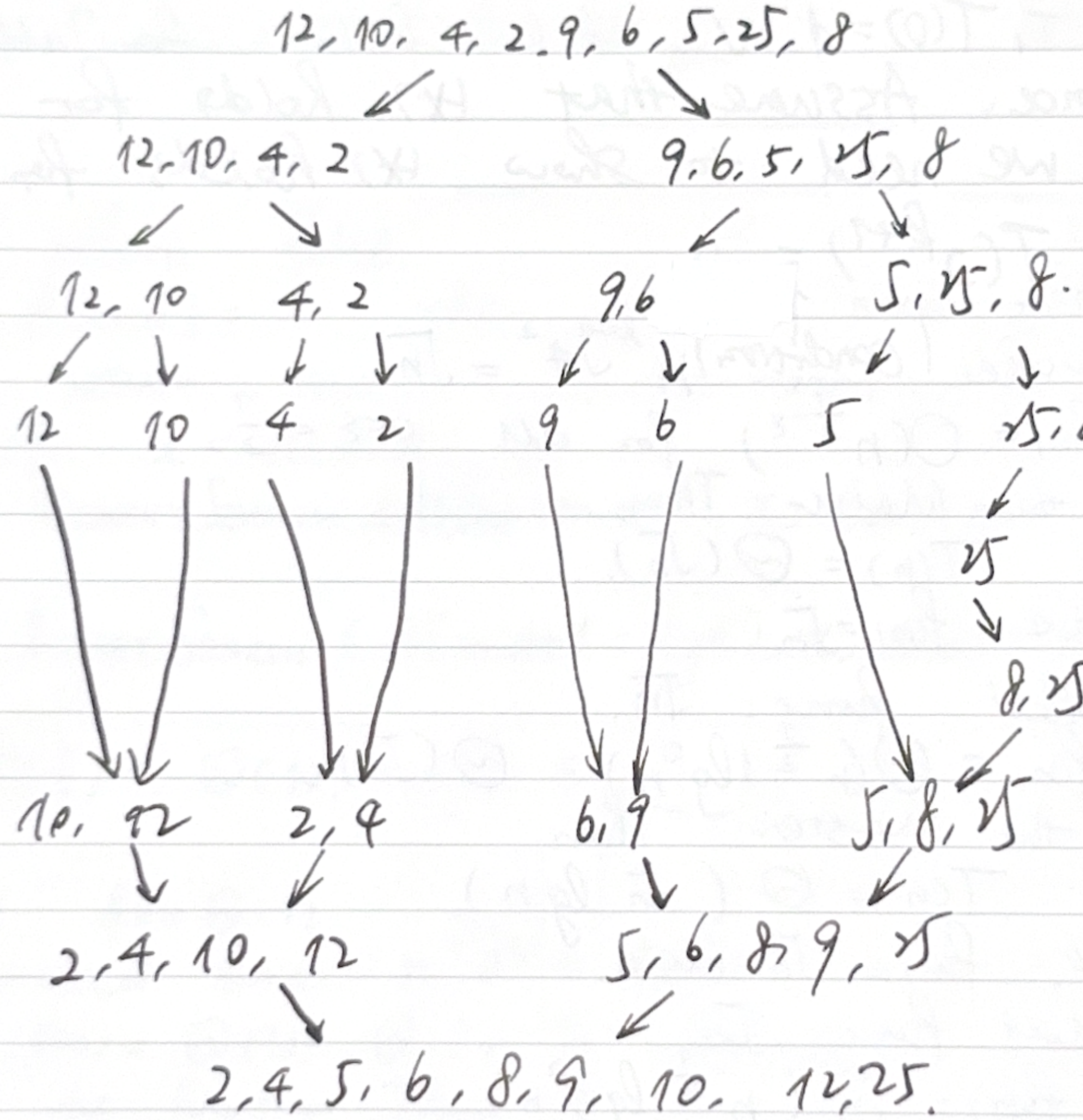
Data Structure and Algorithm Analysis (H).
Lab 03.
12311410 [illegible]

Q3.1

Sol:



Q.3.2

Proof: let n be 2^k for some $k \in \mathbb{N}^*$.

We want to show that

$$T(2^k) = d \cdot 2^k + c \cdot 2^k \cdot k \qquad \text{——— } (*)$$

Induction on k:

① Base case. $k = 0$.

$$T(1) = d \cdot 2^0 + c \cdot 2^0 \cdot 0 \quad \checkmark.$$

② Assume $(*)$ holds for k.
we prove for the case $k+1$. (which must be ≥ 1).

$$T(2^{k+1}) \underset{\uparrow}{=} 2T(2^k) + c \cdot 2^{k+1} \underset{\uparrow}{=} d \cdot 2^{k+1} + c \cdot 2^{k+1} \cdot k + c \cdot 2^{k+1}$$

recurrence relation. induction hypothesis.

$= d \cdot 2^{k+1} + c \cdot 2^{k+1} \cdot (k+1)$ ✓.

Thus, $T(2^k) = d \cdot 2^k + c 2^k \cdot k$ holds for all $k \in \mathbb{N}$. □.

Q3.3.

1. $a=2$, $b=4$, $f(n)=1$.
   watershed func: $n^{\log_4 2} = \sqrt{n}$.
   Since $f(n) = O(n^{\frac{1}{2}-\varepsilon})$ for all $0 < \varepsilon < \frac{1}{2}$.
   by the Master Thm,
   $T(n) = \Theta(\sqrt{n})$.
2. $a=2$, $b=4$, $f(n) = \sqrt{n}$.
   watershed func: $\sqrt{n}$.
   Since $f(n) = \Theta(n^{\frac{1}{2}} \log^0 n) = \Theta(\sqrt{n})$,
   by the Master Thm.
   $T(n) = \Theta(\sqrt{n} \log n)$.
3. $a=2$, $b=4$, $f(n) = \sqrt{n} \log^2 n$.
   watershed func: $\sqrt{n}$.
   Since $f(n) = \Theta(n^{\frac{1}{2}} \log^2 n)$
   by the Master's Thm.
   $T(n) = \Theta(\sqrt{n} \log^3 n)$.
4. $a=2$, $b=4$. $f(n) = n$.
   watershed func: $\sqrt{n}$.
   Since $f(n) = \Omega \log(n^{\frac{1}{2}+\varepsilon})$ for all $0 < \varepsilon < \frac{1}{2}$,
   and $f(n)$ additionally satisfies
   $2 f(n/4) = 2 \cdot \frac{n}{4} = \frac{n}{2} \leq c f(n)$, for all $\frac{1}{2} \leq c < 1$.
   $\forall n$.
   by the Master Thm,
   $T(n) = \Theta(f(n))$
   $= \Theta(n)$.

NO.

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Q.3.4.

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BINARY SEARCH (A, x, low, high).
1. mid = ⌊(low + high)/2⌋.
2. if    high < low   then
3.          return false.
4. if    A[mid] = x   then
5.          return true.
6. else if    x < A[mid]
7.          BINARY SEARCH (A[low : mid-1], x, low, mid-1 ).
8. else
9.          BINARY SEARCH ( A[mid+1 : high], x, mid+1, high ).
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$T(n) = T(\frac{n}{2}) + \Theta(1)$.

$a=1,\ b=2$ $\quad f(n) = \Theta(1)$

watershed func.: $n^{\log_2 1} = n^0 = 1$.

Since $f(n) = \Theta(1) = \Theta(n^{\log_2 1} \cdot \log^0 n)$

by the Master Thm,

$T(n) = \Theta(n^0 \cdot \log^1 n) = \Theta(\log n)$.