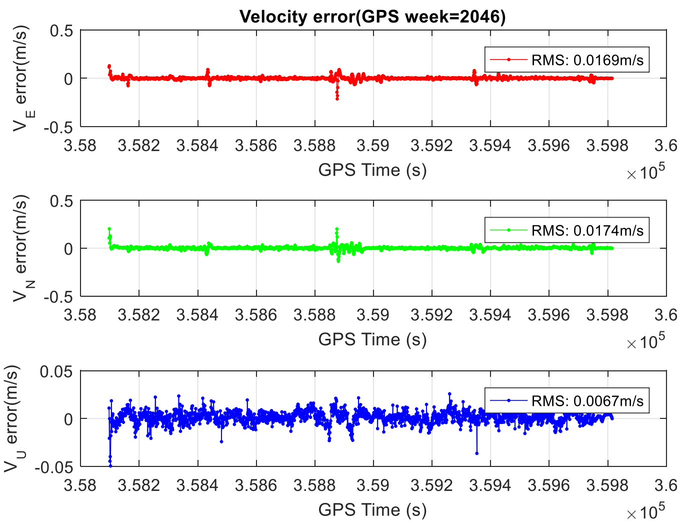


Figure 14 Velocity error of a vehicle in PPK/INS tightly coupled mode

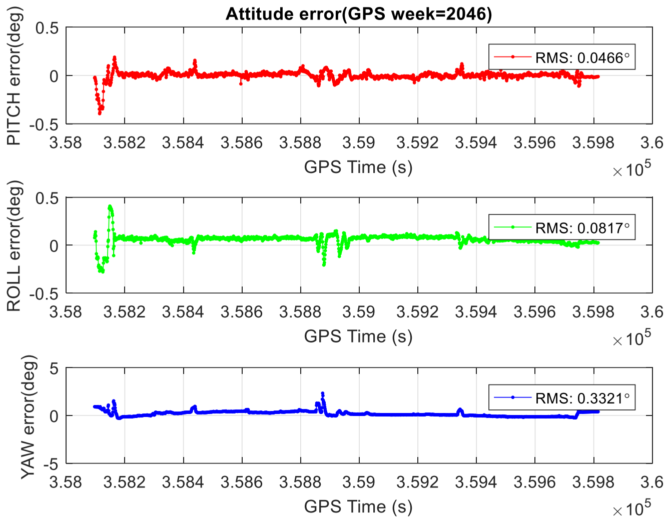


Figure 15 Attitude error of a vehicle in PPK/INS tightly coupled mode

(3). PPP Plot module

PPP Plot module can position error of IGS station when ground truth is known.

The steps are as follows:

a. Select solution file by pushing  button.

b. Select SNX file to obtain the ground truth by pushing  button.

c. Push  button to plot PPP error.

Figure 16 show the shows the position error of HARB station in static PPP mode.

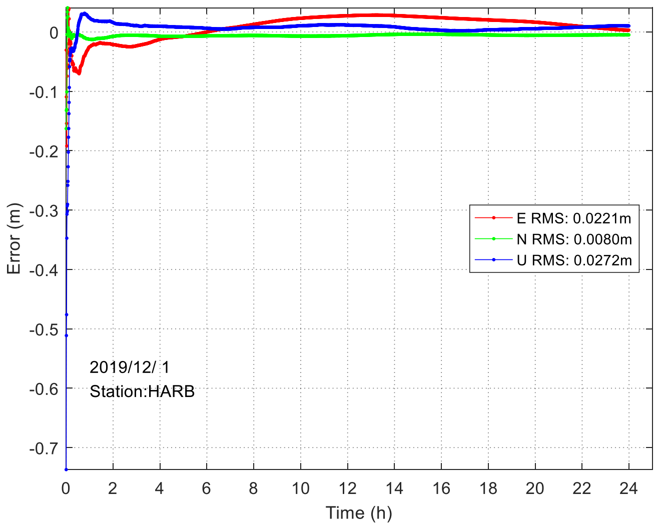


Figure 16 Position error of HARB station in PPP static mode

7. Dataset

GINav provides different dataset to evaluate its performance, you can use the corresponding configuration file in **GINav\conf** folder to perform these datasets. Note that some data are collected from the Internet, if you use the dataset, please cite related paper or indicate the source.

- Dataset "**data_cpt**". This dataset is collected in a suburban environment with vehicle. The data collection platform is equipped with the Trimble R10 receiver and a tactical grade IMU, together with accurate reference solutions from NovAtel-SPAN-CPT system. This dataset can be used to evaluate the performance of GNSS modes (SPP, PPD, PPK and PPP) and GNSS/INS integration modes (SPP/INS LC, PPD/INS LC, PPK/INS LC, PPP/INS LC, SPP/INS TC, PPD/INS TC, PPK/INS TC and PPP/INS TC). You can use the corresponding configuration file in **GINav\conf** folder to perform this dataset, and analyze position, velocity and attitude error using GINavPlot.
- Dataset "**data_tokyo**". This dataset comes from open-source project UrbanNav that can be found at website https://github.com/weisongwen/UrbanNavDataset. The dataset was collected in a typical urban canyon of Tokyo on December 19, 2018. It can be used to assess the availability of GINav for handling urban dataset. You can use the corresponding configuration file in **GINav\conf** folder to achieve SPP, PPK, SPP/INS TC and PPK/INS TC modes, and analyze position, velocity and attitude error using GINavPlot.
- Dataset "**data_mgex**". The GNSS observations are collected from IGS-MGEX stations on December, 1, 2019, with 30s sampling intervals, including station WUH2, JFNG and HARB. The dataset includes all required files that achieve PPP mode. You can use the corresponding configuration file in **GINav\conf** folder to perform this dataset, and analyze PPP error using GNIavPlot.
- Dataset "**data_cu**". The short-baseline GNSS data are collected from GNSS Research Centre at Curtin University (http://gnss.curtin.edu.au/). GNSS observations on March, 6, 2020 from station CUAA and CUBB can be used to evaluate the performance of PPS mode. Moreover, the ground truth of CUBB station is stored in the **data_cu\cubb_pos_ref.mat**. You can use the corresponding configuration file in **GINav\conf** folder to perform this dataset, and analyze position error using GINavPlot.

8. Mathematical model

This section briefly introduces the mathematical model of GNSS/INS integration used in GINav, include SPP/INS TC, PPK/INS TC, PPP/INS TC and GNSS/INS LC model.

8.1 Common part of GNSS/INS dynamic model

GINav provides a variety of GNSS/INS integration mode. Different modes correspond to different dynamic model, but the dynamic model related to INS is universal. The system dynamic equation related to INS includes 15 error states as follows:

$$\boldsymbol{x}_{INS}^{n} = \begin{bmatrix} \boldsymbol{\psi}^{n} & \delta\boldsymbol{v}^{n} & \delta\boldsymbol{r}^{n} & \nabla^{b} & \boldsymbol{\varepsilon}^{b} \end{bmatrix}^{T} \tag{1}$$

where the superscript b and n denotes the body frame (*b-frame*) and navigation frame (*n-frame*), respectively; $\boldsymbol{\psi}^n$, $\delta\boldsymbol{v}^n$ and $\delta\boldsymbol{r}^n$ are the attitude, velocity and position error vector in *n-frame*, respectively; ∇^b and $\boldsymbol{\varepsilon}^b$ are the accelerometer and gyro error vector in *b-frame*, respectively.
System error dynamics equation related INS is shown as follows:

$$\dot{\boldsymbol{x}}_{INS}^{n} = \boldsymbol{F}_{INS}\boldsymbol{x}_{INS}^{n} + \boldsymbol{G}_{INS}\boldsymbol{w}_{INS} \tag{2}$$

where $\boldsymbol{F}_{INS}$ and $\boldsymbol{G}_{INS}$ are state transformation matrix and system noise distribution matrix respectively; $\boldsymbol{w}_{INS}$ is system noise sequence associated with INS.
The expressions of $\boldsymbol{F}_{INS}$, $\boldsymbol{G}_{INS}$ and $\boldsymbol{w}_{INS}$ are shown as follows:

$$\boldsymbol{F}_{SINS}=\begin{bmatrix}\boldsymbol{F}_{aa} & \boldsymbol{F}_{av} & \boldsymbol{F}_{ap} & -\boldsymbol{C}_b^n & \boldsymbol{0}_{3\times3}\\ \boldsymbol{F}_{va} & \boldsymbol{F}_{vv} & \boldsymbol{F}_{vp} & \boldsymbol{0}_{3\times3} & \boldsymbol{C}_b^n\\ \boldsymbol{0}_{3\times3} & \boldsymbol{F}_{pv} & \boldsymbol{F}_{pp} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{F}_{\nabla} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{F}_{\varepsilon}\end{bmatrix} \tag{3}$$

$$\boldsymbol{G}_{INS}=\begin{bmatrix}-\boldsymbol{C}_b^n & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{C}_b^n & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times3}\\ \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3}\end{bmatrix} \tag{4}$$

$$\boldsymbol{w}_{INS}=\begin{bmatrix}\boldsymbol{w}_a & \boldsymbol{w}_g & \boldsymbol{w}_{\nabla} & \boldsymbol{w}_{\varepsilon}\end{bmatrix}^T \tag{5}$$

where

$$\boldsymbol{F}_{aa}=(-\boldsymbol{\omega}_{in}^n\times),\quad \boldsymbol{F}_{av}=\begin{bmatrix}0 & -1/R_{Mh} & 0\\ 1/R_{Nh} & 0 & 0\\ \tan L/R_{Nh} & 0 & 0\end{bmatrix},\quad \boldsymbol{F}_{ap}=\begin{bmatrix}0 & 0 & v_N^n/R_{Mh}^2\\ -\omega_{ie}\sin L & 0 & -v_E^n/R_{Nh}^2\\ \omega_{ie}\cos L+\dfrac{v_E^n\sec^2 L}{R_{Nh}} & 0 & -v_E^n\tan L/R_{Nh}^2\end{bmatrix}$$

$$\boldsymbol{F}_{va}=(\boldsymbol{f}^n\times),\quad \boldsymbol{F}_{vv}=(\boldsymbol{v}^n\times)\boldsymbol{F}_{av}-((2\boldsymbol{\omega}_{ie}^n+\boldsymbol{\omega}_{en}^n)\times),\quad \boldsymbol{F}_{vp}=(\boldsymbol{v}^n\times)\left(\boldsymbol{F}_1+\boldsymbol{F}_2\right)$$

$$\boldsymbol{F}_{pv}=\begin{bmatrix}0 & 1/R_{Mh} & 0\\ \sec L/R_{Nh} & 0 & 0\\ 0 & 0 & 1\end{bmatrix},\quad \boldsymbol{F}_{pp}=\begin{bmatrix}0 & 0 & -v_N^n/R_{Mh}^2\\ v_E^n\sec L\tan L/R_{Nh} & 0 & -v_E^n\sec L/R_{Nh}^2\\ 0 & 0 & 0\end{bmatrix}$$

$$\boldsymbol{F}_{\nabla}=\begin{bmatrix}-1/\tau_{\nabla} & 0 & 0\\ 0 & -1/\tau_{\nabla} & 0\\ 0 & 0 & -1/\tau_{\nabla}\end{bmatrix},\quad \boldsymbol{F}_{\varepsilon}=\begin{bmatrix}-1/\tau_{\varepsilon} & 0 & 0\\ 0 & -1/\tau_{\varepsilon} & 0\\ 0 & 0 & -1/\tau_{\varepsilon}\end{bmatrix}$$

$$\boldsymbol{F}_1=\begin{bmatrix}0 & 0 & v_N^n/R_{Mh}^2\\ -2\omega_{ie}\sin L & 0 & -v_E^n/R_{Nh}^2\\ 2\omega_{ie}\cos L+\dfrac{v_E^n\sec^2 L}{R_{Nh}} & 0 & -v_E^n\tan L/R_{Nh}^2\end{bmatrix},\quad \boldsymbol{F}_2=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ -g_0\sin 2L\left(\beta_1\text{-}4*\beta_2*\cos 2\mathrm{L}\right) & 0 & \beta_3\end{bmatrix}$$

and where the subscript i is the inertial frame (i-frame); $\boldsymbol{\omega}_{in}^n$ is the angular rate of the n-frame with respect to i-frame in n-frame; L is the latitude; $()\times$ is an antisymmetric matrix operator; R_{Mh} is radius of curvature in meridian; R_{Nh} is radius of curvature in prime vertical; ω_{ie} is the earth rotation rate; $\boldsymbol{f}^n$ is the specific force in n-frame; $\boldsymbol{\omega}_{ie}^n$ is the angular rate of the earth frame (e-frame) with respect to the i-frame in n-frame; $\boldsymbol{\omega}_{en}^n$ is the angular rate of the n-frame with respect to e-frame in n-frame; τ_{∇}

and τ_ε are the time constants of first-order Markov processes for accelerometer and gyroscope errors, respectively; g_0 is gravity magnitude at the equatorial sea-surface; $\beta_1 = 5.27094\times10^{-3}$; $\beta_2 = 2.32718\times10^{-3}$; $\beta_3 = 3.086\times10^{-6}$.

8.2 SPP/INS TC model

In SPP/INS TC mode, except the basic dynamics model, i.e., Eq. (2), the receiver clock offset, clock drift and inter-system bias (ISB) that other navigation satellites system with respect to GPS, should be modelled as random walk process, as follows:

$$\begin{cases} d\dot{t}_G = dt_d + w_r \\ d\dot{t}_{GR} = w_{ISB} \\ d\dot{t}_{GE} = w_{ISB} \\ d\dot{t}_{GC} = w_{ISB} \\ d\dot{t}_{GJ} = w_{ISB} \\ d\dot{t}_d = w_d \end{cases} \tag{6}$$

where dt_G and dt_d is the receiver clock offset and drift for GPS, respectively; dt_{GR} , dt_{GE} , dt_{GC} and dt_{GJ} are ISB parameters for GLONASS, GALILEO, BDS and QZSS with respect to GPS. w_r and w_d are the receiver clock white noise sequences. w_{ISB} is the ISB white noise sequences.

By convention, GNSS-SPP mode uses pseudorange measurements to estimate position and receiver offset, and the ionospheric delay and tropospheric delay usually are corrected by using Klobuchar and Saastamoinen model respectively. However, to better estimate the state parameters, the doppler measurements is also used in SPP/INS TC mode. The measurement equation for SPP/INS TC mode can be simply expressed as follows:

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k \tag{7}$$

where $\boldsymbol{z}_k$ is the measurement innovations vector; the subscript k is the kth epoch; $\boldsymbol{H}_k$ is the measurement model; $\boldsymbol{x}_k$ is the states vector, and it includes INS states in Eq. (1) and the receiver clock parameters in Eq. (6); $\boldsymbol{v}_k$ is the innovation noise and obey zero-mean Gaussian normal distribution.

The expression of states vector is:

$$\boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{x}_{INS}^n & dt_G & dt_{GR} & dt_{GE} & dt_{GC} & dt_{GJ} & dt_d \end{bmatrix}^T \tag{8}$$

The measurement innovations vector is the difference between the raw GNSS measurements $\boldsymbol{m}_{GNSS}$ and INS predicted measurements $\tilde{\boldsymbol{m}}_{INS}$, as follow:

$$\boldsymbol{z}_k = \boldsymbol{m}_{GNSS} - \tilde{\boldsymbol{m}}_{INS} \tag{9}$$

$$\begin{cases} \boldsymbol{m}_{GNSS} = \begin{bmatrix} \boldsymbol{P}^{s,f} \\ \dot{\boldsymbol{P}}^{s,f} \end{bmatrix} \\ \tilde{\boldsymbol{m}}_{INS} = \begin{bmatrix} \boldsymbol{\rho}_{INS} \\ \dot{\boldsymbol{\rho}}_{INS} \end{bmatrix} + \begin{bmatrix} \Delta \boldsymbol{dtr}_P + \Delta\boldsymbol{\delta}_P \\ \Delta \boldsymbol{dtr}_{\dot{P}} + \Delta\boldsymbol{\delta}_{\dot{P}} \end{bmatrix} \end{cases} \tag{10}$$

where $\boldsymbol{P}^{s,f}$ and $\dot{\boldsymbol{P}}^{s,f}$ are the raw pseudorange and doppler measurements respectively; the superscript f denotes frequency; the subscript s denotes satellite systems that include GSP, GLONASS, GALILEO, BDS and QZSS; $\boldsymbol{\rho}_{INS}$ and $\dot{\boldsymbol{\rho}}_{INS}$ are the range and range rate predicted by INS from receiver to satellite, respectively; $\Delta \boldsymbol{dtr}_P$ and $\Delta \boldsymbol{dtr}_{\dot{P}}$ are the sum of error corrections associated with the receiver clock for pseudorange and doppler, respectively; $\Delta\boldsymbol{\delta}_P$ and $\Delta\boldsymbol{\delta}_{\dot{P}}$ are the sum of other error corrections for pseudorange and doppler, respectively. Note that only one frequency measurement is usually used in the SPP/INS TC mode, which is consistent with the GNSS-SPP mode.

In addition, the measurement model can be expressed as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{H}_{INS,P}^{s,f} & \boldsymbol{H}_{GNSS,P}^{s,f} \\ \boldsymbol{H}_{INS,\dot{P}}^{s,f} & \boldsymbol{H}_{GNSS,\dot{P}}^{s,f} \end{bmatrix} \tag{11}$$

$$\boldsymbol{H}_{INS,P}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{e}^{s,f} \cdot \boldsymbol{T}_n^e & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{12}$$

$$\boldsymbol{H}_{INS,\dot{P}}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{e}^{s,f} \cdot \boldsymbol{C}_n^e & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{13}$$

$$\boldsymbol{H}_{GNSS,P}^{s,f} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \tag{14}$$

$$\boldsymbol{H}_{GNSS,\dot{P}}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \tag{15}$$

$$\boldsymbol{I} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T \tag{16}$$

where $\boldsymbol{e}^{s,f}$ represents the line of sight (LOS) from receiver to satellite; $\boldsymbol{T}_n^e$ is the transformation matrix of position perturbation error from navigation frame to earth frame; $\boldsymbol{C}_n^e$ is the transformation matrix of velocity error from navigation frame to earth frame.

8.3 PPK/INS TC model

In this section, we briefly introduce the algorithms for PPK/INS TC mode used in GINav. For carrier-based relative positioning with a short length baseline between rover and base-station, the common error, to be specific, satellite and receiver clock bias, and atmospheric delay can be eliminated by double-differenced technology. Therefore, except the basic dynamics model related with INS, only the ambiguity needs to be modelled. The ambiguity can be modelled as the random walk process, as follows:

$$\Delta\dot{N}_{rb} = w_{\Delta N_{rb}} \tag{17}$$

where Δ represents the single-difference (SD) operator; the subscript *r* and *b* denote rover and base-station, respectively; ΔN_{rb} is the SD ambiguity between rover and base-station; $w_{\Delta N_{rb}}$ is the SD ambiguity random noise.

For PPK/INS TC mode, the expression of the measurement equation is the same as that of Eq. (3), but the specific expression of each variable is different from the SPP/INS TC mode. The expression of states vector $\boldsymbol{x}_k$ is:

$$\boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{x}_{INS}^{n} & \Delta \boldsymbol{N}_{rb}^{s,f} \end{bmatrix} \tag{18}$$

where $\Delta \boldsymbol{N}_{rb}^{s,f}$ is the SD ambiguity vector, and it includes multi-GNSS and multi-frequency SD ambiguity. Note that for long baseline, the ionospheric and tropospheric delay for both rover and base-station can be estimated as unknown parameters in GINav.

The measurement innovations $\boldsymbol{z}_k$ can be expressed as follows:

$$\begin{aligned} \boldsymbol{z}_k &= \boldsymbol{m}_{GNSS} - \tilde{\boldsymbol{m}}_{INS} \\ &= \begin{bmatrix} \nabla\Delta \boldsymbol{L}_{rb}^{ij,s,f} \\ \nabla\Delta \boldsymbol{P}_{rb}^{ij,s,f} \end{bmatrix} - \left(\begin{bmatrix} \nabla\Delta \boldsymbol{\rho}_{rb,INS}^{ij} \\ \nabla\Delta \boldsymbol{\rho}_{rb,INS}^{ij} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\lambda}^{s,f} \left(\Delta \boldsymbol{N}_{rb}^{i,s,f} - \Delta \boldsymbol{N}_{rb}^{j,s,f} \right) \\ \boldsymbol{0} \end{bmatrix} \right) \end{aligned} \tag{19}$$

where $\nabla\Delta$ represents double-difference (DD) operator; the superscript *i* and *j* represent the *i*th and *j*th satellites; $\nabla\Delta \boldsymbol{L}_{rb}^{ij,s,f}$ is the DD carrier phase; $\nabla\Delta \boldsymbol{P}_{rb}^{ij,s,f}$ is the DD pseudorange; $\nabla\Delta \boldsymbol{\rho}_{rb,INS}^{ij}$ is the DD geometric range predicted by INS; $\nabla \boldsymbol{N}_{rb}^{i,s,f}$ and $\nabla \boldsymbol{N}_{rb}^{j,s,f}$ are the SD ambiguity of *i*th and *j*th satellites respectively.

The measurement model $\boldsymbol{H}_k$ of PPK/INS TC mode is expressed as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{H}_{INS,\nabla\Delta L}^{s,f} & \boldsymbol{H}_{GNSS,\nabla\Delta L}^{s,f} \\ \boldsymbol{H}_{INS,\nabla\Delta P}^{s,f} & \boldsymbol{0} \end{bmatrix} \tag{20}$$

$$\boldsymbol{H}_{INS,\nabla\Delta L}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{e}_{rb}^{ij,s,f} \cdot \boldsymbol{T}_n^e & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{21}$$

$$\boldsymbol{H}_{INS,\nabla\Delta P}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{e}_{rb}^{ij,s,f} \cdot \boldsymbol{T}_n^e & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{22}$$

$$\boldsymbol{H}_{GNSS,\nabla\Delta L}^{s,f} = \begin{bmatrix} \lambda^{G,f}\boldsymbol{D} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \lambda^{R,f}\boldsymbol{D} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \lambda^{E,f}\boldsymbol{D} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \lambda^{C,f}\boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \lambda^{J,f}\boldsymbol{D} \end{bmatrix} \tag{23}$$

$$\boldsymbol{D} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \tag{24}$$

where $\boldsymbol{e}_{rb}^{ij,s,f}$ is the DD LOS; $\boldsymbol{D}$ is SD matrix.

Additionally, GINav also provide the so-called PPD/INS TC mode, which is based on DD pseudorange, and its model is the same as the pseudorange related model in PPK/INS TC mode.

8.4 PPP/INS TC model

Traditionally, PPP relies on the IGS precise satellite orbit and clock products to eliminate the satellite orbit and clock offsets. IGS precise satellite clock products are generated using the ionospheric-free liner combination (IFLC) measurements. For the measurement model of PPP/INS TC mode used by GINav, we only briefly introduce the usual measurement model based on IFLC measurements, the model based on undifferenced and uncombined (UDUC) GNSS measurements can be found in some other literatures. In PPP based on IFLC measurements, the satellite and receiver antenna phase center offsets and variations, relativistic effect, Sagnac effect, tropospheric hydrostatic delay, earth tides, and phase windup (only for carrier phase) can be corrected by using the corresponding model. Therefore, for PPP/INS TC mode, the state vector comprises the INS states, receiver clock offset, clock drift, ISB parameters, tropospheric zenith wet delay and carrier phase ambiguity. The modelling of the receiver clock offset, clock drift and ISB parameters is the same as the SPP/INS TC mode and is given by Eq. (6). The tropospheric ZWD can be modelled as random walk process, as follows:

$$\dot{T}_W = w_{T_W} \tag{25}$$

where T_W denotes ZWD; w_{T_W} is the ZWD random noise, and the power spectral density (PSD) of 1e-4 m/√s is usually used in GINav.

Additionally, the zero-difference (ZD) ambiguity is estimated in PPP/INS TC mode, which is different from SD ambiguity in PPK/INS TC mode. The ZD ambiguity can be modelled as random walk process, as follows:

$$\dot{N} = w_N \tag{26}$$

where N represents ZD ambiguity; w_N is the ZD ambiguity random noise, and its PSD is usually set at 1e-4 m/√s.

Based on the above discussion, the state vector is expressed as follows:

$$\boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{x}_{INS} & dtr_G & dtr_{GR} & dtr_{GE} & dtr_{GC} & dtr_{GJ} & dtr_d & T_{ZWD} & N^{s,f} \end{bmatrix} \tag{27}$$

where the INS state vector, $\boldsymbol{x}_{INS}$, is given by Eq. (1).

The measurement innovations $\boldsymbol{z}_k$ can be expressed as follows:

$$\begin{aligned} \boldsymbol{z}_k &= \boldsymbol{m}_{GNSS} - \tilde{\boldsymbol{m}}_{INS} \\ &= \begin{bmatrix} \boldsymbol{L}_{IF}^{s,f} \\ \boldsymbol{P}_{IF}^{s,f} \end{bmatrix} - \left(\begin{bmatrix} \boldsymbol{\rho}_{IF,INS} \\ \boldsymbol{\rho}_{IF,INS} \end{bmatrix} + \begin{bmatrix} \Delta \boldsymbol{dtr}_{L_{IF}} + \boldsymbol{M}_w \boldsymbol{T}_w + \boldsymbol{\lambda}^{s,f} N^{s,f} + \Delta \boldsymbol{\delta}_{L_{IF}} \\ \Delta \boldsymbol{dtr}_{P_{IF}} + \boldsymbol{M}_w \boldsymbol{T}_w + \Delta \boldsymbol{\delta}_{P_{IF}} \end{bmatrix} \right) \end{aligned} \tag{28}$$

where subscript IF represents ionospheric-free; $\boldsymbol{L}_{IF}^{s,f}$ and $\boldsymbol{P}_{IF}^{s,f}$ are the IF carrier phase and pseudorange measurements, respectively; $\boldsymbol{\rho}_{IF,INS}$ is the IF geometric range predicted by INS; $\Delta \boldsymbol{dtr}_{L_{IF}}$ and $\Delta \boldsymbol{dtr}_{P_{IF}}$ are the sum of error corrections associated with the receiver clock for IF carrier phase and pseudorange, respectively; $\boldsymbol{M}_w$ the wet mapping function; $\Delta \boldsymbol{\delta}_{L_{IF}}$ and $\Delta \boldsymbol{\delta}_{P_{IF}}$ are the sum of other error corrections for the IF carrier phase and pseudorange.

The measurement model $\boldsymbol{H}_k$ of PPK/INS TC mode is expressed as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{H}_{INS,L_{IF}}^{s,f} & \boldsymbol{H}_{GNSS,L_{IF}}^{s,f} \\ \boldsymbol{H}_{INS,P_{IF}}^{s,f} & \boldsymbol{H}_{GNSS,P_{IF}}^{s,f} \end{bmatrix} \tag{29}$$

$$\boldsymbol{H}_{INS,L_{IF}}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{e}_{IF}^{s,f} \cdot \boldsymbol{T}_n^e & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{30}$$

$$\boldsymbol{H}_{INS,P_{IF}}^{s,f} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{e}_{IF}^{s,f} \cdot \boldsymbol{T}_n^e & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{31}$$

$$\boldsymbol{H}_{GNSS,L_{IF}}^{s,f} = \begin{bmatrix} \boldsymbol{H}_{GNSS,P}^{s,f} & \boldsymbol{M}_w & \boldsymbol{\lambda}^{s,f} \end{bmatrix} \tag{32}$$

$$\boldsymbol{H}_{GNSS,P_{IF}}^{s,f} = \begin{bmatrix} \boldsymbol{H}_{GNSS,P}^{s,f} & \boldsymbol{M}_w & \boldsymbol{0} \end{bmatrix} \tag{33}$$

where $\boldsymbol{e}_{IF}^{s,f}$ is the IF LOS; $\boldsymbol{H}_{GNSS,P}^{s,f}$ is given by Eq. (14).

The above discussion gives the usual IF PPP/INS TC mode. In fact, if you use the UDUC PPP/INS TC mode, the ionospheric delays should be also modelled as unknown parameter. Moreover, GLONASS utilizes FDMA signals, which means that each GLONASS satellite has a different frequency channel and hardware bias. Thus, if you use the GLONASS in PPP/INS TC mode, the inter-frequency bias can be simply modelled as a linear or quadratic polynomial function related to the frequency number.

8.5 GNSS/INS LC model

Compared with the GNSS/INS TC model, the LC model is simpler. It uses the position and velocity estimated by GNSS to correct the system state, so the filter is more stable. In GINav, the GNSS/INS LC integration can be divided into SPP/INS LC, PPD/INS LC, PPK/INS LC and PPP/INS LC mode according to GNSS positioning mode. The difference between these modes is that the position and velocity information used to assist the INS is estimated by the corresponding GNSS positioning mode, but the dynamics and measurement model is universal.

For GNSS/INS LC mode, the state vector $\boldsymbol{x}_k$ only comprises the INS states. Thus,

$$\boldsymbol{x}_k = \boldsymbol{x}_{INS}^n \tag{34}$$

where $\boldsymbol{x}_{INS}^n$ is given by Eq. (1).

The measurement innovations $\boldsymbol{z}_k$ can be expressed as follows:

$$\boldsymbol{z}_k = \begin{bmatrix} \boldsymbol{r}_{GNSS}^n \\ \boldsymbol{v}_{GNSS}^n \end{bmatrix} - \left(\begin{bmatrix} \boldsymbol{r}_{INS}^n \\ \boldsymbol{v}_{INS}^n \end{bmatrix} + \begin{bmatrix} \boldsymbol{T}\boldsymbol{C}_b^n \boldsymbol{L}^b \\ \boldsymbol{C}_b^n \left(\boldsymbol{\omega}_{eb}^n \times \right) \boldsymbol{L}^b \end{bmatrix} \right) \tag{35}$$

where $\boldsymbol{r}_{GNSS}^n$ and $\boldsymbol{v}_{GNSS}^n$ are the rover position and velocity estimated by GNSS in *n-frame*, respectively; $\boldsymbol{r}_{INS}^n$ and $\boldsymbol{v}_{INS}^n$ are the rover position and velocity predicted by INS in *n-frame*, respectively; $\boldsymbol{T}$ is the transformation matrix of cartesian-to-curvilinear position; $\boldsymbol{C}_b^n$ is the transformation matrix from *b-frame* to *n-frame*; $\boldsymbol{L}^b$ is the lever arm from INS to GNSS receiver projected in *b-frame*; $\boldsymbol{\omega}_{eb}^n$ is the angular rate of the *b-frame* with respect to *e*-frame in *n-frame*.

The measurement model $\boldsymbol{H}_k$ of GNSS/INS LC mode is expressed as follows:

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{3\times3} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{3\times3} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{36}$$

where $\boldsymbol{I}_{3\times3}$ is the three-dimensional identity matrix.

Appendix

- GINav does not support the IMU file formats from any manufacturer, please save IMU data in the format GINav requires. You can refer to the format of **cpt_imu.csv** file that provided by GINav and can be found from **data_cpt** folder. **Figure 17** shows the format of IMU file example. Note that the XYZ order of the IMU data corresponds to the RFU-frame (i.e., right-front-up).

	A	B	C	D	E	F	G	H
1	GPS week	GPS sow (s)	Angular rate X (rad/s)	Angular rate Y (rad/s)	Angular rate Z (rad/s)	Acceleration X (m/s^2)	Acceleration Y (m/s^2)	Acceleration Z (m/s^2)
2	2046	357254.3158	-0.000640191	0.000455729	-0.000423177	0.000457764	0.106658936	9.884033203
3	2046	357254.3258	-0.000488281	-0.001106771	-0.000813802	0.087738037	0.051879883	9.680175781
4	2046	357254.3358	-0.000108507	0.000998264	4.34E-05	0.10848999	0.068817139	9.691925049
5	2046	357254.3458	-0.000542535	0.002680122	0.00016276	-0.002288818	0.109863281	9.81628418
6	2046	357254.3558	-0.000607639	-0.001790365	-0.000575087	0.003051758	0.075073242	9.864807129
7	2046	357254.3658	0.00016276	-0.001367188	-0.000412326	0.131530762	0.0390625	9.69543457
8	2046	357254.3758	4.34E-05	0.001996528	0.00031467	0.087738037	0.054016113	9.690093994
9	2046	357254.3858	0.000553385	0.000640191	0.000249566	-0.013275146	0.063323975	9.851379395
10	2046	357254.3958	0.000824653	-0.000455729	8.68E-05	0.027160645	0.073852539	9.805145264
11	2046	357254.4058	-4.34E-05	0.000846354	0.000292969	0.129852295	0.067901611	9.716796875
12	2046	357254.4158	0.00061849	0.001291233	0.000488281	0.051269531	0.054321289	9.774475098
13	2046	357254.4258	0.000390625	-4.34E-05	0.000531684	-0.003204346	0.093688965	9.826660156
14	2046	357254.4358	-0.000282118	0.000683594	0.00046658	0.042572021	0.096740723	9.764709473
15	2046	357254.4458	-0.000705295	0.000249566	0.000358073	0.074005127	0.065765381	9.756011963
16	2046	357254.4558	-0.000249566	-0.001106771	0.000282118	0.048980713	0.053405762	9.786224365
17	2046	357254.4658	-0.000108507	-0.000141059	0.000282118	0.040435791	0.06942749	9.788208008
18	2046	357254.4758	-0.000585938	0	3.26E-05	0.027160645	0.094146729	9.763641357
19	2046	357254.4858	-0.000325521	-0.000585938	-9.77E-05	0.058898926	0.040588379	9.808807373
20	2046	357254.4958	0.000401476	-0.000596788	-0.00031467	0.109405518	0.046844482	9.740142822
21	2046	357254.5058	0.000531684	0.000596788	-0.000282118	0.06362915	0.042877197	9.753417969
22	2046	357254.5158	0.000336372	0.000976563	-0.000325521	0.02456665	0.077209473	9.802703857
23	2046	357254.5258	0.000401476	-0.000249566	-0.000531684	0.063018799	0.086975098	9.772338867
24	2046	357254.5358	0.000401476	0.000130208	-0.000520833	0.101928711	0.07232666	9.730072021
25	2046	357254.5458	8.68E-05	0.001193576	-0.000206163	0.054473877	0.068511963	9.77722168
26	2046	357254.5558	1.09E-05	0.000434028	-0.000303819	0.025024414	0.084381104	9.807128906
27	2046	357254.5658	-0.000184462	-0.000249566	-0.000282118	0.073394775	0.082397461	9.75402832
28	2046	357254.5758	-0.000455729	0.000238715	-3.26E-05	0.088806152	0.061035156	9.745941162

Figure 17 The format of imu file

- If you want to analyze position, velocity, and attitude errors using GINavPlot, please save the reference truth in the format GINav requires. You can refer to the format of **cpt_pva_ref.mat** file that provided by GINav and can be found from **data_cpt** folder. **Figure 18** shows the format of reference truth file example. Note that you must use the "reference" struct defined by GINav to store the GPS week, GPS sow, position, velocity and attitude.

Fields	week	sow	pos	vel	att
1	2046	357453	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.00100000000000000,0.00700000000000000,-1.35238972798213e-11]	[0.774913000000000,0.158687000000000,289.964980000000]
2	2046	357454	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.00100000000000000,0.00600000000000000,-1.14392258686313e-11]	[0.773790000000000,0.158638000000000,289.964222000000]
3	2046	357455	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00100000000000000,0.00400000000000000,-9.40748412329995e-12]	[0.773264000000000,0.158241000000000,289.963268000000]
4	2046	357456	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00100000000000000,0.00200000000000000,-5.23814195144134e-12]	[0.772523000000000,0.157787000000000,289.963267000000]
5	2046	357457	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00100000000000000,0.00200000000000000,-5.23814195144134e-12]	[0.772012000000000,0.157140000000000,289.962574000000]
6	2046	357458	[-2408694.47400000,4698106.45000000,3566698.36500000]	[5.42101086242752e-20,0.00100000000000000,-2.08467097750908e-12]	[0.771376000000000,0.157099000000000,289.962006000000]
7	2046	357459	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-1.06879991510801e-12,2.08467114013941e-12,0.00100000000000000]	[0.770881000000000,0.156624000000000,289.961103000000]
8	2046	357460	[-2408694.47400000,4698106.45000000,3566698.36500000]	[1.06879991510801e-12,-2.08467114013941e-12,-0.00100000000000000]	[0.770628000000000,0.156752000000000,289.960700000000]
9	2046	357461	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00100000000000000,0.00100000000000000,-3.15347097393226e-12]	[0.770210000000000,0.155880000000000,289.960083000000]
10	2046	357462	[-2408694.47400000,4698106.45000000,3566698.36500000]	[1.06879988800296e-12,-0.00100000000208467,-0.000999999997915329]	[0.769850000000000,0.155314000000000,289.959236000000]
11	2046	357463	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-1.06879994221307e-12,-0.000999999997915329,0.00100000000208467]	[0.769508000000000,0.155076000000000,289.958643000000]
12	2046	357464	[-2408694.47400000,4698106.45000000,3566698.36500000]	[1.06879994221307e-12,0.000999999997915329,-0.00100000000208467]	[0.768834000000000,0.154629000000000,289.957796000000]
13	2046	357465	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.00100000000106880,-2.08467097750908e-12,-0.000999999998931200]	[0.768332000000000,0.154487000000000,289.956879000000]
14	2046	357466	[-2408694.47400000,4698106.45000000,3566698.36500000]	[1.06879994221307e-12,0.000999999997915329,-0.00100000000208467]	[0.768236000000000,0.154244000000000,289.956168000000]
15	2046	357467	[-2408694.47400000,4698106.45000000,3566698.36500000]	[2.13759983021603e-12,-4.16934228027882e-12,-0.00200000000000000]	[0.768138000000000,0.153982000000000,289.955599000000]
16	2046	357468	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.000999999998931200,-2.08467124855963e-12,-0.00100000000106880]	[0.768280000000000,0.153836000000000,289.954806000000]
17	2046	357469	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-5.42101086242752e-20,-0.00100000000000000,2.08467097750908e-12]	[0.768809000000000,0.153741000000000,289.954332000000]
18	2046	357470	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00100000000000000,-0.00200000000000000,3.10054206701521e-12]	[0.768748000000000,0.153345000000000,289.953551000000]
19	2046	357471	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-1.08420217248550e-19,-0.00200000000000000,4.16934195501817e-12]	[0.768785000000000,0.152900000000000,289.953263000000]
20	2046	357472	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.00100000000213760,-0.00100000000416934,-0.00199999999684653]	[0.771165000000000,0.149885000000000,289.952263000000]
21	2046	357473	[-2408694.47400000,4698106.45000000,3566698.36500000]	[5.42101086242752e-20,0.00100000000000000,-2.08467097750908e-12]	[0.774957000000000,0.144397000000000,289.950270000000]
22	2046	357474	[-2408694.47400000,4698106.45000000,3566698.36500000]	[5.42101086242752e-20,0.00100000000000000,-2.08467097750908e-12]	[0.778186000000000,0.140403000000000,289.948580000000]
23	2046	357475	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.000999999998931200,0.00100000000208467,0.000999999998984129]	[0.782321000000000,0.137111000000000,289.948448000000]
24	2046	357476	[-2408694.47400000,4698106.45000000,3566698.36500000]	[2.13759988442613e-12,0.00199999999583066,-0.00200000000416934]	[0.767172000000000,0.148655000000000,289.950705000000]
25	2046	357477	[-2408694.47400000,4698106.45000000,3566698.36500000]	[0.00600000000213760,0.00199999999583066,-0.00199999999775654]	[0.809785000000000,0.104186000000000,289.942657000000]
26	2046	357478	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00999999999465600,-0.00400000001042336,-0.00500000000234931]	[0.814991000000000,0.0904560000000000,289.945652000000]
27	2046	357479	[-2408694.47400000,4698106.45000000,3566698.36500000]	[-0.00899999999679360,-0.00400000000625401,-0.00300000000128052]	[0.792704000000000,0.115543000000000,289.943491000000]

Figure 18 The format of reference truth file

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Reference

[1] Groves P 2013 Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd edn (Bostan and London: Artech House)

[2] Chao C and Chang G 2021 PPPLib: An open-source software for precise point positioning using GPS, BeiDou, Galileo, GLONASS, and QZSS with multi-frequency observations GPS Solutions. 25

[3] Kouba J 2009 A guide to using International GNSS Service (IGS) products