

## KFX: A Minimal Core as a Constrained-Path Formal System

### 0. Overview

We define a finite formal system **KFX** in which a “world” is not generated by a deterministic evolution function, but by **locally allowed step rules combined with global trajectory constraints**.

The system is characterised by:

- a minimal finite state space (10 states),
- exactly two local step rules,
- strict global closure constraints at 5 and 10 steps,
- a fully characterisable and finite set of valid trajectories.

The resulting structure constitutes a **minimal rigid core**: a non-trivial system whose admissible evolution collapses to a uniquely determined trajectory pattern.

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### 1. Basic Objects

#### 1.1 Sets

$$B = \{0,1\}, P = \mathbb{Z}_5 = \{0,1,2,3,4\}$$
$$S = B \times P$$

A state is denoted by:

$$s = (b, p), b \in B, p \in P$$

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#### 1.2 Operations

Binary flip:

$$\neg: B \rightarrow B, \neg b = 1 - b$$

Cyclic increment and decrement:

$$\sigma(p) = p + 1 \pmod{5}, \quad \sigma^{\wedge}(-1)(p) = p - 1 \pmod{5}$$

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## 2. Local Step Rules

The system permits **exactly two atomic step relations**, defining a binary relation  $\rightarrow \subseteq S \times S$ :

- **R1 (forward step)**

$$(b, p) \rightarrow (b, \sigma(p))$$

- **R2 (reverse-flip step)**

$$(b, p) \rightarrow (\neg b, \sigma^{-1}(p))$$

No other local transitions are allowed.

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## 3. Trajectories and Legality

### 3.1 Trajectories

A trajectory of length  $n$  is a sequence:

$$\gamma = (s_0, s_1, \dots, s_n)$$

such that  $s_i \rightarrow s_{i+1}$  for all  $i < n$ .

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### 3.2 Valid Trajectories (Global Constraints)

A trajectory  $\gamma = (s_0, \dots, s_{10})$  of length 10 is **valid** if and only if:

- **C1 (five-step flip closure)**

If  $s_0 = (b, p)$ , then

$$s_5 = (\neg b, p)$$

- **C2 (ten-step closure)**

$$s_{10} = s_0$$

The **KFX world** is defined as the set of all valid trajectories:

$$\mathcal{W}_{\text{KFX}} = \{\gamma \mid \gamma \text{ satisfies C1 and C2}\}$$

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## 4. Main Results

### Theorem 1 (Existence)

$$\mathcal{W}_{\text{KFX}} \neq \emptyset$$

#### Construction:

For any initial state  $s_0 = (b, p)$ , the step sequence  $R2^{10}$  produces a valid trajectory.

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### Theorem 2 (Five-Step Rigidity)

For any valid trajectory  $\gamma \in \mathcal{W}_{\text{KFX}}$ , the first five steps must be:

$$R2, R2, R2, R2, R2$$

and consequently the last five steps must also be  $R2^5$ .

#### Proof sketch:

Let the first five steps contain  $m$  occurrences of R1 and  $5 - m$  of R2.

- From C1 (phase closure):  
net displacement  $m - (5 - m) = 2m - 5 \equiv 0 \pmod{5}$ ,  
hence  $m \equiv 0 \pmod{5}$ , so  $m \in \{0, 5\}$ .
- From C1 (binary flip):  
the number of R2 steps,  $5 - m$ , must be odd, excluding  $m = 5$ .

Thus  $m = 0$ , and the first five steps must be all R2.

C2 enforces the same for the final five steps.

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### Theorem 3 (Uniqueness)

$$|\mathcal{W}_{\text{KFX}}| = 1$$

There exists exactly one valid step pattern:

$$R2^{10}$$

Different initial states yield only state-label shifts of the same step pattern and do not produce distinct trajectories.

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## 5. Interpretation (Purely Structural)

KFX realises a minimal formal core with:

- state space:  $2 \times 5$ ,
- local rules: 2,
- global constraints: 2,
- outcome: a **rigid, fully determined valid trajectory set**.

The system marks a boundary point:

any reduction collapses it to triviality, while any relaxation or extension releases additional degrees of freedom.

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## 6. Quotable Definition Block

KFX is a formal system with state space  $S = \{0,1\} \times \mathbb{Z}_5$ .

Two local step rules are permitted:

$$(b, p) \rightarrow (b, p + 1) \text{ and } (b, p) \rightarrow (1 - b, p - 1) \pmod{5}.$$

A length-10 trajectory is valid iff, for  $s_0 = (b, p)$ , one has

$$s_5 = (1 - b, p) \text{ and } s_{10} = s_0.$$

The world  $\mathcal{W}_{\text{KFX}}$  is the set of all valid trajectories.

Then  $|\mathcal{W}_{\text{KFX}}| = 1$ , with the unique valid step pattern  $R2^{10}$ .

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