ADA Mini HW #8

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(1) The total number of flips is 14.

number	0	1	2	3	4	5
read	0000000	1111111	1010101	1110001	1111001	1101001
# of flips	-	7	3	2	1	1

(2) By Aggregate method:

$$\begin{aligned}
\text{total \# of flips} &= \lfloor\frac{n}{1}\rfloor + \lfloor\frac{n}{2}\rfloor + \cdots + \lfloor\frac{n}{n}\rfloor \\
&= \sum_{i=1}^{n}\lfloor\frac{n}{i}\rfloor \\
&\le \sum_{i=1}^{n}\frac{n}{i} \\
&\approx n\int_{1}^{n}\frac{1}{x}dx \\
&= n(ln(n) - ln(1)) \\
&= n\,ln(n)
\end{aligned}$$

$$\Longrightarrow \text{the amortized cost for one counting operation is } \frac{n\,ln(n)}{n} = ln(n) \ \#$$

(3) (a) When we count to x, we flip $a_x, a_{2x}, a_{3x}, \ldots$.

(b) After we count x, a_x and all the lower bits will never be flipped in later operations, because $1 \cdot y$ always greater than $1 \cdot x$ if $y > x$.

(c) Let the counter reads V_x when it count to x, then V_x and V_{x+1} differ from each other at a_{x+1} because the counter flips a_{x+1} when count to $x+1$, and by (b), we know a_{x+1} will no longer be flipped, so V_x will also be different from V_y, for all $y > x$ at the $(x+1)^{th}$ bit (a_{x+1}).

(d) From above observation, we get that $V_x \neq V_y$ for all $1 \le x < y \le n$, which means each pair of reading of the counter when counting from 0 to n is different from each other $\Longrightarrow$ all of them are unique.