4-1

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23 Dezember 2016

$$s^2 = \underbrace{(y - X\beta)'(y - X\hat{\beta})}_{SSE}/(n-K-1)$$

$$\begin{aligned}
SEE &= y'(I - X(X'X)^{-1}X')y \\
E(SEE) &= (tr(I - X(X'X)^{-1}X')\sigma^2 V) + E(y')(I - X(X'X)^{-1}X')E(y) \\
&= \sigma^2 tr(V - X(X'X)^{-1}X'V) + \underbrace{\beta' X'(I - X(X'X)^{-1}X')X\beta}_{=0} \\
&= \sigma^2(tr(V) - tr(X(X'X)^{-1}X'V)) \\
&= \sigma^2(tr((1-\rho)I_n + \rho 1_n 1_n') - tr(X(X'X)^{-1}X'((1-\rho)I_n + \rho 1_n 1_n'))) \\
&= \sigma^2((1-\rho)n + \rho n - (1-\rho)(K+1) + \underbrace{\rho tr(X(X'X)^{-1}X' - 1_n 1_n')}_{non-singular}) \\
&= \sigma^2((1-\rho)n + \rho n - (1-\rho)(K+1) + \rho tr(X(X'X)^{-1}X'X(X'X)^{-1} - I)) \\
&= \sigma^2((1-\rho)n + \rho n - (1-\rho)(K+1) + \rho tr(HH^{-1}I)) \\
&= \sigma^2((1-\rho)n + \rho n - (1-\rho)(K+1) + \rho tr(I)) \\
&= \sigma^2((1-\rho)n + \rho n - (1-\rho)(K+1) + \rho n
\end{aligned}$$

$$\begin{aligned}
E(s^2) &= \frac{\sigma^2(n-K-1)(1-\rho)}{n-K-1} \\
&= \sigma^2(1-\rho)
\end{aligned}$$

$$Cov(y) = \sigma^2 V; V = (1-\rho)I_n + \rho 1_n 1_n'$$

The covariance structure implies the same positive correlation ρ between any two observations and the same variance for all observations.