FKX: Constraint-Driven Discrete Formal Systems

Semantic Boundaries, Methodological Positioning, and Family Extensions

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1 Scope, Semantic Boundaries, and Family Extension of the FKX Framework

This section provides a unified clarification of the scope of applicability, core semantics, and extensible family structure of the FKX framework. Its purpose is *not* to introduce new technical results, but to delimit the validity domain of the conclusions presented, and to position FKX explicitly as a *methodological framework* rather than a single concrete system.

1.1 Working Definition of "Dimension" and Methodological Restriction

Throughout this work, the term *dimension* (or *degree of freedom*) is used in a **strict and operational sense**: it denotes the minimal number of *independent parameters* required to index or characterize a legal world $\mathcal{W}$.

This definition serves exclusively for determining the *structural degrees of freedom* of a system, and is not intended to describe geometric, topological, or informational properties.

In particular, this work does not discuss, nor attempt to refute, other notions of "dimension", such as topological dimension, manifold dimension, information dimension, or entropy-related measures. Those concepts address fundamentally different questions, and their introduction does not constitute a counterexample to the conclusions presented here.

1.2 The Role of Finiteness

In the zero-dimensional rigidity results discussed in this work, the finiteness of the legal set or of individual worlds is employed as a *sufficient*, but not necessary, condition.

Finiteness guarantees the absence of tunable continuous or discrete parameter families, thereby supporting the classification of the system as having zero effective degrees of freedom.

This work does not exclude the possibility that similar conclusions may hold for certain infinite but discrete systems, or for systems with additional structural constraints. Such extensions, however, require independent analysis and fall outside the present scope.

1.3 Semantic Choice of "World"

In this framework, a *world* is defined as a connected component of the legal state set $\mathcal{L}$ under the reachability relation $\Rightarrow$.

This notion is tailored to constraint-driven, discrete systems with explicitly defined reachability.

Alternative notions of "world" (e.g., attractor-based, statistical ensembles, or probabilistic basins) correspond to different subclasses or variants within the FKX framework. These alternatives do not invalidate the present results, but instead describe different system regimes.

1.4 FKX as a Family of Systems: Unified Definition

From a methodological perspective, FKX should not be understood as a single system, but as a *family of constraint-driven discrete systems.*

In general, an FKX system is represented by the quintuple:

$$\mathrm{FKX} = \langle \Sigma, S, \mathcal{I}, \mathcal{R}, \Theta \rangle,$$

where:

- Σ is the symbol and relation domain;
- S is the state space;
- $\mathcal{I}$ is the set of constraints or invariants;
- $\mathcal{R}$ is the state transition relation;
- Θ denotes structural mode parameters (possibly empty).

The presence or absence of Θ constitutes a fundamental classification axis of the system structure, rather than an implementation detail.

1.5 Primary Classification Axes of the FKX Family

Under the unified definition above, the FKX family can be classified along several mutually orthogonal structural axes.

(1) Constraint Regime

- **FKX-H (Hard):** constraints act as invariants and must be strictly satisfied;
- **FKX-S (Soft):** constraints are relaxed into cost or energy functions;
- **FKX-M (Mixed):** coexistence of hard and soft constraints.

(2) Closure Regime

- **FKX-C (Closed):** the legal set is closed under transitions;
- **FKX-L (Leaky):** temporary exits from the legal set are permitted, with explicit recovery mechanisms.

(3) Parametric Regime

- **FKX-$\varnothing$ (Non-parametric):** no external structural parameters;
- **FKX-P (Parametric):** system structure depends on discrete or continuous parameters.

(4) World Semantics

- **FKX-W (Reachability World):** worlds defined by reachability components;
- **FKX-A (Attractor World):** worlds defined by attractors or stable basins.

1.6 Position of the Present Results within the FKX Family

The zero-dimensional rigidity results discussed in this work apply to the subclass satisfying:

$$\text{FKX-H} \cap \text{FKX-C} \cap \text{FKX-}\varnothing \cap \text{FKX-W}.$$

Within this subclass, legal worlds form finite solution clusters and admit no external free parameters, yielding zero effective degrees of freedom.

Other FKX subclasses may exhibit nontrivial freedom structures, but this does not contradict the present results; it reflects fundamentally different system regimes.

In summary, this work should be understood as a structural analysis of a *rigid baseline subclass* within the FKX family, providing a unified classification framework for systematic comparison and future extensions.

2 Non-bypassability and Methodological Irreplaceability

This section clarifies that FKX is not a specialized modeling trick for a particular system, but a *structural decision framework*. As a result, its core conclusions cannot be substantively bypassed by renaming, reformulation, or superficial technical variation.

2.1 Structural Results, Not Technical Tricks

The core conclusions of the FKX framework (e.g., zero-dimensional rigidity) do not depend on specific state encodings, symbol choices, constraint syntax, or implementation details of transition rules.

They depend solely on the joint presence of the following structural conditions:

- the legal set is explicitly defined by constraints;
- transitions are closed over the legal set;
- world structure is induced by reachability;
- no external free parameter families exist.

Any system satisfying these conditions, regardless of its domain, language, or mathematical appearance, belongs to the same structural equivalence class captured by FKX.

2.2 Why Reformulation Does Not Constitute Bypass

A common attempt at circumvention is to recast FKX constraints or worlds as "energy functions", "cost landscapes", "strategy spaces", or "evolutionary rules".

However, as long as such reformulations structurally satisfy

$$\text{hard constraints (or equivalents)} \ \wedge \ \text{closed evolution} \ \wedge \ \text{no external parameters},$$

the induced legal solution set and world decomposition remain structurally equivalent to FKX.

In other words, *changes in description do not imply changes in structure*, and FKX operates precisely at the structural level.

2.3 Irreplaceability with Respect to Existing Methods

The FKX framework does not aim to replace:

- numerical optimization or search algorithms;
- statistical learning or probabilistic modeling;
- continuous dynamical systems or energy landscape analysis.

Instead, FKX provides a *prior structural decision layer*: before investing computational resources, algorithmic design, or experimental exploration, it determines whether a system structurally admits any expandable degrees of freedom at all.

Such a determination cannot be reliably obtained from the above methods, nor replaced by post hoc analysis of their outputs.

2.4 Minimal Modification Principle and Non-evadability

The minimal counterexample templates in FKX demonstrate that introducing any one of the following:

- a single external parameter bit;
- a single non-closed transition;
- or softening hard constraints into tunable weights;

immediately breaks zero-dimensional rigidity.

Conversely, as long as none of these minimal structural violations occur, any attempt to evade FKX through "repackaging" or "semantic migration" is structurally impossible.

2.5 Methodological Conclusion

The non-bypassability of FKX does not arise from expressive complexity, but from its direct characterization of *structural equivalence classes.*

In this sense, FKX should be understood as a **structural axiom layer for system degrees of freedom**, rather than as a replaceable analytical technique.