LiMo WiSe 16/17 Sheet 7: Ex 3

Task:

Let $x = (x_1, \ldots, x_n)$T be a random vector with $xi \overset{iid}{\sim} Po(\lambda)$ for $i = 1, \ldots, n$. The parameter λ should be estimated. Look at the estimator

$$T = T(X) = \sum_{i=1}^{n} x_i.$$

Is T sufficient for λ? Justify your answer.

Solution:

$$\begin{aligned}
T(x) &= \sum_{i=1}^{n} (x_i - \mu)^2 \\
f(x|\lambda) &= \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!} \exp(-\lambda) \\
&= \frac{\lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!} \exp(-\lambda n) \\
&= \underbrace{\lambda^{\sum_{i=1}^{n} x_i} \exp(-\lambda n)}_{g(T(x),\lambda)} \underbrace{\frac{1}{\prod_{i=1}^{n} x_i!}}_{h(\lambda)}
\end{aligned}$$

According to the factorization theorem (of Fisher-Neymann) $T(x)$ is sufficient in λ.