νe kinematics & dynamics

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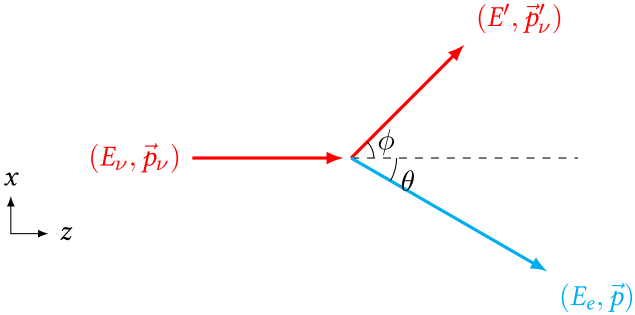

A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have ($\hbar = c = 1$)

$$
\begin{aligned}
\mathbb{p}_\nu &= (E_\nu, 0, 0, E_\nu) && (1)\\
\mathbb{p}_e &= (m, 0, 0, 0) && (2)
\end{aligned}
$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$
\begin{aligned}
\mathbb{p}'_\nu &= (E', E' \sin\phi, 0, E' \cos\phi) && (3)\\
\mathbb{p}'_e &= (E_e, -p\sin\theta, 0, p\cos\theta) && (4)
\end{aligned}
$$

The total 4-momentum before collision is

$$
\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \tag{5}
$$

The total 4-momentum after collision is

$$
\mathbb{P}' = (E' + E_e, -p\sin\theta + E'\sin\phi, 0, p\cos\theta + E'\cos\phi) \tag{6}
$$

Since total 4-momentum is conserved before and after collision, we have equations

$$\begin{aligned} E' + E_e &= E_\nu + m && (7) \\ -p\sin\theta + E'\sin\phi &= 0 && (8) \\ p\cos\theta + E'\cos\phi &= E_\nu && (9) \end{aligned}$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$E'^2 = E_\nu^2 - 2E_\nu p\cos\theta + p^2 \tag{10}$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$p\cos\theta = (E_e - m)\left(1 + \frac{m}{E_\nu}\right) \tag{11}$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\boxed{\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2\theta}} \tag{12}$$

If we choose to eliminate θ and E_e, we get

$$E' = \frac{E_\nu m}{E_\nu(1 - \cos\phi) + m} \tag{13}$$

Rearranging and dividing Eq. (8) by Eq. (9), we have

$$\cot\theta = \frac{\frac{E_\nu}{E'} - \cos\phi}{\sin\phi} = \left(1 + \frac{E_\nu}{m}\right)\tan\left(\frac{\phi}{2}\right) \tag{14}$$

, where we have used Eq. (13) for E_ν/E'.
For a neutrino scattering angle $0 \leq \phi \leq \pi$, the range of the electron scattering angle is

$$\boxed{0 \leq \theta \leq \frac{\pi}{2}} \tag{15}$$

kinematical cut

For small θ, Eq. (11) becomes $E_e\theta^2/2m \approx 1$. However, since we have the *exact* formula for E_e, we can actually plot the function

$$f(\theta) = \frac{E_e\theta^2}{2m} = \frac{\left(1+\frac{m}{E_\nu}\right)^2 + \cos^2\theta}{\left(1+\frac{m}{E_\nu}\right)^2 - \cos^2\theta} \cdot \frac{\theta^2}{2} \tag{16}$$

and see how close it is to 1.