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Precise parametrizations of muon energy losses in water

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Abstract. The description of muon propagation through large depths of matter, based on a concept of the correction factor, is proposed. The results of Monte-Carlo calculations of this correction factor are presented. The parametrizations for continuous energy loss coefficients, valid in the broad interval of muon energies, and for the correction factor are given. The concrete calculations for pure water are presented.

1 Introduction

The existing methods of calculations of muon propagation through thick layers of matter (basically for a standard rock) are largely presented in the literature. There are two completely different approaches in such calculations. In first (historically) works on the subject of muon propagation the integro-differential kinetic equation for muon flux had been formulated and approximately solved using semianalytical methods. We mention here one of the pioneering works of this approach: (Zatsepin and Mikhalchi, 1962), and one of the most recent papers: (Naumov, Sinegovsky, and Bugaev, 1993) (this latter paper contains also the large bibliography). Works of the second approach use Monte Carlo (MC) technique for propagation studies: the main element in such works is simulation of sequences of free flights and interactions during muon passage through a medium. One of the most early works using this method is (Hayman, Palmer, and Wolfendale, 1963), the most recent one is the work of the present authors (Sokalski, Bugaev, and Klimushin, 2000), containing also many bibliographic references.

2 Method

The integral flux of muons in matter with energies above cutoff E_f expected at slant depth R at a zenith angle of θ taking

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into account *fluctuating* character of muon losses is conventionally described by

$$I_{fl}(\geq E_f, R, \theta) = \int_0^\infty P(E_0, R, \geq E_f) D(E_0, \theta) \, dE_0, \quad (1)$$

where $P(E_0, R, \geq E_f)$ is the probability that a muon, having starting energy E_0, after passing of path R will survive with final energy above cut-off E_f and $D(E_0, \theta) = dN(E_0, \theta)/dE_0$ is a sea level differential angular spectrum. In a case of a flat surface the slant depth is expressed by $R = h/\cos\theta$, where h is a vertical depth below the surface.

With the assumption of *continuous* energy loss rate of muon in matter, $L(E) = -dE/dx = a(E) + b(E)E$, the integral flux I_{cl} is derived from the equation

$$I_{cl}(\geq E_f, R, \theta) = \int_{E_s}^\infty D(E_0, \theta) \, dE_0, \quad (2)$$

where the low limit of the integral, E_s, is the value of starting energy E_0 which results, after passing of path R, in the final energy E_f. This value is derived from the solution of the integral equation

$$-\int_{E_s}^{E_f} \frac{dE}{L(E)} = R. \quad (3)$$

The method proposed by us for a calculation of real integral muon flux allowing for loss fluctuations consists in the following. As is shown in (Klimushin, Bugaev, and Sokalski, 2000), if we approximate the muon energy loss function $L(E)$ by a linear energy dependence,

$$L(E) = \alpha + \beta E \quad (4)$$

(where α and β are independent on energy), and if we parametrize the sea level muon spectrum by dependencies of the type

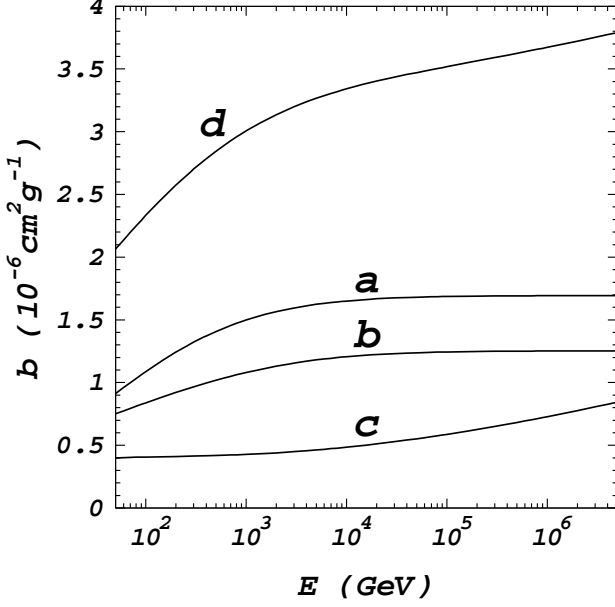$$D(E_0) \propto \frac{E_0^{-\gamma}}{1 + E_0/\varepsilon_0},$$

Fig. 1. The b-terms of muon radiative energy losses in pure water (as MUM code output). Losses due to e^+e^- pair production b_p (curve (a)), bremsstrahlung b_b (b), and photonuclear interaction b_n (c) are shown. Curve (d) is the sum of all b-terms.
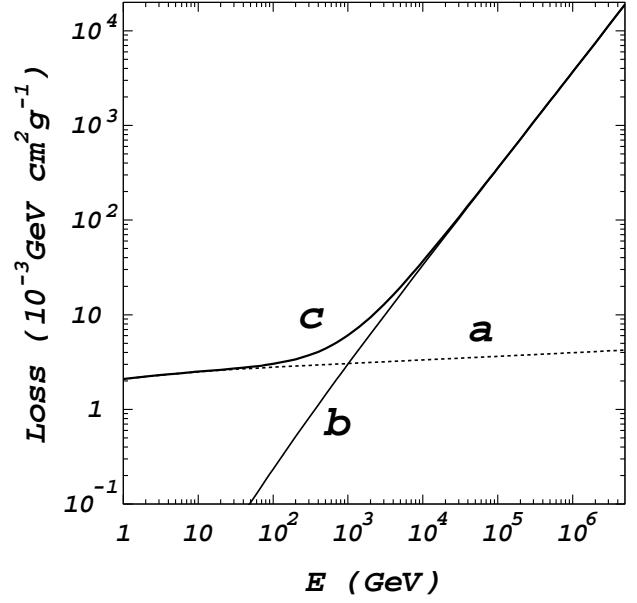
Fig. 2. Muon energy losses in pure water as a function of energy E (as MUM code output). Curve (a) is loss due to ionization $a(E)$ and (b) is total loss due to all radiative processes $b(E)E$. Curve (c) describes total energy losses $a(E) + b(E)E$.

then the integral flux I_{cl} (Eq.6) can be expressed analytically. Further, to calculate a real I_{fl} we must know the *correction factor* defined by the ratio

$$C_f(\geq E_f, R, \theta) = \frac{I_{cl}(\geq E_f, R, \theta)}{I_{fl}(\geq E_f, R, \theta)}. \tag{5}$$

In principle, this factor can be calculated using known codes for muon propagation through matter. In this work we apply for this aim the MUM code described in previous work of the present authors (Sokalski, Bugaev, and Klimushin, 2000). As we will see, this factor, as a function of muon energy and depth, can be easily parametrized by simple expressions. As a result, the real flux I_{fl} is conveniently expressed by the formula, containing continuous energy loss coefficients α and β.

3 Continuous energy losses

For the description of continuous energy losses of muon in water needed for solving the integral equation (3) we have done parametrizations based on output data from MUM code. All formulas for cross sections used in MUM code for muon energy loss computations are described in details in Ref. (Sokalski, Bugaev, and Klimushin, 2000).

The energy loss rate per unit of path x is given, conventionally, by the expression $L(E) = -dE/dx = a(E) + b(E)E$, where $a(E)$ is loss due to ionization and $b(E) = b_p(E) + b_b(E) + b_n(E)$ is the sum of coefficients for all radiative processes: e^+e^- pair production (b_p), bremsstrahlung (b_b) and photonuclear interaction (b_n).

Ionization loss was taken as composition of 2 processes $a(E) = a_c(E) + a_e(E)$, where a_c is classic ionization calculated using Bethe-Bloch formula and a_e results from e-diagrams for bremsstrahlung being treated as a part of ionization process following (Kelner, Kokoulun, and Petrukhin, 1997) (γ-quantum is emitted by atomic electron). Taking into account the last process leads to $1.8\%, 3.4\%, 5.5\%$ increase of ionization loss for 100 GeV, 1 TeV, 10 TeV muons, correspondingly. The approximation formula for a_c is given by

$$a_c(E) = a_{c_0} + a_{c_1} \ln\left(\frac{W_{max}}{m_\mu}\right),$$
$$W_{max} = \frac{E}{1 + m_\mu^2/(2m_e E)}, \tag{6}$$

where W_{max} is maximum energy transferable to the electron and m_μ, m_e are the rest masses of muon and electron. The set of cofficients, in units of $(10^{-3}\text{GeVcm}^2\text{g}^{-1})$,

$a_{c_0} = 2.106, \ a_{c_1} = 0.0950$ for $E \leq 45$ GeV,

$a_{c_0} = 2.163, \ a_{c_1} = 0.0853$ for $E > 45$ GeV,

gives the error for parametrization (6) smaller than 0.2% for $(1$–$10^8)$ GeV range. For a_e the following polynomial approximation (E in units of (GeV)):

$$a_e(E) = 3.54 + 3.785 \ln E + 1.15 \ln^2 E$$
$$+0.0615 \ln^3 E \qquad (10^{-6}\text{GeVcm}^2\text{g}^{-1}), \tag{7}$$

has the error $\leq 0.2\%$ for $(50$–$10^8)$ GeV range. Finally, the sum of (6) and (7) has the error of $a(E)$ approximation smaller than 0.2% for $(1$–$10^8)$ GeV range.

Table 1. Coefficients b_{ij} $(10^{-6}\mathrm{cm}^2\mathrm{g}^{-1})$ of the fitting formula (8) for b-terms of muon energy losses in water. Maximum absolute values of relative errors are shown.

b-term	subscript i	b_{i0}	b_{i1}	b_{i2}	b_{i3}	b_{i4}	Max.err,%
e^+e^- pairs, b_p	p	-11.31×10^{-1}	7.876×10^{-1}	-8.192×10^{-2}	3.763×10^{-3}	-6.437×10^{-5}	0.8
bremsstrahlung, b_b	b	-1.149×10^{-1}	2.963×10^{-1}	-2.165×10^{-2}	5.630×10^{-4}	-2.119×10^{-6}	0.9
photonuclear, b_n	n	3.903×10^{-1}	9.355×10^{-3}	-3.378×10^{-3}	4.913×10^{-4}	-1.216×10^{-5}	0.6

Radiative energy losses of muon in water has been calculated using the cross sections from the following works: (Andreev, Bezrukov, and Bugaev, 1994) for bremsstrahlung, (Kelner, Kokoulin, and Petrukhin, 1999) for e^+e^- pair production, and (Bezrukov and Bugaev, 1980) for photonuclear interaction. Fig. 1 sketches corresponding b-terms of radiative energy losses.

In principle MUM code gives values for all b-terms up to 10^9 GeV. Note that the logarithmic rise with an energy of the photoabsorption cross section, used in the model of Ref. (Bezrukov and Bugaev, 1980) and resulting in corresponding increase of the b_n-term, is experimentally proved already up to the photon energy $\sim 10^5$ GeV.

For parametrization of b-terms we have used the same functional form as in Ref. (Bezrukov and Bugaev, 1981) but increased the power of polynomial to improve accuracy and enlarge the range of application up to 100 PeV without division on energy subintervals (E in units of (GeV)):

$$b_i(E) = \sum_{j=0}^{4} b_{ij} \ln^j E, \qquad \text{where } i = p,\ b,\ n. \qquad (8)$$

Corresponding coefficients of this decomposition are collected in Tab. 1. This fit works with typical errors $\pm(0.2-0.4)\%$ within $(50-7\times10^7)$ GeV for all b-terms. The sum $b(E)$ of fits for b-terms is valid for the energy range $(50-10^8)$ GeV with the relative error $< 0.5\%$.

Fig. 2 shows muon energy loss rate in water. Note that the radiative losses account for approximately 5% of the value of total losses at muon energy 70 GeV, 50% at 1 TeV and dominate as 95% at 20 TeV. The sum of all parametrizations (6), (7) and (8) results in the description of total energy losses $L(E)$ with excellent accuracy $< 0.3\%$ with varying sign of error for energy range $(1-10^8)$ GeV mainly because of b-term error compensations (even for region $(1-50)$ GeV). Thanks to this fact just this parametrization of $L(E)$ has been used for the numerical computation of integral flux I_{cl} defined by (2) and (3).

Further, we realized that the total losses $L(E)$ may be described with an accuracy better than 2.5% (with varying sign of error) for energy range $(3-3\times10^6)$ GeV by 3-slope linear fit

$$L(E) = \alpha + \beta E, \qquad (9)$$

where

$\alpha = \alpha_0 = 2.30, \beta = \beta_0 = 15.50$ for $E \leq E_{01} = 30.0\mathrm{GeV}$,

$\alpha = \alpha_1 = 2.67, \beta = \beta_1 = 3.40$

for $E_{01} < E \leq E_{12} = 35.3\mathrm{TeV}$,

$\alpha = \alpha_2 = -6.50, \beta = \beta_2 = 3.66$ for $E > E_{12} = 35.3\mathrm{TeV}$,

and α are in $(10^{-3}\mathrm{GeVcm}^2\mathrm{g}^{-1})$ and β in $(10^{-6}\mathrm{cm}^2\mathrm{g}^{-1})$. The energy losses expressed by this parametrizations have sense of effective ones, for example in energy region 30 GeV–35 TeV the values $\alpha = 2.67$ and $\beta = 3.40$ represent effective energy losses due to ionization and radiative processes, correspondingly. The availability of the linear dependence (9) leads to the possibility to derive an analytical formula for underwater integral flux (see (Klimushin, Bugaev, and Sokalski, 2001)).

4 Correction factor

The influence of fluctuations of muon losses in matter (mainly due to radiative processes) results in that the real integral flux I_{fl} is generally greater than I_{cl} calculated in the approximation of continuous losses. In this work we propose to allow for the influence of energy loss fluctuations on the value of angular flux in matter by means of correction factor expressed by the ratio (5). In the assumption when the same differential cross sections are used for computing both the numerator and denominator of ratio (5), one may expect that this factor depends only weakly on sea level angular spectrum. For numerical calculations of the correction factor we used total continuous energy losses defined by sum of (6), (7) and (8), and survival probabilities calculated by using MUM code (Klimushin, Bugaev, and Sokalski, 2000). The values of correction factors calculated for the same slant depth R at vertical direction and at zenith angle θ differ weakly. It is illustrated in Fig. 3, where one can see that $C_f(\geq E_f, R, 0°)$ differs from $C_f(\geq E_f, R, \arccos h/R)$ maximum on 3.3% for $E_f > 10$ GeV at vertical depth h of 1.15 km. It appears that with acceptable accuracy the correction factor depends on slant depth R only, rather than on R and θ separately.

The dependencies of correction factor on E_f and R, calculated for sea level spectrum given by expression (5) of Ref. (Klimushin, Bugaev, and Sokalski, 2001), represent the set of rather smooth curves (shown in Fig. 3) and it is possible to approximate this factor by formula

$$C_f(\geq E_f, R, \theta) = \sum_{i=0}^{4}\left(\sum_{j=0}^{4} c_{ij} \log_{10}^j E_f\right)R^i. \qquad (10)$$

Table 2. Coefficients c_{ij} of the fitting formula (10) for correction factor calculated for vertical sea level spectrum given by expression (5) of Ref. (Klimushin, Bugaev, and Sokalski, 2001).

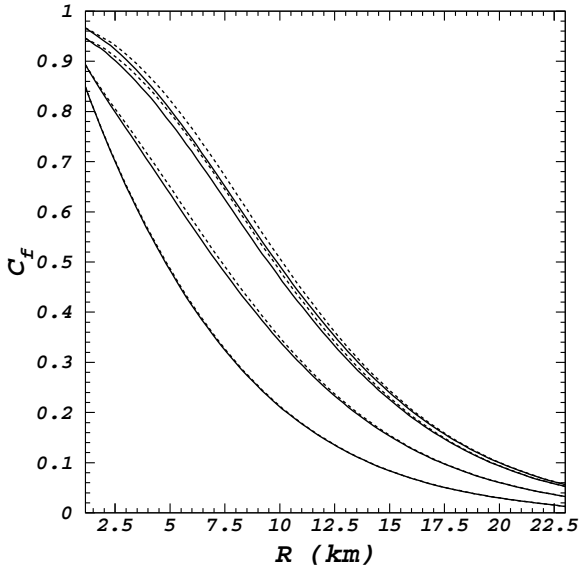| subscript i	c_{i0}	c_{i1}	c_{i2}	c_{i3}	c_{i4}
0	6.3045×10^{-1}	6.6658×10^{-1}	-4.5138×10^{-1}	1.2441×10^{-1}	-1.1904×10^{-2}
1	2.0152×10^{-1}	-4.2990×10^{-1}	3.2532×10^{-1}	-1.0265×10^{-1}	1.0751×10^{-2}
2	-3.3419×10^{-2}	5.1833×10^{-2}	-3.9229×10^{-2}	1.2360×10^{-2}	-1.2911×10^{-3}
3	1.6365×10^{-3}	-2.3645×10^{-3}	1.7775×10^{-3}	-5.5495×10^{-4}	5.7557×10^{-5}
4	-2.6630×10^{-5}	3.7770×10^{-5}	-2.8207×10^{-5}	8.7275×10^{-6}	-8.9919×10^{-7}

Fig. 3. Correction factor C_f as a function of slant depth R in pure water. The results obtained using sea level spectrum defined by expression (5) of Ref. (Klimushin, Bugaev, and Sokalski, 2001) are given. Solid curves correspond to numerical calculations for vertical case $\theta = 0°$. Dashed curves describe the correction factor computed at vertical depth h of 1.15 km for various zenith angles as a function of slant depth defined by $R = h/\cos\theta$. Both solid and dashed curves are shown for four values of cut-off energy E_f: 10 GeV, 100 GeV, 1 TeV, and 10 TeV, from top to bottom.

Here cut-off energy E_f is expressed in (GeV) and slant depth R is in (km) with the coefficients c_{ij} collected in Table 2. When using (10) for cut-off energies $E_f < 10$ GeV one should substitute value of E_f=10 GeV.

Formula (10) can be applied for any geometrical shape of the surface. Right hand side of (10) depends on θ because, generally, $R = R(\theta)$. So, in the particular case of a flat surface the angular dependence of the correction factor appears, in our approximation, only through the relation $R = h/\cos\theta$ (where h is a vertical depth).

The accuracy of formula (10) for E_f=(1–100) GeV is better than $\pm 2\%$ for slant depths R as large as 22 km and is not worse than $\pm 3\%$ for E_f=1 TeV up to R=17 km and for E_f=10 TeV up to R=15 km. Fig. 3 shows that for $E_f < 100$ GeV the total energy loss may be treated as quasi-continuous (at level of $C_f > 0.9$) only for slant depths $R < 2.5$ km but for E_f=10 TeV the fluctuations should be taken into account at level of 15 % already for slant depth as small as R=1 km. For slant depths larger than 10 km I_{fl} and I_{cl} differ more than on a factor of 2.

5   Conclusions

We showed in this paper that the most complicate part of any calculation of muon integral flux at great depths (which is taking into account the energy loss fluctuations) can be reduced to a computation of the correction factor which depends only weakly on the sea level spectrum and is a smooth function of its arguments (muon cut off energy and a slant depth). The method described here permits to carry out fast analytical calculations of underwater and underground fluxes, as is shown in Ref. (Klimushin, Bugaev, and Sokalski, 2001). These analytical calculations are greatly facilitated by the parametrizations of all continuous energy losses (using the most recent expressions for muon interaction cross sections) proposed in the present paper.

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