ADA Mini HW #1

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Answer

$given\ g(n) = \Theta(F(n))$
$\implies \exists\ c_1, c_2, n_1 > 0,\ s.t.\ 0 \le c_1 F(n) \le g(n) \le c_2 F(n),\ \forall n \ge n_1\ \ \ldots(1)$

$given\ h(n) = o(F(n))$
$\implies \forall\ c_3 > 0,\ \exists\ n_2 > 0,\ s.t.\ 0 \le h(n) < c_3 F(n),\ \forall n \ge n_2\ \ \ldots(2)$

$from\ (1)\ ,\ c_1 F(n) \le g(n)$
$from\ (2)\ ,\ h(n) < c_3 F(n)$
$\implies g(n) - h(n) \ge c_1 F(n) - h(n) > c_1 F(n) - c_3 F(n) = (c_1 - c_3)F(n)$
$since\ c_3\ can\ be\ any\ positive\ constant\ ,\ take\ c_3 < c_1$
$Let\ c = (c_1 - c_3) > 0,\ n_0 = max(n_1,\ n_2)$
$\implies 0 \le cF(n) \le g(n) - h(n),\ \forall\ n \ge n_0$
$\implies f(n) = g(n) - f(n) = \Omega(F(n))$
$Q.E.D$