Ex 2

Auf Buch S169:

$$\begin{aligned} L(\beta,\sigma^2;x) &= \frac{1}{(2\pi)^{n/2}|\sigma^2 V|^{\frac{1}{2}}} \exp\left(-\frac{(y-X\beta)'(\sigma^2 V)^{-1}(y-X\beta)}{2}\right) \\ &= \frac{1}{(2\pi\sigma^2)^{n/2}|V|^{\frac{1}{2}}} \exp\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \end{aligned}$$

Herleitung für β:

$$\begin{aligned} L(\beta;x) &= \frac{1}{(2\pi\sigma^2)^{n/2}|V|^{\frac{1}{2}}} \exp\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \\ &\propto \exp\left(-\frac{1}{2\sigma^2}(y-X\beta)'V^{-1}(y-X\beta)\right) \\ l(\beta,x) &= -\frac{1}{2\sigma^2}(y-X\beta)'V^{-1}(y-X\beta) \\ &= -\frac{1}{2\sigma^2}(y'-\beta'X')V^{-1}(y-X\beta) \\ &= -\frac{1}{2\sigma^2}(y'V^{-1}-\beta'X'V^{-1})(y-X\beta) \\ &= -\frac{1}{2\sigma^2}(yy'V^{-1}-\beta'X'V^{-1}y-\beta'X'V^{-1}y+\beta'X'V^{-1}X\beta) \overset{!}{=} 0 \\ \frac{\partial(\beta;x)}{\partial\beta} &= -\frac{1}{2\sigma^2}(-2X'V^{-1}y+\beta'X'V^{-1}X\beta) \\ &\propto -X'V^{-1}y+\beta'X'V^{-1}X\beta \\ \hat{\beta} &= (X'V^{-1}X)^{-1}X'V^{-1}y \end{aligned}$$

Herleitung für σ^2:

$$\begin{aligned} L(\beta,\sigma^2;x) &= \frac{1}{(2\pi)^{n/2}|\sigma^2 V|^{\frac{1}{2}}} \exp\left(-\frac{(y-X\beta)'(\sigma^2 V)^{-1}(y-X\beta)}{2}\right) \\ &= \frac{1}{(2\pi\sigma^2)^{n/2}|V|^{\frac{1}{2}}} \exp\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \\ &= (2\pi)^{-n/2}(\sigma^2)^{-n/2}|V|^{-\frac{1}{2}} \exp\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \\ &\propto (\sigma^2)^{-n/2} \exp\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \\ l(\sigma^2;x) &= \frac{-n}{2}\log(\sigma^2)\left(-\frac{(y-X\beta)'V^{-1}(y-X\beta)}{2\sigma^2}\right) \\ \frac{\partial(\sigma^2;x)}{\partial\sigma^2} &= \frac{-n}{2\sigma^2}\left(-\frac{2(y-X\beta)'V^{-1}(y-X\beta)}{(2\sigma^2)^2}\right) \overset{!}{=} 0 \\ n &= \frac{(y-X\beta)'V^{-1}(y-X\beta)}{\sigma^2} \\ \hat{\sigma}^2 &= \frac{1}{n}(y-X\beta)'V^{-1}(y-X\beta) \end{aligned}$$