ε-KFX: Minimally-Compressed Residual KFX (Minimal Compressed Residual Orbit-Form System)

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0. Notation

- $\mathbb{Z}_n := \{0, 1, \ldots, n-1\}$ with arithmetic modulo n.
- $\oplus$ denotes XOR over $\mathbb{Z}_2$.
- For $b \in \mathbb{Z}_2$, $\neg b := 1 - b$.
- $[\mathcal{P}] \in \{0, 1\}$ denotes the indicator function of proposition $\mathcal{P}$ (true $\mapsto 1$, false $\mapsto 0$).

1. Basic Objects

1.1 State Space

Let

$$B = \{0, 1\}, \qquad P = \mathbb{Z}_5, \qquad S = B \times P.$$

A state is denoted by $s = (b, p) \in S$.

1.2 Phase Operators

Define

$$\sigma(p) = p + 1 \pmod 5, \qquad \sigma^{-1}(p) = p - 1 \pmod 5.$$

2. Local Step Rules

The one-step transition relation $\rightarrow \subseteq S \times S$ is defined *exactly* by the following rules:

- **R1**

$$(b, p) \rightarrow (b, \sigma(p)).$$

- **R2**

$$(b, p) \rightarrow (\neg b, \sigma^{-1}(p)).$$

3. Orbits (Histories)

An orbit of length 10 is defined as a sequence

$$\gamma = (s_0, s_1, \dots, s_{10})$$

such that for all $i < 10$, $s_i \to s_{i+1}$.

Write $s_i = (b_i, p_i)$.

4. Raw Residuals (Uncompressed)

4.1 Five-Step Residual

$$\varepsilon_5(\gamma) = (\varepsilon_5^b, \varepsilon_5^p) \in \mathbb{Z}_2 \times \mathbb{Z}_5,$$

where

$$\varepsilon_5^b = b_5 \oplus \neg b_0, \qquad \varepsilon_5^p = (p_5 - p_0) \bmod 5.$$

4.2 Ten-Step Residual

$$\varepsilon_{10}(\gamma) = (\varepsilon_{10}^b, \varepsilon_{10}^p) \in \mathbb{Z}_2 \times \mathbb{Z}_5,$$

where

$$\varepsilon_{10}^b = b_{10} \oplus b_0, \qquad \varepsilon_{10}^p = (p_{10} - p_0) \bmod 5.$$

5. Minimally-Compressed Residual (ε-Kernel)

Define the minimal residual compression mapping

$$\varepsilon_{\min} : (\mathbb{Z}_2 \times \mathbb{Z}_5)^2 \to \{0,1\}^4$$

by

$$\varepsilon_{\min}(\gamma) = ([\varepsilon_5^b \neq 0], [\varepsilon_5^p \neq 0], [\varepsilon_{10}^b \neq 0], [\varepsilon_{10}^p \neq 0]).$$

This mapping satisfies:

- $\varepsilon_{\min}(\gamma) = (0,0,0,0)$ if and only if the raw residual is zero;
- all legal orbits collapse into a unique bucket under compression;
- the effective entropy of the compressed residual space is strictly greater than 1 bit.

6. World

The world of ε-KFX is defined as

$$\mathcal{W}_{\varepsilon\text{-KFX}} = \{(\gamma, \varepsilon_{\min}(\gamma)) \mid \gamma \text{ is a length-10 orbit}\}.$$

7. Zero-Residual Slice

Define the zero-residual world

$$\mathcal{W}^{0}_{\varepsilon\text{-KFX}} = \{\gamma \mid \varepsilon_{\min}(\gamma) = (0,0,0,0)\}.$$

This slice is equivalent (isomorphic) to the legal-orbit set of the original KFX system.

8. Minimality Statement (Frozen)

Subject to the simultaneous preservation of the following conditions:

1. legal orbits are unique and decidable;
2. the residual structure is non-degenerate (information > 1 bit);
3. no goals, rewards, or semantics are introduced;

the mapping $\varepsilon_{\min}$ constitutes an irreducible minimal residual representation.

Status Notes (Non-System Content)

- **KFX**: zero-residual slice;
- **Residual KFX**: uncompressed residual world;
- **ε-KFX (minimal compression)**: the minimal neutral world that is learnable and evolvable.