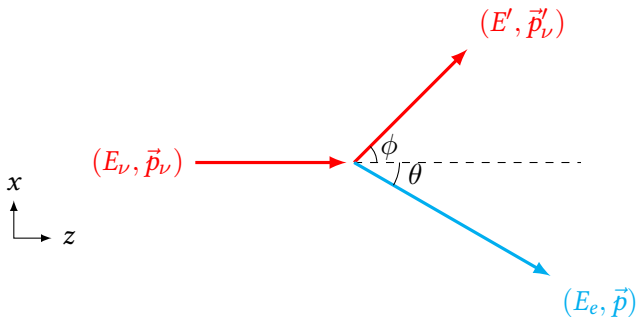


νe kinematics

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A neutrino collides with an electron at rest. Write down the 4-momenta of the neutrino and the electron in the lab frame. Suppose the neutrino is massless. Before collision we have ($\hbar = c = 1$)

$$\mathbb{p}_\nu = (E_\nu, 0, 0, E_\nu) \quad (1)$$

$$\mathbb{p}_e = (m, 0, 0, 0) \quad (2)$$

, where m is the electron rest mass, and E_ν is the total energy of the incident neutrino. After collision, we have

$$\mathbb{p}'_\nu = (E', E' \sin \phi, 0, E' \cos \phi) \quad (3)$$

$$\mathbb{p}'_e = (E_e, -p \sin \theta, 0, p \cos \theta) \quad (4)$$

The total 4-momentum before collision is

$$\mathbb{P} = (E_\nu + m, 0, 0, E_\nu) \quad (5)$$

The total 4-momentum after collision is

$$\mathbb{P}' = (E' + E_e, -p \sin \theta + E' \sin \phi, 0, p \cos \theta + E' \cos \phi) \quad (6)$$

Since total 4-momentum is conserved before and after collision, we have equations

$$E' + E_e = E_\nu + m \quad (7)$$

$$-p \sin \theta + E' \sin \phi = 0 \quad (8)$$

$$p \cos \theta + E' \cos \phi = E_\nu \quad (9)$$

Rearranging equations (8) and (9), squaring, and adding them, we eliminate ϕ and obtain

$$E'^2 = E_\nu^2 - 2E_\nu p \cos \theta + p^2 \quad (10)$$

If we eliminate E' with Eq. (7), and employ the energy-momentum relation $E_e^2 = p^2 + m^2$ once, we obtain

$$p \cos \theta = (E_e - m) \left(1 + \frac{m}{E_\nu} \right) \quad (11)$$

By squaring Eq. (11), and employing the energy-momentum relation again, we arrive at the answer

$$\frac{E_e}{m} = \frac{\left(1 + \frac{m}{E_\nu}\right)^2 + \cos^2 \theta}{\left(1 + \frac{m}{E_\nu}\right)^2 - \cos^2 \theta} \quad (12)$$