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Q1 $P(\{1,a\}) = \{\phi, \{1\}, \{a\}, \{1,a\}\}$

Q2 let $s = \{1,4,9,16,25\}$ $\quad s = \{x \mid x = n^2 (1 \le n \le 5)\}$

Q3. To prove $x = y$, by showing if $a \in x$, then $a \in y$, if $a \notin x$, $a \notin y$

① let $a = (x,y)$ suppose $a \in A \times (B-C)$

therefore $(x,y) \in A \times (B-C)$

therefore $x \in A \wedge y \in (B-C)$

therefore $x \in A \wedge (y \in B \wedge \bar{C})$

$\rightarrow$ $x \in A \wedge (y \in B \wedge y \in \bar{C})$

$\rightarrow$ $(x \in A \wedge y \in B) \wedge (x \in A \wedge y \in \bar{C})$

$\rightarrow$ $(x \in A \wedge y \in B) \wedge (x \in \bar{A} \vee (x \in A \wedge y \in \bar{C}))$

$(x,y) \in (A \times B) \wedge (T \wedge x \in \bar{A} \wedge y \in \bar{C})$

$(x,y) \in (A \times B) \wedge (x \notin A \wedge y \notin C)$

$(x,y) \in (A \times B) \wedge (x,y) \notin A \times C$

$(x,y) \in (A \times B) \wedge (x,y) \in \overline{A \times C}$

$(x,y) \in (A \times B) - (A \times C)$

② let $a = (x,y)$, suppose $a \notin A \times (B-C)$

$(x,y) \notin A \times (B-C)$

$x \notin A \vee y \notin (B-C)$

$x \notin A \vee (y \notin B \wedge y \notin \bar{C})$

$(x \notin A \vee y \notin B) \wedge ((x \notin A) \vee (y \notin \bar{C}))$

$(x,y) \notin A \times B \wedge (x,y) \in A \times \bar{C}$

$(x,y) \notin A \times B \wedge (x,y) \notin \overline{A \times C}$

$(x,y) \notin A \times B - A \times C$.

Q4 $f(x) = 8$

Q5 To prove f is not injective ~~⟺~~

$\Leftrightarrow \exists a \subseteq P(U) \exists b \subseteq P(U) (a \neq b \wedge f(a) = f(b))$

let $a = \{1, 2, 7\}$ $b = \{2, 3, 7\}$

$f(a) = f(b) \{7\}$, $a \neq b$ $\therefore f$ is not injective.

Q6 To prove f is not surjective.

$\Leftrightarrow \exists b \in P(U) \forall a \in P(U) (f(a) \neq b)$

let ~~b=~~ b equals to a set which has more than 1 element there are no a that $f(a) = b$, because the result of $f(x)$ is a set that only contain one element, therefore we can find a set X ~~that~~ $\forall b \in P(U) \exists X \in P(U) (f(X) = b)$

$\therefore f$ is not surjective.

Q7 To prove $f(x) = \{$~~max~~ ~~max(a1~~ max(a1, $a_2, \cdots a_m)\}$ for all $X = \{a_1, \cdots a_m\}$

let $m = \max\{a_1, a_2 \cdots a_m\}$ $n \in \{a_1, a_2 \cdots a_m\}$ and $n \neq m$. $X \subseteq U$

By way of contradiction, suppose $f(x) = \{n\}$

$\because f(x) = \{x \in X : \forall y \in X, x \geq y\}$

$\therefore \forall a \in X$, ~~$a \geq n$~~ $n \geq a$.

$\because m \neq n$, $n < m$ $\therefore m \geq a$ contradicts to $n < m$

$\therefore f(x) = \{\max(a_1, a_2 \cdots a_m)\}$ for all $X = \{a_1, a_2 \cdots a_m\} \subseteq U$

Q8

For R_1

since $|x-y| \leq 2 \Leftrightarrow |y-x| \leq 2$ $\therefore (y,x) \in R$, R_1 is symmetric

since $(1,3),(3,1) \in R$, $1 \neq 3$ R_1 is not antisymmetric

since $(1,2),(2,4) \in R$, $(1,4) \notin R$, R_1 is not transitive

since $|a-a| = 0 \leq 2$, $\forall a \in A$ $(a,a) \in R$. R_1 is reflexive.

For R_2

since $x < 2x$ $(x \in A)$ R_2 is reflexive.

since $(1,2) \in R$, $(2,1) \notin R$, R_2 is not symmetric

since $(3,4),(4,3) \in R$. $4 \neq 3$, R_2 is not antisymmetric.

~~for all~~ R_2 is ~~transitive~~ transitive. by checking every pair

$R_2 = \{(1,1),(1,2),(1,3),(1,4),(2,4),(2,3),(2,2),(3,3),(3,2)$

$(3,4),(4,3),(4,4)\}$

Q9 $U = B \circ P$, $G = P \circ P$

Q10 $G = P \circ P$

= {(Harry, John), (Alice, Bob), (Alice, Greg), ~~(Harry, Mary)~~

(Harry, Mary)}