最低稳定壳：从奇偶到对偶的离散结构统一

——一种以 $\mathbb{Z}_2$为下界的工程—数学骨架

The Minimal Stable Shell: A Parity-to-Duality Unification of Discrete Structures

— A $\mathbb{Z}_2$-Lower-Bound Framework Bridging Mathematics and Engineering

摘要

本文不试图统一全部离散数学，而是刻意压缩到**最低仍然稳定的结构层**。
在不引入顺序、度量、权重与方向的前提下，仅保留奇偶、核—像、正交与对偶。
结果是一套闭合、可计算、可工程化的统一壳：一切可在 $\mathbb{Z}_2$上成立的组合—图—系统结构，皆被同构进同一骨架。

Abstract

This work does not aim to unify all of discrete mathematics.
Instead, it deliberately compresses structure to the **lowest layer that remains stable**.
By discarding order, metric, weight, and orientation, and retaining only parity, kernel–image, orthogonality, and duality, we obtain a closed, computable, and engineerable shell.
All combinatorial, graphical, and systemic structures that survive over $\mathbb{Z}_2$are unified within this framework.

引言

本工作并不试图给出一种覆盖全部离散数学对象的统一理论。
相反，它采取一种**反向策略**：不断削减假设，直至结构即将崩塌，但尚未崩塌。

统一在此并不意味着“表达能力的最大化”，而意味着**在最弱代数条件下仍然保持同构关系**。
凡需要顺序、度量、权重、方向或精确计数的结构，均被有意排除在讨论范围之外。

在这一策略下，唯一被保留下来的代数基底是 $\mathbb{Z}_2$。
其原因并非简化，而是稳定性：
$\mathbb{Z}_2$是在不引入额外结构的前提下，仍允许定义分解、不变量、核与像、正交与对偶的最低层级。

由此得到的不是一套新的数学对象，而是一种**结构壳**：
所有能够在模 2 意义下成立的组合结构、图结构与系统结构，均可被压缩并同构到同一骨架中。

该骨架由以下要素构成:

- 对称差作为唯一加法
- 奇偶作为唯一可用的不变量
- 边界算子作为结构映射
- 核与像作为自由度与约束的分离
- 正交补作为对偶的定义方式

在此框架内，组合论中的配对消失、图论中的环与割、同调中的不可填充残差，以及工程系统中的守恒与控制，均不再作为不同领域的结论出现，而是同一结构在不同语义下的投影。

需要强调的是:
本文并不声称这些结构穷尽了离散数学。
相反，本文的结论是一个**边界判断**——指出哪些结构在假设继续削弱时必然消失，哪些结构构成了最后一个不碎的统一层。

因此，本工作的目标不是扩展，而是定位。
不是给出更多内容，而是给出一个**不能再向下压缩的共同底座**。

Introduction

This work does not attempt to present a theory that unifies all of discrete mathematics.
Instead, it adopts a **reverse strategy**: systematically removing assumptions until the structure is about to collapse—but does not.

Here, unification does not mean maximal expressive power.
It means **structural isomorphism preserved under the weakest possible algebraic assumptions**.
Any structure requiring order, metric, weight, orientation, or exact counting is deliberately excluded.

Under this strategy, the only surviving algebraic base is $\mathbb{Z}_2$.
This choice is not motivated by simplicity, but by stability.
$\mathbb{Z}_2$ is the lowest level at which decomposition, invariants, kernel–image separation, orthogonality, and duality remain definable without introducing additional structure.

The result is not a new class of mathematical objects, but a **structural shell**.
All combinatorial, graphical, and systemic structures that remain valid modulo 2 can be compressed and identified within a single framework.

This framework is built upon:

- symmetric difference as the sole additive operation,
- parity as the only invariant,
- boundary operators as structural mappings,
- kernels and images as the separation of freedom and constraint,
- orthogonal complements as the definition of duality.

Within this shell, pairing arguments in combinatorics, cycles and cuts in graph theory, non-fillable residues in homology, and conservation versus control in engineering systems no longer appear as domain-specific results, but as projections of the same underlying structure.

It must be emphasized that this framework does not exhaust discrete mathematics.
On the contrary, its primary contribution is a **boundary determination**: identifying which structures inevitably vanish as assumptions are weakened, and which constitute the last non-fragmenting layer of unification.

The goal of this work is therefore not expansion, but localization.
Not to add content, but to identify a **common base that cannot be further reduced**.

符号约定

为避免语义漂移，本文采用最小且固定的符号体系。
所有符号在全文中保持一致，不随语境重载。

集合与代数

- U, V, E, F：有限集合
- $\mathcal{P}(X)$：集合 X的幂集
- $\oplus$：对称差运算
- $\emptyset$：空集

所有集合运算默认在 $\mathbb{Z}_2$意义下进行。

向量空间同构

- $(\mathcal{P}(X), \oplus) \cong (\mathbb{Z}_2)^{|X|}$
- 集合元素与基向量一一对应

- 子集与向量的等价关系不再区分说明

映射与算子

- $\pi(A) = |A| \bmod 2$：奇偶投影
- ∂：边界算子（上下文决定阶数）
- $\ker\partial$：算子核
- $\operatorname{im}\partial$：算子像

内积与正交

- $\langle A, B \rangle := |A \cap B| \bmod 2$
- $W^{\perp}$：子空间 W的正交补

图论对应

- 图 $G = (V, E)$仅指无向、无权、有限图
- 子图默认指边集子集
- 顶点度数、连通性均在模 2 意义下讨论，除非另行声明

Notation

To prevent semantic drift, a minimal and fixed notation system is adopted.
All symbols retain a single meaning throughout the text.

Sets and Algebra

- U, V, E, F: finite sets
- $\mathcal{P}(X)$: power set of X
- $\oplus$: symmetric difference
- $\emptyset$: empty set

All set operations are understood modulo $\mathbb{Z}_2$.

Vector Space Identification

- $(\mathcal{P}(X), \oplus) \cong (\mathbb{Z}_2)^{|X|}$
- Set elements correspond bijectively to basis vectors
- No distinction is made between subsets and vectors unless required

Maps and Operators

- $\pi(A) = |A| \ mod 2$: parity projection
- ∂: boundary operator (order determined by context)
- $\ker \partial$: kernel
- $\mathrm{im}\, \partial$: image

Inner Product and Orthogonality

- $\langle A, B \rangle := |A \cap B| \ mod 2$
- $W^{\perp}$: orthogonal complement of W

Graph-Theoretic Conventions

- A graph $G = (V, E)$is finite, undirected, and unweighted
- A subgraph is identified with a subset of edges
- Degree and connectivity are considered modulo 2 unless stated otherwise

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方法论约束

本文的推演受以下约束条件严格限制。这些条件不是技术选择，而是结构边界。

1. 假设削减原则

任何结构若其定义依赖于以下要素之一，则不被纳入核心推演：

- 顺序
- 度量
- 权重
- 方向
- 精确计数

这些要素仅在后续“换域”讨论中作为对比出现。

2. 不引入新对象原则

全文不通过引入更高阶或更复杂的对象来获得统一结果。
所有结论均来自以下操作之一：

- 重命名

- 等价变换
- 商结构
- 正交补

3. 最低稳定性判据

若某一结构在以下任一操作下失效，则该结构不属于最低稳定壳：

- 去除方向
- 去除权重
- 去除大小信息，仅保留奇偶

4. 计算可达性约束

所有核心结构必须满足：

- 可在有限步骤内构造
- 可在多项式时间内判定
- 不依赖无限过程或极限

该约束用于保证工程可迁移性。

5. 解释最小化

本文避免领域叙事与直觉类比。
所有解释以结构等价与算子关系为终点，不再向外延伸。

Methodological Constraints

All derivations in this work are subject to strict constraints.
These constraints are not technical preferences but structural boundaries.

1. Assumption Reduction Principle

Any structure whose definition depends on one of the following is excluded from the core development:

- order
- metric
- weight
- orientation

- exact counting

Such features appear only in later comparisons under domain extension.

2. No-New-Objects Principle

Unification is not achieved by introducing higher-order or more complex objects.
All results arise from:

- renaming
- equivalence transformations
- quotient constructions
- orthogonal complements

3. Minimal Stability Criterion

A structure is excluded from the minimal stable shell if it fails under any of the following:

- removal of orientation
- removal of weights
- removal of magnitude information, retaining only parity

4. Computational Accessibility

All core structures must be:

- constructible in finite steps
- decidable in polynomial time
- independent of infinite processes or limits

This ensures engineering transferability.

5. Minimal Interpretation Policy

Narrative explanations and intuitive metaphors are avoided.
All explanations terminate at structural equivalence or operator relations.

第一章　有限集合与对称差结构

1.1 有限基集

设 U为有限集合，$|U|=n$。
本文中不对 U施加任何额外结构：

无顺序、无度量、无标号优先级。

所有后续对象均由 U派生。

1.2 幂集作为状态空间

定义幂集:

$$\mathcal{P}(U) = \{A \mid A \subseteq U\}$$

$\mathcal{P}(U)$被视为**全部可能状态的全集**。
本文不区分“对象”“配置”“状态”“结构”，均以子集表示。

1.3 对称差运算

在 $\mathcal{P}(U)$上定义二元运算:

$$A \oplus B := (A \cup B) \setminus (A \cap B)$$

该运算满足:

1. 封闭性
2. 交换律
3. 结合律
4. 单位元：$\emptyset$
5. 自反性：$A \oplus A = \emptyset$

因此:

$$(\mathcal{P}(U), \oplus)$$

构成交换群。

1.4 与 $\mathbb{Z}_2$向量空间的同构

定义映射:

$$\phi: \mathcal{P}(U) \to (\mathbb{Z}_2)^n$$

对每个 $e_i \in U$，规定：

$$(\phi(A))_i = \begin{cases} 1, & e_i \in A \\ 0, & e_i \notin A \end{cases}$$

则 ϕ为群同构。
对称差对应逐位模 2 加法。

由此，本文在集合语言与线性代数语言之间不再作区分。

1.5 原子与最小分解

定义原子为单元素集合：

$$\{e_i\} \subseteq U$$

任意 $A \subseteq U$可唯一表示为：

$$A = \bigoplus_{e_i \in A} \{e_i\}$$

该分解在模 2 意义下唯一。
不存在更小的非零分解单元。

1.6 最小性说明

在当前假设下：

- 无法定义大小顺序
- 无法比较两个非空子集的“多寡”
- 无法区分重复出现

对称差是唯一在不引入额外结构的前提下仍然闭合的“加法”。

因此，本章构造构成全文的**代数下界**。

1.1 Finite Base Set

Let U be a finite set with $|U| = n$.
No additional structure is imposed on U:
no order, no metric, no priority labeling.

All subsequent objects are derived from U.

1.2 Power Set as State Space

Define the power set:

$$\mathcal{P}(U) = \{A \mid A \subseteq U\}$$

$\mathcal{P}(U)$ is regarded as the **universe of all possible states**.
No distinction is made between "object," "configuration," "state," or "structure"; all are represented as subsets.

1.3 Symmetric Difference

Define a binary operation on $\mathcal{P}(U)$:

$$A \oplus B := (A \cup B) \setminus (A \cap B)$$

This operation satisfies:

1. closure
2. commutativity
3. associativity
4. identity element: $\emptyset$
5. involution: $A \oplus A = \emptyset$

Hence:

$$(\mathcal{P}(U), \oplus)$$

forms an abelian group.

1.4 Isomorphism with a $\mathbb{Z}_2$ Vector Space

Define the map:

$$\phi: \mathcal{P}(U) \to (\mathbb{Z}_2)^n$$

For each $e_i \in U$, set:

$$(\phi(A))_i = \begin{cases} 1, & e_i \in A \\ 0, & e_i \notin A \end{cases}$$

Then ϕ is a group isomorphism.
Symmetric difference corresponds to coordinate-wise addition modulo 2.

Henceforth, set-theoretic and linear-algebraic languages are used interchangeably.

1.5 Atoms and Minimal Decomposition

Define atoms as singleton sets:

$$\{e_i\} \subseteq U$$

Any $A \subseteq U$ admits a unique representation:

$$A = \bigoplus_{e_i \in A} \{e_i\}$$

This decomposition is unique modulo 2.
No smaller nonzero decomposition units exist.

1.6 Minimality

Under the present assumptions:

- no size ordering can be defined,
- no comparison of magnitude between nonempty subsets is possible,
- multiplicity cannot be distinguished.

Symmetric difference is the only additive operation that remains closed without introducing additional structure.

This construction therefore constitutes the **algebraic lower bound** of the entire development.

第二章　奇偶投影与不变量

2.1 奇偶投影的定义

定义映射:

$$\pi: \mathcal{P}(U) \to \mathbb{Z}_2, \pi(A) = |A| \bmod 2$$

该映射称为**奇偶投影**。
其值域仅包含两种状态：0（偶）与 1（奇）。

2.2 同态性质

对任意 $A, B \subseteq U$，有:

$$\pi(A \oplus B) = \pi(A) + \pi(B)$$

因此，π 是从 $(\mathcal{P}(U), \oplus)$到 $(\mathbb{Z}_2, +)$的群同态。

奇偶不作为集合的属性出现，而作为**结构的投影结果**出现。

2.3 奇偶层分解

由 π将 $\mathcal{P}(U)$划分为两个互不相交的层:

- 偶层：$\mathcal{P}_0 = \{A \mid \pi(A) = 0\}$
- 奇层：$\mathcal{P}_1 = \{A \mid \pi(A) = 1\}$

两层之间不存在连续过渡，仅能通过奇扰动发生翻转。

2.4 扰动模型

考虑变换:

$$A \mapsto A \oplus T$$

其中 $T \subseteq U$称为扰动集。

有:

$$\pi(A \oplus T) = \pi(A) + \pi(T)$$

由此得到判据:

- 若 $\pi(T) = 0$，则奇偶层不变
- 若 $\pi(T) = 1$，则奇偶层翻转

2.5 奇偶不变量

设 $\mathcal{T}$为一类允许的扰动集合。
若对所有 $T \in \mathcal{T}$有 $\pi(T) = 0$，则:

$$\pi(A) \text{ 在所有允许变换下保持不变}$$

该不变量不依赖于 A的具体结构，仅依赖于扰动的奇偶性质。

2.6 配对消失原理

设 Ω为有限集合，定义映射:

$$f: \Omega \to \Omega$$

若满足:

1. $f(f(x)) = x$
2. $f(x) \neq x$

则 Ω可被分解为若干互不相交的二元对。

因此:

$$|\Omega| \equiv 0 (\mathrm{mod} 2)$$

2.7 例外集与残差

若存在不动点集合:

$$F = \{x \in \Omega \mid f(x) = x\}$$

则:

$$|\Omega| \equiv |F| \pmod 2$$

奇偶信息完全由不可配对的残余决定。

2.8 结构性结论

奇偶不变量的获得不依赖于:

- 元素的排列
- 元素的数值大小
- 配对的具体实现方式

其唯一来源是模 2 的可加结构。

Chapter 2. Parity Projection and Invariants

2.1 Definition of the Parity Projection

Define the map:

$$\pi: \mathcal{P}(U) \to \mathbb{Z}_2, \pi(A) = |A| \ mod 2$$

This map is called the **parity projection**.
Its codomain contains only two states: 0 (even) and 1 (odd).

2.2 Homomorphism Property

For any $A, B \subseteq U$,

$$\pi(A \oplus B) = \pi(A) + \pi(B)$$

Thus, $\pi$is a group homomorphism from $(\mathcal{P}(U), \oplus)$to $(\mathbb{Z}_2, +)$.

Parity appears not as a property of sets, but as a **projection of structure**.

2.3 Parity Layer Decomposition

The map πpartitions $\mathcal{P}(U)$into two disjoint layers:

- even layer: $\mathcal{P}_0 = \{A \mid \pi(A) = 0\}$
- odd layer: $\mathcal{P}_1 = \{A \mid \pi(A) = 1\}$

There is no continuous transition between layers; only parity-flipping perturbations connect them.

2.4 Perturbation Model

Consider transformations of the form:

$$A \mapsto A \oplus T$$

where $T \subseteq U$is a perturbation set.

Then:

$$\pi(A \oplus T) = \pi(A) + \pi(T)$$

Hence:

- if $\pi(T) = 0$, the parity layer is preserved;
- if $\pi(T) = 1$, the parity layer is flipped.

2.5 Parity Invariants

Let $\mathcal{T}$be a class of admissible perturbations.
If $\pi(T) = 0$for all $T \in \mathcal{T}$, then:

$$\pi(A) \text{ is invariant under all admissible transformations}$$

This invariant depends only on the parity of perturbations, not on the internal structure of A.

2.6 Pairing Cancellation Principle

Let Ω be a finite set and define a map:

$$f: \Omega \to \Omega$$

If:

1. $f(f(x)) = x$,
2. $f(x) \neq x$,

then Ω decomposes into disjoint pairs.

Thus:

$$|\Omega| \equiv 0 \pmod 2$$

2.7 Exceptional Sets and Residues

If a fixed-point set exists:

$$F = \{x \in \Omega \mid f(x) = x\},$$

then:

$$|\Omega| \equiv |F| \pmod 2$$

All parity information is carried by the unpaired residue.

2.8 Structural Consequence

Parity invariants do not depend on:

- ordering of elements,
- numerical magnitude,
- the concrete realization of pairings.

Their sole source is the additive structure modulo 2.

第三章　图结构化与边界算子

3.1 图的集合化表示

设图 $G=(V,E)$，其中：

- V为有限顶点集
- $E\subseteq\binom{V}{2}$为无向边集

本文中，图不被视为几何对象，而被视为**边集的状态空间**。

任意子图由其边集子集 $F\subseteq E$唯一确定。

3.2 边集空间

定义边集空间：

$$C_1:=\mathcal{P}(E)$$

在对称差运算下，(C_1,⊕) 构成 $\mathbb{Z}_2$向量空间。

边的存在与否仅以模 2 意义记录，不区分多重出现。

3.3 顶点空间

定义顶点空间：

$$C_0:=\mathcal{P}(V)$$

顶点集合被视为另一层状态空间，用于承载边界信息。

3.4 边界算子

定义映射：

$$\partial_1\colon C_1\to C_0$$

对任意 $F\subseteq E$，令：

$$\partial_1(F)=\{\,v\in V\mid \deg_F(v)\equiv 1(\mathrm{mod}2)\,\}$$

该映射称为**边界算子**。

3.5 线性性

对任意 $F, G \subseteq E$，有：

$$\partial_1(F \oplus G) = \partial_1(F) \oplus \partial_1(G)$$

因此，∂_1 为 $\mathbb{Z}_2$-线性映射。

3.6 奇偶约束

对任意 $F \subseteq E$，成立：

$$| \partial_1(F) | \equiv 0 (\mathrm{mod} 2)$$

该约束不依赖于 F的具体结构，仅依赖于边—点关联关系。

3.7 Euler 判据的结构形式

设 $F \subseteq E$为一组边。

- 若 $\partial_1(F) = \emptyset$，则所有顶点度数为偶
- 若 $| \partial_1(F) | = 2$，则恰有两个奇度顶点

其余情况均违反奇偶约束。

3.8 补边与背景依赖

设全集边集为 E，定义补边：

$$F^{\backslash *} := E \setminus F$$

则：

$$\partial_1(F^{\backslash *}) = \partial_1(E) \oplus \partial_1(F)$$

奇偶性质是否保持，取决于背景图 E的边界。

3.9 本章结论

图论中的度数、路径与回路条件，可完全压缩为：

- 边集空间
- 边界算子
- 奇偶约束

几何与连通叙事在该层面不参与结构判定。

Chapter 3. Graph Structurization and the Boundary Operator

3.1 Set-Based Representation of Graphs

Let $G = (V, E)$be a graph, where:

- $V$is a finite vertex set,
- $E \subseteq \binom{V}{2}$is an undirected edge set.

Graphs are treated not as geometric objects, but as **state spaces of edge sets**.

Any subgraph is uniquely determined by a subset $F \subseteq E$.

3.2 Edge Space

Define the edge space:

$$C_1 := \mathcal{P}(E)$$

Under symmetric difference, $(C_1, \oplus)$forms a $\mathbb{Z}_2$vector space.

Edges are recorded only modulo 2; multiplicity is ignored.

3.3 Vertex Space

Define the vertex space:

$$C_0 := \mathcal{P}(V)$$

Vertex sets serve as the codomain for boundary information.

3.4 Boundary Operator

Define the map:

$$\partial_1 : C_1 \to C_0$$

For any $F \subseteq E$, let:

$$\partial_1(F) = \{ v \in V \mid \deg_F(v) \equiv 1 (\mathrm{mod} 2) \}$$

This map is called the **boundary operator**.

3.5 Linearity

For any $F, G \subseteq E$,

$$\partial_1(F \oplus G) = \partial_1(F) \oplus \partial_1(G)$$

Hence, $\partial_1$is $\mathbb{Z}_2$-linear.

3.6 Parity Constraint

For any $F \subseteq E$,

$$| \partial_1(F) | \equiv 0 (\mathrm{mod} 2)$$

This constraint depends only on the incidence structure between edges and vertices.

3.7 Structural Form of Euler Criteria

Let $F \subseteq E$be a set of edges.

- If $\partial_1(F) = \emptyset$, all vertices have even degree.

- If $|\partial_1(F)| = 2$, exactly two vertices have odd degree.

All other cases violate the parity constraint.

3.8 Complement and Background Dependence

Let Ebe the full edge set and define:

$$F^{\backslash *} := E \setminus F$$

Then:

$$\partial_1(F^{\backslash *}) = \partial_1(E) \oplus \partial_1(F)$$

Whether parity properties are preserved depends on the boundary of the background graph.

3.9 Chapter Conclusion

Degree, path, and cycle conditions in graph theory reduce entirely to:

- the edge space,
- the boundary operator,
- parity constraints.

Geometric or connectivity narratives do not participate at this level of structural determination.

第四章　核与像：Cycle Space 与 Cut Space

4.1 核的定义

给定边界算子:

$$\partial_1 : C_1 \to C_0$$

定义其核:

$$\ker\,\partial_1 = \{F \subseteq E \mid \partial_1(F) = \emptyset\}$$

该集合称为 **Cycle Space**。

4.2 Cycle Space 的结构性质

对任意 $F, G \in \ker\, \partial_1$，有：

$$\partial_1(F \oplus G) = \emptyset$$

因此，$\ker \partial_1$ 在 $\oplus$下构成 $\mathbb{Z}_2$向量子空间。

Cycle Space 中的元素仅满足“无奇度顶点”条件，不依赖几何或嵌入信息。

4.3 Cycle 的生成性

设 $T \subseteq E$为生成树的边集。
对任意非树边 $e \in E \setminus T$，存在唯一简单环 $C_e \subseteq E$。

集合 $\{C_e\}$生成 $\ker \partial_1$。

该生成方式不依赖于环的形状，仅依赖于树–非树分解。

4.4 像的定义

定义边界算子的像：

$$\mathrm{im}\ \partial_1 = \{\partial_1(F) \mid F \subseteq E\} \subseteq C_0$$

像空间中的元素表示**可实现的奇度顶点集合**。

4.5 像的奇偶约束

对任意 $S \in \mathrm{im}\ \partial_1$，成立：

$$\mid S \mid \equiv 0 (\mathrm{mod} 2)$$

因此：

$$\mathrm{im}\ \partial_1 \subseteq \{S \subseteq V \mid\mid S \mid \text{ 为偶}\}$$

4.6 连通图情形

若图 G连通，则：

$$\operatorname{im} \partial_1 = \{S \subseteq V \mid |S| \text{ 为偶}\}$$

即任意偶大小的顶点集合，均可由某个边集实现为边界。

4.7 Cut Space 的定义

对任意顶点划分 $S \subseteq V$，定义对应割边集：

$$\delta(S) = \{ e = \{u, v\} \in E \mid u \in S, v \in V \setminus S \}$$

所有此类割边集在 $\oplus$下生成的空间称为 **Cut Space**。

4.8 Cut Space 与像的关系

Cut Space 与 $\operatorname{im} \partial_1$通过顶点划分一一对应。

在模 2 意义下，Cut Space 表征所有可实现的边界注入模式。

4.9 维数关系

设图 G具有：

- $n = |V|$个顶点
- $m = |E|$条边
- c个连通分量

则：

$$\dim \ker \partial_1 = m - n + c$$
$$\dim \operatorname{im} \partial_1 = n - c$$

并有:

$$m = (m - n + c) + (n - c)$$

4.10 本章结论

边集空间被自然分解为:

- 无边界的自由部分（Cycle Space）
- 可注入的约束部分（Cut Space）

该分解不依赖于度量、方向或嵌入。

Chapter 4. Kernel and Image: Cycle Space and Cut Space

4.1 Definition of the Kernel

Given the boundary operator:

$$\partial_1 : C_1 \to C_0,$$

define its kernel:

$$\ker\, \partial_1 = \{F \subseteq E \mid \partial_1(F) = \emptyset\}.$$

This set is called the **cycle space**.

4.2 Structural Properties of the Cycle Space

For any $F, G \in \ker\, \partial_1$,

$$\partial_1(F \oplus G) = \emptyset.$$

Hence, $\ker \partial_1$forms a $\mathbb{Z}_2$vector subspace.

Elements of the cycle space satisfy only the even-degree condition, independent of geometry or embedding.

4.3 Generation of Cycles

Let $T \subseteq E$ be the edge set of a spanning tree.
For each non-tree edge $e \in E \setminus T$, there exists a unique simple cycle $C_e \subseteq E$.

The family $\{C_e\}$ generates $\ker \partial_1$.

4.4 Definition of the Image

Define the image of the boundary operator:

$$\mathrm{im}\ \partial_1 = \{\partial_1(F) \mid F \subseteq E\} \subseteq C_0.$$

Elements of the image represent **realizable sets of odd-degree vertices**.

4.5 Parity Constraint on the Image

For any $S \in \mathrm{im}\ \partial_1$,

$$|S| \equiv 0 (\mathrm{mod} 2).$$

Thus:

$$\mathrm{im}\ \partial_1 \subseteq \{S \subseteq V \mid |S| \text{ is even}\}.$$

4.6 The Connected Case

If G is connected, then:

$$\mathrm{im}\ \partial_1 = \{S \subseteq V \mid |S| \text{ is even}\}.$$

Every even-sized vertex set is realizable as a boundary.

4.7 Definition of the Cut Space

For any vertex partition $S \subseteq V$, define the cut:

$$\delta(S) = \{ e = \{u, v\} \in E \mid u \in S, v \in V \setminus S \}.$$

The space generated by all such cuts under $\oplus$ is called the **cut space**.

4.8 Relation Between Cut Space and Image

The cut space corresponds bijectively to $\operatorname{im} \partial_1$ via vertex partitions.

Modulo 2, the cut space represents all admissible boundary injection patterns.

4.9 Dimension Relations

Let G have:

- $n = |V|$ vertices,
- $m = |E|$ edges,
- c connected components.

Then:

$$\begin{aligned}\dim\ \ker\ \partial_1 &= m - n + c,\\ \dim\ \operatorname{im}\ \partial_1 &= n - c,\end{aligned}$$

with:

$$m = (m - n + c) + (n - c).$$

4.10 Chapter Conclusion

The edge space decomposes naturally into:

- a boundary-free component (cycle space),
- an injectable constraint component (cut space).

This decomposition is independent of metric, orientation, or embedding.

第五章　链复形与同调

5.1 链群的引入

在前述结构之上，引入分层对象：

- 0-链群：

$$C_0 := \mathcal{P}(V)$$

- 1-链群：

$$C_1 := \mathcal{P}(E)$$

在具备面结构的情形下，可进一步引入：

- 2-链群：

$$C_2 := \mathcal{P}(F)$$

所有链群均在 $\mathbb{Z}_2$意义下理解。

5.2 边界算子的层级化

已定义的一阶边界算子为：

$$\partial_1 : C_1 \to C_0$$

若存在 2-链群，则定义二阶边界算子：

$$\partial_2 : C_2 \to C_1$$

∂_2将每个面映射为其边界边集（模 2）。

5.3 边界的边界恒等式

在上述定义下，成立恒等式：

$$\partial_1 \circ \partial_2 = 0$$

即：

任意边界的边界为空集。

该恒等式不依赖于几何直觉，仅依赖于模 2 的偶数抵消。

5.4 链复形

由此得到链复形：

$$C_2 \xrightarrow{\partial_2} C_1$$

并满足：

$$\partial_k \circ \partial_{k+1} = 0$$

链复形并非附加结构，而是边界算子可组合性的直接结果。

5.5 同调的定义

定义一阶同调群：

$$H_1 := \ker\ \partial_1 / \mathrm{im}\ \partial_2$$

其中：

- $\ker \partial_1$：所有闭合链（cycle）
- $\mathrm{im}\, \partial_2$：所有可填充的边界

5.6 同调的结构意义

H_1的元素表示：

- 闭合
- 非边界
- 不可由更高阶结构消去

的奇偶残差。

同调并不描述形状，而描述**结构不可压缩性**。

5.7 无面情形

若不存在 2-链群，则：

$$\mathrm{im}\ \partial_2 = \{0\}$$

此时：

$$H_1 \cong \ker\ \partial_1$$

即所有闭合结构均为非平凡同调类。

5.8 本章结论

同调并非拓扑附加物，而是由以下三点强制出现：

- 模 2 加法
- 边界算子
- 边界可组合性

它刻画的是在当前假设下**无法继续压缩的结构自由度**。

Chapter 5. Chain Complexes and Homology

5.1 Chain Groups

On top of the previous structures, introduce layered objects:

- 0-chain group:

$$C_0 := \mathcal{P}(V)$$

- 1-chain group:

$$C_1 := \mathcal{P}(E)$$

When faces are present, further introduce:

- 2-chain group:

$$C_2 := \mathcal{P}(F)$$

All chain groups are interpreted over $\mathbb{Z}_2$.

5.2 Hierarchy of Boundary Operators

The first boundary operator is:

$$\partial_1 : C_1 \to C_0$$

If a 2-chain group exists, define:

$$\partial_2 : C_2 \to C_1$$

∂_2maps each face to its boundary edge set modulo 2.

5.3 Boundary-of-Boundary Identity

Under these definitions, the identity holds:

$$\partial_1 \circ \partial_2 = 0$$

That is:

the boundary of any boundary is empty.

This identity relies solely on even cancellation modulo 2.

5.4 Chain Complex

This yields the chain complex:

$$C_2 \xrightarrow{\partial_2} C_1$$

satisfying:

$$\partial_k \circ \partial_{k+1} = 0.$$

The chain complex is not an added structure but a direct consequence of composable boundaries.

5.5 Definition of Homology

Define the first homology group:

$$H_1 := \ker\ \partial_1 / \mathrm{im}\ \partial_2.$$

Where:

- $\ker \partial_1$: all closed chains (cycles),
- $\mathrm{im}\, \partial_2$: all fillable boundaries.

5.6 Structural Meaning of Homology

Elements of H_1 represent parity residues that are:

- closed,
- not boundaries,
- irreducible by higher-order structures.

Homology captures **structural incompressibility**, not shape.

5.7 The No-Face Case

If no 2-chain group exists, then:

$$\mathrm{im}\ \partial_2 = \{0\}.$$

Hence:

$$H_1 \cong \ker\ \partial_1.$$

All closed structures are nontrivial homology classes.

5.8 Chapter Conclusion

Homology is not a topological add-on.

It is forced by:

- modulo-2 addition,
- boundary operators,
- composability of boundaries.

It measures the degrees of freedom that **cannot be further compressed** under the present assumptions.

第六章　对偶与正交结构

6.1 边空间上的内积

在边集空间

$$C_1 = \mathcal{P}(E)$$

上定义双线性形式：

$$\langle F, G\rangle := |F \cap G| \ mod 2$$

该形式对称、非退化，取值于 $\mathbb{Z}_2$。
其语义仅记录**冲突的奇偶性**。

6.2 正交补的定义

对任意子空间 $W \subseteq C_1$，定义其正交补：

$$W^{\perp} := \{x \in C_1 \mid \langle x, w\rangle = 0, \qquad \forall w \in W\}$$

正交补在 $\mathbb{Z}_2$意义下唯一确定。

6.3 Cycle Space 的正交补

设

$$\mathcal{C} := \ker\ \partial_1$$

为 cycle space。

则存在以下结论:

$$\mathcal{C}^{\perp} = \mathcal{K}$$

其中 $\mathcal{K}$为由所有割边集生成的子空间（cut space）。

6.4 正交性的结构理由

对任意 $C \in \mathcal{C}$与任意割 $\delta(S)$，成立:

$$\langle C, \delta(S) \rangle = 0$$

该等式源于以下事实:
闭合结构穿越任意割的次数必为偶数。

6.5 维数互补性

设图 G有 m条边，则:

$$\dim\ \mathcal{C} + \dim\ \mathcal{K} = m$$

结合正交性，可得:

$$C_1 = \mathcal{C} \oplus \mathcal{K}$$

该分解在 $\mathbb{Z}_2$上是正交直和。

6.6 对偶的定义

在本文中，“对偶”不指对象替换，而指以下操作:

通过内积，将一类结构映射为其正交不相容类。

因此，对偶是**关系层面的操作**，而非几何或构造层面的变换。

6.7 平面图对偶的地位

在平面嵌入的情形下：

- cut 对应于对偶图中的 cycle
- cycle 对应于对偶图中的 cut

该对应关系仅是正交对偶在可视化条件下的实例化。

6.8 本章结论

Cycle 与 Cut 并非并列概念，而是同一空间中的正交对偶。
对偶关系由内积唯一决定，不依赖于嵌入、方向或权重。

Chapter 6. Duality and Orthogonal Structures

6.1 Inner Product on the Edge Space

On the edge space

$$C_1 = \mathcal{P}(E),$$

define a bilinear form:

$$\langle F, G\rangle := | F \cap G | \ mod\, 2.$$

This form is symmetric and non-degenerate over $\mathbb{Z}_2$.
Its semantics record only the parity of conflicts.

6.2 Definition of Orthogonal Complements

For any subspace $W \subseteq C_1$, define its orthogonal complement:

$$W^{\perp} := \{x \in C_1 \mid \langle x, w\rangle = 0, \qquad \forall w \in W\}.$$

The orthogonal complement is uniquely determined modulo $\mathbb{Z}_2$.

6.3 Orthogonal Complement of the Cycle Space

Let

$$\mathcal{C} := \ker\ \partial_1$$

be the cycle space.

Then:

$$\mathcal{C}^{\perp} = \mathcal{K},$$

where $\mathcal{K}$ denotes the subspace generated by all cuts (the cut space).

6.4 Structural Basis of Orthogonality

For any $C \in \mathcal{C}$ and any cut $\delta(S)$,

$$\langle C, \delta(S) \rangle = 0.$$

This follows from the fact that any closed structure crosses a cut an even number of times.

6.5 Dimensional Complementarity

If the graph G has m edges, then:

$$\dim\ \mathcal{C} + \dim\ \mathcal{K} = m.$$

Together with orthogonality:

$$C_1 = \mathcal{C} \oplus \mathcal{K},$$

as an orthogonal direct sum over $\mathbb{Z}_2$.

6.6 Definition of Duality

In this work, "duality" does not denote object replacement.
It denotes the following operation:

mapping a class of structures to its orthogonally incompatible class via an inner product.

Duality is thus a **relational operation**, not a geometric transformation.

6.7 Status of Planar Duality

In the presence of planar embeddings:

- cuts correspond to cycles in the dual graph,
- cycles correspond to cuts in the dual graph.

This correspondence is merely a visualization of orthogonal duality.

6.8 Chapter Conclusion

Cycles and cuts are not parallel notions but orthogonal duals within the same space. Duality is uniquely determined by the inner product and is independent of embedding, orientation, or weight.

第七章　工程语义映射：守恒、驱动与交换

7.1 语义映射原则

本章不引入新的数学结构。
仅对既有结构施加工程语义解释。

映射遵循以下原则:

- 不改变代数关系
- 不引入额外假设
- 不增加自由度

7.2 边变量与状态向量

对每条边 $e \in E$，引入变量 $x_e \in \mathbb{Z}_2$。
由此得到边状态向量:

$$x \in C_1$$

边变量不区分方向，仅表示“是否激活”。

7.3 守恒条件（KCL 的结构形式）

定义守恒条件为:

$$\partial_1(x) = 0$$

该条件等价于:

- 每个顶点 incident 边的激活数为偶
- 无净注入或泄漏

在工程语义中，该条件对应**节点守恒**。

7.4 守恒解空间

所有满足守恒条件的状态构成:

$$\ker \partial_1$$

即 cycle space。

工程上，该空间描述**内部自洽的运行模式**。

7.5 驱动模式（Cut 的语义）

设 $k \in \mathcal{K}$为 cut space 中的元素。

k表示一组跨越系统边界的激活边。
其语义为**外部注入或控制接口**。

7.6 正交条件（KVL 的结构形式）

对任意守恒状态 $x \in \ker \partial_1$与任意驱动模式 $k \in \mathcal{K}$，有:

$$\langle x, k \rangle = 0$$

该正交条件等价于:

- 内部守恒模式不消耗接口驱动

- 驱动仅作用于非守恒方向

7.7 交换量的结构定义

定义交换量为内积：

$$\langle x, k \rangle$$

在 $\mathbb{Z}_2$语义下，该量仅记录交换是否发生，而不记录大小。

7.8 结构分解的工程含义

边空间的分解：

$$C_1 = \mathcal{C} \oplus \mathcal{K}$$

在工程语义下对应：

- $\mathcal{C}$：系统内部自由运行空间
- $\mathcal{K}$：系统外部控制与注入空间

两者结构上正交，不可混用。

7.9 本章结论

工程系统中的守恒、驱动与交换，不是附加规则，
而是 cycle–cut 正交对偶在工程语义下的直接读法。

Chapter 7. Engineering Semantics: Conservation, Driving, and Exchange

7.1 Principle of Semantic Mapping

No new mathematical structures are introduced in this chapter.
Only engineering semantics are assigned to existing structures.

The mapping obeys:

- preservation of algebraic relations,
- no additional assumptions,

- no increase in degrees of freedom.

7.2 Edge Variables and State Vectors

For each edge $e \in E$, introduce a variable $x_e \in \mathbb{Z}_2$.
This yields an edge state vector:

$$x \in C_1.$$

Edge variables are unoriented and record only activation.

7.3 Conservation Condition (Structural KCL)

Define the conservation condition:

$$\partial_1(x) = 0.$$

This is equivalent to:

- each vertex having an even number of active incident edges,
- no net injection or leakage.

In engineering terms, this corresponds to **node conservation**.

7.4 Space of Conserved States

All states satisfying conservation form:

$$\ker\, \partial_1,$$

the cycle space.

This space represents **internally self-consistent operating modes**.

7.5 Driving Modes (Semantic Role of Cuts)

Let $k \in \mathcal{K}$be an element of the cut space.

Such a $k$represents a set of edges crossing a system boundary.

Its semantic role is **external injection or control**.

7.6 Orthogonality Condition (Structural KVL)

For any conserved state $x \in \ker \partial_1$ and any driving mode $k \in \mathcal{K}$,

$$\langle x, k \rangle = 0.$$

This orthogonality expresses:

- conserved internal modes do not consume external drive,
- driving acts only along non-conserved directions.

7.7 Structural Definition of Exchange

Define exchange as the inner product:

$$\langle x, k \rangle.$$

Over $\mathbb{Z}_2$, this quantity records only whether exchange occurs, not its magnitude.

7.8 Engineering Meaning of the Decomposition

The decomposition:

$$C_1 = \mathcal{C} \oplus \mathcal{K}$$

corresponds in engineering terms to:

- $\mathcal{C}$: internal free-operation space,
- $\mathcal{K}$: external control and injection space.

The two are structurally orthogonal and non-interchangeable.

7.9 Chapter Conclusion

Conservation, driving, and exchange in engineered systems are not additional rules.
They are direct semantic readings of the cycle–cut orthogonal duality.

第八章　换域与稳定性分析

8.1 域变换的动机

此前所有结构均在 $\mathbb{Z}_2$上建立。
本章考察当代数域发生变化时，哪些结构保持，哪些结构必然断裂。

域变换不被视为推广，而被视为**稳定性测试**。

8.2 从 $\mathbb{Z}_2$到一般域

设底层代数结构替换为一般域 $\mathbb{F}$。

以下结构仍可定义：

- 线性空间
- 边界算子
- 链复形
- 核与像
- 同调

但需引入额外假设：

- 边的定向
- 系数符号
- 加权内积

8.3 稳定性代价

在一般域上：

- 奇偶抵消不再自动成立
- 正交性依赖于选定的内积
- Cycle–Cut 分解不再唯一

结构的保持需要显式维护，而非自然出现。

8.4 $\mathbb{Z}_2$的极限地位

$\mathbb{Z}_2$具有以下特性:

- 无符号
- 无方向
- 无大小信息

在此条件下，仍可定义:

- 分解
- 不变量
- 同调
- 对偶

因此，$\mathbb{Z}_2$ 构成**最低稳定层**。

8.5 半环情形

设代数结构为半环（如 $\mathbb{N}$、$\max,+$）。

则以下结构失效:

- 加法逆元
- 正交补
- 核–像商结构
- 同调定义

可保留的仅为:

- 非负守恒约束
- 优化问题形式

8.6 断裂判据

若代数结构满足以下任一条件，则对偶与同调必然失效:

- 不存在加法逆元
- 内积不可定义

- 奇偶信息无法表达

该判据与具体对象无关，仅依赖代数性质。

8.7 稳定性排序

在不引入额外结构的前提下，稳定性由高到低排列为：

$$\mathbb{Z}_2 \ > \ 一般域 \ > \ 半环$$

排序依据为结构自发保持能力。

8.8 本章结论

$\mathbb{Z}_2$不是简化版本，而是结构仍能自洽存在的下界。
越过该下界，统一性必须依赖额外假设维持。

Chapter 8. Domain Changes and Stability Analysis

8.1 Motivation for Domain Changes

All previous structures were built over $\mathbb{Z}_2$.
This chapter examines which structures persist and which necessarily break when the underlying algebraic domain changes.

Domain change is treated not as extension, but as a **stability test**.

8.2 From $\mathbb{Z}_2$to General Fields

Let the underlying algebraic structure be replaced by a general field $\mathbb{F}$.

The following structures remain definable:

- linear spaces,
- boundary operators,
- chain complexes,
- kernels and images,
- homology.

However, additional assumptions become necessary:

- orientation of edges,
- sign conventions,
- weighted inner products.

8.3 Cost of Stability

Over general fields:

- parity cancellation is no longer automatic,
- orthogonality depends on a chosen inner product,
- cycle–cut decomposition is no longer canonical.

Structural persistence requires explicit maintenance.

8.4 The Limiting Status of $\mathbb{Z}_2$

$\mathbb{Z}_2$ has the following properties:

- no sign,
- no orientation,
- no magnitude information.

Yet it still supports:

- decomposition,
- invariants,
- homology,
- duality.

Thus, $\mathbb{Z}_2$ constitutes the **minimal stable layer**.

8.5 The Semiring Case

Let the algebraic structure be a semiring (e.g., $\mathbb{N}$, $\max, +$).

Then the following fail:

- additive inverses,
- orthogonal complements,
- kernel–image quotients,
- homology definitions.

What remains are:

- non-negative conservation constraints,
- optimization formulations.

8.6 Breakage Criteria

If an algebraic structure satisfies any of the following, duality and homology necessarily fail:

- absence of additive inverses,
- inability to define an inner product,
- inability to express parity.

These criteria are algebraic, not object-dependent.

8.7 Stability Ordering

Without introducing additional structure, stability decreases in the order:

$$\mathbb{Z}_2 > \text{general fields} > \text{semirings.}$$

The ordering reflects the ability of structures to remain self-maintaining.

8.8 Chapter Conclusion

$\mathbb{Z}_2$ is not a simplified version, but the lowest level at which structural coherence survives. Below this level, unification can only be preserved through added assumptions.

第九章　算法侧：结构的可执行形式

9.1 算法视角的限定

本章不引入新的判定目标。
仅讨论此前结构在**有限步骤内的可判定性与可构造性**。

算法被视为结构的执行形式，而非独立对象。

9.2 奇偶判定

给定有限集合 $A \subseteq U$，奇偶判定定义为计算：

$$\pi(A) = |A| \bmod 2$$

该操作在输入规模线性时间内完成。
其结果为单比特信息，不可再压缩。

9.3 边界判定

给定边集 $F \subseteq E$，计算：

$$\partial_1(F)$$

该计算可通过一次边–点扫描完成，时间复杂度为：

$$O(|V| + |E|)$$

该判定决定 F是否属于 $\ker \partial_1$。

9.4 Cycle 判定

定义 cycle 判定为验证：

$$\partial_1(F) = \emptyset$$

该判定不涉及路径搜索或连通性分析，仅依赖度数奇偶统计。

9.5 Cycle Space 的构造

设 $T \subseteq E$为生成树。

对每条非树边 $e \in E \setminus T$，构造唯一基本环 C_e。

集合 $\{C_e\}$构成 $\ker \partial_1$的一组基。

该构造的时间复杂度为:

$$O(|V|+|E|)$$

9.6 Cut 判定

给定边集 $K \subseteq E$，其是否属于 cut space 可通过以下任一方式判定:

- 存在顶点划分 $S \subseteq V$，使 $K = \delta(S)$;
- 对所有 $C \in \ker \partial_1$，有 $\langle C, K \rangle = 0$。

第二种方式为纯代数判定。

9.7 同调判定

在存在 2-链群的情形下，给定 cycle C，判断其是否为边界:

$$C \in \operatorname{im} \partial_2$$

该问题等价于模 2 线性方程组的可解性问题。

9.8 复杂度下界

所有核心判定均满足:

- 不依赖全局搜索
- 不依赖指数枚举
- 不依赖连续近似

其复杂度下界由输入规模线性或多项式给出。

9.9 算法稳定性

上述算法在以下削减操作下保持正确性:

- 删除边权
- 删除方向
- 删除大小信息，仅保留奇偶

该稳定性直接来源于 $\mathbb{Z}_2$结构。

9.10 本章结论

在最低稳定层上，
结构判定与构造均可在低复杂度内完成。

算法复杂性在此不构成主要障碍，
限制来自结构本身而非计算资源。

Chapter 9. Algorithmic Perspective: Executable Forms of Structure

9.1 Scope of the Algorithmic View

No new decision problems are introduced in this chapter.
Only the **decidability and constructibility** of previously defined structures are considered.

Algorithms are treated as executable realizations of structure.

9.2 Parity Testing

Given a finite set $A \subseteq U$, parity testing computes:

$$\pi(A) = |A| \bmod 2.$$

This operation completes in linear time in the input size.
Its output is a single bit and cannot be further compressed.

9.3 Boundary Evaluation

Given an edge set $F \subseteq E$, compute:

$$\partial_1(F).$$

This requires a single edge–vertex scan, with time complexity:

$$O(|V| + |E|).$$

The result determines whether $F \in \ker \partial_1$.

9.4 Cycle Testing

Cycle testing verifies:

$$\partial_1(F) = \emptyset.$$

No path search or connectivity analysis is required; only parity counts of degrees.

9.5 Construction of the Cycle Space

Let $T \subseteq E$ be a spanning tree.

For each non-tree edge $e \in E \setminus T$, construct the unique fundamental cycle C_e.

The set $\{C_e\}$ forms a basis of $\ker \partial_1$.

This construction runs in:

$$O(|V| + |E|).$$

9.6 Cut Detection

Given an edge set $K \subseteq E$, membership in the cut space can be tested by either:

- existence of a vertex partition $S \subseteq V$ such that $K = \delta(S)$;
- verification that $\langle C, K \rangle = 0$ for all $C \in \ker \partial_1$.

The latter is a purely algebraic test.

9.7 Homology Testing

When a 2-chain group exists, given a cycle C, test whether:

$$C \in \operatorname{im} \partial_2.$$

This reduces to solvability of a linear system over $\mathbb{Z}_2$.

9.8 Complexity Lower Bounds

All core procedures satisfy:

- no global search,
- no exponential enumeration,
- no reliance on continuous approximation.

Their complexity bounds are linear or polynomial in input size.

9.9 Algorithmic Stability

The above algorithms remain valid under:

- removal of edge weights,
- removal of orientation,
- removal of magnitude information, retaining only parity.

This stability derives directly from the $\mathbb{Z}_2$structure.

9.10 Chapter Conclusion

At the minimal stable layer,
structural decisions and constructions admit low-complexity algorithms.

Algorithmic complexity is not the limiting factor;
the limitation lies in the structure itself.

第十章　工程侧：系统级映射

10.1 工程系统的抽象前提

工程系统在本文中被抽象为:

- 有限组件集

- 有限连接关系
- 离散状态切换

连续量、噪声模型与控制律不参与本章结构判定。

10.2 拓扑稳定性判据

设系统连接关系形成图 $G = (V, E)$。

若:

$$\ker \partial_1 \neq \{0\},$$

则系统存在内部自由循环模式。

该模式在无额外约束下不可被外部观测唯一确定。

10.3 环的工程语义

在系统级语义中:

- cycle 表示内部自洽但不可判定的状态流
- cycle 的存在意味着状态回溯与因果定位的不确定性

因此,环并非冗余的同义词。

10.4 割的工程语义

cut 表示系统与外界的接口集合。

cut 的存在对应:

- 外部注入
- 控制点
- 切换点

所有可控行为必须落在 cut 空间中。

10.5 结构性冗余条件

工程冗余在结构上要求：

- 不引入新的 cycle 自由度
- 仅在 cut 空间中增加可切换路径

违反该条件的冗余会增加系统不可判定性。

10.6 配置与一致性

系统配置变更被视为边集扰动。

若扰动路径形成 cycle，则：

- 无全局一致状态
- 无唯一回滚路径

一致性要求配置变更可表示为 cut 扰动。

10.7 告警与事件传播

事件传播在结构上为边激活。

若传播路径包含 cycle，则可能发生：

- 奇偶抵消
- 重复触发
- 事件消失

告警汇聚点必须位于 cut 对应接口。

10.8 升级与回滚流程

升级流程被视为状态图。

若状态图包含 cycle，则存在不可回滚中间态。

必要条件：

$$\text{状态图为 DAG}$$

10.9 工程侧结构总表

在最低稳定层上：

- cycle = 不可判定自由度
- cut = 可控接口
- 正交 = 控制不干扰内部守恒

10.10 本章结论

工程系统中的稳定性、可控性与可回滚性，
均可追溯至 cycle–cut 的结构分解。

Chapter 10. Engineering Systems: System-Level Mapping

10.1 Abstraction of Engineering Systems

Engineering systems are abstracted as:

- finite component sets,
- finite connection relations,
- discrete state transitions.

Continuous quantities, noise models, and control laws are excluded.

10.2 Topological Stability Criterion

Let the system connectivity form a graph $G = (V, E)$.

If:

$$\ker \partial_1 \neq \{0\},$$

then the system contains internal free cyclic modes.

Such modes are not uniquely observable without additional constraints.

10.3 Engineering Semantics of Cycles

At the system level:

- cycles represent internally consistent but indeterminate state flows,
- the presence of cycles implies ambiguity in rollback and causal attribution.

Thus, cycles are not synonymous with redundancy.

10.4 Engineering Semantics of Cuts

Cuts represent interfaces between the system and its environment.

Cuts correspond to:

- external injection,
- control points,
- switching points.

All controllable actions must lie in the cut space.

10.5 Structural Condition for Redundancy

Structural redundancy requires:

- no introduction of new cycle degrees of freedom,
- additional switchable paths only within the cut space.

Violation of this condition increases system indeterminacy.

10.6 Configuration and Consistency

Configuration changes are treated as edge-set perturbations.

If a perturbation path forms a cycle, then:

- no globally consistent state exists,
- no unique rollback path exists.

Consistency requires configuration changes to be representable as cut perturbations.

10.7 Alarms and Event Propagation

Event propagation is modeled as edge activation.

If propagation paths contain cycles, the following may occur:

- parity cancellation,
- duplicate triggering,
- event disappearance.

Alarm aggregation points must lie on cut-defined interfaces.

10.8 Upgrade and Rollback Processes

Upgrade processes are modeled as state graphs.

If the state graph contains cycles, irreversible intermediate states exist.

Necessary condition:

$$\text{state graph is a DAG.}$$

10.9 Structural Summary for Engineering

At the minimal stable layer:

- cycles = indeterminate internal freedom,
- cuts = controllable interfaces,
- orthogonality = control does not interfere with internal conservation.

10.10 Chapter Conclusion

Stability, controllability, and rollback feasibility in engineered systems
reduce directly to the cycle–cut structural decomposition.

结语

本文并未提出新的数学对象。
所做工作仅为一件事：确定**在假设不断削弱的过程中，哪些结构最后仍然存在**。

结论是明确的：

在不引入顺序、度量、权重、方向与精确计数的前提下，
奇偶、边界、核与像、同调与对偶构成一个不可再压缩的统一壳。

该壳不是离散数学的全部，
但它是离散结构在完全失去装饰之后仍能保持一致性的最后层级。

Conclusion

This work introduces no new mathematical objects.
Its sole task has been to identify **which structures survive as assumptions are progressively removed**.

The conclusion is unambiguous:
without order, metric, weight, orientation, or exact counting,
parity, boundaries, kernels and images, homology, and duality form a non-reducible unifying shell.

This shell does not encompass all of discrete mathematics,
but it is the last layer at which discrete structures remain coherent after all ornamentation is stripped away.
