


# The KFX-12 Core

## A Twelve-Step Closed Minimal Formal System

Kaifan XIE 

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### 1 Overview

The KFX-12 Core is a twelve-step closed formal system defined on a discrete state space. While maintaining extremely simple local transition rules, the system introduces global constraints in the form of a fixed midpoint inversion-return condition and terminal closure.

It is the first minimal closed core within the KFX framework that generates a non-trivial set of step patterns (400 in total).

This document presents only the core definition of KFX-12 and does not address any endogenous slicing or continuous extensions.

### 2 Basic Objects

#### 2.1 State Space

Define the binary set:

$$B := \{0, 1\}$$

Define the modulo-five phase ring:

$$\mathbb{Z}_5 := \{0, 1, 2, 3, 4\}$$

Define the state space:

$$S := B \times \mathbb{Z}_5$$

A state is denoted as:

$$s = (b, p), \quad b \in B, \quad p \in \mathbb{Z}_5$$

Define the bit inversion operator:

$$\neg b := 1 - b$$

### 3 Local Transition Rules

Define a one-step transition relation  $\rightarrow \subseteq S \times S$ . Only the following two atomic rules are allowed:

- **R1 (Forward Step)**

$$(b, p) \rightarrow (b, p + 1 \bmod 5)$$

- **R2 (Backward Inversion Step)**

$$(b, p) \rightarrow (-b, p - 1 \bmod 5)$$

No other local evolution rules exist in the system.

## 4 Orbit Definition

An orbit of length 12 is a state sequence:

$$\gamma = (s_0, s_1, \dots, s_{12})$$

such that for all  $i = 0, 1, \dots, 11$ :

$$s_i \rightarrow s_{i+1}$$

## 5 Global Constraints (Definition of the KFX-12 Core)

Given an initial state:

$$s_0 = (b, p)$$

An orbit  $\gamma$  is called *valid* if and only if the following two global constraints are simultaneously satisfied.

### 5.1 Midpoint Inversion-Return Constraint

At step 6, the state must satisfy:

$$s_6 = (-b, p)$$

That is:

- the phase returns to its initial value;
- the bit undergoes exactly one inversion.

### 5.2 Terminal Closure Constraint

At step 12, the orbit must close:

$$s_{12} = s_0$$

### 5.3 World Set

Define the world set of the KFX-12 Core as:

$$W_{12} := \{\gamma \mid \gamma \text{ is a length-12 orbit satisfying both constraints}\}$$

## 6 Structural Consequences and Completeness

**Theorem 1** (Completeness of KFX-12 Step Patterns). *For the KFX-12 Core defined by the state space  $S = B \times \mathbb{Z}_5$ , local transition rules R1 and R2, and the global midpoint inversion-return and terminal closure constraints, all valid length-12 orbits are exactly characterized by step sequences in which:*

$$\#R1 = 3, \quad \#R2 = 3$$

*on both the first and second halves of the orbit.*

*The total number of distinct valid step patterns is therefore:*

$$|M_{12}| = 400$$

*Proof.* Consider an action sequence of length  $n$ . Let  $m$  be the number of applications of rule R1, and  $n - m$  the number of applications of rule R2.

### Net Phase Displacement

The net phase change is:

$$\Delta p \equiv m - (n - m) = 2m - n \pmod{5}$$

### Bit Inversion Parity

The number of bit inversions is  $n - m$ . Its parity determines whether the bit flips.

### Unique Solution of the Halfway Constraint

For the halfway point  $n = 6$ , the midpoint inversion-return constraint is equivalent to:

$$\begin{cases} 2m - 6 \equiv 0 \pmod{5} \\ 6 - m \equiv 1 \pmod{2} \end{cases}$$

The unique solution is:

$$m = 3$$

Thus:

$$\#R1 = 3, \quad \#R2 = 3$$

### Step Pattern Count

For the first six steps, the number of valid action permutations is:

$$\binom{6}{3} = 20$$

The same holds for the second half. Therefore, the total number of valid step patterns is:

$$|M_{12}| = 20 \times 20 = 400$$

□

## 7 Properties of the Core

- The KFX-12 Core is a fully discrete, finite, and enumerable formal system.
- Local rules are minimal, yet global constraints induce non-trivial structure.
- This system serves as the reference core for all subsequent KFX extensions (endogenous slicing, soft constraints, length generalization, etc.).

## 8 Non-Circumventability Declaration

### 8.1 Irreducibility of Global Constraints

The midpoint inversion-return constraint and the terminal closure constraint are defined at the full-orbit scale. No finite-step local transition rule admits an equivalent replacement.

Any attempt to absorb these global constraints into local evolution rules through rule rewriting, state expansion, or time reparameterization is not considered an equivalent formulation of the KFX-12 Core.

### 8.2 Incompressibility of the Step Pattern Set

The 400 valid step patterns arise directly from the discrete state space and global constraints. Under a fixed core definition, this set cannot be compressed into a lower-dimensional parameterization.

### 8.3 Exclusivity of the Core Definition

The term *KFX-12 Core* refers exclusively to the formal system jointly determined by the state space, local rules, and global constraints defined above. Any system that omits, weakens, or absorbs any of these elements is not considered the KFX-12 Core.

## 9 Judgement Scale (Engineering Definition)

### 9.1 Definition

Let an orbit have length  $L$ , and let its validity be determined by a judgement function  $J$ . If the judgement depends on states at time indices  $\{t_1, t_2, \dots, t_k\}$ , define the judgement scale as:

$$JS(J) := \max\{t_1, t_2, \dots, t_k\}$$

### 9.2 Interpretation

- Smaller  $JS$ : validity is locally or early decidable.
- $JS$  close to  $L$ : judgement is deferred to the full scale, preserving local freedom prior to decision.

### 9.3 Judgement Scale of the KFX-12 Core

In the KFX-12 Core, validity depends explicitly on:

$$t = 6 \quad (\text{midpoint inversion-return}), \quad t = 12 \quad (\text{terminal closure})$$

Therefore:

$$JS(\text{KFX-12}) = 12$$

## Conclusion

The KFX-12 Core marks the minimal transition from frozen paths to structural multiplicity. Its complexity arises not from rule proliferation, but from the closure effect of global constraints at a finite scale.