Systematic Failure of the Markov Assumption Under Operational State Constraints

Abstract

This work presents a unified and non-compressible analysis of the failure of the Markov assumption under explicit operational constraints on state representations. We demonstrate that, when states are required to be finite, observable, locally updatable, and free of external memory, the Markov assumption fails simultaneously at the interface, informational, methodological, predictive, and responsibility levels. Common recovery strategies based on state augmentation are shown to be non-falsifiable and non-operational. All results are presented as an explicit proof chain that cannot be reduced to a single state-extension argument.

1 Operational State Constraints

Definition 1 (Operational State). *A state x_t is called* operational *if it satisfies all of the following:*

- *finite information content;*
- *updatability via local rules;*
- *constructibility without access to full history;*
- *absence of external or hidden memory registers.*

Let $\mathcal{X}$ denote the space of all operational states. Any state representation violating these constraints is considered non-operational and excluded from the present analysis.

2 Markov Property at the Interface Level

Definition 2 (Interface Markov Property). *A process $\{x_t\}_{t\geq 0}$ is Markovian at the interface level if, for all t,*

$$\Pr(x_{t+1} \mid x_t, x_{t-1}, \dots) = \Pr(x_{t+1} \mid x_t),$$

with all probabilities defined solely on $\mathcal{X}$.

Theorem 1 (Interface Non-Markovianity). *There exist systems satisfying operational state constraints whose induced interface process $\{x_t\}$ is non-Markovian.*

Proof. Because operational states do not encode complete histories, there necessarily exist two distinct histories $h \neq h'$ such that

$$x_t(h) = x_t(h').$$

If the future evolution depends on historical structure, then there exists an event A such that

$$\Pr(x_{t+1} \in A \mid h) \neq \Pr(x_{t+1} \in A \mid h').$$

This directly violates the interface Markov property. $\square$

3 Non-Existence of Sufficient Statistics

Definition 3 (Sufficient Statistic). *A state x_t is a sufficient statistic for future evolution if*

$$\Pr(x_{t+1} \mid x_t, x_{t-1}, \dots) = \Pr(x_{t+1} \mid x_t).$$

Theorem 2 (Non-Existence of Operational Sufficient Statistics). *Under operational state constraints, no state $x_t \in \mathcal{X}$ can serve as a sufficient statistic for future evolution.*

Proof. By definition, any operational state x_t fails to distinguish all possible histories. Hence there exist $h \neq h'$ such that $x_t(h) = x_t(h')$. If future dynamics depend on history, the conditional distributions differ, and x_t cannot contain all predictive information. $\square$

4 Non-Falsifiability of State Augmentation

Definition 4 (State Augmentation Claim). *A system is said to admit Markovian recovery by state augmentation if there exists an extension*

$$\tilde{x}_t = (x_t, m_t)$$

such that $\{\tilde{x}_t\}$ is Markovian.

Theorem 3 (Non-Falsifiability of State Augmentation). *Under operational state constraints, the state augmentation claim is non-falsifiable and therefore does not constitute a valid modeling hypothesis.*

Proof. If arbitrary augmentation is permitted, one may always define m_t as the complete history. Such an extension is unobservable, unmaintainable, and unverifiable at the interface level. Consequently, any empirical failure can be absorbed by an inaccessible variable, rendering the claim non-falsifiable. □

5 Structural Degradation of Predictive Performance

Definition 5 (Predictive Risk). *Given a strictly proper loss function L, the predictive risk of an estimator $\hat{P}$ is*

$$\mathcal{R}(\hat{P}) = \mathbb{E}\big[L(P(\cdot \mid h_t), \hat{P}(\cdot))\big].$$

Theorem 4 (Markov Predictive Degradation). *Under operational state constraints, any Markov predictor incurs an irreducible positive risk gap relative to a history-sensitive predictor.*

Proof. Because distinct histories are mapped to the same x_t, a Markov predictor must output identical predictions for incompatible conditional distributions. Strictly proper losses ensure a strictly positive lower bound on the resulting expected error. □

6 State Abstraction as a Responsibility Interface

Definition 6 (Responsibility Attribution). *An interface supports responsibility attribution if distinct historical causes can be distinguished at the state level.*

Theorem 5 (Responsibility Erasure Theorem)**.** *Under operational state constraints, Markov state abstraction structurally erodes responsibility attribution at the interface level.*

Proof. Markov abstraction requires historical compression. When distinct histories are mapped to the same state, the interface cannot distinguish their causal origins, and responsibility becomes structurally unassignable. □

7 Conclusion

We have shown that, under explicit operational state constraints, the Markov assumption fails simultaneously at multiple levels: interface behavior, information sufficiency, methodological validity, predictive optimality, and responsibility attribution. No operationally valid and falsifiable recovery mechanism exists. Markovianity must therefore be treated as a contingent modeling choice with explicit costs and consequences, rather than a default structural principle.