$\nu_\mu e$ kinematics & dynamics

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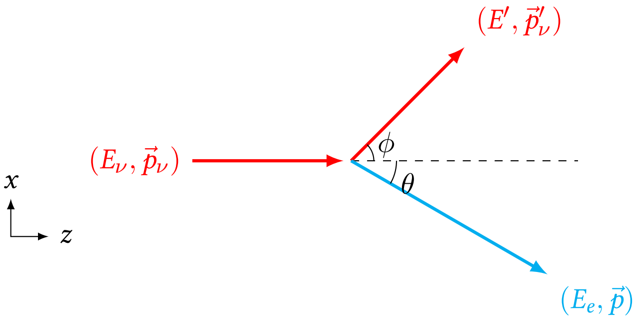


A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have ($\hbar = c = 1$)

$$
\begin{aligned}
\mathbb{p}_\nu &= (E_\nu, 0, 0, E_\nu) && (1)\\
\mathbb{p}_e &= (m, 0, 0, 0) && (2)
\end{aligned}
$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$
\begin{aligned}
\mathbb{p}'_\nu &= (E', E' \sin\phi, 0, E' \cos\phi) && (3)\\
\mathbb{p}'_e &= (E_e, -p\sin\theta, 0, p\cos\theta) && (4)
\end{aligned}
$$

The total 4-momentum before collision is

$$
\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \tag{5}
$$

The total 4-momentum after collision is

$$
\mathbb{P}' = (E' + E_e, -p\sin\theta + E'\sin\phi, 0, p\cos\theta + E'\cos\phi) \tag{6}
$$

Since total 4-momentum is conserved before and after collision, we have equations

$$\begin{aligned} E' + E_e &= E_\nu + m & (7) \\ -p\sin\theta + E'\sin\phi &= 0 & (8) \\ p\cos\theta + E'\cos\phi &= E_\nu & (9) \end{aligned}$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$E'^2 = E_\nu^2 - 2E_\nu p\cos\theta + p^2 \tag{10}$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$p\cos\theta = (E_e - m)\left(1 + \frac{m}{E_\nu}\right) \tag{11}$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\boxed{\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2\theta}} \tag{12}$$

If we choose to eliminate θ and E_e, we get

$$E' = \frac{E_\nu m}{E_\nu(1 - \cos\phi) + m} \tag{13}$$

Rearranging and dividing Eq. (8) by Eq. (9), we have

$$\cot\theta = \frac{\frac{E_\nu}{E'} - \cos\phi}{\sin\phi} = \left(1 + \frac{E_\nu}{m}\right)\tan\left(\frac{\phi}{2}\right) \tag{14}$$

, where we have used Eq. (13) for E_ν/E'.
For a neutrino scattering angle $0 \le \phi \le \pi$, the range of the electron scattering angle is

$$\boxed{0 \le \theta \le \frac{\pi}{2}} \tag{15}$$

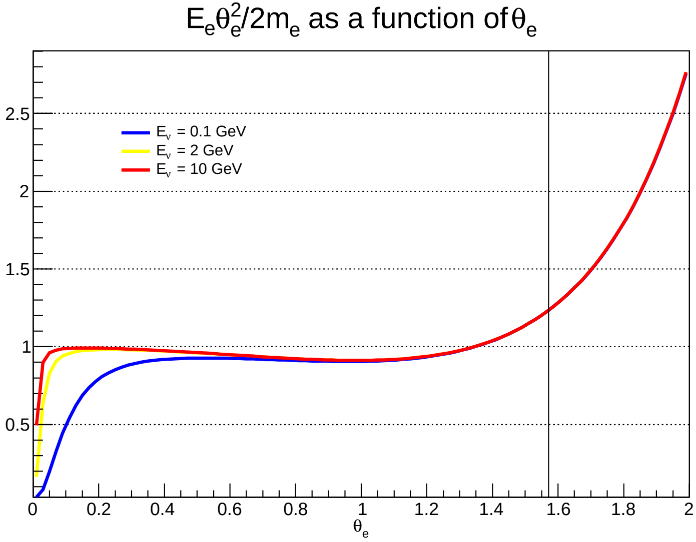
kinematical cut I

For small θ, Eq. (12) becomes $E_e\theta^2/2m \approx 1$. However, since we have the *exact* formula for E_e, we can actually plot the function

$$f(\theta) = \frac{E_e\theta^2}{2m} = \frac{\left(1+\frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1+\frac{m}{E_\nu}\right)^2 - \cos^2\theta} \cdot \frac{\theta^2}{2} \tag{16}$$

and see how close it is to 1.

kinematical cut II



kinematics: squared momentum transfer I

Suppose before collision the electron has four-momentum $p_2 = (m, \vec{0})$. After collision it has four-momentum $p_4 = (E, \vec{p}_4)$. The squared momentum transfer is

$$
\begin{aligned}
Q^2 &\equiv -(p_2 - p_4)^2 \\
&= -(p_2^2 + p_4^2 - 2p_2 \cdot p_4) \\
&= 2mE - 2m^2 \\
&= 2m(E - m)
\end{aligned}
$$

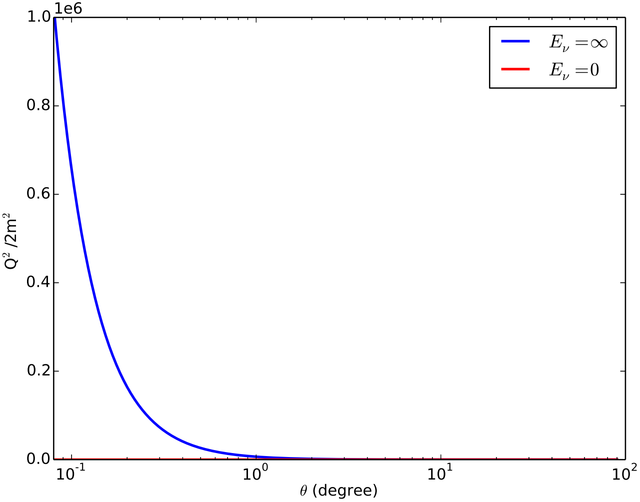
Therefore,

$$\boxed{Q^2 = 2mT} \tag{17}$$

, where T is the kinetic energy of the outgoing electron. This is a renowned result for fixed target collisions. Write Eq. (17) as a function of θ, we have

$$\frac{Q^2}{2m^2} = \frac{2\cos^2\theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2\theta} \tag{18}$$

kinematics: squared momentum transfer II



$\nu_\mu e$ dynamics I

The differential cross section of $\nu_\mu e$ scattering is[1]

$$\frac{d\sigma}{d\cos\theta} = \sigma_0 \frac{4E_\nu^2 (m_e + E_\nu)^2 \cos\theta}{\left[(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta\right]^2} \times \left[g_1^2 + g_2^2 \left(1 - \frac{2m_e E_\nu \cos^2\theta}{(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta}\right)^2 - g_1 g_2 \frac{2m_e^2 \cos^2\theta}{(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta}\right] \tag{19}$$

, where

$$\begin{aligned} \sigma_0 &= \frac{2G_F^2 m_e^2}{\pi} \approx 88.06 \times 10^{-46}\ \mathrm{cm}^2 \\ g_1 &= -\frac{1}{2} + \sin^2\theta_W \approx -0.27 \\ g_2 &= \sin^2\theta_W \approx 0.23 \end{aligned}$$

[1] Giunti C. and Kim C. W., *Fundamentals of Neutrino Physics and Astrophysics*, 2007

$\nu_\mu e$ dynamics II - angular cut efficiency

Define the angular cut efficiency as

$$\epsilon_\theta(\theta) = \frac{\int_{\cos\theta}^{1} \frac{d\sigma}{d\cos\theta'}\, d\cos\theta'}{\int_0^1 \frac{d\sigma}{d\cos\theta'}\, d\cos\theta'} \tag{20}$$

$\nu_\mu e$ dynamics III