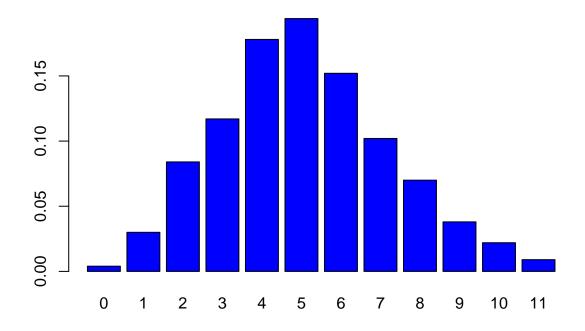
Grid Approximation and Highest Density Interval

```
set.seed(1999)
n<-1000
lambda <- 5
data<-rpois(n, lambda)
barplot(table(data)/n,col="blue",main="Poisson distribution n=1000")</pre>
```

Poisson distribution n=1000



```
Poisson_Dist<-function(x,lambda){
  p<-1
  for(i in 0:max(x)){
    p<-p*exp(-lambda)*lambda^i/factorial(i)
  }
  p</pre>
```

Likelihood

Dividing the data by the sum of data so it sums to $1? \rightarrow$ for simpler calculation for posterior

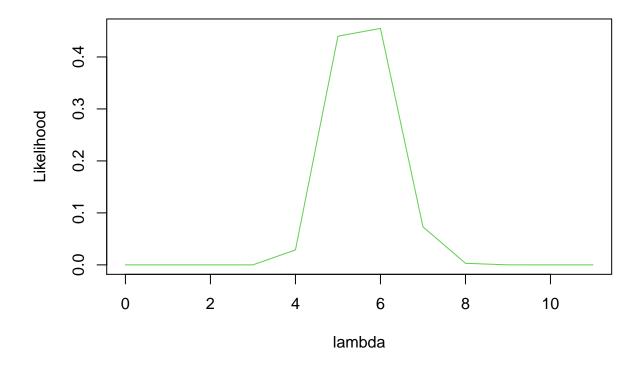
```
Likelihood<-function(x){
    likeli<-NULL
    for( i in 0:max(x)){
        p<-Poisson_Dist(x,lambda=i)
            likeli<-rbind(likeli,c(i,p))
}

# note
likeli[,2]<-likeli[,2]/sum(likeli[,2])

likeli<-data.frame(likeli)
    names(likeli)<-c("lambda","Likelihood")
    likeli
}

LkPoi<-Likelihood(x=data)
plot(LkPoi,col=3,type="l",main="Likelihood")</pre>
```

Likelihood



LkPoi

```
##
      lambda
               Likelihood
## 1
           0 0.000000e+00
## 2
           1 2.278110e-26
           2 1.032810e-11
## 3
## 4
           3 2.657724e-05
## 5
           4 2.876953e-02
## 6
           5 4.400072e-01
## 7
           6 4.548710e-01
```

```
## 8 7 7.325589e-02
## 9 8 3.025362e-03
## 10 9 4.418857e-05
## 11 10 2.842959e-07
## 12 11 9.422237e-10
```

Prior

```
shape: Alpha rate: Beta
```

```
prior<-dgamma(x=0:max(data),shape=5,rate=1)
plot(x=0:max(data),y=prior,col=2,type="l", main="Prior",xlab="lambda")</pre>
```

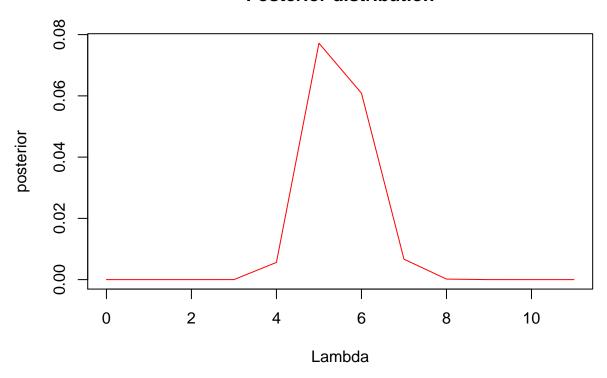


```
\#\# Posterior
```

```
Posterior<-function(x,a){
  i<-seq(0,max(x))
  p<-Likelihood(x)$Likelihood*dgamma(i,shape=a,rate=1)
  Post<-data.frame(i,p)
  names(Post)<-c("Lambda","posterior")
  Post
}</pre>
```

```
Post <- Posterior(x=data, a=lambda)
plot(Post,col="red",type="1", main= "Posterior distribution")</pre>
```

Posterior distribution



```
library(bayestestR)
map_estimate(Post)

## MAP Estimate
##
## Parameter | MAP_Estimate
## -----
## Lambda | 5.48
## posterior | 0.00

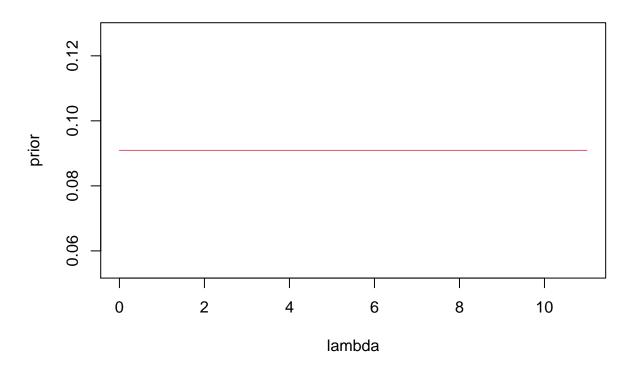
ci_hdi <- ci(Post$Lambda, method = "ETI")
ci_hdi

## 95% ETI: [0.28, 10.72]</pre>
```

Prior 2

```
prior<-dunif(x=0:max(data),0,max(data))
plot(x=0:max(data),y=prior,col=2,type="l", main="Prior",xlab="lambda")</pre>
```

Prior

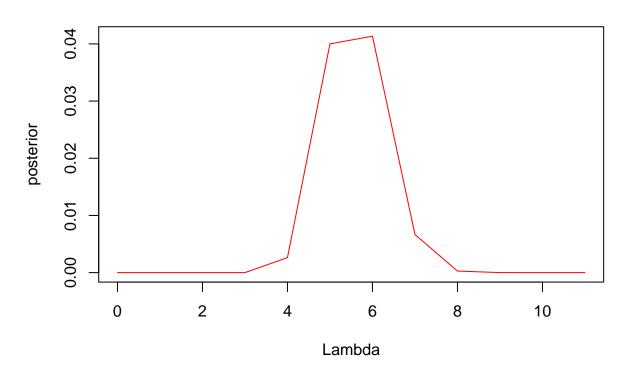


Posterior

```
Posterior<-function(x,a){
  i<-seq(0,max(x))
  p<-Likelihood(x)$Likelihood*dunif(x=0:max(x),0,max(x))
  Post<-data.frame(i,p)
   names(Post)<-c("Lambda","posterior")
  Post
  }

Post <- Posterior(x=data, a=lambda)
  plot(Post,col="red",type="l", main= "Posterior distribution")</pre>
```

Posterior distribution



```
library(bayestestR)
map_estimate(Post)

## MAP Estimate
##
## Parameter | MAP_Estimate
## ------
## Lambda | 5.48
## posterior | 4.04e-05

ci_hdi <- ci(Post$Lambda, method = "HDI")

## Warning in .hdi(x, ci = i, verbose = verbose): `ci` is too large or x does not
## contain enough data points, returning NAs.
ci_hdi
## 95% HDI:</pre>
```