$\nu_\mu e$ cross-section error propagation

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Let $\sin^2 \theta_W = x$ and ignore the multiplicative term $\frac{G_F^2 m_e E_\nu}{2\pi}$, we want to know the standard deviation in $f = 1 - 4x + \frac{16}{3}x^2$. The error propagation formula reads

$$\sigma_f^2 = \left(\left. \frac{df}{dx} \right|_{x=x_0} \right)^2 \sigma_x^2 \tag{1}$$

where x_0 is the central value of x.

Given $x_0 = 0.23126$ and $\sigma_x = 2.2 \times 10^{-4}$, $f'(x_0) = -1.5332267$, and

$$
\begin{aligned}
\sigma_f^2 &= (-1.5332267)^2 \times (2.2 \times 10^{-4})^2 \\
&= 0.000000117 \\
\Rightarrow \sigma_f &= 0.000341731
\end{aligned}
$$

Given $f(x_0) = 0.360193001$, the relative uncertainty in f is $\frac{\sigma_f}{f(x_0)} = 0.09\%$.