Abstract

Image segmentation has many applications in the field of computer vision, as well as in areas such as medical imaging, autonomous vehicles and satellite imagery. This paper introduces a program that uses Principal Component Analysis to optimize the Gaussian Mixture Models and Expectation-Maximization Algorithm to achieve image segmentation. This algorithm greatly reduces the time for segmentation, making it ~25x faster than only using EM iterations and GMM, and has high accuracy.

1 Problem/Mathematical Setup

PCA: Principal component analysis is a technique for exploring high-dimensional data structures. It is mainly used for dimensionality reduction of data. It can find features that are easier for people to understand and speed up the processing of valuable information of samples. In addition, it can also be applied to visualization (down to two-dimensional) and denoising.

Covariance and divergence matrices:

Sample mean: $\underline{x} = \frac{1}{n}\sum_{i=1}^{N} x(i)$

Sample covalent: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x(i) - \underline{x})^2$

Covariance of sample X and sample Y:

$$Cov(X,Y) = \frac{1}{n-1}\sum_{i=1}^{n} (x(i) - \underline{x})(y(i) - \underline{y})$$

Principle of eigenvalue decomposition matrices:

$Av = \lambda v$ (λ is the eigenvalue corresponding to the eigenvector v, and a set of eigenvectors of a matrix is a group of orthogonal vectors.)

$A = Q\Sigma Q^{-1}$ (Where Q is the matrix of the eigenvectors of matrix A, and Σ is a diagonal matrix, the elements on the diagonal are the eigenvalues).

K-Means Clustering: This is a simple clustering algorithm that only computes local optima.

Select initialized k samples as initial cluster centers:

$$a = a1, a2, a3, \ldots ak$$

Then,computing cluster center:

$$a(j) = \frac{1}{|c(i)|}\sum_{x \in c(i)} x$$

2 Numerical Methods and Algorithms

The optimal parameters of the Gaussian Mixture Models can be estimated by iteration with Expectation-Maximization algorithm, and the discrete data can be clustered. But in the case of a large data sample, EM iterations can take a very long time. In image processing, the pixel colors of all images are composed of three dimensions of RGB values. If we want to cluster pixels, we need a three-dimensional GMM, which will undoubtedly cost more computing time. So we decided to use the PCA algorithm shown above to reduce the 3D pixel features to 2D. Then the two-dimensional data are clustered. The PCA algorithm has many advantages, for example, for all points in the sample set, the reconstructed point's distance from the original point of the sum of errors is the smallest. The projection of the sample in the low-dimensional space is as far as possible, so not only can it remove the noise of the data, but also improve the data clustering.

Even if the pixel features are reduced, traversing a 512 × 512 image still requires very large time and space complexity. We used a similar structure to pyramid based sliding windows, inspired by the original. First, we reduce the image scale. The reduced image and the original image have the same color distribution, so we can determine k cluster centers by the EM algorithm iterations. After confirming the cluster center, we use k-mean clustering instead of EM iteration to traverse each pixel in the original image, find the closest cluster center from the pixel features, and then label the pixel. We repeat the determination of the cluster centers by finding the k-means value of the pixels with the same label, and repeat until the cluster centers do not change. Since the color distribution of an image is not much different from the original image after the image is scaled down, the k-mean clustering algorithm needs only one iteration. This calculation speeds up the processing of images by 25 times on average.

3 Experiments and Results

Figure: *K=8, segmented labeled (left) and averaged color images (right)*

We need to scale down the image to speed up the iteration time as with larger images there is much more information to go over. We experiment with the factor by which we scale down to find the fastest iteration time, found to be 7. With this optimization, the speed is much faster, e.g. labeling the test image takes 4.07s when K=3. Next, we determine the maximum number of iterations for the GM process, observing it never crosses 300 iterations, hence setting the maximum iterations parameter to 300.

Our algorithm performs well under testing upon various images. By observation, we see the results are accurate in terms of the mean colors and the segmentations are accurate as well, with similar colors averaged to make the same segments, giving smooth output images. The speed is dependent on image size, so if image size is 512 x 512 the average operating time is 6.17 seconds, for example.

Bibliography

[1] Mudrov, M. PRINCIPAL COMPONENT ANALYSIS IN IMAGE PROCESSING PCA Use for Image Compression. *no*, *3*, 2-5.

[2] Likas, A., Vlassis, N., & Verbeek, J. J. (2003). The global k-means clustering algorithm. *Pattern recognition*, *36*(2), 451-461.