xkf: A Formal Minimal Shell for Continuity

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Purpose

This document defines **xkf**, a formal minimal shell whose sole purpose is to fix and freeze the *legitimate sources of continuity.* It is not a replacement for analysis, topology, or measure theory. Rather, it serves as a *boundary object* that constrains which continuity claims are admissible.

Any theory invoking limits, continuity, derivatives, or integrals must be reducible to the gates explicitly defined herein in order to be considered valid within the xkf shell.

1 Primitive Objects

- A countable set $\mathbb{N}$.
- The derived sets $\mathbb{Z}$ and $\mathbb{Q}$ constructed from $\mathbb{N}$.
- A binary function $d : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}_{\geq 0}$.

No uncountable objects are assumed as primitive.

2 Axioms

Axiom 1 (Arithmetic Kernel)**.** $\mathbb{N}$ *satisfies the Peano axioms. From* $\mathbb{N}$*, the structures* $\mathbb{Z}$ *and* $\mathbb{Q}$ *are constructed.* $(\mathbb{Q}, +, \cdot, <)$ *forms an ordered field.*

Axiom 2 (Metric Gate)**.** *Define, on* $\mathbb{Q}$*,*

$$d(x, y) = |x - y|.$$

The function d *satisfies:*

1. $d(x, y) = 0 \iff x = y$,
2. $d(x, y) = d(y, x)$,
3. $d(x, z) \leq d(x, y) + d(y, z)$.

Axiom 3 (Cauchy Gate)**.** *A sequence* $(q_n) \subset \mathbb{Q}$ *is Cauchy if and only if*

$$\forall \varepsilon \in \mathbb{Q}_{>0},\ \exists N \in \mathbb{N},\ \forall m, n > N :\ d(q_m, q_n) < \varepsilon.$$

Axiom 4 (Completion Gate)**.** *Define an equivalence relation on Cauchy sequences:*

$$(q_n) \sim (p_n) \iff d(q_n, p_n) \to 0.$$

Define

$$\mathbb{R} := \mathbb{Q}_{Cauchy}/\sim .$$

3 Derived Structures

Within $(\mathbb{R}, d)$, the following notions may be defined via standard constructions reducible to the above axioms:

- limits,
- continuity,
- derivatives (as difference-quotient limits),
- integrals (Riemann or constructions provably equivalent at this level).

No additional structures are taken as primitive.

4 Boundary Conditions

1. xkf does not take topology, measure, linear spaces, or set-theoretic axioms as primitives.
2. Any such structures, if used, must be explicitly constructed on top of xkf.
3. No appeal to geometry, physics, intuition, or semantics is admissible as a substitute.

5 Non-Circumvention Theorems

Theorem 1 (Continuity Source Uniqueness). *Any valid notion of continuity within xkf must be reducible to the Metric Gate and the Completion Gate. No alternative source of continuity exists within the shell.*

Theorem 2 (Continuity Exclusion). *Any system invoking continuity, limits, or derivatives without an explicit reduction to the axioms of xkf is invalid within the shell.*

Theorem 3 (Escape Clause Failure). *Appeals to topology, smoothness, geometry, physical realism, or "intuitive limits" do not constitute admissible extensions unless explicitly reconstructed from xkf axioms. Such appeals do not bypass the shell.*

6 Frozen Decision Sentence

Theorem 4 (Membership Criterion). *A system belongs to xkf if and only if all continuity claims are explicitly derivable from the Metric Gate and the Completion Gate. All other systems are excluded.*

Status

The xkf shell is minimal and irreducible. Any further compression destroys the ability to distinguish legitimate continuity from illegitimate invocation.

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