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Answer for question 1.

$$\begin{aligned}(p \wedge q) \rightarrow \neg(q \wedge r) &= \neg(p \wedge q) \vee \neg(q \wedge r) \\ &= (p \vee \neg q) \vee (\neg q \vee r) \\ &= p \vee \neg q \vee r\end{aligned}$$

$$q \rightarrow (p \vee r) = \neg q \vee (p \vee r) = p \vee \neg q \vee r$$

\therefore logically equal

Answer for question 2.

p	q	r	$\neg p$	$\neg p \wedge q$	$p \oplus q$	$p \oplus q \rightarrow r$	$(p \wedge q) \vee (p \oplus q \rightarrow r)$	$r \rightarrow p$
T	T	T	F	F	F	F	F	T
T	T	F	F	F	F	T	T	F
T	F	T	F	F	T	T	T	T
T	F	F	F	F	T	F	F	F
F	T	T	T	T	T	T	T	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	F	T	T	T
F	F	T	T	F	F	F	F	T

Q3: Contingent

Q4

$P(x)$ = "x likes sweetcorn" $Q(x)$ = "x likes peas"

$M(x)$ = "x likes chips" the domain of x is the group of friends

$$a) \forall x (P(x) \rightarrow Q(x)) \quad b) \neg \forall x (Q(x) \rightarrow M(x))$$

$$c) \neg \forall x (M(x) \wedge P(x)) = \exists x (\neg M(x) \vee \neg P(x))$$

Q5: according to b, $\neg \forall x (P(x) \rightarrow M(x))$

$$\neg (\forall x (P(x) \rightarrow M(x))) \quad \text{Universal Instantiation}$$

$$\neg M(c) \wedge P(c)$$

$$\exists x (\neg M(x) \wedge P(x))$$

which is contradictory to $\exists x (M(x) \wedge \neg Q(x)) \therefore$ impossible

$$Q6: \forall x (P(x) \rightarrow Q(x)) \quad c \text{ is in the domain}$$

$$P(c) \rightarrow Q(c)$$

$$\neg \forall x (Q(x) \rightarrow M(x))$$

$$\neg (Q(c) \rightarrow M(c))$$

$$Q(c) \wedge \neg M(c)$$

$$Q(c)$$

$$P(c)$$

$$\neg M(c)$$

$$\neg M(c) \vee \neg P(c)$$

$$\therefore \exists x (\neg M(x) \vee \neg P(x))$$