Fragmented Evolution Theory

A Formal Structure Based on Scale-Dependent Differences

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Abstract

This paper presents a formal decision-based framework in which *decidable difference* is treated as the only decision gate, in order to characterize under what structural conditions a system appears as "evolving" at a given scale, and under what conditions it collapses into "halt". Differences are treated as scale-dependent objects. Evolution is defined as the persistence of decidable differences over time intervals; dead loops are characterized as absorbing states in which all future differences collapse to zero; fragmentation is defined as the existence of a positive lower bound on differences, thereby structurally excluding the existence of global terminal attractors.

Scale is further treated as an operator acting on difference structures. Under this view, we formally derive the claims that zero difference may be a scale-induced artifact, that evolution is scale-relative, that complexity is a scale function rather than an ontological property, and that irreversibility can be understood as the structural cost of the decision framework itself.

No assumptions of optimization, progress, monotonic growth, or continuity are introduced. The goal is to provide a reusable decision structure and a list of structural consequences, serving system classification, boundary annotation, and cross-scale formal alignment.

Keywords: decidability; scale operator; difference function; dead loop; fragmentation; complexity; irreversibility

1 Introduction

In many disciplinary contexts, "evolution" is implicitly assumed to exist, and discussion is often shifted toward questions such as the direction of evolution, whether evolution represents progress, or whether complexity increases. However, at a more fundamental decision level, a more prior question is frequently skipped: under what structural conditions does a system's change remain decidable?

If, at a certain analytical scale, all state differences of a system are fully compressed to zero, then the question of "whether change occurs" loses its decision meaning at that scale. Conversely, if differences diverge beyond the resolving capacity of that scale, decision likewise fails.

Fragmented Evolution Theory does not take "evolution necessarily occurs" as a premise. Instead, it takes "under what conditions evolution can or cannot be decided" as its starting point, and establishes a formal framework centered on scale-dependent differences.

The main contributions of this paper are:

1. a unified formal structure of difference, decidability, dead loop, evolution, fragmentation, and complexity;
2. a set of derived structural consequences without introducing optimization or value narratives, including: fragmentation excludes terminal attractors; evolution is scale-relative; zero difference can be scale-induced; complexity is a scale function; irreversibility as a cost of decision structure;
3. a finite semantic model (Appendix) serving as a consistency and satisfiability witness, enabling reproducibility and conceptual alignment.

2 Preliminaries: Object Levels and Notation

2.1 Time Domain and State Space

Definition 1 (Time Domain). Let T be a set of time indices. This paper only assumes the existence of a symbolic description of temporal advancement: for any $t \in T$, a "successor time" $t + \delta$ can be discussed, where $\delta > 0$ denotes the minimal advancement unit. No assumptions are made regarding continuity, discreteness, or topology of T, unless explicitly stated.

Definition 2 (State Space). Let X be a set of states. No algebraic, topological, or metric structure on X is assumed. If additional structure is required in specific applications, it may be introduced without altering the decision gate of this framework.

Definition 3 (System). A system is defined as a mapping of states over time:

$$S : T \to X, \quad t \mapsto x_t.$$

A system is treated primarily as a state trajectory rather than a static object.

2.2 Scale Parameters and Scale-Dependent Objects

Definition 4 (Scale Parameter Set). Let Σ be a set of scale parameters, with $\sigma \in \Sigma$. Scale delimits the structural boundaries of observability, operability, and decidability. Scale is not interpreted as "perspective", but as a structural condition determining how difference structures are revealed or collapsed.

3 Difference and Decidability

Definition 5 (Difference Function, Scale-Dependent). For each scale $\sigma \in \Sigma$, define a difference function

$$\Delta_\sigma : X \times X \to \mathbb{R}_{\geq 0}.$$

$\Delta_\sigma(x, y)$ denotes the decidable difference between states x and y under scale σ.

Definition 6 (Decidability Predicate). Define the predicate $\mathrm{Dec}_\sigma(x, y)$ as

$$\mathrm{Dec}_\sigma(x, y) \iff \Delta_\sigma(x, y) > 0,$$

and correspondingly

$$\neg\mathrm{Dec}_\sigma(x, y) \iff \Delta_\sigma(x, y) = 0.$$

In this framework, decidability is entirely determined by whether the difference function evaluates to zero.

Remark 1. The deliberate choice of "$\Delta_\sigma = 0$" as the sole trigger for undecidability preserves the reusability of the decision gate. Threshold-based variants (e.g. $\Delta_\sigma > \varepsilon_\sigma$) may be viewed as conservative extensions by replacing Δ_σ with $\max\{0, \Delta_\sigma - \varepsilon_\sigma\}$.

4 Dead Loop and Evolution

Definition 7 (Dead Loop, Absorbing State). A system S enters a dead loop at time $t \in T$ under scale σ if and only if

$$\forall \delta > 0,\ \Delta_\sigma(x_t, x_{t+\delta}) = 0.$$

A dead loop indicates that no future temporal advancement produces decidable differences at that scale.

Definition 8 (Evolution Interval). A system S evolves over an interval $I \subset T$ under scale σ if and only if

$$\forall t \in I,\ \Delta_\sigma(x_t, x_{t+\delta}) > 0.$$

Evolution is an interval property rather than a point event: its essence lies in the persistence of decidable differences through temporal advancement.

Lemma 1 (Incompatibility of Dead Loop and Evolution). If a system S enters a dead loop at time t under scale σ, then no interval $I \subset T$ exists such that $t \in I$ and S evolves on I.

Proof. Dead loop requires $\Delta_\sigma(x_t, x_{t+\delta}) = 0$ for all $\delta > 0$, while evolution requires $\Delta_\sigma(x_t, x_{t+\delta}) > 0$. Contradiction. □

Corollary 1 (Absorbing Property of Dead Loop). Once a system enters a dead loop under scale σ, no evolution interval containing any later time exists under that scale.

5 Fragmentation Structure

Definition 9 (Fragmentation, Lower Bound of Difference). A system S is fragmented under scale σ if and only if there exists a constant $c_\sigma > 0$ such that

$$\forall t \in T,\ \Delta_\sigma(x_t, x_{t+\delta}) \geq c_\sigma.$$

Fragmentation expresses a positive lower bound on differences, not system defect or noise.

Lemma 2 (Fragmentation Excludes Dead Loop). If a system S is fragmented under scale σ, then no dead loop time exists under that scale.

Proof. Fragmentation implies $\Delta_\sigma(x_t, x_{t+\delta}) \geq c_\sigma > 0$ for all t, contradicting the dead loop condition. □

Proposition 1 (Fragmentation Excludes Global Terminal Attractors). If a system

S is fragmented under scale σ, then no state $x \in X$ and time $t_0 \in T$ exist such that

$$\forall t \geq t_0, \ \Delta_\sigma(x_t, x) = 0.$$

Proof. If such x, t_0 existed, then for sufficiently large t, both x_t and $x_{t+\delta}$ would be indistinguishable from x, yielding $\Delta_\sigma(x_t, x_{t+\delta}) = 0$, contradicting fragmentation. □

Remark 2. If one wishes to avoid implicit consistency assumptions, Proposition 1 may be reformulated as the exclusion of eventual collapse of adjacent differences, which directly contradicts fragmentation.

6 Scale as Operator and Scale-Induced Zero Difference

Definition 10 (Scale Operator). Scale σ acts as a structural condition determining the form of Δ_σ; different scales induce different difference structures.

Proposition 2 (Scale-Induced Zero Difference). For a given state pair (x, y), $\Delta_\sigma(x, y) = 0$ under some scale σ does not imply $\Delta_{\sigma'}(x, y) = 0$ for all scales σ'.

Proposition 3 (Scale Relativity of Evolution). There exist a system S and a scale pair (σ, σ') such that:

1. S has an evolution interval under scale σ;
2. S has no evolution interval under scale σ'.

7 Complexity as Scale Function

Definition 11 (Complexity). The complexity of a system S under scale σ, denoted $C_\sigma(S)$, represents the density or summary of decidable difference structures under that scale.

Proposition 4 (Complexity Is a Scale Function). $C_\sigma(S)$ varies with σ; complexity is not an intrinsic property of the system.

Corollary 2 (No Absolute Complexity). No scale-invariant absolute complexity $C(S)$ exists such that $C(S) = C_\sigma(S)$ for all σ.

8 Scale Window, Researchability, and Irreversibility

Definition 12 (Scale Window). A subset $\Sigma' \subseteq \Sigma$ is called a scale window of system S if there exists $\sigma \in \Sigma'$ such that S has an evolution interval under σ.

Proposition 5 (Researchability Criterion). A system S is researchable if and only if its scale window is nonempty.

Proposition 6 (Irreversibility as Decision Cost). Under a fixed scale σ, once a system enters an undecidable state, its history cannot be fully recovered under the same decision structure. Irreversibility is thus an intrinsic cost of the decision framework.

Remark 3. This proposition does not rely on thermodynamic assumptions, but only states irrecoverability under a fixed decision gate.

9 Discussion: Boundaries and Minimal Notes

This framework does not address:

1. whether evolution implies progress;
2. whether complexity grows monotonically;
3. whether a unique correct scale exists;
4. specific dynamics or control strategies.

10 Conclusion

This paper establishes a decision-based formal framework centered on scale-dependent differences. Evolution is characterized as the persistence of decidable differences; dead loops as absorbing zero-difference states; fragmentation as a structural lower bound excluding terminal attractors. Scale induces relativity of evolution, complexity becomes a scale function, researchability is characterized by scale windows, and irreversibility emerges as a structural decision cost.

A Appendix: Finite Semantic Model (kfx-10 Example)

This appendix provides a finite model as a semantic witness of consistency and satisfiability.

A.1 Model Definition

Let $X = \mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$, $T = \mathbb{Z}_5$, and define the system trajectory as

$$x_t = t \pmod 5, \quad x_{t+1} = x_t + 1 \pmod 5.$$

Define a difference function under a fixed scale σ as

$$\Delta_\sigma(x, y) := (y - x) \pmod 5,$$

with 0/nonzero interpreted as undecidable/decidable.

A.2 Property Verification

For all t,

$$\Delta_\sigma(x_t, x_{t+1}) = 1 \neq 0,$$

hence evolution exists and no dead loop occurs. Moreover, $\Delta_\sigma(x_t, x_{t+1}) \geq 1$, so fragmentation holds.

A.3 Illustration of Scale Relativity

Under a coarse-grained scale σ', define $\Delta_{\sigma'}(x, y) = 0$ for all (x, y). Then the same trajectory exhibits no evolution interval under σ', illustrating scale-relative evolution.