Operation Research Applied to Work-Life Balance

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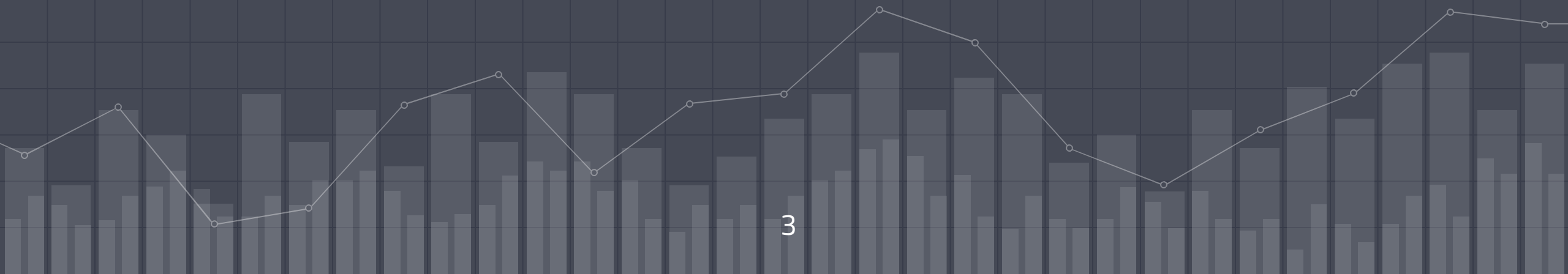
Agenda

1. Defining problems
2. Constraints and Utilities
3. Solving the first model
4. Improving Further
5. Refine the model



What's the problem with sleeping?

- Everybody gets 24 hours a day. We want to efficiently utilize our time to plan the best schedule.
- How to evaluate a schedule?

The Problem

Impactful Influence Close to Our Life

Physical strength = Battery (%)

Balance

Test Utility

+

long-term physical strength

We Assume...

1. Events are taken as given.
2. Exams of a day are taken right after you wake up.
3. Events are done instantaneously.
4. A day begins when you go to bed.

Variable Meanings

$$Days = \text{number of days}$$
$$S = \text{number of subjects}$$
$$Tired_i = \text{tiredness of subject } i \quad \forall i \in S$$
$$Credit_i = \text{credit of subject } i \quad \forall i \in S$$
$$\overline{L} = \text{upper bound of } L$$

$$C_{study_{it}} = \begin{cases} 1, & \text{if } t \text{ is before Test date of subject } i \\ 0, & \text{o/w} \end{cases} \qquad \forall i \in S, t \in Days$$

$$C_{test_{it}} = \begin{cases} 1, & \text{if } t \text{ is the Test date of subject } i \\ 0, & \text{o/w} \end{cases} \qquad \forall i \in S, t \in Days$$

Variable Meanings

C_α = sensitivity of study efficiency

C_{weight} = to balance the influence of TU and Long-term strength

C_{spirit} = lower bound of L to take test without penalty

C_r = slope of $f(X)$

L_t = one's power before sleeping at day t

TU = Test utility

$\mathcal{L}$ = Long-term strength

β_{sleep} = how important one person evaluate sleeping

β_{study} = how important one person evaluate studying

β_{demand} = how important one person evaluate taking course

x_t = the power waking up at day t $\quad \forall t \in Days$

w_{TU_i} = the penalty we get from the test $\quad \forall i \in S$

w_{L_t} = the difference bewtween L_t and $\overline{L}$ $\quad \forall t \in Days$

D_t = the demand at day t $\quad \forall t \in Days$

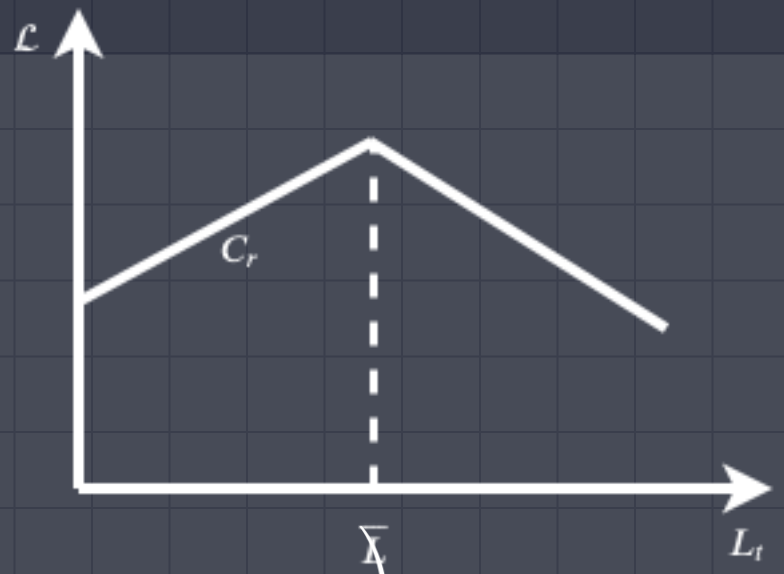
$Study_i$ = the total study time for subject i $\quad \forall i \in S$

$$subjcet_{it} = \begin{cases} 1 & \text{if subject } i \text{ is at day } t \\ 0 & \text{o/w} \end{cases} \quad \forall t \in Days \quad \forall i \in S$$

Objective Function

$$\max \quad \mathcal{L} + C_{weight} \cdot TU$$

$$\begin{aligned}\mathcal{L} &= \Sigma_{t=1}^{Days} f(L_t) \\ &= \Sigma_{t=1}^{Days} \overline{L} \cdot C_r - |L_t - \overline{L}| \cdot C_r \\ &= \Sigma_{t=1}^{Days} \overline{L} \cdot C_r - w_{Lt} \cdot C_r\end{aligned}$$



$$TU = \sum_{i=0}^{S} \left(\frac{C_\alpha \cdot Study_i}{Tired_i \cdot Credit_i} + min(x_i - C_{spirit}, 0) \cdot \sum_{t \in Days} C_{testit} \right)$$

$$= \sum_{i=0}^{S} \left(\frac{C_\alpha \cdot Study_i}{Tired_i \cdot Credit_i} + w_{TU_i} \cdot \sum_{t \in Days} C_{testit} \right)$$

Constraints

$$L_t = L_{t-1} + sleep_t\beta_{sleep} - D_t\beta_{demand} - \sum_{i \in S} study_{it}\beta_{study}, \qquad \forall t \in Days$$

$$L_t \geq 0 \qquad \forall t \in Days$$

$$L_0 = 0$$

$$\begin{cases} w_{L_t} \geq L_t - \overline{L} \\ w_{L_t} \geq \overline{L} - L_t \end{cases} \quad \forall t \in Days \qquad \begin{cases} w_{TU_i} \leq x - C_{spirit} \\ w_{TU_i} \leq 0 \end{cases} \quad \forall i \in S$$

$$D_t = \sum_{i \in S} Subject_{it} \cdot (0.5 + 0.5 \cdot \frac{Tired_i}{5}), \qquad \forall t \in Days$$

$$24 \geq sleep_t \geq 0, \qquad \forall t \in Days$$

$$24 \geq study_{it} \geq 0, \qquad \forall i \in S, t \in Days$$

$$24 = sleep_t + \sum_{i \in S} Subject_{it} + \sum_{i \in S} study_{it}, \qquad \forall t \in Days$$

$$sleep_t\beta_{sleep} \geq D_t\beta_{demand} \qquad \forall t \in Days$$

Constraints

$$x_t = L_{t-1} + sleep_t \cdot \beta_{sleep} \quad \forall t \in Days$$

$$Study_i = \sum_{t=1}^{Days} study_{it} \cdot C_{study_{it}} \quad \forall i \in S$$

Decision Variable

$$sleep_t = \text{the sleep time at day } t \quad \forall t \in Days$$

$$study_{it} = \text{the study time for subject } i \text{ at day } t \quad \forall t \in Days \quad \forall i \in S$$

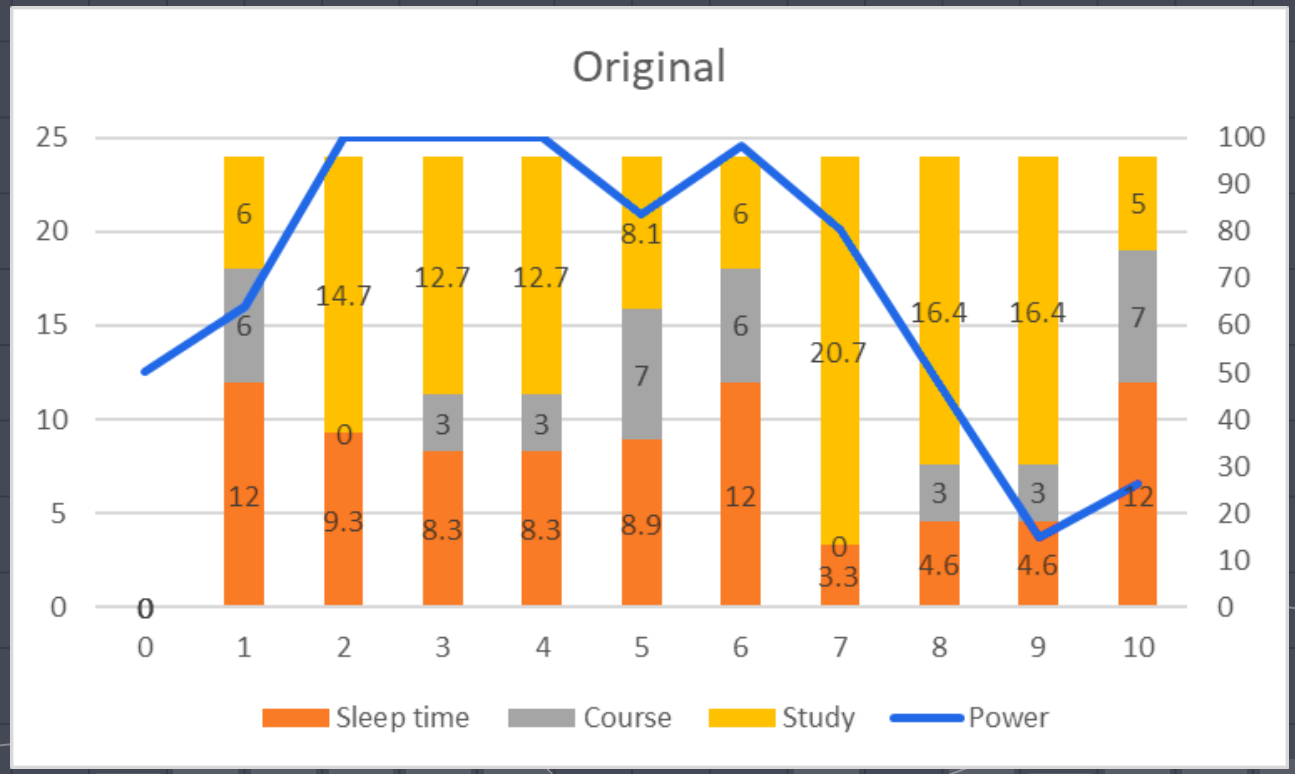
Time Table

Course Number
$(Credit, Tiredness, Hardness)$

	Monday	Tuesday	Wednesday	Thursday	Friday
08:10-09:00					Course D (1,2,0)
09:10-10:00	Course A (3,3,3)		Course B (3,4,3)		Course C (3,1,1)
10:20-11:10					
11:20-12:10					
12:20-13:10					
13:20-14:10					
14:20-15:10	Course E (3,3,3)			Course F (3,4,4)	
15:30-16:20					Course G (3,4,2)
16:30-17:20					
17:30-18:20					
18:30-19:20					
19:30-20:20					
20:30-21:20					

The Base Solution

- Study time increases...
- Test Utility is linear.
- LT is summed in every season
- Study (Test) v.s. Sleep (LT)



Refining the model

Improving Further

Study Related

- Switching Cost
- Setup Cost
- Marginal Utility

Activity Based

- Garbage Time
- Caffeine Boost

Diminishing Marginal Utility

The longer we study, the less we get…

Diminishing Marginal Utility

$$\sum_{i=0}^{S} \frac{C_\alpha \cdot Study_i}{Tired_i \cdot Credit_i} + w_{TU} \cdot \sum_{t \in Days} C_{testit}$$

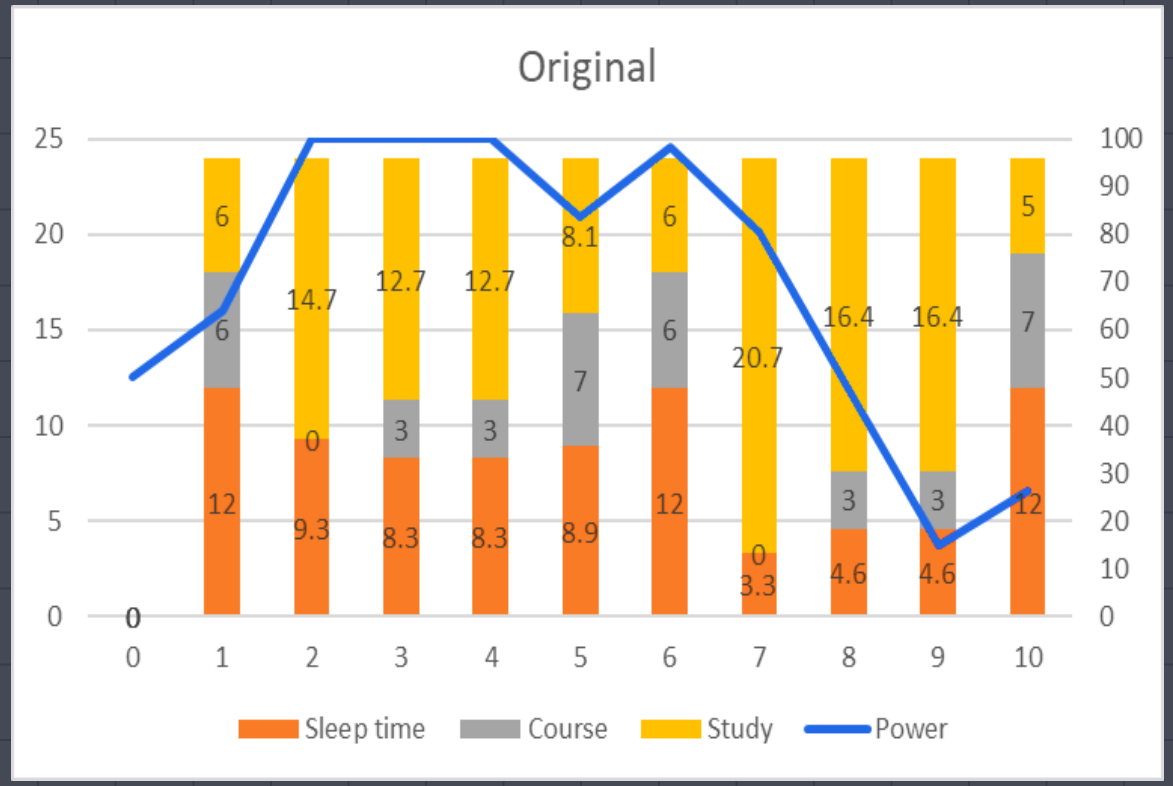
Now Becomes → $\sum_{i=0}^{S} \left(\left(\frac{C_\alpha \cdot g(Study_i)}{Tired_i \cdot Credit_i} \right) + w_{TU_i} \cdot \sum_{t \in Days} C_{testit} \right)$

Where $g(x) = C_k \cdot ln(x+1)$

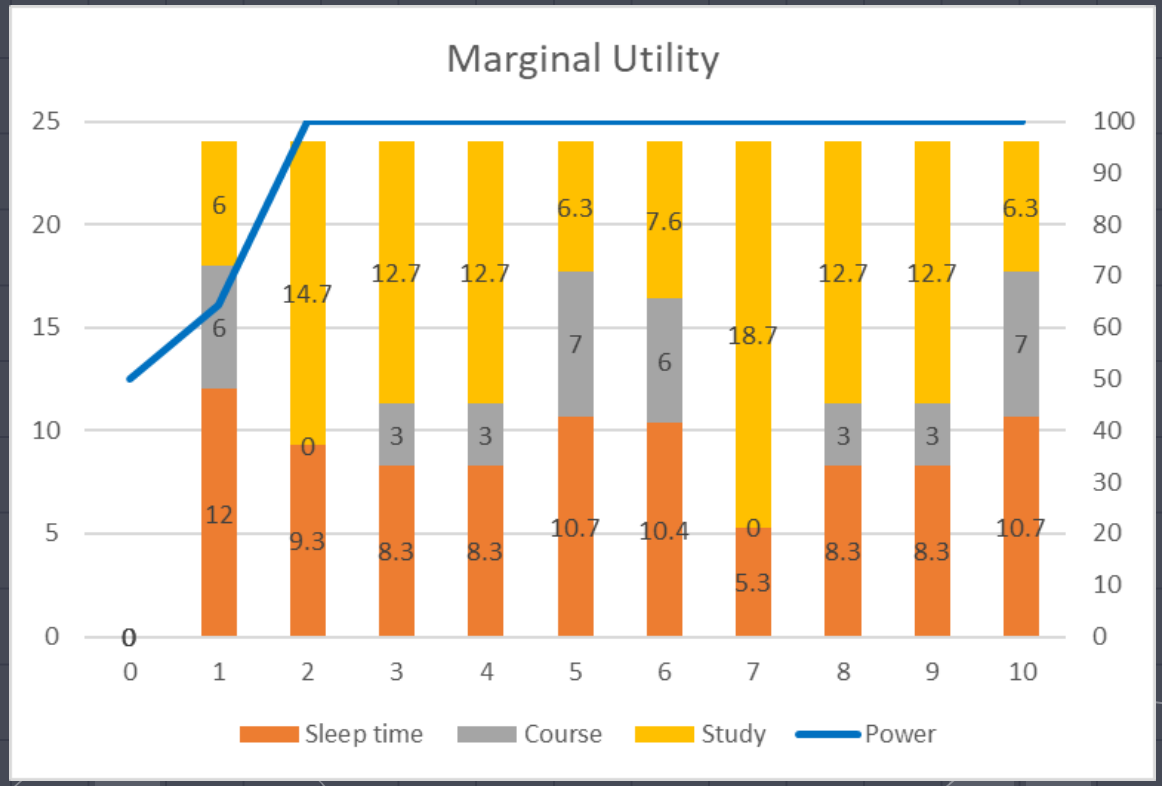
It is an increasing Convex Function:

$$\frac{\partial g}{\partial x} = \frac{C_k}{x+1} \quad x \geq 0$$

Results



Objective 895.0
L = 71.6
TU = 823.4



Objective 192.7
L= 96.4
TU = 96.2

Don’t study before test.

Sleep time is increased compared to base model.

Garbage time

It is impossible for a normal person to keep alive when his/her life consists of studying, sleeping, having classes and tests.

All work and no play makes Jack a dull boy

Garbage time

Objective function should include $+\sum_{t}^{Days} f(garbage_t)$ Where $f(x) = \left(-x^3 + 20x\right) 0.07$

Modified function $L_t = L_{t-1} + sleep_t\beta_{sleep} - D_t\beta_{demand} - \sum_{i \in S} study_{it}\beta_{study}$

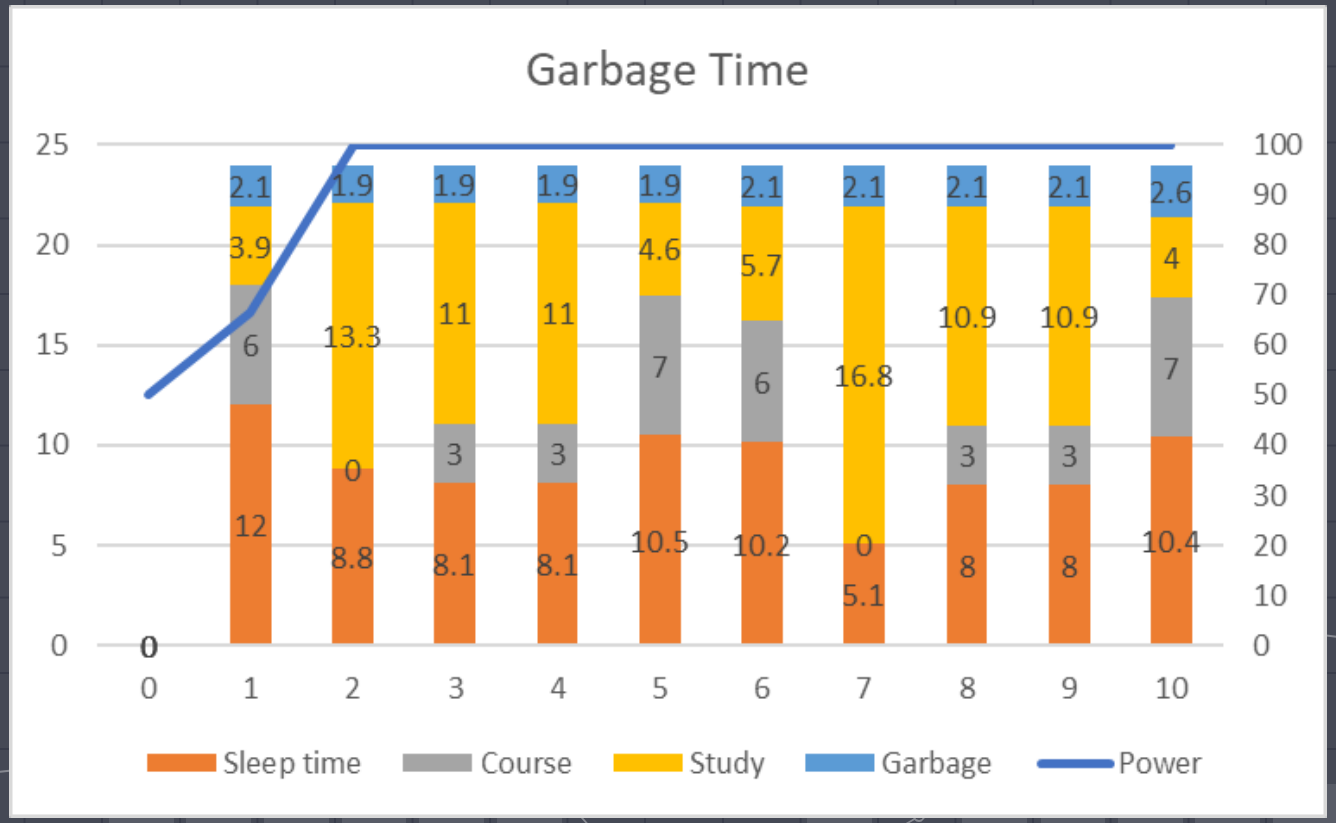
$$- \beta_{garbage} \cdot garbage_t$$

Results

- Garbage Time comes from study time.
- Test Utility drops, but increases Garbage Time.
- Objective Value is increased.
- Previous result:

  Obj Val: 192.7 L= 96.4 TU= 96.2
- New result:

  Obj Val: 199.4 L= 96.7 TU= 91.5 G= 11.3



Caffeine Boost

When the final is coming.....

When the project is coming...

You need to stay AWAKE !

Caffeine Boost

Additional constraints $cafe\_record_t = cafe\_record_{t-1} + cup_t \cdot \beta_{cafe} - cafe\_ret_t \geq 0 \quad \forall t \in Days$

Modified function $L_t = L_{t-1} + sleep_t \beta_{sleep} - D_t \beta_{demand} - \sum_{i \in S} study_{it} \beta_{study} - \beta_{garbage} \cdot garbage_t$

$$+ cup_t \cdot \beta_{cafe} - \beta_{cafe\_ret} \cdot cafe\_ret_t$$

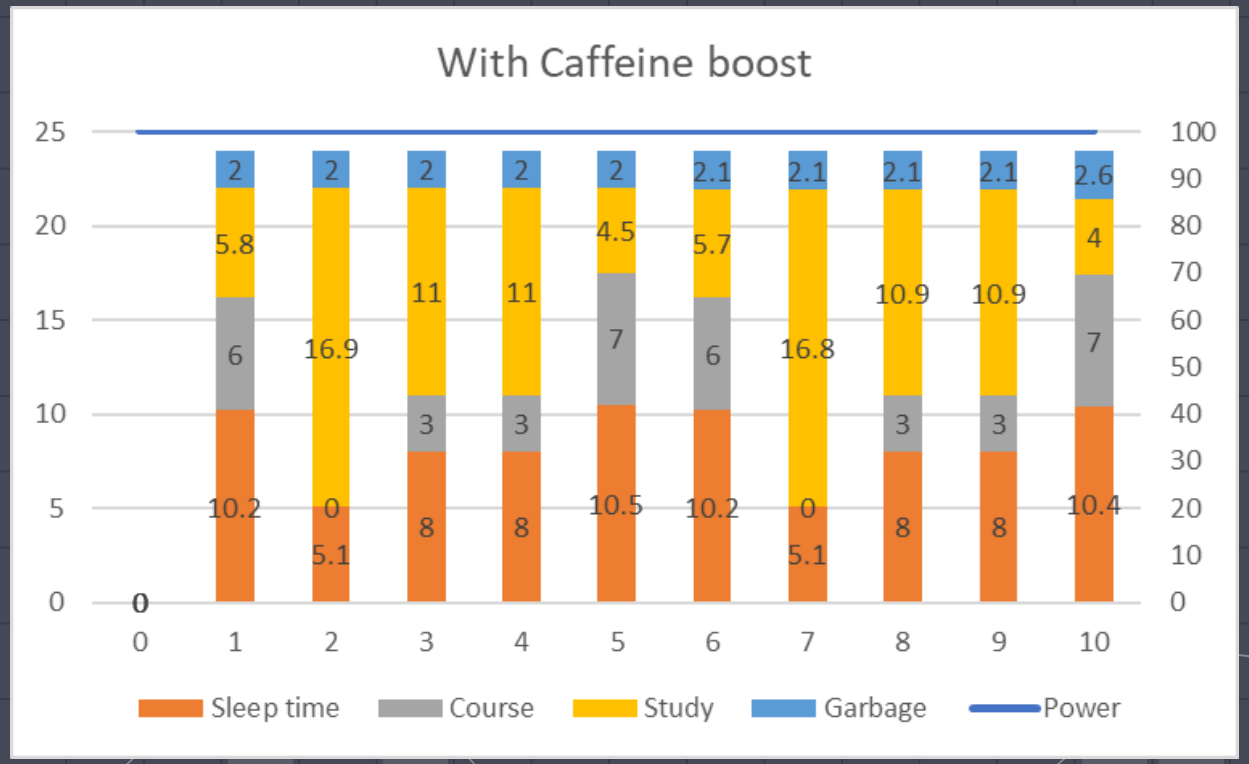
Additional variables $cup_t = \text{cups of coffee taken at day } t \quad cup_t \in \mathbb{Z}_0^+, \forall t \in Days.$

$cafe\_ret_t = \text{Coffee Debt returned at day } t$

$cafe\_record_t = \text{Accumulated coffee debt at day } t$

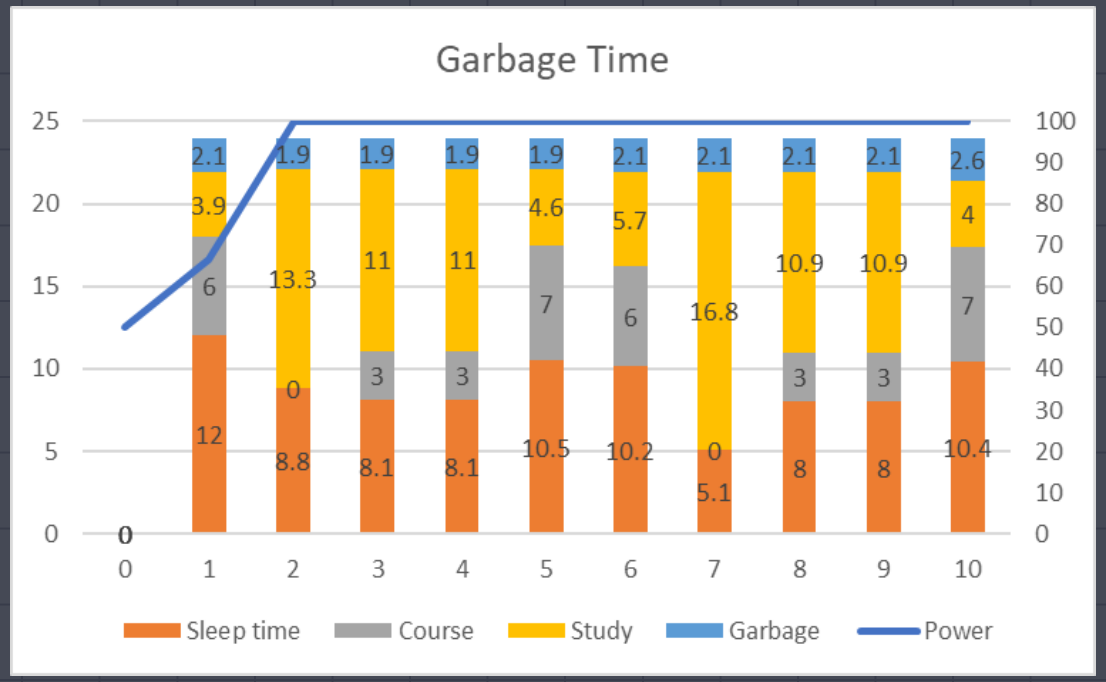
Results

- Drinking at day 1?

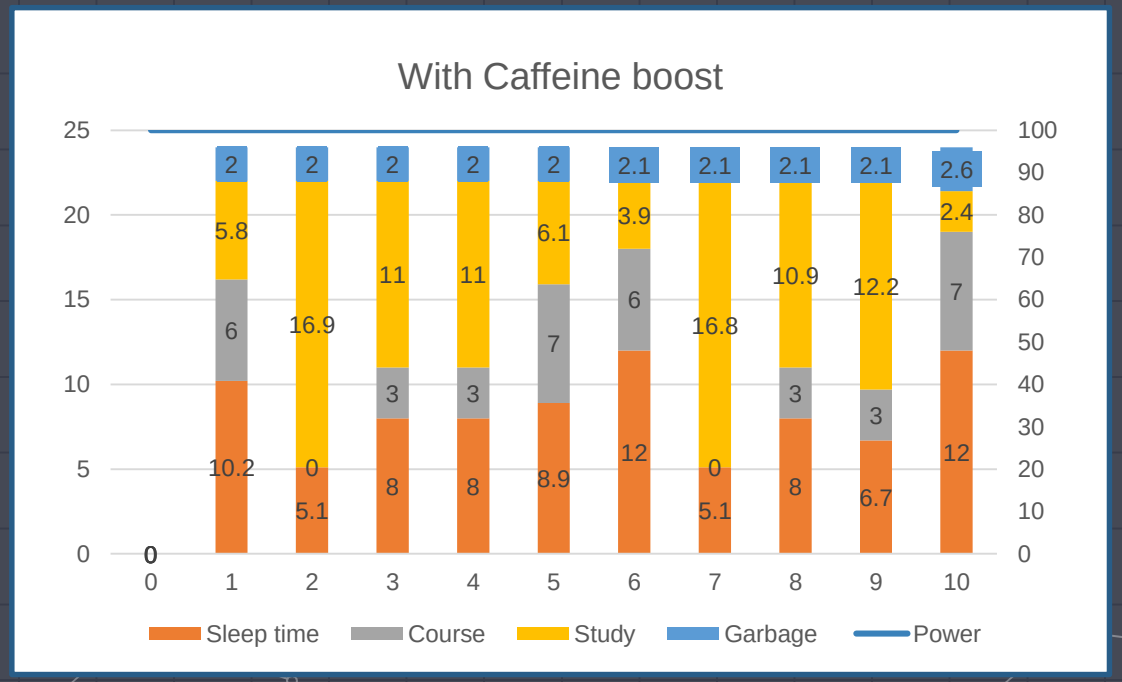


Drinks 2.0 cups of coffee at day 1
Drinks 1.1 cups of coffee at day 5
Drinks 1.1 cups of coffee at day 9

Comparison



Obj Val: 199.412, L = 96.7, TU1 = 91.5, G = 11.3



Drinks 2.0 cups of coffee at day 1
Drinks 1.1 cups of coffee at day 5
Drinks 1.1 cups of coffee at day 9

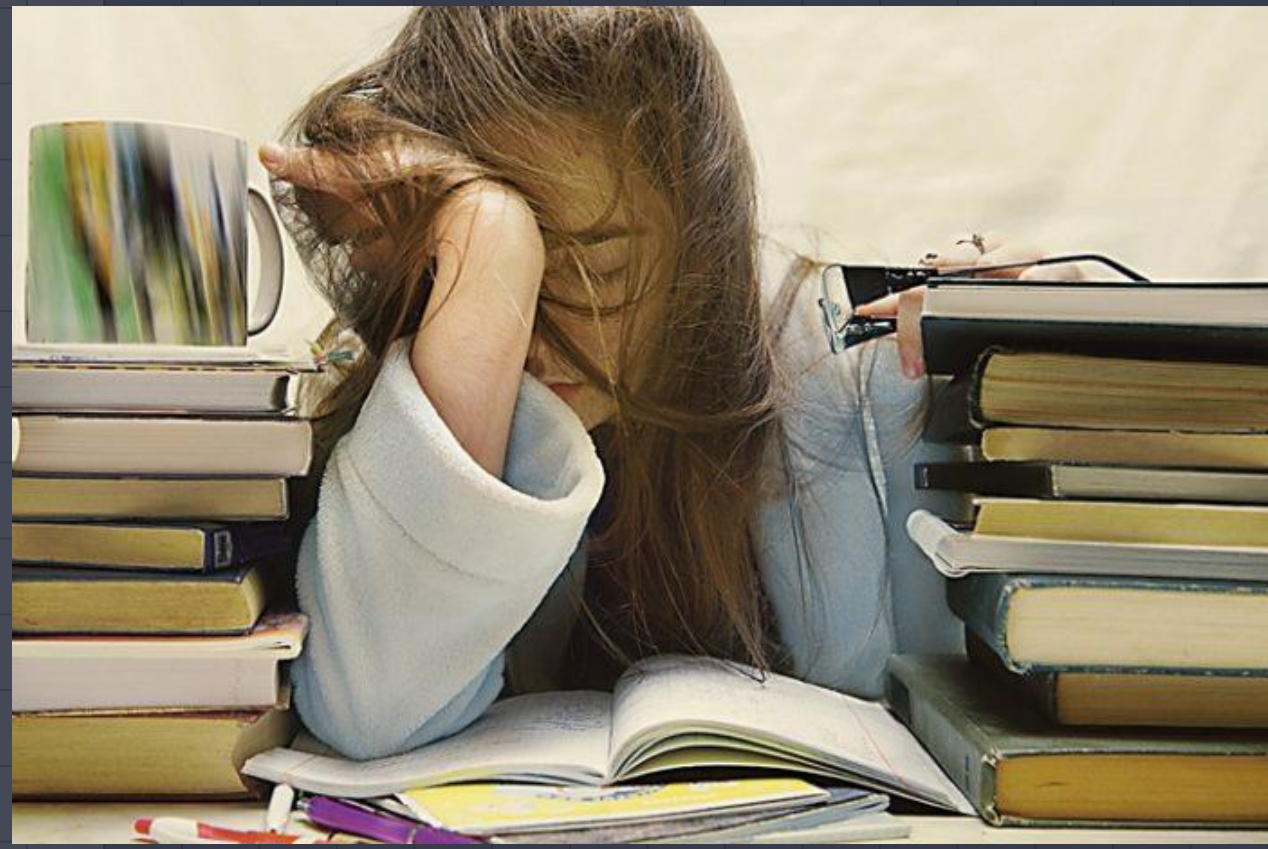
Obj Val: 205.0587125 , L = 100.0 , TU1 = 93.7 , G = 11.4

Test Utility slightly increased.
Objective got greater.
Sleep time grew.

Switching / Setup Cost

- Switching Cost :
  When we change the subject we study, we need to reallocate our brain CPU, we need to open up new textbook.....
  It is energy consuming!

- Setup Cost :
  When we decide to study, the pain will weary us in a flash.
  Mental destruction -> energy consuming



Setup Cost

Objective function should include $-\sum_{t \in Days} w_{setup_t} \cdot C_{setup}$

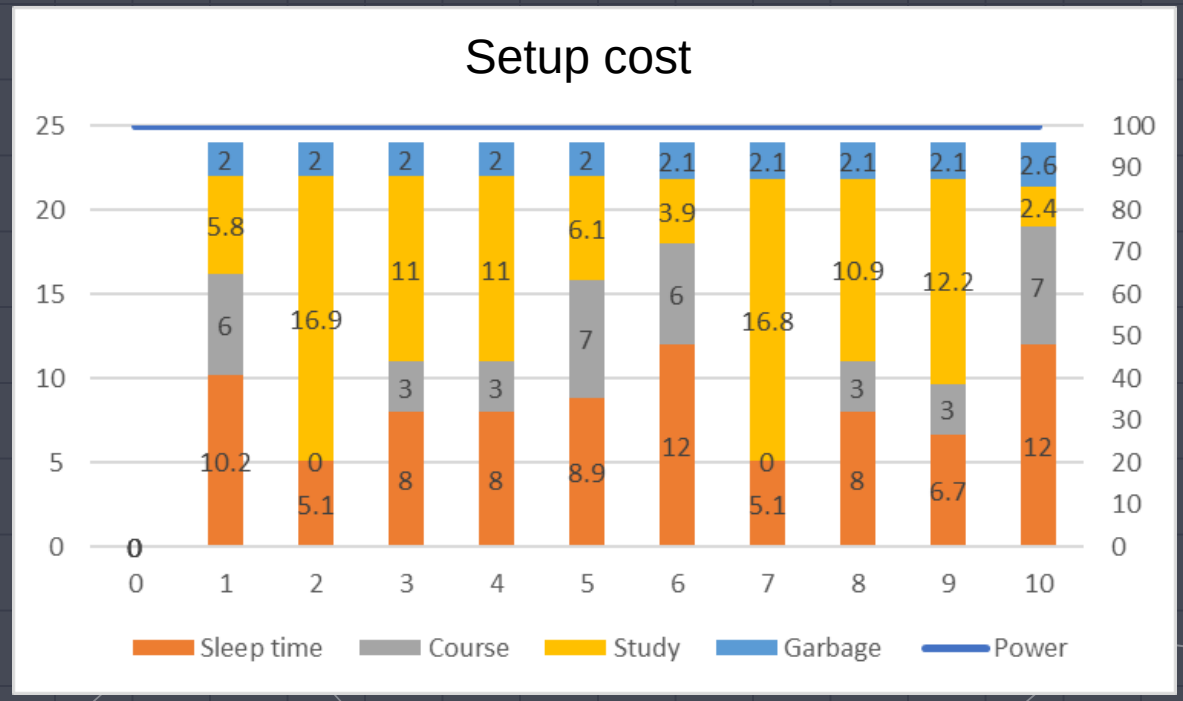
Constraint should include $study\_count_t \leq S \cdot w_{setup_t}, \forall t \in Days$

Variable(s) should include $w_{setup_t} = \begin{cases} 1, & \text{if } study\_count_t > 0 \\ 0, & \text{o/w} \end{cases} \quad \forall t \in Days$

Result (setup cost)

- same as previous?

objective 185.1
L = 100.0
TU1 = 93.7
G = 11.4
Setup = 20.0



Drinks 1.4 cups of coffee at day 5
Drinks 1.2 cups of coffee at day 9

Switching cost

Objective function should include $-\sum_{t \in Days} \left(max\{\left(study\_count_t - 1\right), 0\}\right) \cdot C_{switching}$

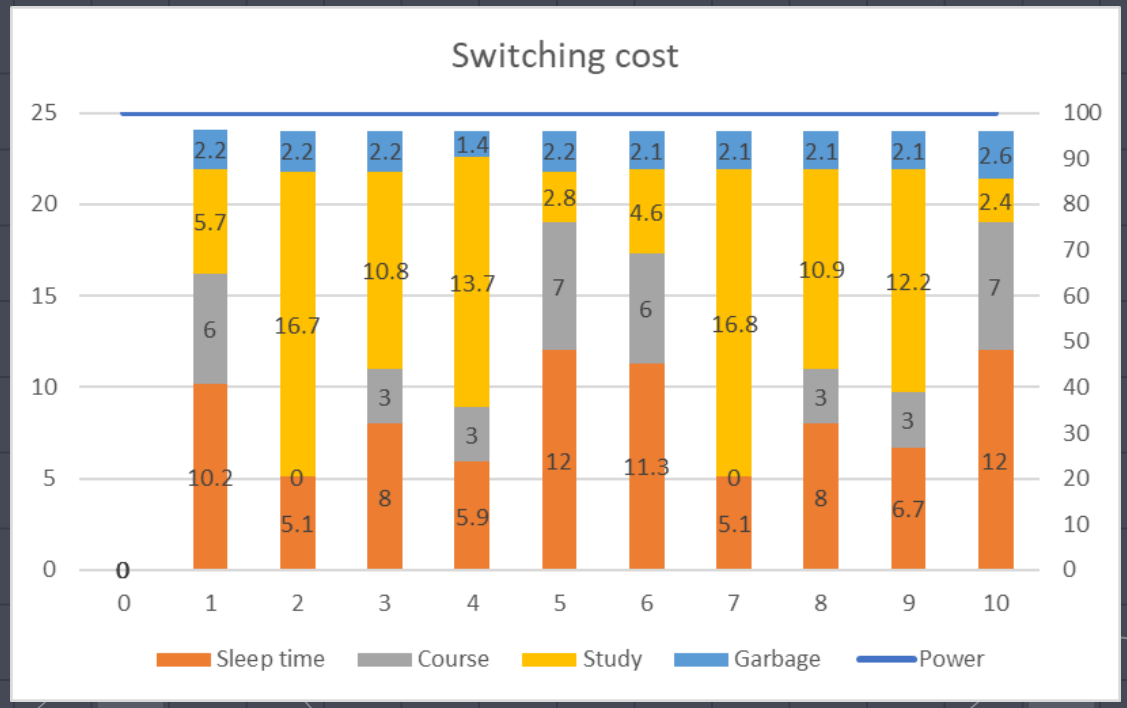
Constraint should include $study_{it} \leq M \cdot w_{study_{it}},\ M = 24$

Variable(s) should include $w_{study_{it}} = \begin{cases} 1, & \text{if } study_{it} > 0 \\ 0, & \text{ø/w} \end{cases} \quad \forall i \in S, t \in Days$

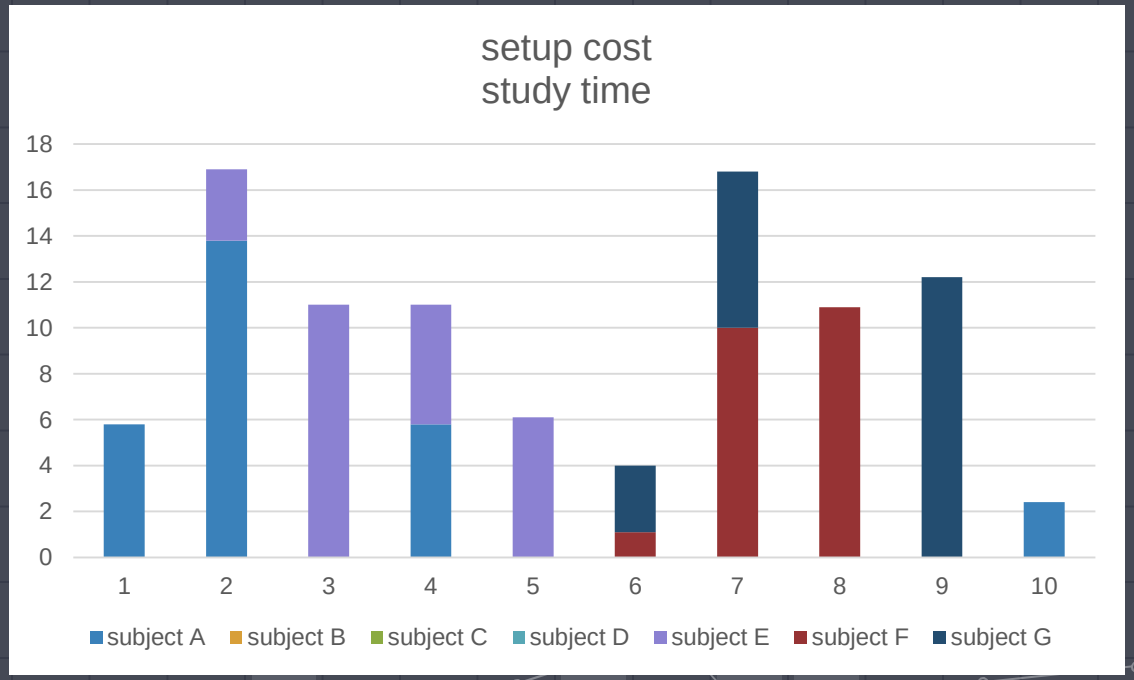
Result (switching cost)

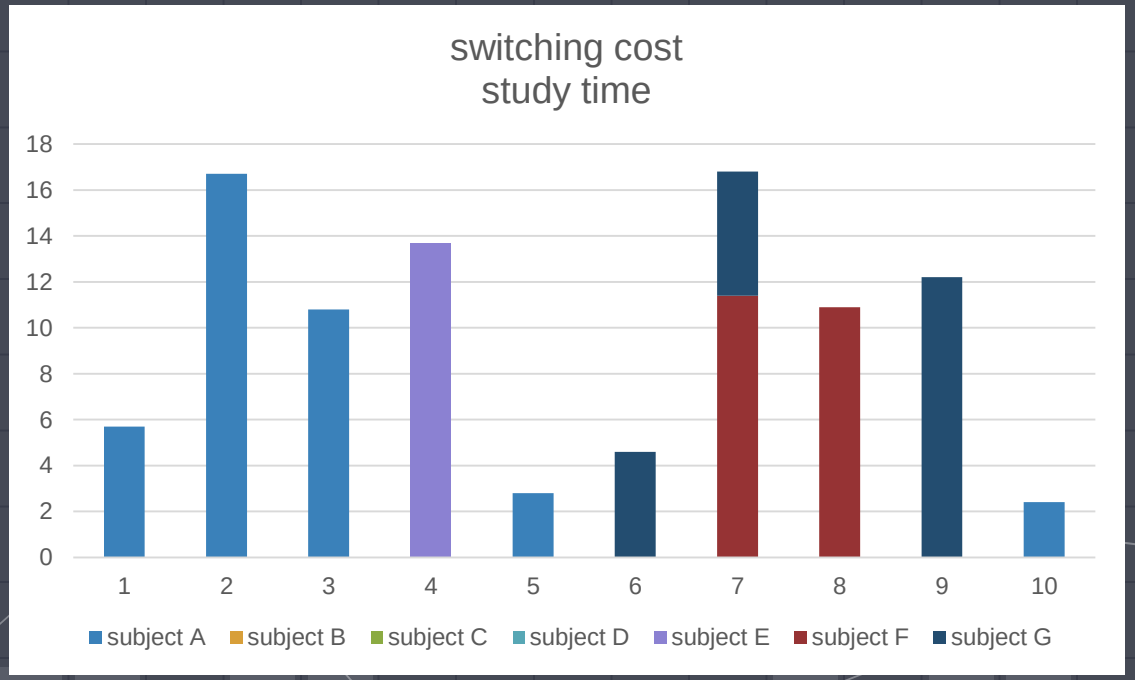
- Switching cost vs Setup cost
- Reduced Test Utility?

objective 183.2
L = 100.0
TU1 = 91.8
G = 11.3
Switching = 0.0
Setup = 20.0



Drinks 2 cups of coffee at day 4,
Drinks 1.2 cup of coffee at day 9


setup cost
study time
18
16
14
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10
subject A
subject B
subject C
subject D
subject E
subject F
subject G


switching cost
study time
18
16
14
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10
subject A
subject B
subject C
subject D
subject E
subject F
subject G

Thanks for watching