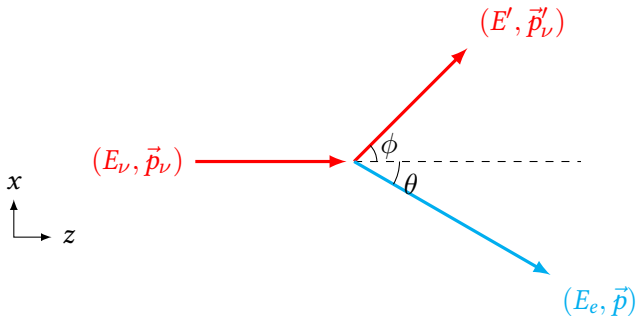


$\nu_\mu e$ kinematics & dynamics

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A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have ($\hbar = c = 1$)

$$\mathbb{p}_\nu = (E_\nu, 0, 0, E_\nu) \quad (1)$$

$$\mathbb{p}_e = (m, 0, 0, 0) \quad (2)$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$\mathbb{p}'_\nu = (E', E' \sin \phi, 0, E' \cos \phi) \quad (3)$$

$$\mathbb{p}'_e = (E_e, -p \sin \theta, 0, p \cos \theta) \quad (4)$$

The total 4-momentum before collision is

$$\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \quad (5)$$

The total 4-momentum after collision is

$$\mathbb{P}' = (E' + E_e, -p \sin \theta + E' \sin \phi, 0, p \cos \theta + E' \cos \phi) \quad (6)$$

Since total 4-momentum is conserved before and after collision, we have equations

$$E' + E_e = E_\nu + m \quad (7)$$

$$-p \sin \theta + E' \sin \phi = 0 \quad (8)$$

$$p \cos \theta + E' \cos \phi = E_\nu \quad (9)$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$E'^2 = E_\nu^2 - 2E_\nu p \cos \theta + p^2 \quad (10)$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$p \cos \theta = (E_e - m) \left(1 + \frac{m}{E_\nu} \right) \quad (11)$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\boxed{\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2 \theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2 \theta}} \quad (12)$$

If we choose to eliminate θ and E_e , we get

$$E' = \frac{E_\nu m}{E_\nu(1 - \cos \phi) + m} \quad (13)$$

Rearranging and dividing Eq. (8) by Eq. (9), we have

$$\cot \theta = \frac{\frac{E_\nu}{E'} - \cos \phi}{\sin \phi} = \left(1 + \frac{E_\nu}{m}\right) \tan \left(\frac{\phi}{2}\right) \quad (14)$$

, where we have used Eq. (13) for E_ν/E' .

For a neutrino scattering angle $0 \leq \phi \leq \pi$, the range of the electron scattering angle is

$$\boxed{0 \leq \theta \leq \frac{\pi}{2}} \quad (15)$$

kinematical cut I

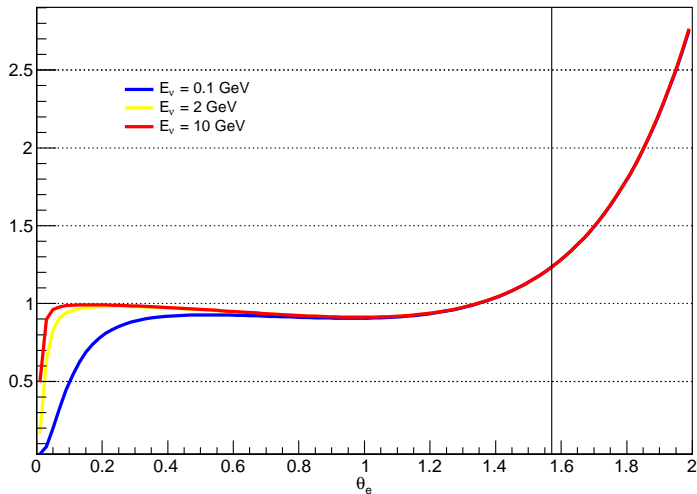
For small θ , Eq. (12) becomes $E_e \theta^2 / 2m \approx 1$. However, since we have the *exact* formula for E_e , we can actually plot the function

$$f(\theta) = \frac{E_e \theta^2}{2m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2 \theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2 \theta} \cdot \frac{\theta^2}{2} \quad (16)$$

and see how close it is to 1.

kinematical cut II

$E_e \theta_e^2 / 2m_e$ as a function of θ_e



kinematics: squared momentum transfer I

Suppose before collision the electron has four-momentum $p_2 = (m, \vec{0})$. After collision it has four-momentum $p_4 = (E, \vec{p}_4)$. The squared momentum transfer is

$$\begin{aligned} Q^2 &\equiv -(p_2 - p_4)^2 \\ &= -(p_2^2 + p_4^2 - 2p_2 \cdot p_4) \\ &= 2mE - 2m^2 \\ &= 2m(E - m) \end{aligned}$$

Therefore,

$$\boxed{Q^2 = 2mT} \tag{17}$$

, where T is the kinetic energy of the outgoing electron. This is a renowned result for fixed target collisions. Write Eq. (17) as a function of θ , we have

$$\frac{Q^2}{2m^2} = \frac{2 \cos^2 \theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2 \theta} \tag{18}$$

$\nu_\mu e$ dynamics I

The differential cross section of $\nu_\mu e$ scattering is¹

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} &= \sigma_0 \frac{4E_\nu^2 (m_e + E_\nu)^2 \cos\theta}{[(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta]^2} \\ &\times \left[g_1^2 + g_2^2 \left(1 - \frac{2m_e E_\nu \cos^2\theta}{(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta} \right)^2 - g_1 g_2 \frac{2m_e^2 \cos^2\theta}{(m_e + E_\nu)^2 - E_\nu^2 \cos^2\theta} \right] \end{aligned} \quad (19)$$

, where

$$\begin{aligned} \sigma_0 &= \frac{2G_F^2 m_e^2}{\pi} \approx 88.06 \times 10^{-46} \text{ cm}^2 \\ g_1 &= -\frac{1}{2} + \sin^2\theta_W \approx -0.27 \\ g_2 &= \sin^2\theta_W \approx 0.23 \end{aligned}$$

¹Giunti C. and Kim C. W., *Fundamentals of Neutrino Physics and Astrophysics*, 2007

$\nu_\mu e$ dynamics II - angular cut efficiency

Define the angular cut efficiency as

$$\epsilon_\theta(\theta) = \frac{\int_{\cos \theta}^1 \frac{d\sigma}{d \cos \theta'} d \cos \theta'}{\int_0^1 \frac{d\sigma}{d \cos \theta'} d \cos \theta'} \quad (20)$$

$\nu_\mu e$ dynamics III

