This code is quite standard and follows our other code in our BVAR webpage. The only nonstandard thing is the way the different heterogeneity restrictions are indexed in the panel VAR. In order to explain what is the idea behind this code, let's examine again the restrictions in a panel VAR with $N = 3$, $G = 2$ and $p = 1$. Assume for simplicity that the $G = 2$ macro variables are GDP and CPI, i.e. $y_t^1 = \left[gdp_t^{1\prime}, cpi_t^{1\prime}\right]'$, $y_t^2 = \left[gdp_t^{2\prime}, cpi_t^{2\prime}\right]'$ and $y_t^3 = \left[gdp_t^{3\prime}, cpi_t^{3\prime}\right]'$ for countries 1, 2 and 3, respectively.This VAR can be written in the following unrestricted form:

$$y_t = Ay_{t-1} + \varepsilon_t, \tag{1}$$

where A_1 is a $NG \times NG$ matrix. We can "unfold" the country dimension and write it as

$$\left[\begin{array}{c} y_t^1 \\ y_t^2 \\ y_t^3 \end{array}\right] = \left[\begin{array}{ccc} A^{11} & A^{12} & A^{13} \\ A^{21} & A^{22} & A^{23} \\ A^{31} & A^{32} & A^{33} \end{array}\right] \left[\begin{array}{c} y_{t-1}^1 \\ y_{t-1}^2 \\ y_{t-1}^3 \end{array}\right] + \varepsilon_t, \tag{2}$$

where A_1^{ij} are $G \times G$ (i.e. 2×2) matrices. We can also unfold the macro variable dimension and write the VAR in analytical form

$$\left[\begin{array}{c} gdp_t^1 \\ cpi_t^1 \\ gdp_t^2 \\ cpi_t^2 \\ gdp_t^3 \\ cpi_t^3 \end{array}\right] = \left[\begin{array}{cccccc} \alpha_{11}^{11} & \alpha_{12}^{11} & \alpha_{11}^{12} & \alpha_{12}^{12} & \alpha_{11}^{13} & \alpha_{12}^{13} \\ \alpha_{21}^{11} & \alpha_{22}^{11} & \alpha_{21}^{12} & \alpha_{22}^{12} & \alpha_{21}^{13} & \alpha_{22}^{13} \\ \alpha_{11}^{21} & \alpha_{12}^{21} & \alpha_{11}^{22} & \alpha_{12}^{22} & \alpha_{11}^{23} & \alpha_{12}^{23} \\ \alpha_{21}^{21} & \alpha_{22}^{21} & \alpha_{21}^{22} & \alpha_{22}^{22} & \alpha_{21}^{23} & \alpha_{22}^{23} \\ \alpha_{11}^{31} & \alpha_{12}^{31} & \alpha_{11}^{32} & \alpha_{12}^{32} & \alpha_{11}^{33} & \alpha_{12}^{33} \\ \alpha_{21}^{31} & \alpha_{22}^{31} & \alpha_{21}^{32} & \alpha_{22}^{32} & \alpha_{21}^{33} & \alpha_{22}^{33} \end{array}\right] \left[\begin{array}{c} gdp_{t-1}^1 \\ cpi_{t-1}^1 \\ gdp_{t-1}^2 \\ cpi_{t-1}^2 \\ gdp_{t-1}^3 \\ cpi_{t-1}^3 \end{array}\right] \varepsilon_t, \tag{3}$$

where now α_{kl}^{ij} **is a scalar which denotes the coefficient of variable** l **of country** j**, which appears on the equation of variable** k **of country** i**.** When estimating the VAR, we need convert the 6×6 coefficient matrix A into the following vector (see Technical Appendix of our paper)

$$\alpha = \left[\begin{array}{cccccccccccccccc} \alpha_{11}^{11} & \alpha_{12}^{11} & \alpha_{11}^{12} & \alpha_{12}^{12} & \alpha_{11}^{13} & \alpha_{12}^{13} & \alpha_{21}^{11} & \alpha_{22}^{11} & ... & \alpha_{12}^{33} & \alpha_{21}^{31} & \alpha_{22}^{31} & \alpha_{21}^{32} & \alpha_{22}^{32} & \alpha_{21}^{33} & \alpha_{22}^{33} \end{array}\right]',$$

i.e. we stack each row of A as it appears in equation (3).

Dynamic interdependencies are tested by checking whether $A^{12}, A^{13}, A^{21}, A^{23}, A^{31}$ or A^{32} are zero.

Cross sectional heterogeneities are present when $A^{11} = A^{22}$, $A^{11} = A^{33}$ or $A^{22} = A^{33}$. In order to test if the whole matrices are equal, we test whether each element of the matrices is equal to the respective element of the other matrix. For example, we test whether

$$A^{11} = \left[\begin{array}{cc} \alpha_{11}^{11} & \alpha_{12}^{11} \\ \alpha_{21}^{11} & \alpha_{22}^{11} \end{array}\right] = \left[\begin{array}{cc} \alpha_{11}^{22} & \alpha_{12}^{22} \\ \alpha_{21}^{22} & \alpha_{22}^{22} \end{array}\right] = A^{22}, \tag{4}$$

by checking individually the restrictions $\alpha_{11}^{11} = \alpha_{11}^{22}, \alpha_{12}^{11} = \alpha_{12}^{22}, \alpha_{21}^{11} = \alpha_{21}^{22}$ and $\alpha_{22}^{11} = \alpha_{22}^{22}$. If, say, $\alpha_{22}^{11} = \alpha_{22}^{22}$ then countries 1 and 2 have the same AR(1) coefficient for CPI (in "economic terms" this means that in each country inflation persistence is similar which is not a completely unrealistic assumption to check; in statistical terms we just test a restriction that saves degrees of freedom in a heavily parametrized model). If $\alpha_{11}^{11} = \alpha_{11}^{22}$ then both countries have also the same AR coefficient for GDP (which can be the result of synchronization of business cycle dynamics).

After this "smooth" introduction, let's go to the code. The code from lines 66 to lines 77 creates a variable `index_restriction` which denotes the place of where the elements of the matrices A^{11}, A^{12}, A^{13}... are in the vector α. For example,

```
>> index_restriction(:,:,1)
ans =
```

```
1 2
7 8
```

gives the position that the elements $\begin{pmatrix} \alpha_{11}^{11} & \alpha_{12}^{11} & \alpha_{21}^{11} & \alpha_{22}^{11} \end{pmatrix}$ of the matrix A^{11} have in the VAR coefficient vector α. If we type `index_restriction(:,:,2)` we get the position of the elements of A^{12}, `index_restriction(:,:,3)` we get the position of the elements of A^{13}, and for `index_restriction(:,:,9)`, we get the position of the elements of A^{33}. The code finds immediately the position of each element of the $G \times G$ matrices A^{ij} for any definition of G and N.

Subsequently, **dynamic interdependencies** are easy to check since we have the position of each scalar coefficient α_{kl}^{ij} in the large $(NG)^2 \times 1$ vector a. We only need to pick the relevant G^2 elements of the matrix A^{ij} (these matrices are picked in line 86 using the variable `index_restr_DI`) and test whether all of them are jointly zero or not. This is done in the lines 162-181 of the code by evaluating the likelihood when $A^{ij} = 0$ as opposed to evaluating the likelihood for $A^{ij} \neq 0$.

For **cross-sectional heterogeneity** things are a little bit more complicated. We now have the index (position) of the matrices of interest A^{11}, A^{22} and A^{33} but also define all the pairs of scalar coefficients we want to check. This is done in lines 92-101 where I take all these pairs and I index them accordingly. In our example we have three such restrictions to check ($A^{11} = A^{22}$, $A^{11} = A^{33}$ or $A^{22} = A^{33}$) but if we also check individually restrictions on the G^2 elements of these matrices, then we have in total 12 restrictions. The variable `index_restr_CS` is a 3×1 MATLAB cell array with each cell being a 4×2 matrix of pairs of restrictions. For example, if we type

```
>> index_restr_CS{1,1}
ans =
 1   15
 7   21
 2   16
 8   22
```

then we get the positions in the vector α of the sub-restrictions $\alpha_{11}^{11} = \alpha_{11}^{22}, \alpha_{12}^{11} = \alpha_{12}^{22}, \alpha_{21}^{11} = \alpha_{21}^{22}$ and $\alpha_{22}^{11} = \alpha_{22}^{22}$. If we type `index_restr_CS{2,1}` we get the indexes of the restrictions $A^{11} = A^{33}$, and if we type `index_restr_CS{2,1}` we get indexes of the restrictions $A^{22} = A^{33}$ (which gives in total $3 \times 4 = 12$ restrictions). The code is able to also adapt automatically to the different positioning of elements α_{kl}^{ij} in the vector α implied by various choices of N and G. Lines 182-204 evaluate these 12 cross-sectional restrictions by evaluating the likelihood and drawing from a Bernoulli density.

The matrix `GAMMA` corresponds to the matrix Γ in the appendix of the paper. The matrices `gamma_DI` and `gamma_CS` are vectors with the Dynamic Interdependency and Cross-Sectional Heterogeneity restrictions, respectively. The respective posterior draws are stored in the vectors `gammaDI_draws` and `gammaCS_draws`. For example, if we get posterior mean:

```
>> mean(gammaDI_draws)
ans =
 1 0 1 0 0 0
```

this means that $A^{12} \neq 0$ and $A^{21} \neq 0$ (with probability 1), while $A^{13} = A^{23} = A^{31} = A^{32} = 0$. Similarly, if we type the posterior mean:

```
>> mean(gammaCS_draws)
ans =
 0 0.28 0.31 0 0 0.27 0.39 0.01 0 0.24 0.33 0
```

then $\alpha_{11}^{11} = \alpha_{11}^{22}, \alpha_{22}^{11} = \alpha_{22}^{22}$, $\alpha_{11}^{11} = \alpha_{11}^{33}$, $\alpha_{22}^{11} = \alpha_{22}^{33}$ and $\alpha_{11}^{22} = \alpha_{11}^{33}$, $\alpha_{22}^{22} = \alpha_{22}^{33}$ with certainty (posterior mean of restriction is 0), while we have a $100 - 28 = 72\%$ that $\alpha_{12}^{11} = \alpha_{12}^{22}$, or we have a $100 - 33 = 67\%$ probability that $\alpha_{12}^{22} = \alpha_{12}^{33}$.