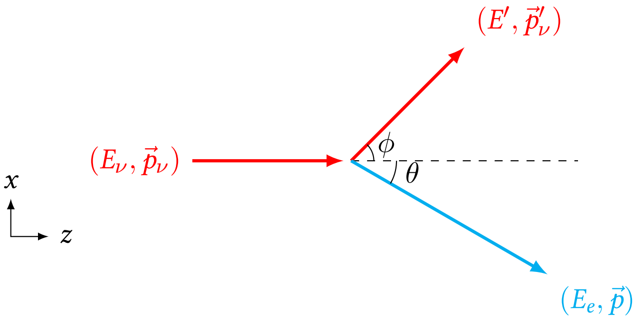
νe kinematics



A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have $(\hbar = c = 1)$

$$\mathbb{p}_\nu = (E_\nu, 0, 0, E_\nu) \tag{1}$$
$$\mathbb{p}_e = (m, 0, 0, 0) \tag{2}$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$\mathbb{p}'_\nu = (E', E' \sin\phi, 0, E' \cos\phi) \tag{3}$$
$$\mathbb{p}'_e = (E_e, -p\sin\theta, 0, p\cos\theta) \tag{4}$$

The total 4-momentum before collision is

$$\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \tag{5}$$

The total 4-momentum after collision is

$$\mathbb{P}' = (E' + E_e, -p\sin\theta + E'\sin\phi, 0, p\cos\theta + E'\cos\phi) \tag{6}$$

Since 4-momentum is conserved before and after collision, we have equations

$$\begin{aligned} E' + E_e &= E_\nu + m && (7) \\ -p \sin\theta + E' \sin\phi &= 0 && (8) \\ p\cos\theta + E' \cos\phi &= E_\nu && (9) \end{aligned}$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$E'^2 = E_\nu^2 - 2E_\nu p\cos\theta + p^2 \tag{10}$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$p\cos\theta = (E_e - m)\left(1 + \frac{m}{E_\nu}\right) \tag{11}$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2\theta} \tag{12}$$