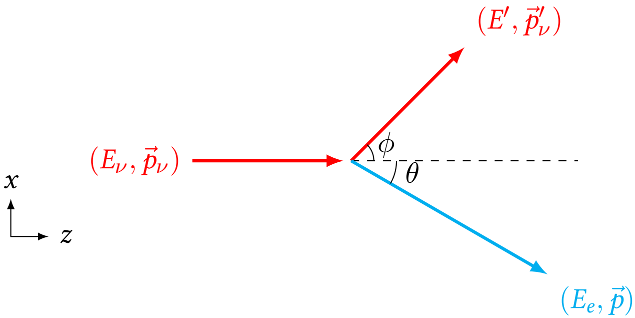
νe kinematics



A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have $(\hbar = c = 1)$

$$
\begin{aligned}
\mathbb{p}_\nu &= (E_\nu, 0, 0, E_\nu) & (1)\\
\mathbb{p}_e &= (m, 0, 0, 0) & (2)
\end{aligned}
$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$
\begin{aligned}
\mathbb{p}'_\nu &= (E', E' \sin\phi, 0, E' \cos\phi) & (3)\\
\mathbb{p}'_e &= (E_e, -p \sin\theta, 0, p \cos\theta) & (4)
\end{aligned}
$$

The total 4-momentum before collision is

$$
\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \tag{5}
$$

The total 4-momentum after collision is

$$
\mathbb{P}' = (E' + E_e, p \sin\theta - E' \sin\phi, 0, p \cos\theta + E' \cos\phi) \tag{6}
$$

Since 4-momentum is conserved before and after collision, we have equations

$$
\begin{aligned}
E' + E_e &= E_\nu + m && (7)\\
-p \sin\theta + E' \sin\phi &= 0 && (8)\\
p \cos\theta + E' \cos\phi &= E_\nu && (9)
\end{aligned}
$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$
E'^2 = E_\nu^2 - 2E_\nu p \cos\theta + p^2 \tag{10}
$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$
p \cos\theta = (E_e - m)\left(1 + \frac{m}{E_\nu}\right) \tag{11}
$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2\theta} \tag{12}$$