KFX-R: Residual KFX
(Residual Orbit-Form System)

Kaifanxie

February 4, 2026

0. Notation

- $\mathbb{Z}_n := \{0, 1, \ldots, n-1\}$ with arithmetic modulo n.
- $\oplus$ denotes XOR over $\mathbb{Z}_2$ (exclusive OR).
- For $b \in \mathbb{Z}_2$, $\neg b := 1 - b$.

1. Basic Objects

1.1 Sets

Let

$$B = \{0, 1\}, \qquad P = \mathbb{Z}_5,$$

and define the state space

$$S = B \times P.$$

A state is denoted by $s = (b, p) \in S$.

1.2 Shift Operators

Define

$$\sigma(p) = p + 1 \pmod 5, \qquad \sigma^{-1}(p) = p - 1 \pmod 5.$$

2. Local Step Rules (Atomic Steps)

The one-step transition relation $\rightarrow \subseteq S \times S$ allows *exactly* the following two rules:

- **R1 (Forward Step)**

$$(b, p) \rightarrow (b, \sigma(p)).$$

- **R2 (Backward-Flip Step)**

$$(b, p) \rightarrow (\neg b, \sigma^{-1}(p)).$$

3. Orbits (Histories)

An orbit of length 10 is a sequence

$$\gamma = (s_0, s_1, \ldots, s_{10})$$

such that for all $i < 10$, $s_i \to s_{i+1}$.

Write $s_i = (b_i, p_i)$.

4. Residuals

For any orbit γ of length 10, define the following residual quantities.

4.1 Five-Step Residual

$$\varepsilon_5(\gamma) = \big(\varepsilon_5^b(\gamma), \varepsilon_5^p(\gamma)\big) \in \mathbb{Z}_2 \times \mathbb{Z}_5,$$

where

$$\varepsilon_5^b(\gamma) = b_5 \oplus \neg b_0, \qquad \varepsilon_5^p(\gamma) = (p_5 - p_0) \bmod 5.$$

4.2 Ten-Step Residual

$$\varepsilon_{10}(\gamma) = \big(\varepsilon_{10}^b(\gamma), \varepsilon_{10}^p(\gamma)\big) \in \mathbb{Z}_2 \times \mathbb{Z}_5,$$

where

$$\varepsilon_{10}^b(\gamma) = b_{10} \oplus b_0, \qquad \varepsilon_{10}^p(\gamma) = (p_{10} - p_0) \bmod 5.$$

4.3 Total Residual

$$\varepsilon(\gamma) = \big(\varepsilon_5(\gamma), \varepsilon_{10}(\gamma)\big) \in (\mathbb{Z}_2 \times \mathbb{Z}_5)^2.$$

5. World

The *world* of Residual KFX is defined as the set

$$\mathcal{W}_{\text{KFX-R}} = \{(\gamma, \varepsilon(\gamma)) \mid \gamma \text{ is a length-10 orbit}\}.$$

6. Relation to the Original KFX (Zero-Residual Slice)

Define the zero-residual subset

$$W^0_{\text{KFX}} = \{\gamma \mid \varepsilon_5(\gamma) = (0,0) \ \wedge \ \varepsilon_{10}(\gamma) = (0,0)\}.$$

Then the legal-orbit world of the original KFX system is exactly this zero-residual slice: the same underlying set of objects, expressed under a different representation.