ADA Mini HW #7

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Let the N pieces of land denoted as $v_1, v_2, \dots, v_N$. Construct undirected graph $G = \langle V, E \rangle$ as follow:

1. $V = S \cup \{v_1, v_2, \dots, v_N\}$
2. $\forall\ i \in \{1, 2, \dots, N\}$, there is an edge weighted W_i connecting S and v_i.
3. $\forall\ (i, j) \in \{1, 2, \dots, N\}^2$ *and* $i \neq j$, there is an edge weighted P_{ij} that connects v_i and v_j.

Then we run *Kruskal's algorithm* on this graph to compute the *Minimum Spanning Tree* and the answer would be the total weight of the result *Minimum Spanning Tree*.

Explanation of Correctness: The edges between S and v_i can be seen as "build a reservoir" on land v_i. And the construction of minimum spanning tree ensures that all nodes have a simple path to S (i.e. connected to a land with a reservoir).

Time Complexity Analysis: $|V| = N + 1 = O(N)$, and there is an edge for each distinct pair of vertices, so $|E| = C_2^{N+1} = \frac{N(N+1)}{2} = \frac{N^2}{2} + \frac{N}{2} = O(N^2)$. The time complexity of *Kruskal's algorithm* where *disjoint-set-forest* is implemented with *union-by-rank* only is $O(E \, log \, V) = O(N^2 \, log \, N)$, which satisfies the requirement.