ADA Mini HW #9

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I will construct a polynomial time transformation f such that $x \in \text{HAM-PATH} \iff f(x) \in \text{TSP}$

$f =$ "on input $< G = (V, E) >$:

1. construct V' such that $V' = V \cup \{v^*\}$

2. construct E' such that $E' = E \cup \{ \ (v^*, v_i) \mid v_i \in V \ \}$

3. Let $G' = (V', E')$

4. construct cost function c where:

(1) $c(v_i, v_j) = 0$ if $(v_i, v_j) \in E$
(2) $c(v_i, v_j) = 1$ if $(v_i, v_j) \notin E$
(3) $c(v^*, v_i) = 0 \ \forall \ v_i \in V$

5. output $< G', \ c \ , \ 0 >$"

*** Proof of Correctness:**

- Let $x =< G >$ where $G = (V, E)$
- Let $f(x) =< G', c, 0 >$ where $G' = (V', E')$

1. $\Longrightarrow$:

Let $h =< v_1, v_2, \ldots, v_n >$ be a Hamiltonian path in x
$\Longrightarrow (v^*, v_1) \in E'$ and $(v^*, v_n) \in E'$ by construction
$\Longrightarrow h' =< v^*, v_1, v_2, \ldots, v_n, v^* >$ is a tour in G'
$\Longrightarrow c(v^*, v_1) = c(v_n, v^*) = 0$ by construction
$\Longrightarrow c(v_i, v_{i+1}) = 0$ because $(v_i, v_{i+1}) \in E, \ \forall \ i \in \{1, 2, \ldots, n-1\}$
$\Longrightarrow$ the cost of $h' = 0 \leq 0 = k$
$\Longrightarrow f(x) \in \text{TSP}$

2. $\Longleftarrow$:

Let $h' =< v^*, v_1, v_2, \ldots, v_n, v^* >$ be a tour whose cost ≤ 0 in $f(x)$
$\Longrightarrow$ all edges in h' must have 0 cost since there is no negative cost by construction
$\Longrightarrow c(v_i, v_{i+1}) = 0, \ \forall \ i \in \{1, 2, \ldots, n-1\}$
$\Longrightarrow (v_i, v_{i+1}) \in E, \ \forall \ i \in \{1, 2, \ldots, n-1\}$
$\Longrightarrow h =< v_1, v_2, \ldots, v_n >$ is a path that visits all vertices only once in x
$\Longrightarrow x \in \text{HAM-PATH}$