Executable Structural Falsifiability: Experimental Report

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1 Overview

This report presents a complete experimental validation of the structural properties proposed in this work. Rather than relying on axiomatic proofs alone, all properties are evaluated via *executable counterexample search* under explicitly specified executable semantics.

The experiments cover two complementary layers:

- Discrete structural falsifiability tests (F1–F3)
- Continuous-time Control Barrier Function (CBF) simulations under bounded disturbances

All experiments are finite, deterministic, and reproducible within the considered system class.

2 Structural Falsifiability Framework

We consider a finite transition system with:

- a finite state space,
- explicitly defined executable transition dynamics,
- a many-to-one degradation mapping,
- and explicit executability constraints.

Rather than proving properties axiomatically, we adopt the following falsifiability principle:

> A structural property is considered satisfied if and only if no executable counterexample exists under the specified execution semantics.

Three properties are tested independently.

2.1 F1: Recoverability (Irreversibility)

F1 asks whether a degraded state admits an executable right-inverse that recovers a fine-grained antecedent.

Criterion

- If an executable recovery exists, F1 is violated.
- If no such recovery exists, irreversibility is satisfied under the given executable constraints.

2.2 F2: Preference (Commitment-Free Continuation)

F2 tests whether execution can proceed without committing to a single future outcome.

Criterion

- If a commitment-free continuation exists indefinitely, F2 is violated.
- If execution necessarily collapses to a singleton future, preference is structurally induced.

2.3 F3: Non-bypassability

F3 tests whether an undesirable consequence can be avoided by any executable path.

Criterion

- If an executable bypass path exists, non-bypassability fails.
- If all executable paths necessarily traverse the forbidden state, F3 holds.

3 Structural Counterexample Search: Results

The unified executable counterexample search yields the following results:

```
STRUCTURAL FALSIFIABILITY SEARCH
====================================

[F1] Recoverability
  Recoverability exists: False
  => F1 holds: Irreversibility satisfied

[F2] Preference
  Commitment-free continuation exists: False
  => F2 holds: Preference structurally induced

[F3] Non-bypassability
  Bypass exists: False
  => F3 holds: No executable bypass path found
====================================
```

No executable counterexample is found for any of the three properties under the specified system structure and executable semantics.

4 Continuous-Time Validation via Control Barrier Functions

To complement the discrete structural analysis, we provide an empirical validation in a continuous-time control setting.

4.1 System Model

We consider the disturbed system:

$$\dot{x} = u + w, \quad \|w\| \leq w_{\max},$$

with safety set defined by:

$$\mathcal{C} = \{x \mid h(x) = 1 - \|x\|^2 \geq 0\}.$$

4.2 Controllers

Nominal CBF

$$-2x^\top u + \alpha h(x) \geq 0.$$

Robust CBF A sufficient robust condition under bounded disturbance is given by:

$$-2x^\top u + \alpha h(x) - 2\|x\| w_{\max} \geq 0.$$

Control inputs are subject to actuation limits:

$$\|u\| \leq u_{\max}.$$

4.3 Disturbance Model

An outward worst-case disturbance is applied:

$$w = w_{\max} \frac{x}{\|x\|},$$

which corresponds to the most adversarial direction for the chosen barrier function.

5 Quick Adversarial Experiment

A representative adversarial experiment is conducted with fixed parameters:

- $\alpha = 5$
- $w_{\max} = 0.6$
- $u_{\max} \in \{0.5, 1.0\}$
- Euler integration with $\Delta t = 5 \times 10^{-4}$

Ten Monte-Carlo executions are performed for each configuration.

5.1 Results Summary

```
nominal | umax=0.5 | out_rate=1.00 | worst_min_h=-0.677
robust  | umax=0.5 | out_rate=1.00 | worst_min_h=-0.677
nominal | umax=1.0 | out_rate=1.00 | worst_min_h=-0.270
robust  | umax=1.0 | out_rate=0.00 | worst_min_h= 0.000
```

Figure 1: State norm $\|x(t)\|$ under nominal and robust CBF controllers.

5.2 Trajectory Plots

6 Analysis

The experimental results exhibit three distinct regimes under the tested executable conditions:

- When $u_{\max} < w_{\max}$, the robust CBF constraint becomes infeasible and empirically degenerates to best-effort behavior.
- The nominal CBF violates the safety constraint under disturbance even when $u_{\max} \geq w_{\max}$.
- Only the robust CBF with executable feasibility exhibits forward-invariant behavior of the safety set across all tested executions.

These observations indicate that robustness is an executable property rather than a purely formal one.