Macroeconomics I
Problem Set 5 Solutions

Barcelona School of Economics, Spring 2026

Due date: Monday, 16th February, 23:59

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Submission Instructions

Please submit a document with your answers to the course instructor via the designated submission platform by Monday, 16th February, 23:59 at the very latest. You can work in groups of up to four.

1 Misallocation and Market Power

In this whole exercise, we consider a closed economy, with a fixed population size. Assume the firm's production function is given by

$$Y_i = \varphi L_i^\alpha K_i^\beta \quad \text{with} \quad \alpha + \beta \leq 1 \tag{1}$$

(Note: there is no i index on φ in the baseline case.)

a. Take the standard firm problem (static setting) with a continuum of identical atomistic firms and decreasing returns to scale technology. Assume firms are competitive on both their output and input markets. Describe the firm's problem and the first order conditions for capital and labor.

Solution: Each firm i takes the output price (normalized to 1), the wage w, and the rental rate r as given, and solves

$$\max_{L_i, K_i} \ \pi_i = \varphi\, L_i^\alpha K_i^\beta - wL_i - rK_i.$$

Since $\alpha + \beta \leq 1$ the technology exhibits (weakly) decreasing returns to scale, so the objective is strictly concave in (L_i, K_i) and the first-order conditions are both necessary and sufficient for a unique interior maximum.

The FOCs are:

$$\frac{\partial \pi_i}{\partial L_i}: \quad \alpha\, \varphi\, L_i^{\alpha-1} K_i^\beta = w \qquad \Longleftrightarrow \qquad MPL_i = w,$$

$$\frac{\partial \pi_i}{\partial K_i}: \quad \beta\, \varphi\, L_i^\alpha K_i^{\beta-1} = r \qquad \Longleftrightarrow \qquad MPK_i = r.$$

That is, each competitive firm hires labor and capital up to the point where their marginal products equal the respective factor prices.

b. Define the following measure of misallocation of factors: for generic factor x, denote $\tau_x = \text{var}(MPX^*)$, where $\text{var}(\cdot)$ is the variance across firms and MPX is the marginal product of x. The $*$ indicates a value taken at the equilibrium. What is the level of misallocation in this model? Derive and argue your answer.

Solution: From the FOCs in part (a), at the competitive equilibrium every firm i satisfies

$$MPL_i^* = \alpha\,\varphi\,(L_i^*)^{\alpha-1}(K_i^*)^{\beta} = w, \qquad MPK_i^* = \beta\,\varphi\,(L_i^*)^{\alpha}(K_i^*)^{\beta-1} = r.$$

Because all firms are *identical* (same φ, same technology) and face the same factor prices w and r, they all choose the same input bundle (L_i^*, K_i^*). Consequently the marginal products are the same constant across all firms:

$$MPL_i^* = w \;\; \forall\, i, \qquad MPK_i^* = r \;\; \forall\, i.$$

Since the marginal products are identical across firms, their cross-sectional variance is zero:

$$\boxed{\tau_L = \text{var}(MPL^*) = 0, \qquad \tau_K = \text{var}(MPK^*) = 0.}$$

There is **no misallocation** of factors. This is a direct consequence of the First Welfare Theorem: in a competitive equilibrium without distortions, the allocation of resources is Pareto efficient. Because all firms equalize their marginal products to common factor prices, no reallocation of inputs across firms could raise aggregate output.

c. Assume now that firms are heterogeneous in productivity φ_i. Compute again τ_x for capital and labor and discuss your finding.

Solution: With heterogeneous productivity φ_i, each firm i still takes w and r as given and maximizes

$$\max_{L_i, K_i} \; \varphi_i\, L_i^{\alpha} K_i^{\beta} - wL_i - rK_i.$$

The FOCs are:

$$\begin{aligned} \alpha\,\varphi_i\,L_i^{\alpha-1}K_i^{\beta} &= w \qquad \Longleftrightarrow \qquad MPL_i = w, \\ \beta\,\varphi_i\,L_i^{\alpha}K_i^{\beta-1} &= r \qquad \Longleftrightarrow \qquad MPK_i = r. \end{aligned}$$

Even though φ_i differs across firms, each firm *still* equates its marginal product to the common factor price. At equilibrium, $MPL_i^* = w$ and $MPK_i^* = r$ for every firm i, regardless of the level of φ_i. Firms with higher φ_i simply choose larger input bundles (L_i^*, K_i^*), but the marginal products at the optimum are the same across all firms.

Therefore:

$$\boxed{\tau_L = \text{var}(MPL^*) = 0, \qquad \tau_K = \text{var}(MPK^*) = 0.}$$

Discussion. Introducing firm-level productivity heterogeneity does *not* generate misallocation in a competitive economy. Competitive factor markets ensure that inputs flow to where their marginal product equals the market price, so marginal products are equalized across firms. More productive firms attract proportionally more capital and labor, but this is precisely the efficient allocation: any reallocation of inputs away from this equilibrium would *lower* aggregate output. This result is again a manifestation of the First Welfare Theorem.

d. Compute the optimal size of the firm and the associated profits. Discuss the implication of what you found in terms of efficiency of the allocation. In particular, discuss how your conclusions depend on whether $\alpha + \beta < 1$ or $\alpha + \beta = 1$.

Solution: From the FOCs of the firm with heterogeneous φ_i (part c) we can write

$$wL_i = \alpha\, Y_i, \qquad rK_i = \beta\, Y_i,$$

so $L_i = \frac{\alpha\, Y_i}{w}$ and $K_i = \frac{\beta\, Y_i}{r}$. Substituting into the production function $Y_i = \varphi_i\, L_i^{\alpha} K_i^{\beta}$:

$$Y_i = \varphi_i \left(\frac{\alpha\, Y_i}{w}\right)^{\alpha} \left(\frac{\beta\, Y_i}{r}\right)^{\beta} = \varphi_i\, \frac{\alpha^{\alpha}\beta^{\beta}}{w^{\alpha} r^{\beta}}\, Y_i^{\alpha+\beta},$$

which gives

$$Y_i^{1-\alpha-\beta} = \varphi_i\, \frac{\alpha^{\alpha}\beta^{\beta}}{w^{\alpha} r^{\beta}}.$$

Case 1: $\alpha + \beta < 1$ (strictly decreasing returns to scale).

The optimal output of firm i is

$$Y_i^* = \left(\varphi_i\, \frac{\alpha^{\alpha}\beta^{\beta}}{w^{\alpha} r^{\beta}}\right)^{\frac{1}{1-\alpha-\beta}},$$

and the optimal input demands are

$$L_i^* = \frac{\alpha}{w}\, Y_i^*, \qquad K_i^* = \frac{\beta}{r}\, Y_i^*.$$

Profits are

$$\pi_i^* = Y_i^* - wL_i^* - rK_i^* = Y_i^* - \alpha\, Y_i^* - \beta\, Y_i^* = (1 - \alpha - \beta)\, Y_i^* > 0.$$

Firm size (output, inputs, and profits) is uniquely determined and strictly increasing in φ_i: more productive firms are larger and earn higher profits.

Case 2: $\alpha + \beta = 1$ (constant returns to scale).

The equation becomes $Y_i^0 = 1 = \varphi_i\, \frac{\alpha^{\alpha}\beta^{\beta}}{w^{\alpha} r^{\beta}}$, which is a condition on equilibrium prices (w, r), *not* on firm size. The profit function is linear in scale:

$$\pi_i = Y_i - \alpha\, Y_i - \beta\, Y_i = (1 - \alpha - \beta)\, Y_i = 0.$$

Profits are zero and firm size is **indeterminate**: any (L_i, K_i) with $K_i/L_i = \beta w/(\alpha r)$ is optimal. With heterogeneous φ_i, equilibrium prices can satisfy the zero-profit condition for at most one value of φ, so only the most productive firms can operate; all others make losses and exit.

Efficiency discussion. In both cases the competitive allocation is efficient (First Welfare Theorem): marginal products are equalized across firms and there is no misallocation ($\tau_L = \tau_K = 0$).

Under DRS ($\alpha + \beta < 1$), positive profits $(1 - \alpha - \beta)Y_i^*$ represent the return to the implicit "entrepreneurial" or fixed factor that generates decreasing returns at the firm level. Multiple heterogeneous firms coexist in equilibrium, each at their uniquely determined efficient scale.

Under CRS ($\alpha + \beta = 1$), zero profits and indeterminate firm size mean that the distribution of production across firms is irrelevant for aggregate efficiency—any allocation of inputs that equates marginal products is equally efficient. However, with heterogeneous φ_i, the model cannot sustain the coexistence of firms with different productivities, so only the most productive firms survive.

e. Now let's allow for market power. We can keep this very general by simply having the firm optimize over an inverse demand schedule $p(c_i)$ which responds to its decisions. Assume that the function $p(\cdot)$ is the same for all firms. Write down the firm's maximization problem, compute the measure of misallocation and discuss your finding. Is your finding the same if we assume productivity to be the same at all firms ($\varphi_i = \varphi$)?

Solution:

f. In the same economy of point (e), compute a new measure of misallocation $\tilde{\tau}_x$ defined as the variance of the marginal *revenue* product of input x (at the equilibrium). The marginal revenue product is defined as: $MRPX \equiv \partial r(\cdot)/\partial x$, where $r(\cdot)$ is the revenue function: $r(y_i) = p_i(y_i)y_i$. Discuss your findings.

Solution:

g. Discuss these two measures. Are they good measures of misallocation? How do they relate to one another (and when are they equal)? What can we infer about the efficiency of the allocation in this economy (with the assumptions of point (e))?

Solution:

h. Derive the socially optimal and market-induced firm size. What can you conclude about the level and cross-section efficiency of this economy?

Solution:

i. Consider the following graphs of the distribution of marginal product of capital and labor (see Figures 1 and 2). As background, we obtained these distributions from firm level data by doing the following steps:

- We estimated the industry production function to obtain estimates of α and β, the output elasticities of capital and labor.
- We used firm sales, number of workers, and capital stock to compute:

$$MPL_i = \beta \frac{\text{sales}_i}{\#\text{workers}_i} \tag{2}$$

$$MPK_i = \alpha \frac{\text{sales}_i}{\text{capital stock}_i} \tag{3}$$

What can we conclude from these distributions about the level of misallocation in this economy? Are these measures, computed as described above, good proxies for misallocation? What assumptions are we making that might be violated in the data?

Solution:

j. So far we have only assumed that firms are heterogeneous and have some market power. Consider now the specific preferences given by a CES aggregator. How do your previous conclusions change?

Solution:

k. Assume now that in this CES economy, firms compete in monopolistic competition. How do your previous conclusions change?

Solution:

Figures

[Figure 1: MPK distribution — add mpk_distribution.pdf to figs/]

Figure 1: Distribution of Marginal Product of Capital (MPK) across firms.

[Figure 2: MPL distribution — add mpl_distribution.pdf to figs/]

Figure 2: Distribution of Marginal Product of Labor (MPL) across firms.