

## Assignment 05

### Solution by Kaifa Lu

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#### 1. Goal

This assignment05 aims to hand design a neural network whose decision surfaces can cluster data samples of  $x, y$  within the range  $(-10:10)$  into three classes, shown as “UF”. One scatter plot is required to show the results from neural network where  $x, y$  describe the location of data samples and the corresponding color differentiates different classes.

#### 2. Implementation

##### (1) Input

The inputs of the hand design neural network are one two-dimensional vector  $(x, y)$ . In this assignment, we generate one  $6561 * 2$  data array as inputs. Each value in the data array ranges from -10 to 10 in both  $x$  and  $y$  coordinates with a step size of 0.25.

##### (2) Output

The outputs of the hand design neural network are one of three two-dimensional vectors:  $[0,0]$ ,  $[1,1]$  and  $[0,1]$ , which separately correspond to the “U” class in blue color, the “F” class in orange color and neither of the two “U” and “F” classes in white color. For convenience, we remap the three vectors:  $[0,0]$ ,  $[1,1]$  and  $[0,1]$  into three scalars: 0, 1, -1 as the final outputs of the hand design neural network.

##### (3) Activate Function

In this assignment, we define a total of four activate functions used in different neurons of the hand design neural network as follows:

*Activate Function 1:*

$$f_1: \quad \phi(v)_1 = \begin{cases} 1, & 0 \leq v \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

*Activate Function 2:*

$$f_2: \quad \phi(v)_2 = \begin{cases} 1, & 0 < v \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Activate Function 3:

$$f_3: \quad \phi(v)_3 = \begin{cases} 1, & 0 < v < 1 \\ 0, & \text{otherwise} \end{cases}$$

Activate Function 4:

$$f_4: \quad \phi(v)_4 = v$$

#### (4) Neural Network Structure

Table 1 shows one summary of the structure of the hand design neural network, in terms of the number of layers, the number of neurons in each layer and the activation functions. The hand design neural network has 1 input layer, 3 hidden layers and 1 output layer. More details about the structure of the hand design neural network are illustrated in Figure 1, particularly specifying the weights and biases for each neuron.

**Table 1** Summary of the structure of the hand design neural network

Layer	Neuron	Activate Function
Input Layer 1	2	N/A
Hidden Layer 1	12	(1) Neuron 1,3,5,7,8,10,12: $f_1$ (2) Neuron 6: $f_2$ (3) Neuron 2,4,9,11: $f_3$
Hidden Layer 2	5	$f_2$
Hidden Layer 3	2	$f_2$
Output Layer 1	1	$f_4$

From Input Layer to Hidden Layer 1:

$$\omega_1 = [[0.5, 0.25, 0.5, 1/3, 0.5, 0.5, 0.5, 0, 0, 0, 0], \\ [0, 0, 0, 0, 0, 0, 1/3, 1, 0.5, 0.5, 0.5]]$$

$$b_1^T = [4.5, 1.75, 1.5, 1/3, -1, -2, -3.5/3, 2, 0.5, -0.5, -1.5]$$

From Hidden Layer 1 to Hidden Layer 2:

$$\omega_2^T = [[0, 0, 0, 0, 0, 0.5, 0, 0, 0, 0.5, 0, 0.5], \\ [0, 0, 0, 0, 0, 0, 0.5, 0, 0, 0, 0.5], \\ [0, 0, 0, 0, 0.5, 0, 0, 0.5, 0.5, 0.5, 0.5, 0.5], \\ [0.5, 0, 0.5, 0, 0, 0, 0, 0.5, 0.5, 0.5, 0.5, 0.5], \\ [0, 0.5, 0, 0, 0, 0, 0, 0.5, 0, 0, 0, 0]]$$

$$b_2^T = [-0.5, -0.5, -0.5, -0.5, -0.5]$$

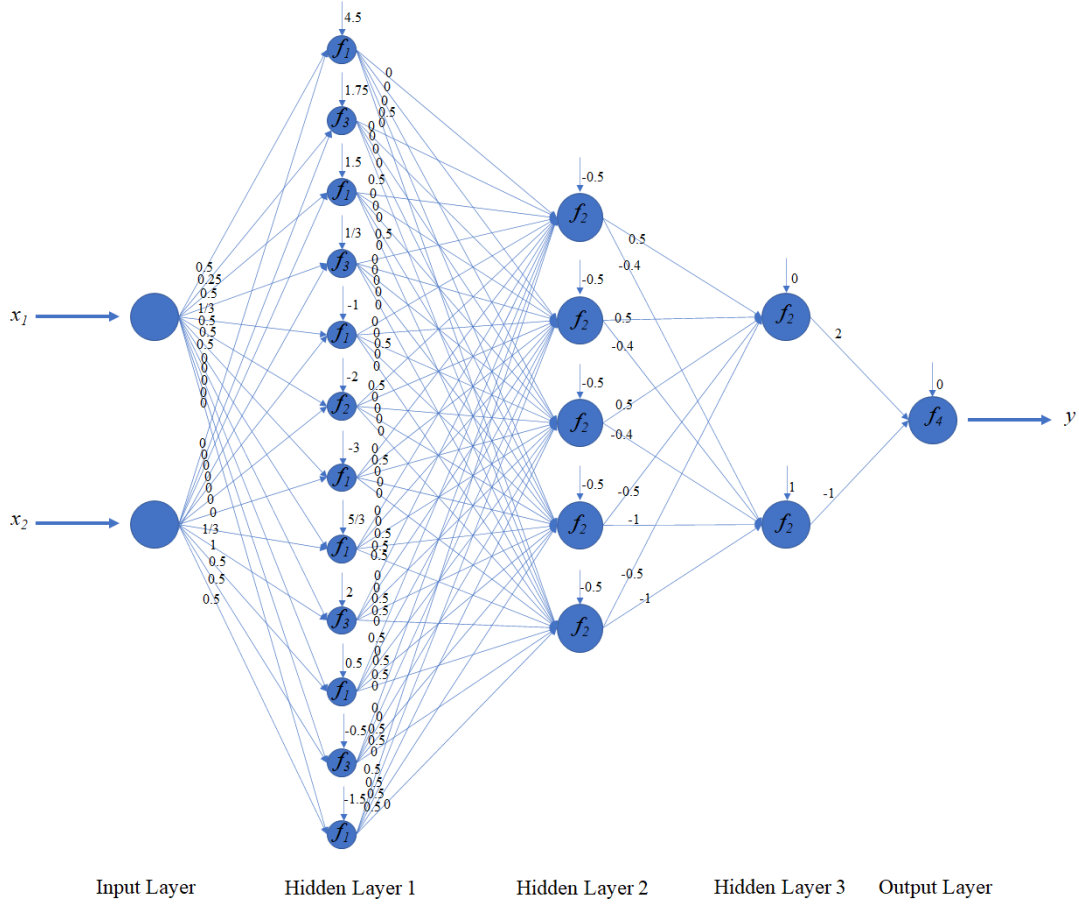
From Hidden Layer 2 to Hidden Layer 3:

$$\omega_3^T = [[0.5, 0.5, 0.5, -0.5, -0.5], \\ [-0.4, -0.4, -0.4, -1, -1]]$$

$$b_3^T = [0, 1]$$

From Hidden Layer 3 to Output Layer:

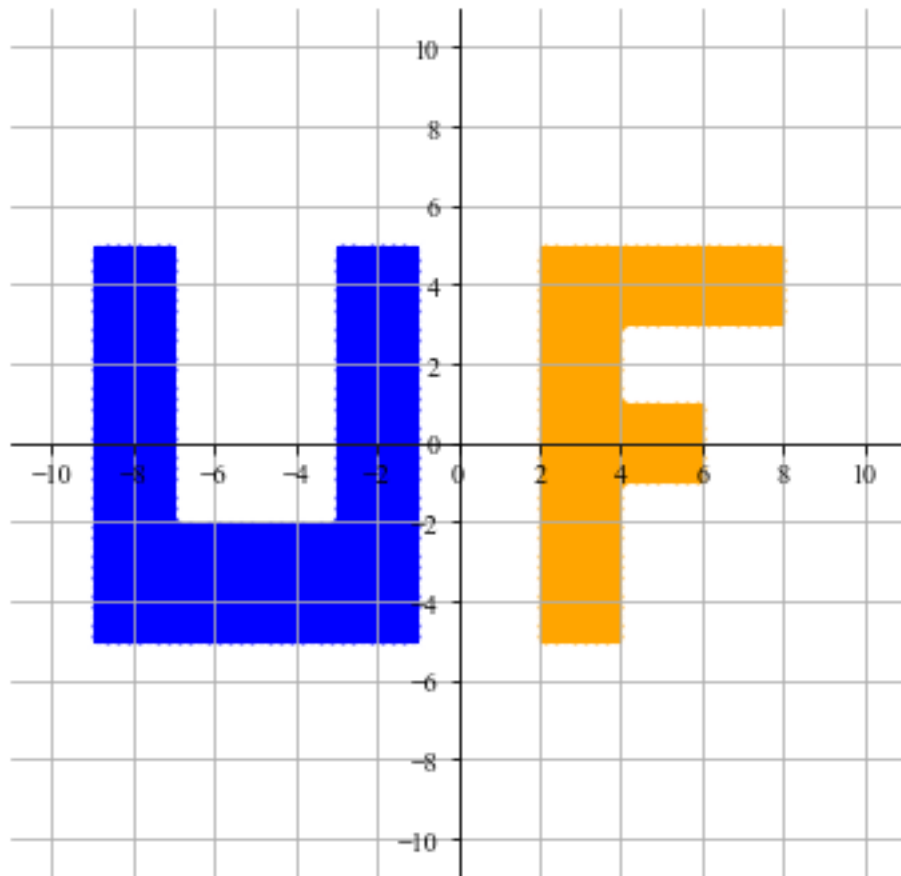
$$\omega_4^T = [2, -1] \\ b_4^T = [0]$$



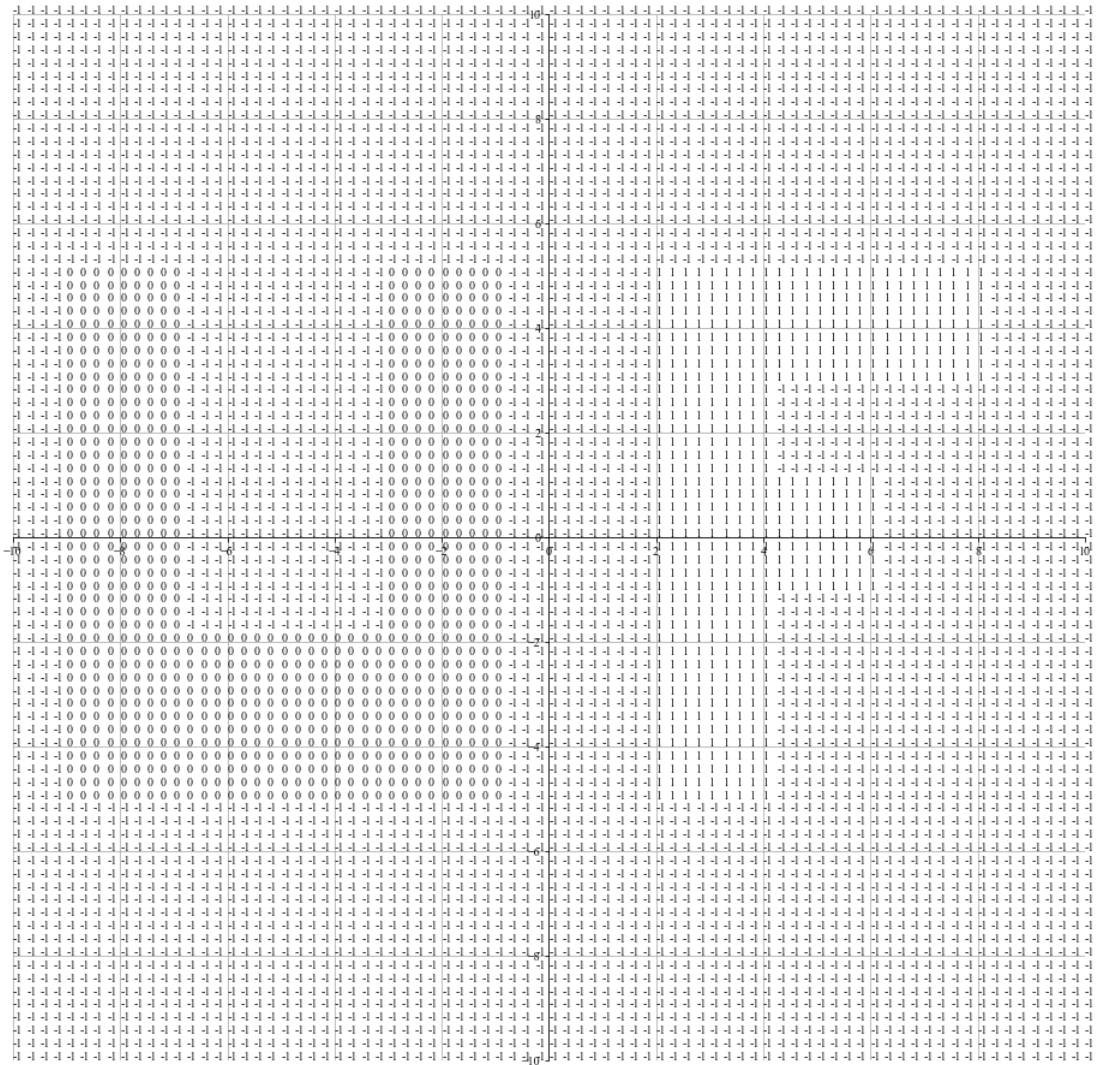
**Figure 1** Structure of the hand design neural network

### 3. Results

Figure 2 shows the scatter plot of output classes from the hand design neural network, while Figure 3 visualizes the image of the whole output matrix of the hand design neural network.



**Figure 2** Scatter plot of outputs from the hand design neural network



**Figure 3** Visualization of the whole output matrix of the hand design neural network