Forced Structures and Canonical Constants: A Minimal Survivability Criterion

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Abstra

This paper introduces a methodological criterion for distinguishing **forced constants** from constants that are merely chosen, measured, or favored.

We call this criterion the **Minimal Survivability Criterion (MSC).**

Rather than predicting numerical values, MSC asks whether **removing an object causes irreducible breakdowns in determinational consistency, composability, or irreversibility.**

By systematically testing a broad range of mathematical and physical constants, we establish a reusable classification framework that separates necessary survivors, conditional survivors, and failures, while clearly stating the scope and limits of the method.

Keywords

Necessity of constants; Minimal survivability criterion; Path independence; Composability; Methodology

Introduction (Chapter 0) | The Question Is Not What Constants Are, but Why They Must Be

0.1 The Longstanding Puzzle of Constants

In mathematics and physics, certain constants are repeatedly used and labeled as fundamental or natural.
In mathematics, objects such as e,π and i are egarded as structurally intrinsic;
in physics, constants such as $\hbar$, c, G and k_B are treated as basic parameters of nature.

Yet a more fundamental question has remained largely unaddressed:

> Are these constants **logically necessary** at the level of consistency,
> or do they merely reflect contingent choices of description or features of our particular world?

0.2 Limitations of Traditional Answers

Traditional discussions typically adopt one of three positions:

1.Definitional: constants are given by definition and require no further explanation;

2.Empirical: constants are fixed by measurement and must be accepted by theory;

3.Metaphysical: constants reflect deeper symmetries or aesthetic principles.

While each position has merit, all avoid a crucial question:
Is there a theory-independent criterion that determines whether an object must be introduced at all?

0.3 Position of This Work

This work adopts the following position:

Before discussing numerical values, existence must be tested.

Rather than asking what value a constant takes,
we introduce a criterion to ask:

Can the system remain consistently operable if the constant is absent?

If the answer is no, and if the failure cannot be repaired by reparameterization, rescaling, or path selection, then the object is said to be **forced.**

0.4 Scope and Boundary Statement

This paper does not propose a new object-level mathematical or physical theory.
It introduces no new dynamical equations and does not attempt to explain numerical values.

Its sole contribution is a **methodological minimal criterion** for distinguishing:

forced constants or structures,
conditionally necessary interface objects,
structurally free parameters,
and failed objects.

0.5 Structure of the Paper

The paper is organized as follows:

Chapter 1 introduces the Minimal Survivability Criterion;
Chapter 2 analyzes mathematical survivor objects;
Chapter 3 examines failed mathematical constants;
Chapter 4 discusses physical decision gates;
Chapter 5 analyzes conditional coupling constants;
Chapters 6 and 7 address structurally free parameters and derived objects;
Chapter 8 presents the unified classification and methodological summary.

Chapter 1 | The Minimal Survivability Criterion (MSC)

1.1 Determinational Consistency as the Minimal Requirement

Consider a mathematical or physical description in which an object X is introduced to support determination, computation, or explanation.
If, after removing X, the system remains consistent, decidable, and extendable under its established rules, then X is not necessary.

We reduce necessity to a minimal standard:

If removing an object leads to inconsistent or undefined determination, then the object is necessary under the given frozen assumptions.

This standard concerns neither numerical precision nor interpretive depth, but solely whether **determination collapses**.

1.2 The Three Core Constraints

The Minimal Survivability Criterion consists of three core constraints.
Violation of any one constitutes a survivability failure.

1.Path Independence
The determination of an object must not depend on construction paths, approximation schemes, or subdivision orders.

2.Composability
When multiple decidable subsystems are combined, the resulting system must remain decidable without introducing new ambiguities.

3.Irreversibility
Once a determinate state is reached, resolving inconsistencies must not require rolling back and altering fundamental rules.

These three constraints form the minimal common basis for all subsequent tests.

1.3 Definition of Survivability Failure

In this work, failure does not imply that an object is meaningless, useless, or unimportant.
Failure simply denotes the following outcome:

> After removal, the system violates **none** of path independence, composability, or irreversibility.

Failure is thus a **methodological conclusion**, not a value judgment.

1.4 Interfaces and Gates

In some cases, objects do not exist independently but appear as interfaces between different descriptive levels, dimensions, or determination schemes.
Such objects are referred to here as **gates**.

If removing an interface object prevents consistent determination across levels, then the object is considered **conditionally necessary** under the corresponding frozen assumptions.

1.5 Statement of Forced Objects

Combining the above definitions, we adopt the following criterion:

> Under given frozen assumptions,
>
> if removing an object X violates path independence, composability, or irreversibility,
>
> then X is a **forced constant or structure**.

This statement is independent of numerical value, physical interpretation, or historical origin.

1.6** Chapter **Summary

This chapter presented the non-formal version of the Minimal Survivability Criterion, established three minimal consistency constraints, and defined forced objects.
All subsequent chapters will strictly operate within this framework.

Chapter 2 | Surviving Mathematical Objects

2.1 The Exponential Constant e: The Unique Survivor Under Continuous Composition

The constant e is commonly introduced via limits, derivatives, or compound interest.

Here, we avoid any functional priors and retain only the following frozen assumptions:

Continuous processes can be subdivided

Subdivided processes can be recombined

Determination does not depend on subdivision scheme

Under these conditions, if exponential growth depends on partition size or recomposition order, determination fails.

The only way to avoid path bifurcation is for the growth rule to satisfy multiplicative composability and converge, under dimensionless normalization, to a fixed base.
That base is e.

Removing e does not merely make expressions inelegant—it causes **the same process to yield different outcomes under different partitions**, violating path independence.

Thus, e is a **forced constant** under continuous composability.

2.2 The Constant π: Path Independence in Geometric Closure

The constant π is often regarded as a geometric definition artifact.
Within the MSC framework, we freeze the following assumptions:

Two-dimensional geometry exists

Path length can be approximated

Different approximation paths must yield consistent results

Consider inner and outer approximations of the same closed path.
If their limits differ, the determination of "length" itself collapses.

Under Euclidean freezing, the only way to avoid this bifurcation is for both approximations to share a common proportional constant.
That constant is π.

Thus, π does not arise from the definition of a circle, but from the requirement that **length determination of closed paths be path independent.**

2.3 The Imaginary Unit i: Minimal Extension for Algebraic Consistency

The imaginary unit i is typically defined by $i^2 = -1$.
Here, we avoid that definition and freeze only:

Polynomial algebraic operations are valid

Factorization is repeatable and composable

Determination does not depend on factorization path

Over the reals, the polynomial $x^2 + 1$ is irreducible.
Yet its square participates in higher-order compositions.

If factorability depends on the algebraic path chosen, determination collapses.

The **minimal cost** to avoid this bifurcation is a two-dimensional extension in which all quadratic polynomials factor consistently.
The minimal extension element is denoted i.

Thus, is not an ad hoc invention, but a **surviving structure required for algebraic consistency.**

2.4 Chapter Summary

The constants e, π, and the unit i belong to analysis, geometry, and algebra respectively, yet under MSC they exhibit isomorphic survivability:
removing any of them induces path-dependent determination.

Chapter 3 | Failed Mathematical Constants (γ and φ)

3.1 The Euler–Mascheroni Constant γ: A Regularization Residue, Not a Survivor

The Euler–Mascheroni γ constant is defined as

$$\gamma = \lim_{n \to \infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \ln n \right)$$

Formally, it appears as the limit of the difference between two divergent quantities.
Under MSC, the question is not whether the limit exists, but:

Does the value remain invariant under all admissible construction paths?

We freeze:

Summation and limit operations are valid

Reordering and regrouping are allowed

Determination must not depend on the choice of reference term

Under these conditions, γ fails irreparably on three fronts.

First, path dependence.
Different regroupings or truncations yield different limiting values.
No construction enforces convergence to a unique number across all paths.

Second, non-unique reference.
Subtracting $\ln n$, $\ln(n+c)$, or higher-order corrections are all admissible regularizations, each producing a different constant.
The choice is not forced.

Third, rollback is required.
Once drift is detected, correction demands revisiting and altering the reference choice, violating irreversibility.

Removing γ does not cause collapse;
keeping it requires non-necessary auxiliary choices.

Hence, γ is not a survivor, but a **regularization residue.**

3.2 The Golden Ratio : Morphological Preference, Not Determinative Necessity

The golden ratio φ is typically introduced via

$$x = 1 + \frac{1}{x}$$

or as the limit of the Fibonacci sequence.

Under MSC, we freeze:

Recursive and limit operations are valid

Alternative recursions are equally admissible

Determination must not rely on aesthetics or optimization

Under these conditions, φ triggers no survivability pressure.

First, no path exclusivity.
Replacing the recursion with

$$x = a + \frac{1}{x}$$

for any $a > 0$ yields a self-consistent system without conflict.

Second, composability is unaffected.
Structures built using φ、$\sqrt{2}$, or other ratios remain composable.

Third, no rollback violation.
Switching ratios upon dissatisfaction introduces no logical collapse.

Thus, φ governs which forms are preferred or stable,not whether determination is possible.

The conclusion is unavoidable:
the golden ratio is a **morphological preference**, not a forced constant.

3.3 Failure as Diagnostic Power, Not Defect

The failure of
γ and φ does not weaken MSC; it establishes its diagnostic power.
MSC does not promise universal explanation—it distinguishes:

Forced objects (removal causes collapse)
Conditionally forced objects (freeze-dependent)
Replaceable objects (removal causes no collapse)

Both γ and φ belong unequivocally to the third class.

Chapter 4 | Physical Decision Gates ($\hbar$ and C)

4.1 The Planck Constant $\hbar$: A Dimensional Gate for Phase Determination

In this section, we do not begin from experiment or history.
We freeze only:

The existence of interference

Composability and divisibility of processes

Path-independent determination

No rollback correction

Under these constraints, the system encounters a direct structural conflict.
Interference requires **phase**.
Phase must be **dimensionless** and **additive**.
Yet among all path-accumulative physical quantities, the only naturally composable one is the **action**

$$S = \int L\, dt$$

whose dimension is energy $\times$ time.
This creates an unavoidable mismatch:

Determinative quantity: phase (dimensionless)

Accumulative quantity: action (dimensionful)

Without additional structure, phase assignment becomes unit-dependent and thus path-dependent.

The only repair that avoids path bifurcation is:

Introduce a dimensional gate such that

$$\phi = \frac{S}{\hbar}$$

Where $\hbar$ carries the same dimension as S.

Crucially, introducing some constant is insufficient;
exactly one such gate must exist to preserve normalization uniqueness.

Thus:

The existence of $\hbar$ is forced

The numerical value of $\hbar$ lies outside MSC's jurisdiction

Conclusion:

$\hbar$ is not an empirical add-on, but the **minimal cost required for phase determination**.

4.2 The Speed of Light C: A Gate for Causal Comparability

We freeze:

Existence of causal ordering
Multi-path information propagation
Path-independent determination
Irreversibility of history

Under these constraints, the absence of a speed limit causes fatal bifurcation.

Consider events A and B :

Path 1: direct propagation

Path 2: detoured propagation

If arbitrarily fast paths are allowed, different routes may yield contradictory temporal orderings.

This implies:

Causal order becomes path-dependent

"Earlier" ceases to be a determinate concept

This is collapse at the level of determination, not physical detail.
The only repair is to introduce a **global speed limit C**, such that:

All admissible paths obey the same bound

All observers apply the same causal criterion

Thus, causal ordering regains path-independence.

Hence:

The existence of C is forced

The numerical value of C depends on frozen units and conventions

Conclusion:

C is not a property of light, but the **minimal structure required for causal comparability.**

4.3 The Unified Role of Decision Gates

Though arising from different theories, $\hbar$ and C are isomorphic under MSC:

$\hbar$: prevents phase bifurcation

C: prevents causal bifurcation

Shared properties:

Removal causes determinative collapse

Independent of specific dynamics

Enforce existence, not numerical value

They form the family of **decision gates.**

Chapter 5 | Conditionally Necessary Coupling Constants(G and k_B)

5.1 The Gravitational Constant G : A Conditional Gate Between Energy and Geometry

Unlike $\hbar$ and c, the necessity of G does not lie at the decision level but at the **response level.**

We freeze:

Geometry is responsive, not rigid
Energy–momentum exists and is composable/divisible
Determination must be path-independent
No rollback correction

Under these conditions, a dimensional inconsistency arises.
Geometric response (e.g., curvature) has dimension:

$$[curvature] = L^{-2}$$

Energy density has dimension:

$$[\rho] = \frac{M}{LT^2}$$

Without a coupling gate, "energy causing geometric change" becomes dependent on:

Unit choices

Coarse-graining procedures

This violates path-independence and compositional consistency.

The only repair is to introduce a **dimensional gate** G:

$$geometryresponse = G \times energydensity$$

Thus enforcing unique geometric determination across representations.

Hence:

The **existence of** G is forced under this freeze

The **uniqueness** of G is not

The **numerical value** of G lies outside MSC

5.2 Three Escape Routes and Conditional Necessity

Unlike decision gates, G admits explicit escape routes:

1.Freeze removal
If geometry is rigid, G is unnecessary.

2.Multi-parameter coupling
If energy–geometry relations involve multiple fields or parameters.

3.Emergent gravity
If gravity is emergent, G is merely an effective parameter.

Thus, G is **conditionally necessary**, not logically inevitable.

5.3 The Boltzmann Constant k_B: A Conditional Gate Between Micro and Macro

We freeze:

Microscopic states are countable

Macroscopic states are determinable

Entropy exists as a counting function

Determination must be path-independent

The natural entropy definition is:

$$S = ln\Omega$$

which is dimensionless.

Yet macroscopic thermodynamics involves dimensional energy and temperature.

Without an interface constant, the relation

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

The only repair is to introduce a dimensional gate:

$$S_{\text{phys}} = k_B \ln \Omega$$

Thus:

The **existence** of k_B is forced under this dual freeze

The **numerical value** of k_B is historically contingent

5.4 Decision Gates vs. Coupling Gates

At this stage, gate types clearly bifurcate:

Decision gates:

$\hbar$,c— removal causes determinative collapse

Coupling gates:

G,k_B— dependent on freeze, with escape routes

This separation decomposes "physical constants" into structural necessity plus physical choice.

Chapter 6 | Excluded Objects and Failure Criteria (γ, φ, α)

6.1 The Euler–Mascheroni Constant γ: A Regularization Residue, Not a Survivor

We freeze the following to test whether γ satisfies MSC:

Objects should arise naturally from composition and refinement

Determination must be path-independent

No rollback or substitution of reference terms

A standard construction is:

$$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln n\right)$$

This construction relies on **difference regularization**.
The critical issue is non-uniqueness of the reference term:

$$\ln n$$

$$\ln(n+c)$$

$$\ln n + \frac{a}{n}$$

Different but equally admissible choices yield different limits, without any determinative collapse.

Hence:

The value of γ is **path-dependent**
Substitution does **not** trigger logical failure
Rollback is structurally permitted

Verdict:

γ violates path-independence

γ lacks minimality

γ is a regularization residue, not a survivor

6.2 The Golden Ratio φ: Morphological Preference, Not Logical Necessity

The golden ratio is commonly defined by:

$$x = 1 + \frac{1}{x} \;\Rightarrow\; x = \varphi$$

or as the limit of certain recursions (e.g., Fibonacci).
We test its necessity under MSC.
Key observations:

1.Path substitutability
Rewriting as

$$x = a + \frac{1}{x}$$

yields different but equally consistent limits for different a.

2.No compositional collapse
Using φ or alternative ratios does not cause determinative failure.

3.Rollback permitted
Changing the ratio affects aesthetic or stability properties, not determinability.

Thus:

φ triggers none of MSC's forcing conditions

The system remains self-consistent without φ

Verdict:

φ is a morphological preference,not a logically necessary object

6.3 The Fine-Structure Constant α: Existential Need Without Numerical Forcing

The fine-structure constant α is a dimensionless physical coupling strength.

Under MSC, we separate two questions:

1.Must some dimensionless coupling exist?

2.Is its numerical value forced?

Freeze:

Existence of electromagnetic interaction

Quantum decision ($\hbar$) and causal decision (c) already frozen

Path-independent determination

Under this freeze:

Existence:
Some dimensionless coupling is required to compare interaction strength.

Value:
Continuously varying the coupling does not cause determinative collapse.

Thus:

The **existence** of α is admissible

The **numerical value** of α is not forced by MSC

Verdict:

α is a **world parameter**
Its value reflects contingent physical choice, not logical necessity

6.4 Summary of Failure Classes

At this point, excluded objects fall into clear classes:

Regularization residues: γ
Morphological preferences: φ

World parameters: α

Shared traits:

Removal does not cause determinative collapse

Substitutable, rollback-permissive, continuously deformable

Fail the minimal survivability criterion

7.1 Full Statement of the Minimal Survivability Criterion

The Minimal Survivability Criterion (MSC) determines whether an object is structurally forced rather than introduced by definition, preference, or measurement.

An object X is classified as a survivor iff all of the following hold:

MSC-1 | Path Independence
Its construction or determination does not depend on equivalent paths.

MSC-2 | Compositional Consistency
Under composition and refinement, its determination remains invariant.

MSC-3 | No Rollback
Removing it causes irreparable determinative collapse, not mere degradation.

MSC-4 | Minimality
It is the minimal repair; weaker structures cannot restore consistency.
Failure of any condition excludes the object from survivorship.

7.2 Unified Classification of Survivors

Under MSC, surviving objects fall into clear classes:

(A) Mathematical Survivors (Dimensionless)

Base e — unique normalization of continuous composition

Constant π — unique limit of path-independent closure

Imaginary unit i— minimal extension for factorization consistency

Traits:

Purely structural

Independent of physical freezes

Fixed numerical values

(B) Physical Decision Gates (Logical Layer)

$\hbar$ — phase determination gate

c — causal comparability gate

Traits:

Removal causes determinative collapse

Numerical values not fixed by MSC

Existentially unavoidable

(C) Conditionally Necessary Coupling Gates (Response Layer)

G — energy–geometry coupling

k_B — micro–macro interface

Traits:

Freeze-dependent

Escape routes exist

Existentially conditional, numerically contingent

7.3 Systematic Classification of Excluded Object

MSC also yields a clear taxonomy of failures:

Regularization residues: γ

Morphological preferences: φ

World parameters: α, mass ratios

Unit conventions: $N_A, \varepsilon_0, \mu_0$

Shared property:
Their removal does not induce determinative collapse.MSC does not predict numerical values.
Its outputs are strictly:

1.Necessarily existent

2.Conditionally necessary

3.Unconstrained

Thus, the objection “explanatory but not predictive” is a category error.

MSC answers:

> Why certain constants **must exist**,
> not what their values are.

7.5 Chapter Conclusion

Via MSC:

- “Forced constants” gain a strict definition
- Mathematical and physical constants are unified
- Necessity, conditional necessity, and contingency are cleanly separated

This prepares the ground for the final methodological discussion.

Chapter 8 | Methodological Boundaries, Misuse Clarifications, and Open Problems

8.1 What MSC Is Not

Before concluding, it is essential to state clearly what MSC does not claim to be.

(1) MSC is not a numerical prediction theory

MSC outputs no numerical values.

- It does not compute $e = 2.718\ldots$
- It does not derive $\alpha \approx 1/137$
- It does not fix the magnitude of G

Any demand for numerical prediction constitutes a **category error.**

(2) MSC is not a dynamical theory

MSC provides no equations of motion, evolution laws, or time dynamics.

It does not replace:

Quantum mechanics

Relativity

Thermodynamics or field theory

MSC operates prior to dynamics, addressing determinability, not evolution.

(3) MSC is not a universal explainer

MSC does not aim to explain all constants or structures.

This work explicitly documents multiple failures:

γ

φ

numerical α

mass ratios, cosmological parameters

A framework unable to say “failure” is not credible.

8.2 Common Misuses and Misplaced Expectations

MSC applies only to questions of:

Existential necessity

Collapse under removal

Explicitly frozen layers

MSC does not apply to:

Optimization problems

Aesthetic or stability criteria

Continuously tunable world parameters

Pure conventions or unit definitions

Extending MSC beyond these bounds constitutes misuse.

8.4 Open Problems

The present work leaves open:

1. Are there further decision gates beyond $\hbar$, c?

2. Do conditional necessities admit higher-order classification?

3. Can MSC be reformulated for discrete or computational systems?

4. Is there a formal correspondence with type theory or program verification?

These lie beyond the current scope but mark possible extensions.

8.5 Chapter Conclusion

This chapter delineates MSC's:

Capability boundaries

Failure modes

Non-extendable regions

Thereby establishing it as a **limited yet reliable analytical tool.**

Conclusion | Forced Constants and the Boundary of Structural Necessity

This work introduces and systematically applies the **Minimal Survivability Criterion (MSC)** to distinguish mathematical and physical constants that are **structurally forced** from those introduced by definition, optimization preference, or empirical measurement.

The central question addressed by MSC is not "what is the numerical value of a constant", but:

Does the system remain determinable if the object is removed?

Under this criterion, the following conclusions are established:

1. A class of strictly forced constants exists

Including e, π, i in mathematics, and the physical decision gates $\hbar$ and c.

Their shared features are:

Removal induces determinative collapse

Existence does not depend on specific dynamics or empirical input

2.A class of conditionally necessary coupling constants exists

Including G and k_B
These are forced under specific freezes but admit explicit escape routes, reflecting necessity at the response layer rather than the logical determination layer.

3.Many familiar constants fail MSC

Such as γ, φ, and the numerical value of the fine-structure constant α.
Their removal does not induce structural collapse and thus classifies them as regularization residues, morphological preferences, or world parameters rather than necessities.

This work emphasizes that **MSC does not predict numerical values,** nor does it aim to replace existing physical theories.
Its sole function is to determine **whether existence itself is structurally forced.**

Accordingly, the criticism that MSC "explains but does not predict" is a category error:
MSC provides **structural necessity judgments**, not numerical forecasts.

By explicitly documenting successes, conditional cases, and failures, this work demonstrates that MSC is neither a universal explainer nor a post-hoc rationalization, but a **bounded, falsifiable analytical framework**.

A formal definition of MSC, including precise criteria and terminology, is provided in **Appendix A**.