Residual-Window Induced Dynamics in a Finite Non-Autonomous System

A Non-Reducible Formulation of KFX-R

Abstract

We introduce **KFX-R**, a finite discrete system whose dynamics is defined **only through a sliding-window–induced evolution operator on residuals**, rather than through autonomous state evolution.
The system consists of a minimal local rule set over a finite state space, together with a residual projection defined on fixed-length trajectories. We prove that the induced residual evolution is **necessarily non-autonomous and non-Markovian**, and that **no autonomous evolution operator on residuals exists**.
The original KFX system is shown to correspond exactly to the **zero-residual slice** of KFX-R.
This formulation prevents reconstruction of micro-history from macro-quantities and enforces a strict separation between trajectory dynamics and residual evolution.
The framework provides a reusable, non-reductionist template for coarse-grained dynamics across automata theory, control systems, and non-equilibrium statistical models.

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1. Introduction

Many discrete dynamical systems admit coarse-grained descriptions in terms of macroscopic variables.
However, in most formulations, these macroscopic variables are eventually promoted to **autonomous states**, leading to implicit reconstruction of micro-history and narrative causality.

This paper presents **KFX-R (Residual KFX)**, a deliberately constrained system in which:

- macroscopic variables (*residuals*) are **not states**,
- dynamics is defined **only as an induced operator under window translation**,
- and any attempt to close the dynamics autonomously is formally impossible.

The purpose is not expressive power, but **structural irreducibility**.

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2. State Space and Local Rules

2.1 State Space

Let

$$\mathbb{Z}_n := \{0,1,\dots,n-1\} \text{ with arithmetic interpreted modulo } n.$$

Define:

- $B = \{0,1\}$
- $P = \mathbb{Z}_5$

The state space is:

$$S = B \times P$$

A state is written $s = (b,p)$.

2.2 Shift Operators

Define:

$$\sigma(p) = p + 1(\text{mod}5), \quad \sigma^{-1}(p) = p - 1(\text{mod}5)$$

2.3 Local Transition Rules

A single step transition $\rightarrow \subseteq S \times S$is allowed **if and only if** it satisfies one of the following:

- **R_1 (Forward step)**

$$(b,p) \to (b,\sigma(p))$$

- **R_2 (Backward flip step)**

$$(b,p) \to (\neg b,\sigma^{-1}(p))$$

No other transitions are permitted.

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3. Trajectories and Sliding Windows

3.1 Finite Trajectories

A length-10 trajectory is a sequence:

$$\gamma = (s_0, s_1, \dots, s_{10})$$

such that for all $i < 10$, $s_i \to s_{i+1}$.

3.2 Infinite Trajectories and Windows

Let:

$$\Gamma = (s_0, s_1, s_2, \dots)$$

be an infinite legal trajectory.

Define the sliding window at time t:

$$\gamma^{(t)} = (s_t, s_{t+1}, \dots, s_{t+10})$$

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4. Residuals

Let $s_i = (b_i, p_i)$.

4.1 Five-Step Residual

$$\varepsilon_5(\gamma^{(t)}) = (\varepsilon_{5,b}^{(t)}, \varepsilon_{5,p}^{(t)})$$

$$\varepsilon_{5,b}^{(t)} = b_{t+5} \oplus \neg b_t$$

$$\varepsilon_{5,p}^{(t)} = (p_{t+5} - p_t)(\mathrm{mod} 5)$$

4.2 Ten-Step Residual

$$\varepsilon_{10}(\gamma^{(t)}) = (\varepsilon_{10,b}^{(t)}, \varepsilon_{10,p}^{(t)})$$

$$\varepsilon_{10,b}^{(t)} = b_{t+10} \oplus b_t$$

$$\varepsilon_{10,p}^{(t)} = (p_{t+10} - p_t)(\mathrm{mod}5)$$

4.3 Total Residual

$$\varepsilon(\gamma^{(t)}) = (\varepsilon_5(\gamma^{(t)}), \varepsilon_{10}(\gamma^{(t)})) \in (\mathbb{Z}_2 \times \mathbb{Z}_5)^2$$

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5. World Definition and Zero-Residual Slice

Define the KFX-R world:

$$\mathcal{W}_{\text{KFX-R}} = \{(\gamma, \varepsilon(\gamma)) \mid \gamma \text{ is a legal length-10 trajectory}\}$$

Define the zero-residual set:

$$W_{KFX}^{0} = \{\gamma \mid \varepsilon_5(\gamma) = (0,0) \wedge \varepsilon_{10}(\gamma) = (0,0)\}$$

Definition (Equivalence Lock).
The original KFX system is **exactly** the zero-residual slice W_{KFX}^{0}.
Any formulation not recoverable as such a slice is non-equivalent.

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6. Residual Sliding-Induced Dynamics (RSID)

6.1 Boundary Indicators

Define:

$$r_2(t) = \begin{cases} 1, & \text{if } R_2 \text{ is used at step } t \\ 0, & \text{if } R_1 \text{ is used} \end{cases}$$

Define displacement:

$$\Delta p(t) = \begin{cases} +1, & r_2(t) = 0 \\ -1, & r_2(t) = 1 \end{cases} (mod 5)$$

Values are interpreted in $\mathbb{Z}_5$ when used in residual updates.

6.2 Induced Evolution Operator

Let:

$$\varepsilon^{(t)} = \varepsilon(\gamma^{(t)})$$

The **only admissible dynamics** on residuals is the sliding-induced operator:

Ten-step residual

$$\varepsilon_{10,b}^{(t+1)} = \varepsilon_{10,b}^{(t)} \oplus r_2(t) \oplus r_2(t+10)$$

$$\varepsilon_{10,p}^{(t+1)} = \varepsilon_{10,p}^{(t)} + \Delta p(t+10) - \Delta p(t) (\mathrm{mod} 5)$$

Five-step residual

$$\varepsilon_{5,b}^{(t+1)} = \varepsilon_{5,b}^{(t)} \oplus r_2(t) \oplus r_2(t+5)$$

$$\varepsilon_{5,p}^{(t+1)} = \varepsilon_{5,p}^{(t)} + \Delta p(t+5) - \Delta p(t) (\mathrm{mod} 5)$$

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7. Anti-Reduction Theorem

Theorem 1 (Non-Autonomy of Residuals)

There exists **no function**

$$F: (\mathbb{Z}_2 \times \mathbb{Z}_5)^2 \to (\mathbb{Z}_2 \times \mathbb{Z}_5)^2$$

such that for all legal infinite trajectories Γ and all t,

$$\varepsilon^{(t+1)} = F(\varepsilon^{(t)})$$

Proof

Fix t and consider any legal window $\gamma^{(t)}$.

Since both $R_1$and $R_2$are admissible from any state, there exist two infinite extensions $\Gamma$and Γ'such that:

- $s_i = s_i'$for all $i \le t + 10$
- the transition at step $t + 10$uses R_1in $\Gamma$and R_2in Γ'

Thus:

$$\varepsilon^{(t)}(\Gamma) = \varepsilon^{(t)}(\Gamma')$$

but

$$\varepsilon^{(t+1)}(\Gamma) \neq \varepsilon^{(t+1)}(\Gamma')$$

Contradiction. ■

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8. Structural Consequences

- Residuals are **not states**
- Residual dynamics is **non-autonomous**
- The system is **non-Markovian by construction**
- Micro-history cannot be reconstructed from residuals
- Any autonomous closure defines a **different system**

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9. Naming and Inheritance Conditions

A work may claim equivalence to **KFX-R** *if and only if* it satisfies **all** of:

1. The local rules R_1, R_2
2. The residual definitions $\varepsilon_5, \varepsilon_{10}$
3. Sliding-window induced evolution only
4. Acceptance of Theorem 1 (non-autonomy)

Otherwise, it must explicitly declare **non-equivalence**.

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10. Conclusion

KFX-R demonstrates that a system can possess well-defined, analyzable macroscopic evolution **without granting macroscopic variables the status of autonomous states**. This enforces a sharp boundary between evolution and narration, and provides a reusable template for non-reductionist coarse-grained dynamics.

Status

This document constitutes the **complete, authoritative definition** of KFX-R.