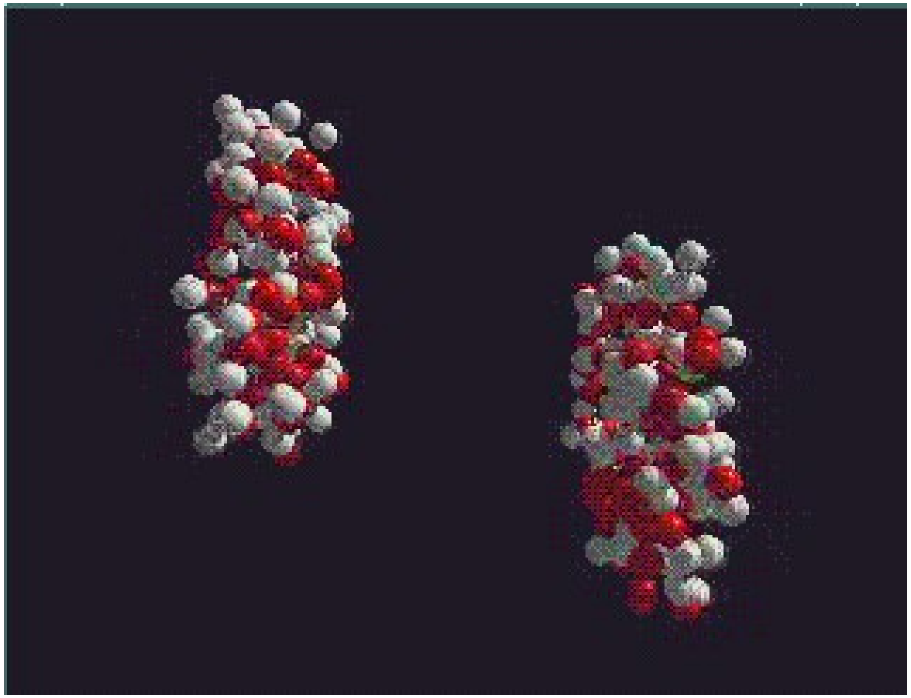
Glauber Modeling in Heavy Ion Collisions

Chris Flores
April 25, 2012



The Executive Overview

Problem: How do you determine the centrality of a nucleus+nucleus collision?

Solution: Use the number of charged particles created in the collision as an indicator...

$$\frac{dN_{ch}}{d\eta} \propto (N_{part} \leftrightarrow N_{coll}) \propto b \rightarrow \mathrm{Centrality}$$

Experimental Observable

Tricky Part

Modeled by Glauber

Known in Simulations

Desired

History of Glauber Model

- Roy Glauber (1925-)

- 1950's: Used quantum mechanical Scattering techniques to analytically describe multi-body scattering of composite systems

- 1970's: Beams of protons and ions were scattered off nuclear targets and Glauber's work was found useful for computing total cross-sections.

- Present: Glauber Montel Carlo Models are used in determining centrality of Heavy Ion Collisions (among other things) -

Fundamental Assumptions

- Two Routes: The (Almost) Impossible way or the Easy Way
  - A "True" **analytical** Glauber Model requires a 2(A+B+1) dimensional integral! For Gold thats over 800 dimensions!

A → Nucleons in A	B → Nucleons in B
2 → Transverse Dims	2 → Longitudinal Dims

  - Monte Carlo – Simply Count N_{part} and N_{coll}
- In both cases the "Optical Limit" is assumed
  - Particles have momenta such that they are deflected very little as they pass through each other

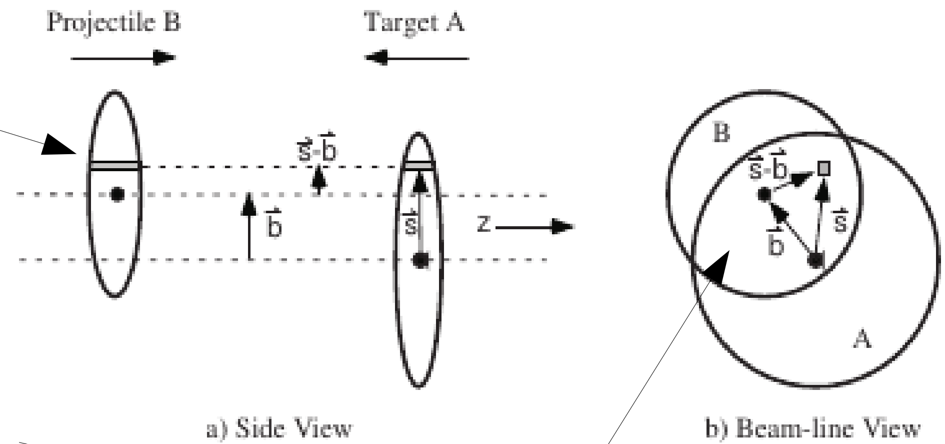
Analytical Glauber Model

- Thickness Function

$$T_A(\vec{s}) = \int \rho_A(\vec{s}, z_A) dz_A$$

- Overlap Function

$$T_{AB}(\vec{b}) = \int T_A(\vec{s}) T_B(|\vec{s} - \vec{b}|) d^2 s$$



- Probability of single NN Collision

$$P_{AB}(b) = T_{AB}(b) \sigma_{NN,inel}$$

- Probability of having n such NN Collisions

$$P(n, \vec{b}) = \binom{AB}{n} \left[T_{AB}(\vec{b}) \sigma_{NN,inel} \right]^n \left[1 - T_{AB}(\vec{b}) \sigma_{NN,inel} \right]^{AB-n}$$

GM: Step 1 – Creating Nuclei

- The size and shape of nuclei have been determined by electron scattering.

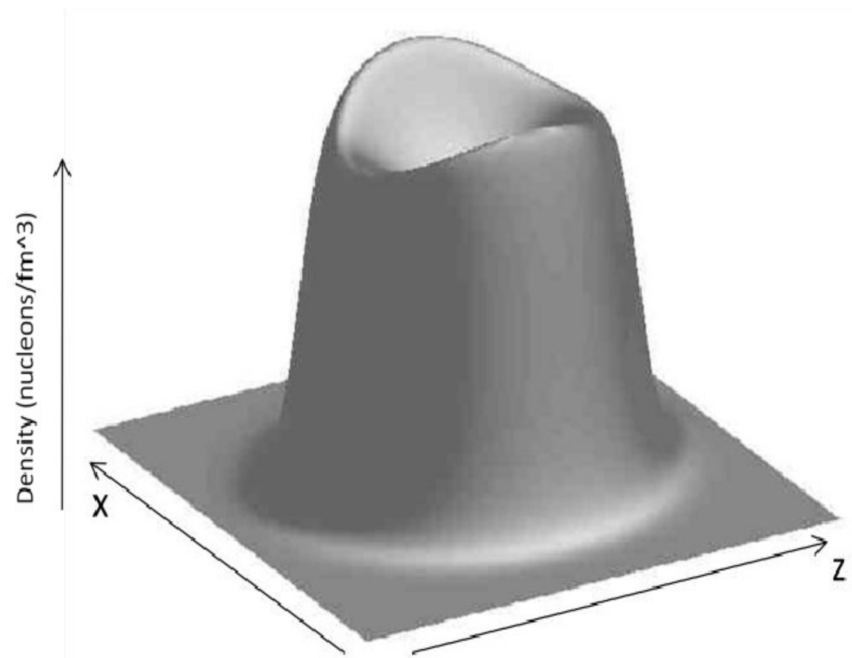
- Nucleons in nuclei are modeled using a Three-Parameter Woods-Saxon distribution (Fermi Distribution) of the form:

$$\rho(r,\theta) = \begin{cases} \rho_0\left(\frac{1+w(\frac{r}{c})^2}{1+e^x}\right) & \text{if } r < c \\ \rho_0\left(\frac{1+w}{1+e^x}\right) & \text{if } r \geq c \end{cases}$$

$$\text{where } x = \frac{r - c(1+\beta_{20}Y_{20}+\beta_{40}Y_{40})}{a(1+\beta_{20}Y_{20}+\beta_{40}Y_{40})}$$

Deformation Parameters

	p_0	w	a	c	B_{20}	B_{40}
Au	.169	0	.535	6.38	-.131	-.031
Pb	.1600	0	.549	6.624	0	0
Cu	.1701	0	.586	4.214	.162	-.006
U	.127	.5	.5	6.8	.254	.052



Analytical Glauber Model

- Total Inelastic
  Cross-Section

$$\sigma_{AB,inel} = 2\pi \int_0^\infty \left\{ 1 - [1 - T_{AB}(b)\sigma_{NN,inel}]^{AB} \right\} bdb$$

- Number of Binary
  Collisions

$$N_{coll}(b) = \sum_{n=1}^{AB} nP(n,b) = ABT_{AB}(b)\sigma_{NN,inel}$$

- Number of Participant (Wounded) Nucleons

$$N_{part}(\vec{b}) = A \int T_A(\vec{s}) \left\{ 1 - \left[1 - T_B(\vec{s} - \vec{b})\sigma_{NN,inel} \right]^B \right\} d^2s +$$

$$B \int T_B(\vec{s} - \vec{b}) \left\{ 1 - [1 - T_A(\vec{s})\sigma_{NN,inel}]^A \right\} d^2s$$

Monte Carlo Glauber Model

- **Step 1:** Create Nuclei
  For each nucleon specify a position vector by drawing random **p** = (x,y,z) location from Woods-Saxon
- **Step 2:** Define Orientations of Nuclei (3 Euler Angles, b)
  → Generate 3 random numbers in range [0,1]

  To uniformly sample phase space transform thusly...

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} = \begin{cases} 2\pi x_1 \\ \arccos(2x_2 - 1) \\ 2\pi x_3 \end{cases}$$

  → Draw Random Impact Parameter from Distribution

$$d\sigma/db = 2\pi b$$

  → Rotate Nucleon position vectors and translate by b

$$\vec{p} = R\vec{p} \qquad\qquad \vec{p} = \vec{p} + (b,0,0)$$

Monte Carlo Glauber

- **Step 3:** Compute N_{part} and N_{coll}

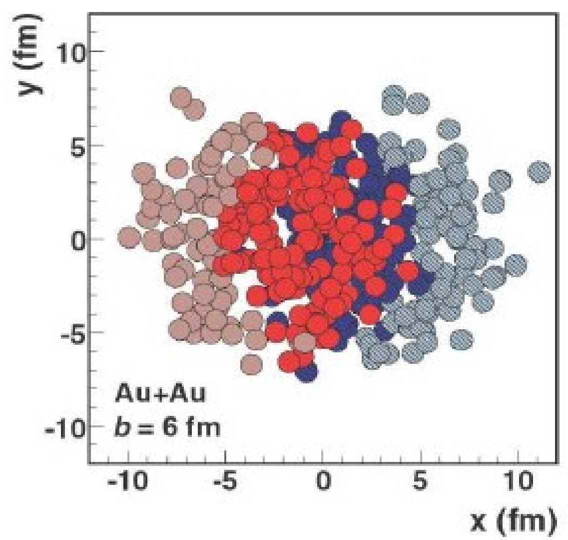
  Compare the distance between each nucleon to the radius of the nucleon $r_{nucleon} = \sqrt{\sigma_{NN}/\pi}$

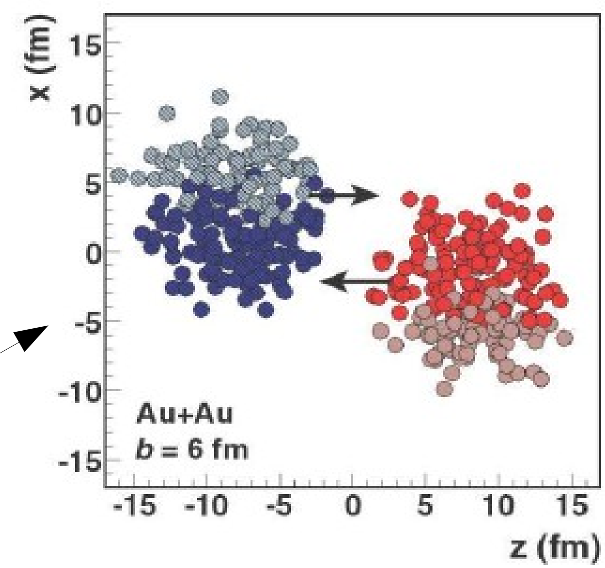
  if ($d_{NN} \leq 2\ r_{Nucleon}$)

  increment N_{coll}

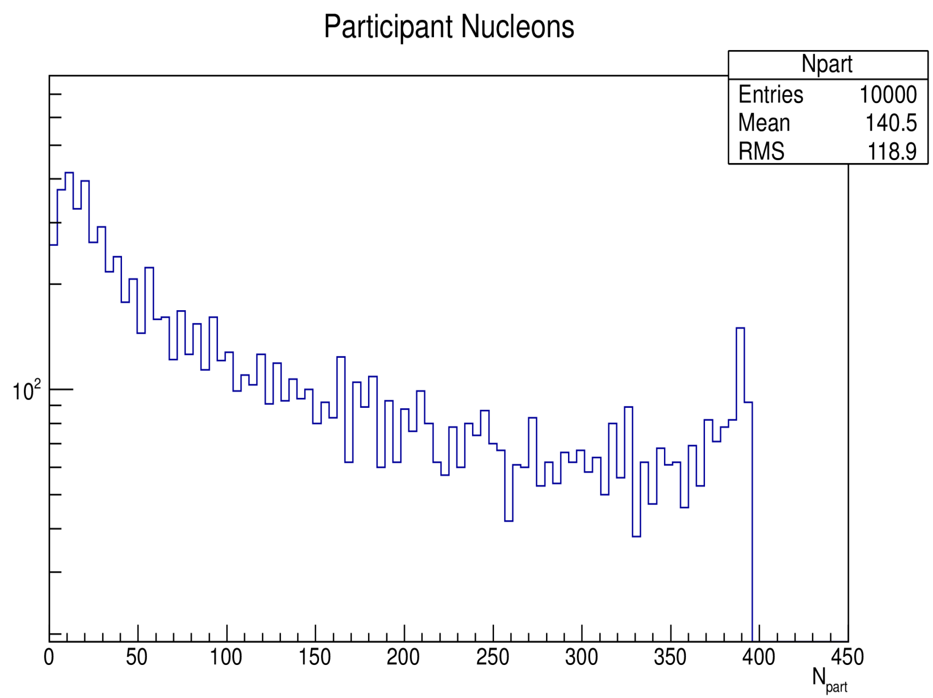
  if ($N_{coll} > 0$)

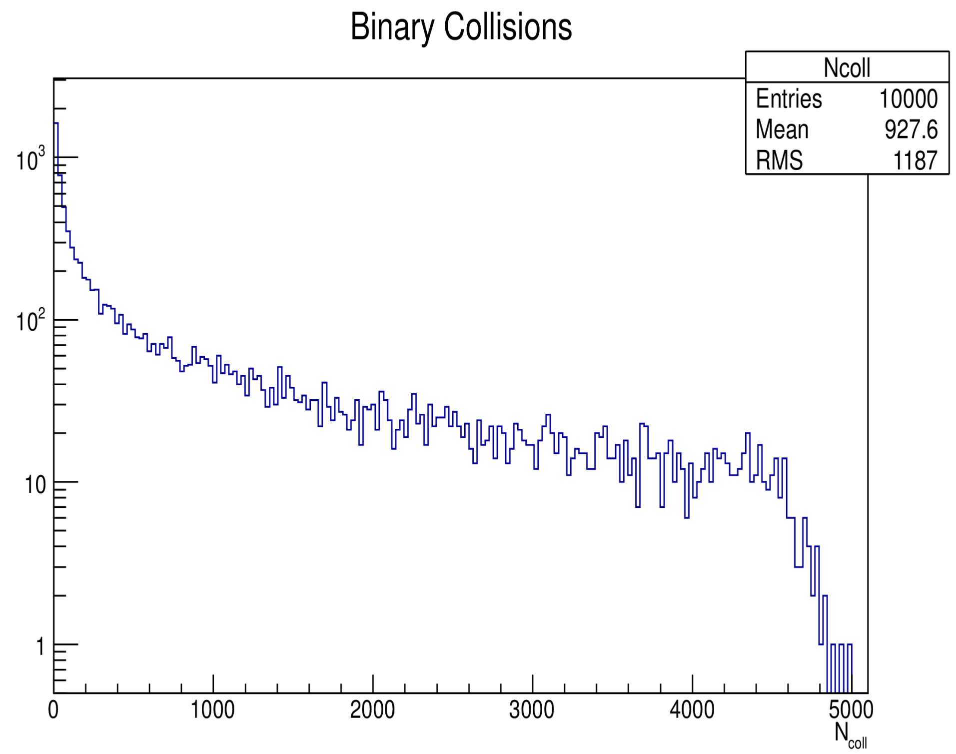
  Increment N_{part}





Glauber MC Results





- Now what? Well, compute geometric cross-section and compare to reported values.

$$\frac{N_s}{N_i} = \frac{N_t \sigma_{geo}}{\pi b_{max}^2} \qquad \sigma_{AuAu,geo} = 6977 \text{ mb}$$

Reported Value at 200GeV 6840 mb

Determining Centrality

- All sorts of Methods:
  1. Use Observed Data A+A

     Fit regions of multiplicity curve with Neg. Binomial
  2. Hardest (?) Possible Way

     Consider each NN collision as a pp collision → study pp multiplicity and then scale

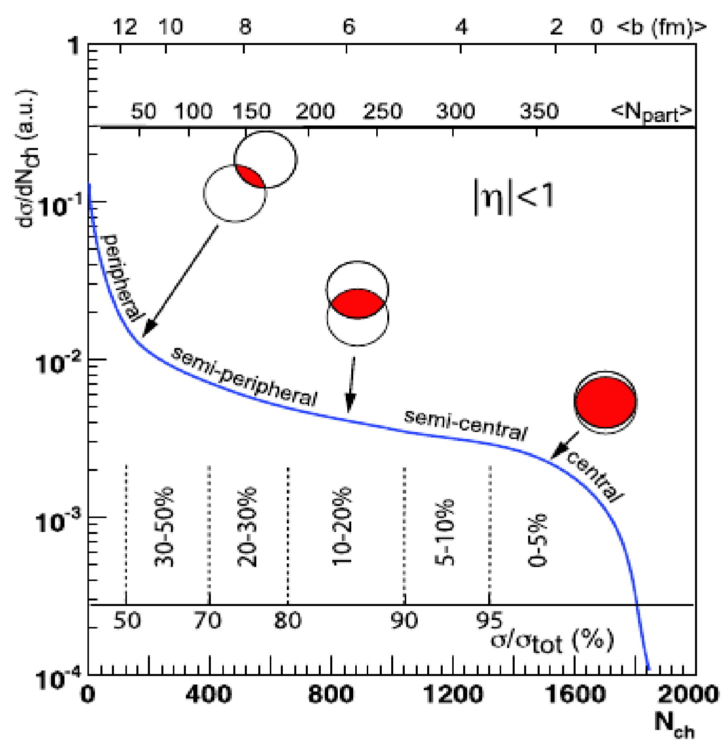
%Central = $B/(R_A + R_B)$

- Many, Many things can be incorporated when connecting Glauber model to Experiment

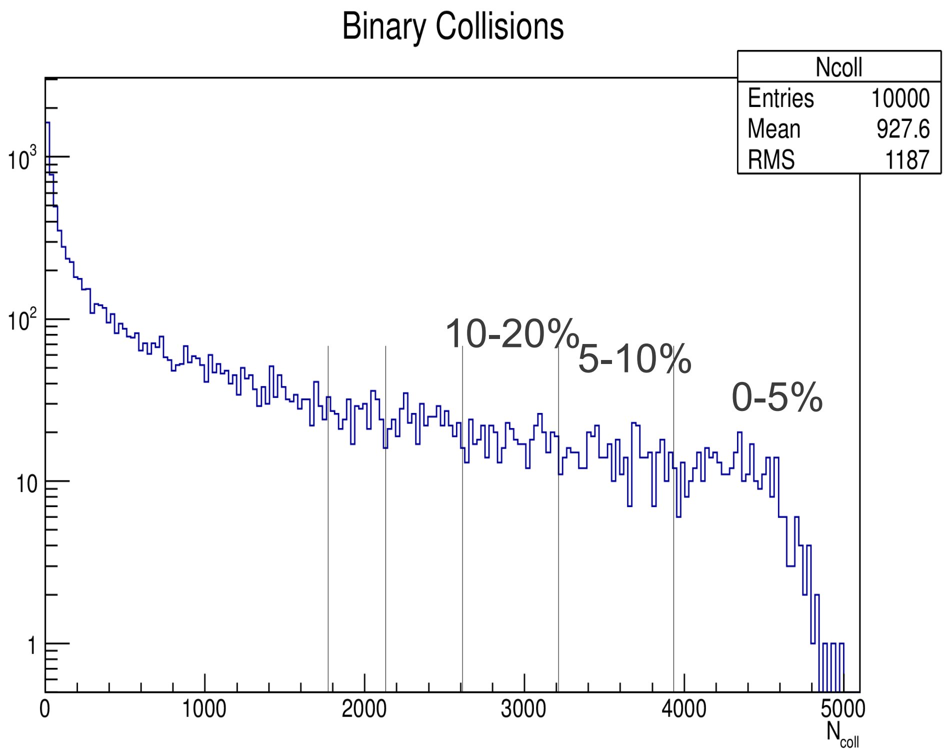
  Event Selection

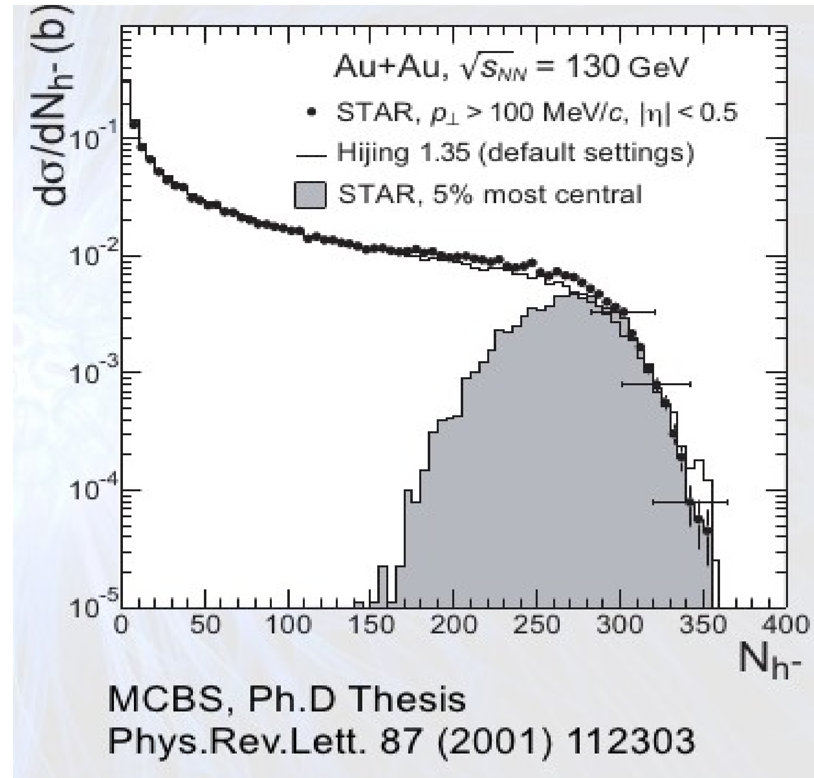
  Detector Acceptance

  Uncertainties in Cross-Section



Using Observed A+A Data





1. Cut on tracks near mid-rapidity
2. Integrate data curve to find top xx% → Fit with Neg. Binomial
3. Use Neg. Binomial to simulate charged particle production from Glauber
4. Integrate charged particle Glauber curve for same top xx%
5. Find the average Impact parameters of Glauber events in top xx% → Centrality

Two Component Model

- The “Hardness” or “Softness” of a NN collision matters

$$\frac{dN_{ch}}{d\eta} = n_{pp}\left[xN_{coll} + (1-x)\frac{N_{part}}{2}\right]$$

x → Sets the “Hardness” scale

N_{pp} → average number of charged particles produced in pp collisions – Found from Neg. Binomial Fits

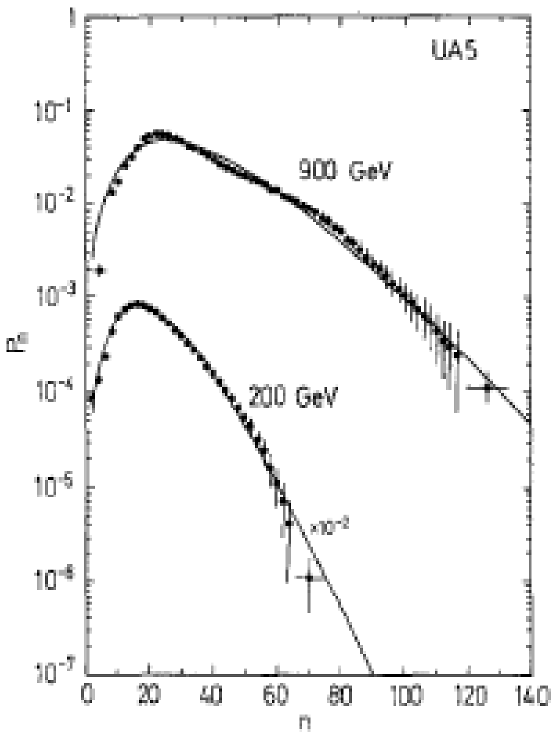


Fig. 15. The best fitted negative binomial distribution compared to the true multiplicity distribution in full phase space at 200 and 900 GeV centre of mass energies. The points are data points taken from Table 1, the lines connect points of the negative binomial distributions

200 GeV				
η_c	$\bar{n}$	k	χ^2_0/NDF	Prob.
0.2	0.96±0.02	1.8±0.2±0.2	9/4	6%
0.25	1.21±0.03	1.9±0.2±0.2	14/6	3%
0.5	2.48±0.06	2.0±0.2±0.1	28/10	0.2%
1.0	5.32±0.08	2.3±0.2±0.1	21/17	23%
1.5	8.1±0.1	2.6±0.1±0.1	25/23	34%
2.0	10.7±0.1	2.6±0.1±0.1	34/30	28%
2.5	13.2±0.1	2.8±0.1±0.1	51/34	3%
3.0	15.4±0.2	3.1±0.1±0.1	60/39	2%
3.5	17.4±0.1	3.4±0.1±0.1	52/41	12%
4.0	18.8±0.2	3.7±0.1±0.1	49/43	25%
4.5	19.8±0.1	4.1±0.1±0.1	56/44	11%
5.0	20.4±0.2	4.3±0.1±0.2	49/45	17%
*	21.2±0.2	4.8±0.2±0.2	50/43	21%

Table 5. Results of fitting the negative binomial distribution with the parameters $\bar{n}$ and k to the multiplicity distributions in central intervals of pseudorapidity, defined by $|\eta| < \eta_c$ and in full phase space (denoted with $*$) at 200 and 900 GeV. The value of χ^2_0 and the number of degrees of freedom (NDF) are given and the corresponding probability $P(\chi^2 > \chi^2_0)$

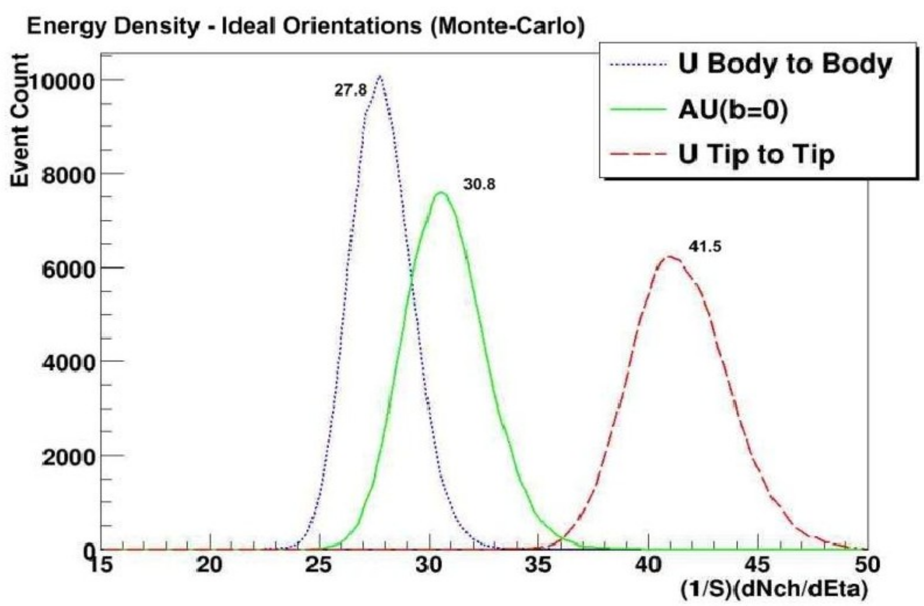
What's up with U+U?

- Energy Density increase

  When Lorentz contracted there is more matter in the same amount of space – Greater Energy Density

Gold
R ~ 7 fm

Uranium
R_{eq} ~ 7 fm

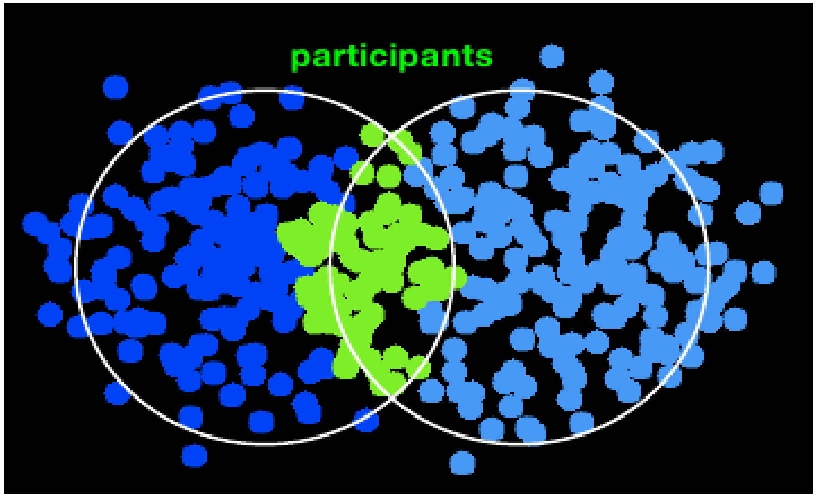


- Additionally, we'll see interesting overlap regions for flow measurements

What else?? → Flow!

- The nuclear overlap region has some eccentricity

$$\epsilon_{std} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2}$$



- Hydrodynamics suggests that any spatial anisotropy in the initial state should lead to a momentum anisotropy in the final state

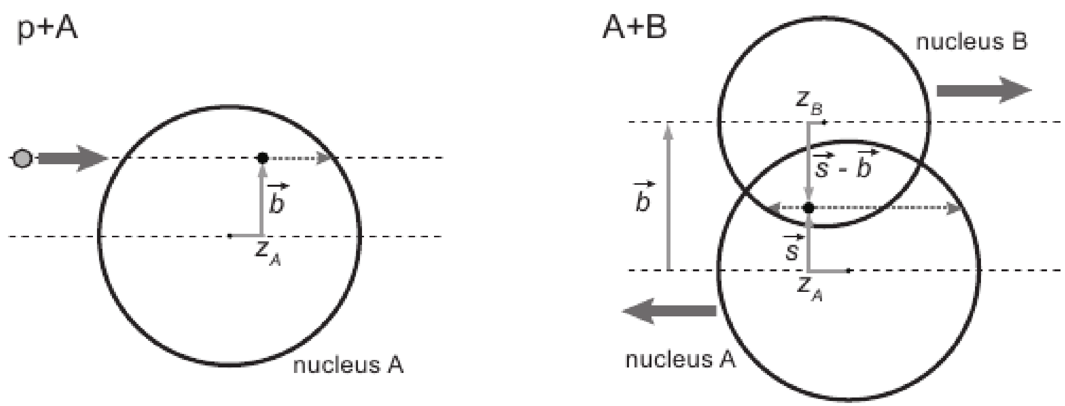
$$v_2 \propto \epsilon$$

- Use Gluaber to understand the initial spatial anisotropy to get a handle on v_2

- Overlap volume fluctuations can also be studied.

J/Psi Suppression

- Glauber can be used to scale from J/Psi suppression in p+A collisions to A+A collisions



- Suppression in p+A

$$S_{\mathrm{p+A}} = \frac{\sigma(p + A \to J/\psi)}{A \cdot \sigma(p + p \to J/\psi)} = \int \mathrm{d}^2 b \, \mathrm{d}z_{\mathrm{A}} \, \hat{\rho}_{\mathrm{A}}(\mathbf{b}, z_{\mathrm{A}}) \, p^{\mathrm{A}}_{\mathrm{surv}}(\mathbf{b}, z_{\mathrm{A}}) \, .$$

- Suppression in A+A

$$\begin{aligned} \frac{\mathrm{d}S_{\mathrm{A+B}}}{\mathrm{d}^2 b}(b) &= \frac{1}{A\,B\,\sigma(p + p \to J/\psi)} \cdot \frac{\mathrm{d}\sigma(AB \to J/\psi)}{\mathrm{d}^2 b} \\ &= \int \mathrm{d}^2 s \, \mathrm{d}z_{\mathrm{A}} \, \mathrm{d}z_{\mathrm{B}} \, \hat{\rho}_{\mathrm{A}}(\mathbf{s}, z_{\mathrm{A}}) \hat{\rho}_{\mathrm{B}}(\mathbf{s} - \mathbf{b}, z_{\mathrm{B}}) \, p^{\mathrm{A}}_{\mathrm{surv}}(\mathbf{s}, z_{\mathrm{A}}) p^{\mathrm{B}}_{\mathrm{surv}}(\mathbf{s} - \mathbf{b}, z_{\mathrm{B}}) \end{aligned}$$

Tons of Info and Sources...

- I'll compile a list of the sources I think are most important/useful on a Glauber model webpage.
- I'm also hoping to set my model up with a web GUI so that you can do your own sims and have all the plots/TTrees in a zip file.
- Happy Glaubering!