A Scale-Gated Structural Paradigm:
Difference, Equivalence, Irreversibility, and Responsibility

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Abstract

This work proposes a structural paradigm in which objecthood, dynamics, preference, and responsibility are not treated as primitive or normative notions, but as consequences of scale-relative equivalence and irreversible execution.

The framework begins from a deliberately minimal assumption: reversible difference without intrinsic meaning, value, or direction. A notion of scale is introduced as an operational constraint on distinguishable differences. Scale induces equivalence relations via connectivity rather than axiomatic identification. Objects arise only as equivalence classes relative to a chosen scale.

Transitions between scales generally induce many-to-one mappings that discard fine-scale distinctions. When execution must proceed under such degradation, reversibility is structurally lost. Irreversibility is therefore not postulated as a physical or metaphysical principle, but emerges from executable constraints under scale transition.

Preference is shown to arise whenever irreversible degradation collapses multiple antecedent states into a single admissible continuation. Responsibility is defined as the necessity of committing to a continuation under irreversible information loss, rather than as a moral attribution.Explicit falsifiability conditions are provided in Section 12

Structural Overview

This paper proceeds as follows. Primitive difference and scale are introduced first, followed by a derivation of scale-induced equivalence and objecthood. Scale transitions and degradation mappings are then analyzed, leading to a structural account of irreversibility. Finally, preference, non-bypassability, and responsibility are derived as forced consequences of irreversible execution.

1 Primitive Difference

We begin from the weakest possible structural assumption. No semantic, normative, or teleological primitives are introduced at this level.

The foundational object of the framework is difference, understood in a purely relational and reversible sense. Difference is not equipped with magnitude, direction, preference, or meaning. It does not presuppose identity, persistence, or relevance.

Definition 1 (Primitive Difference). *A primitive difference is a reversible relation between distinguishable states, carrying no intrinsic ordering, valuation, or semantic interpretation.*

Reversibility here refers to the absence of structural constraints that privilege one direction of transition over another.

Primitive difference alone does not generate objecthood. Without additional structure, there is no basis for identifying equivalence, stability, or persistence across variations.

1.1 Formal Structure of Primitive Difference

In order to avoid implicit assumptions, the notion of primitive difference is made explicit at a minimal formal level.

Let $\mathcal{S}$ denote a set of states. No topology, metric, order, or algebraic structure is assumed on $\mathcal{S}$.

Definition 2 (Difference Relation). *A* difference relation *is a binary relation $\Delta \subseteq \mathcal{S} \times \mathcal{S}$ such that $(x, y) \in \Delta$ indicates that states x and y are distinguishable at the primitive level.*

No symmetry, transitivity, or reflexivity is assumed *a priori*. However, reversibility of primitive difference is captured by the following minimal condition.

Definition 3 (Reversibility). *A difference relation Δ is said to be reversible if $(x, y) \in \Delta$ implies $(y, x) \in \Delta$.*

Reversibility expresses the absence of intrinsic directionality. It does not introduce temporal symmetry, causal ordering, or evaluative structure.

Remark 1. *Reversibility is strictly weaker than equivalence. In particular, it does not imply transitivity and therefore does not induce partitioning of the state space.*

Proposition 1. *Primitive difference equipped with reversibility does not induce object identity.*

Proof. Without transitivity or equivalence closure, no partition of $\mathcal{S}$ is induced. Hence no notion of objecthood, persistence, or stability can be derived from primitive difference alone. $\square$

This formalization establishes primitive difference as a minimal structural substrate. Any emergence of objecthood, preference, or responsibility must therefore arise from additional constraints introduced at later stages.

2 Scale as an Operational Constraint

Scale is introduced as a constraint on admissible distinctions rather than as a physical size or epistemic limitation.

Definition 4 (Scale). *A scale σ is a specification of which primitive differences are treated as operationally distinguishable within a given context of execution. Differences not admitted by σ are treated as indistinguishable.*

Scale is not a property of states themselves but of admissible operations. Different scales impose different structural organizations over the same set of underlying states.

Multiple scales may coexist without contradiction. No single scale is assumed to be privileged.

2.1 Scale-Induced Connectivity

Once a scale is fixed, indistinguishability induces connectivity among states.

Definition 5 (Scale-Induced Connectivity). *Given a scale σ, two states $x, y \in \mathcal{S}$ are said to be connected if there exists a finite sequence $(x_0, \ldots, x_n)$ such that $x_0 = x$, $x_n = y$, and each adjacent pair differs only by differences indistinguishable at scale σ.*

Connectivity formalizes the idea that intermediate distinctions cannot be resolved under the given scale.

2.2 Scale-Induced Equivalence

Connectivity induces equivalence relations.

Definition 6 (Scale-Induced Equivalence). *Two states are equivalent at scale σ if they are connected under scale-induced connectivity.*

Proposition 2. *Scale-induced equivalence is an equivalence relation.*

Proof. Reflexivity follows from the trivial sequence. Symmetry follows from reversibility of primitive difference. Transitivity follows from concatenation of finite sequences. $\square$

The equivalence classes induced by scale partition the state space into sets of mutually indistinguishable states.

3 Objecthood as Scale-Relative Equivalence

Objecthood is not assumed as a primitive notion. Instead, it emerges from scale-induced equivalence.

Theorem 1 (Scale-Relative Objecthood). *At a given scale σ, objects correspond to equivalence classes induced by scale-induced indistinguishability.*

An object is therefore not an intrinsic entity but a structural artifact of a chosen scale.

Object identity is relative rather than absolute. Changing the scale alters the induced equivalence relation and therefore alters the object structure.

Object persistence is likewise conditional. What appears as a single persistent object at one scale may fragment into multiple objects at a finer scale or collapse into a single object at a coarser scale.

Remark 2. *No semantic interpretation is attached to objects at this level. Any meaning attributed to objects must be introduced at later stages and cannot be derived from scale-relative structure alone.*

4 Scale Transitions

Scales rarely operate in isolation. Execution in real systems typically involves movement between multiple scales, from fine-grained distinctions to coarser representations.

A scale transition formalizes this movement.

Definition 7 (Scale Transition)**.** *A* scale transition *is a mapping*

$$\pi_{\sigma_f \to \sigma_c} : \mathcal{S}_{\sigma_f} \to \mathcal{S}_{\sigma_c}$$

from a finer scale σ_f to a coarser scale σ_c such that every distinction admitted at σ_c is also admitted at σ_f, but not conversely.

Scale transitions capture abstraction, aggregation, and representation change. They are not assumed to preserve injectivity or invertibility.

4.1 Properties of Scale Transitions

Several structural properties follow immediately.

Proposition 3. *A scale transition $\pi_{\sigma_f \to \sigma_c}$ is generally many-to-one.*

Proof. Since σ_c admits fewer distinctions than σ_f, multiple fine-scale states may become indistinguishable at the coarser scale. Hence injectivity is not preserved in general. □

Remark 3. *Injective scale transitions are possible only in degenerate cases where no distinctions are discarded. Such transitions do not constitute genuine abstraction.*

4.2 Composition of Scale Transitions

Scale transitions compose naturally.

Proposition 4. *Given scales $\sigma_1 \prec \sigma_2 \prec \sigma_3$, the composition*

$$\pi_{\sigma_2 \to \sigma_3} \circ \pi_{\sigma_1 \to \sigma_2}$$

is a scale transition from σ_1 to σ_3.

Proof. Each transition discards distinctions. Their composition discards at least as many distinctions as either component. □

This establishes a partial order over scales induced by refinement.

5 Degradation Mappings

Scale transitions that discard information give rise to degradation.

Definition 8 (Degradation Mapping)**.** *A* degradation mapping *is a scale transition in which two or more distinct fine-scale states map to a single coarse-scale state.*

Degradation is not an error or approximation artifact. It is a structural consequence of operating at a coarser scale.

5.1 Types of Degradation

Different forms of degradation can be distinguished.

Definition 9 (Representational Degradation)**.** *Representational degradation occurs when distinctions are eliminated due to limited representational capacity at the coarser scale.*

Definition 10 (Access Degradation)**.** *Access degradation occurs when fine-scale distinctions exist in principle but are inaccessible during execution.*

Definition 11 (Historical Degradation)**.** *Historical degradation occurs when past distinctions cannot be reconstructed from the present coarse-scale state.*

These types are not mutually exclusive and may co-occur.

5.2 Information Loss

Proposition 5. *Every degradation mapping induces information loss relative to the finer scale.*

Proof. By definition, multiple fine-scale states correspond to a single coarse-scale state. Therefore, the coarse-scale representation cannot uniquely determine the fine-scale antecedent. □

Information loss here is defined structurally, without reference to entropy or probability.

6 Executable Constraints

Whether degradation results in irreversibility depends on executable constraints.

Definition 12 (Executable Constraint)**.** *An* executable constraint *is a limitation on admissible operations during execution, including but not limited to memory, time, access to prior states, or available representations.*

Executable constraints distinguish mathematical possibility from operational realizability.

6.1 Non-Recoverability

Definition 13 (Non-Recoverability)**.** *A degradation mapping is said to be non-recoverable under a set of executable constraints if no admissible operation reconstructs the discarded distinctions.*

Non-recoverability is relative to both scale and execution.

Proposition 6. *If a degradation mapping is non-recoverable under executable constraints, then the corresponding scale transition is operationally irreversible.*

Proof. Non-recoverability implies that no operation exists to reconstruct fine-scale antecedents. Therefore inversion of the scale transition is impossible under execution. □

7 Irreversibility

Irreversibility is introduced as a derived structural notion.

Definition 14 (Operational Irreversibility)**.** *A process is operationally irreversible if, under the given executable constraints, no admissible operation reconstructs its fine-scale antecedents.*

Operational irreversibility does not presuppose temporal asymmetry, entropy, or physical laws. It arises from the interaction between degradation and executable constraint.

Theorem 2 (Irreversibility from Degradation)**.** *Any non-recoverable degradation mapping induces operational irreversibility.*

Proof. By non-recoverability, discarded distinctions cannot be reconstructed. Hence the inverse transition is not executable. □

7.1 Consequences of Irreversibility

Once irreversibility is present, several consequences follow.

First, historical information is structurally inaccessible. Second, execution becomes directionally constrained. Third, future continuations must proceed without reference to eliminated alternatives.

These consequences are structural and do not rely on semantic interpretation.

8 Preference as Forced Selection

At the level of primitive difference, no preference exists. All distinctions are reversible and no continuation is privileged.

Preference emerges only when irreversibility constrains continuation.

Definition 15 (Forced Selection)**.** *A* forced selection *occurs when, under operational irreversibility, multiple fine-scale antecedent states correspond to a single coarse-scale state, and execution must proceed forward.*

Forced selection is not an act of choice in a normative sense. It is a structural necessity imposed by irreversible execution.

8.1 Structural Emergence of Preference

Theorem 3 (Emergence of Preference)**.** *Whenever forced selection occurs under operational irreversibility, preference is structurally induced over admissible continuations.*

Proof. Operational irreversibility prevents recovery of antecedent distinctions. Execution must therefore commit to one of the admissible continuations without access to eliminated alternatives. This commitment induces a preference structure. □

Preference here is not psychological, intentional, or value-based. It is a consequence of execution under constraint.

Remark 4. *Structural preference may later be interpreted normatively or semantically, but such interpretations are external to the framework.*

9 Non-Bypassability

Irreversibility not only induces preference but also restricts the space of admissible continuations.

Definition 16 (Non-Bypassability)**.** *A constraint is said to be* non-bypassable *if, once imposed under operational irreversibility, no executable continuation exists that avoids its consequences.*

Non-bypassability is not a prohibition. It is a statement about the absence of executable alternatives.

9.1 Necessary Conditions

Proposition 7 (Necessary Condition for Non-Bypassability)**.** *If a constraint eliminates distinctions required to separate alternative continuations, then it is non-bypassable under operational irreversibility.*

Proof. If eliminated distinctions are required to distinguish continuations, their absence prevents reconstruction of alternative paths. Hence no executable bypass exists. □

9.2 Sufficient Conditions

Proposition 8 (Sufficient Condition for Non-Bypassability)**.** *If a degradation mapping is non-recoverable and constrains all admissible continuations, then the induced constraint is non-bypassable.*

Proof. Non-recoverability prevents restoration of discarded distinctions. If all admissible continuations depend on those distinctions, bypass is structurally impossible. □

9.3 Boundary Cases

Not all irreversible processes are non-bypassable. If alternative continuations remain distinguishable despite degradation, bypassability may persist.

This boundary clarifies that non-bypassability is stronger than irreversibility.

10 Structural Commitment

Non-bypassability implies commitment.

Definition 17 (Structural Commitment)**.** *A* structural commitment *is the binding of future execution to the consequences of an irreversible and non-bypassable constraint.*

Structural commitment does not presuppose agency or intention. It applies equally to mechanical systems, algorithms, and socio-technical processes.

10.1 Commitment Without Agency

Commitment arises whenever execution is constrained by past irreversible operations, regardless of whether an agent is present.

This observation separates commitment from notions of will or intention.

11 Responsibility

Responsibility is introduced as a structural notion rather than a moral one.

Definition 18 (Structural Responsibility)**.** *Responsibility is the necessity of committing to a continuation under conditions of irreversible degradation and non-bypassable constraint.*

11.1 Conditions for Responsibility

Structural responsibility arises whenever the following conditions hold:

- Degradation is operationally irreversible.
- Multiple antecedent distinctions are collapsed.
- Execution must continue forward.
- No bypassable alternative exists.

Theorem 4 (Emergence of Responsibility)**.** *Structural responsibility emerges whenever forced selection occurs under non-bypassable irreversible degradation.*

Proof. Forced selection requires commitment to a continuation. Non-bypassability prevents circumvention of the resulting consequences. Therefore responsibility is structurally imposed by execution itself. □

Remark 5. *This notion of responsibility is compatible with, but distinct from, moral or legal responsibility. Those interpretations may be layered on top of the structural substrate.*

12 Explicit Falsifiability Conditions

This framework is falsifiable in a strong structural sense. The core claims of irreversibility, preference, non-bypassability, and structural responsibility can be directly refuted by executable counterexamples. We list explicit failure conditions below.

12.1 Failure Condition F1: Reversible Reconstruction under Non-Recoverable Degradation

Claim under test. Non-recoverable degradation under executable constraints implies operational irreversibility.

Refutation criterion (F1). If there exists a process that satisfies all of the following:

- a degradation mapping collapses multiple fine-scale antecedents into a single coarse-scale state;
- the discarded distinctions are declared inaccessible under the stated executable constraints;
- nevertheless, there exists an admissible (executable) operation that reconstructs a unique fine-scale antecedent from the coarse-scale state,

then Theorem 2 (Irreversibility from Degradation) is false under the stated constraints.

12.2 Failure Condition F2: Continuation without Commitment after Irreversible Collapse

Claim under test. Forced selection under operational irreversibility induces structural preference.

Refutation criterion (F2). If there exists an executable mechanism such that, after irreversible degradation, execution can continue without committing to any particular continuation (e.g., by maintaining all alternatives without informational or operational loss, or by a provably commitment-free continuation semantics), then Theorem 3 (Emergence of Preference) is not structurally necessary.

12.3 Failure Condition F3: Bypass after Irreversible Degradation

Claim under test. Non-bypassability holds when irreversible degradation eliminates distinctions required to separate alternative continuations.

Refutation criterion (F3). If, after an irreversible and non-recoverable degradation event, there exists an admissible continuation that avoids the downstream consequences of the induced constraint (i.e., a bypass path exists in the executable continuation space), then Definition 16 (Non-Bypassability) does not hold for the proposed condition set.

12.4 Failure Condition F4: Responsibility without Structural Commitment

Claim under test. Structural responsibility emerges whenever forced selection occurs under non-bypassable irreversible degradation.

Refutation criterion (F4). If all of the following hold simultaneously:

- degradation is operationally irreversible;
- multiple antecedent distinctions are collapsed;
- execution must proceed forward;
- no bypassable alternative exists;

yet the system admits an executable continuation that requires no commitment to consequences (i.e., there exists an admissible continuation semantics in which future execution is not bound by the irreversible collapse), then Theorem 4 (Emergence of Responsibility) is false under the stated definitions.

12.5 Interpretation

The above conditions define a concrete refutation interface: any executable system meeting one of F1–F4 constitutes a direct counterexample to the corresponding claim. Therefore, the framework does not rely on unfalsifiable interpretive assertions; it can be invalidated by explicit structural constructions under stated constraints.

13 Discussion

The framework developed in this work operates at a structural level, subject to the explicit refutation interface defined in Section 12, and is orthogonal to implementation details. It does not prescribe objectives, utilities, or values. Instead, it characterizes the conditions under which objecthood, preference, and responsibility necessarily arise. No reinterpretation or semantic reformulation avoids these conditions.

By grounding these notions in scale-relative equivalence and irreversible execution, the framework avoids importing semantic or normative assumptions at the foundational level.

This separation clarifies which aspects of decision-making arise from structure and which arise from interpretation layered on top of that structure.

13.1 Why Scale Matters

Scale is often treated implicitly in formal models. Here it is made explicit and operational.

Changing scale alters admissible distinctions, equivalence classes, and therefore the structure of objects and continuations. Many apparent disagreements between models can be traced to unacknowledged differences in scale.

13.2 Irreversibility Without Metaphysics

Irreversibility is derived without appeal to entropy, time asymmetry, or physical laws. It arises whenever degradation interacts with executable constraint.

This structural notion applies uniformly across physical, computational, and organizational systems.

14 Relation to Control Theory

In control theory, abstraction and constraint enforcement are often treated as design choices. Barrier functions, projection operators, and admissible sets restrict system behavior to ensure safety or feasibility.

Within the present framework, such mechanisms can be interpreted as explicit scale transitions. They restrict admissible distinctions and induce equivalence classes over system states.

Irreversibility arises when constraint enforcement eliminates information that cannot be recovered during execution. Under such conditions, non-bypassability is not a design preference but a structural consequence.

This perspective clarifies why certain constraints, once activated, cannot be bypassed without violating feasibility.

15 Relation to Learning and Abstraction

Learning systems routinely operate across scales. Feature extraction, representation learning, and policy abstraction all involve systematic degradation of fine-grained distinctions.

From the present perspective, learning-induced abstraction is a form of scale transition. When training or deployment constraints prevent access to discarded distinctions, learning dynamics become operationally irreversible.

Preference in learning systems—for example, in policy commitment or model selection—can therefore be interpreted as forced selection under irreversible abstraction rather than as intrinsic optimization preference.

This interpretation aligns learning dynamics with structural constraints rather than with normative objective functions.

16 Socio-Technical Responsibility

Responsibility attribution in socio-technical systems is often framed as a moral, legal, or organizational problem. The present framework isolates a structural substrate beneath such debates.

Whenever decisions are executed under irreversible degradation and non-bypassable constraints, responsibility is structurally imposed, regardless of later intentions or interpretations.

This observation does not resolve normative disputes. However, it clarifies which aspects of responsibility arise from execution structure and which arise from interpretive overlay.

Such separation is essential for analyzing responsibility in automated and human-in-the-loop systems.

17 Limitations

The framework deliberately abstracts away semantic content and normative criteria. As a result, it does not by itself prescribe optimal actions or moral judgments.

Identification of scale and executable constraints is context dependent. Different modeling choices may induce different equivalence structures and irreversibility profiles.

These limitations are not deficiencies but boundaries. They delineate the scope within which the framework provides structural clarity.

18 Conclusion

This work has presented a structural paradigm grounded in primitive difference, scale-relative equivalence, and irreversible execution.

Objecthood emerges as equivalence under scale. Irreversibility arises from degradation under executable constraints. Preference and responsibility are forced by the necessity of continuation under irreversible collapse of distinctions.

By deriving these notions structurally rather than assuming them axiomatically, the framework provides a unified foundation for abstraction, control, learning, and responsibility without appealing to teleology or normativity.