Journal of Empirical Finance 58 (2020) 356–368

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

A comparison of non-Gaussian VaR estimation and portfolio construction techniques

David Allen [a,b], Colin Lizieri [c,*], Stephen Satchell [d,e]

[a] Plato Investment Management, 35/60 Margaret Street, Sydney, NSW 2000, Australia
[b] Honorary Associate of the University of Technology, Sydney, Australia
[c] Department of Land Economy, University of Cambridge, 19 Silver Street, Cambridge, CB3 1EP, United Kingdom
[d] The University of Sydney Business School, Codrington Building, The University of Sydney, NSW 2006, Australia
[e] Trinity College, University of Cambridge, Trinity College Cambridge, CB2 1TQ, United Kingdom

ARTICLE INFO

Keywords:
VaR
Tail-risk
Skewed-t copula
Portfolio optimisation
Risk management

ABSTRACT

We propose a multivariate model of returns that accounts for four of the stylised facts of financial data: heavy tails, skew, volatility clustering, and asymmetric dependence with the aim of improving the accuracy of risk estimates and increasing out-of-sample utility of investors' portfolios. We accommodate volatility clustering, the generalised Pareto distribution to capture heavy tails and skew, and the skewed-t copula to provide for asymmetric dependence. The proposed approach produces more accurate VaR estimates than seven competing approaches across eight data sets encompassing five asset classes. We show that this produces portfolios with higher utility, and lower downside risk than alternative approaches including mean–variance. We confirm that investors can substantially increase utility by accounting for departures from normality.

1. Introduction

It has long been established that financial data depart from the idealised normal distribution. Mandelbrot (1963) showed that return distributions of real and financial assets tend to have heavy tails relative to the normal distribution (leptokurtosis), have more extreme negative values than positive values (negative skew), and that volatility tends to cluster (heteroskedasticity).[1] More recently, it has been shown that asset returns exhibit asymmetric tail dependence where correlations increase in times of market stress (Erb et al., 1994; Karolyi and Stulz, 1996; Longin and Solnik, 2001; Alcock and Satchell, 2018). It has also been accepted, at least since the development of *prospect theory* by Kahneman and Tversky (1979), that individuals have non-zero preferences for moments higher than the variance. Critics argue that the mean–variance approach of Markowitz (1952) is therefore inappropriate, and can lead to the mismanagement of risk and substantial reductions in investor welfare.

Several alternatives to mean–variance have been proposed that incorporate higher moments, downside-risk measures, or more realistic utility functions (*inter alia,* Markowitz, 1959; Fama, 1965; Kraus and Litzenberger, 1976; Sortino and van der Meer, 1991). None, however, have managed to dislodge the mean–variance approach from its central role in practice. Indeed, the mean–variance approach remains almost ubiquitous in the financial sector (Fabozzi et al., 2007) despite its perceived shortcomings.

Chen and Fan (2006) formalise a family of models which specify the conditional mean and conditional variance of a multivariate time series parametrically, but specify the multivariate distribution of the standardised innovation semi-parametrically as a parametric copula evaluated at non-parametric marginal densities. The authors coin the phrase semiparametric copula-based

* Corresponding author.
E-mail addresses: david.allen@plato.com.au (D. Allen), cml49@cam.ac.uk (C. Lizieri), ses999gb@yahoo.co.uk (S. Satchell).

[1] See also the special issue of *Journal of Empirical Finance* edited by Dufour et al. (2010).

https://doi.org/10.1016/j.jempfin.2020.07.001
Received 14 August 2018; Received in revised form 25 January 2020; Accepted 1 July 2020
Available online 8 July 2020

multivariate dynamic models (SCOMYD) and provide some insightful results on the large sample properties and model selection for this class of models. While Chen and Fan (2006) use the empirical cumulative distribution function for the univariate marginals, we use kernel smoothing with Pareto tails. We also use the skewed-t copula which enables us to model asymmetric dependence patterns in high dimensions.

We build on and extend previous work in the following ways. We propose an approach to risk management and portfolio construction that accommodates all four stylised facts identified above and that can be utilised even when the number of assets is large.

We use Value at Risk (VaR) as an example to illustrate these more general principles of risk management. Most existing methods that capture all four stylised fact employ copula functions that do not readily generalise to higher dimensions. While there is, for example, a well-developed literature comparing VaR estimation approaches, there is a paucity of evidence evaluating the comparative efficacy of different risk modelling approaches when applied to portfolio construction. We compare our proposed approach to seven techniques prominent in the literature or the financial industry and, in contrast to most prior work, test our findings against eight separate data sets, incorporating five asset classes, providing results relevant to practical investment situations. We show that the proposed approach produces significant uplifts in utility certainty equivalence relative to competing methods, including mean–variance, with important implications for market practice.

This paper proceeds as follows. In Section 2, we provide an overview of alternative non-Gaussian approaches that have been used for portfolio construction and a brief overview of the VaR estimation literature. The third section develops a scalable model that accounts for the four stylised facts. Section 4 discusses our evaluation methodologies, benchmark approaches, and data. Section 5 examines the VaR forecasting performance of the respective models and presents the results of the out-of-sample portfolio rebalancing analysis. Section 6 concludes and sets out practical implications of the results.

2. VaR estimation approaches and non-Gaussian portfolio construction

While used extensively in the financial sector, VaR estimation approaches have been heavily criticised. For example, Artzner et al. (1997) argue that VaR is not a coherent measure of risk. Further, VaR does not account for the fact that the same dollar loss can have very different economic evaluations, depending on business conditions (Aït-Sahalia and Lo, 2000). There is a consensus that models that account for stochastic volatility outperform static models (Berkowitz and O'Brien, 2002; McAleer and Da Veiga, 2008; Skoglund et al., 2010; Moreira and Muir, 2017) and that models that employ non-Gaussian innovations tend to outperform Gaussian models (McAleer and Da Veiga, 2008). Further, there is a general consensus that single index approaches tend to outperform portfolio approaches (Berkowitz and O'Brien, 2002; Brooks and Persand, 2003; Bauwens et al., 2006; Christoffersen, 2009; McAleer, 2009).[2] Interestingly, survey evidence suggests that relatively simplistic historical simulation and its variant. the filtered historical simulation approach. are the most widely used methods at commercial banks (Christoffersen, 2009; Pérignon and Smith, 2008; Gurrola-Perez and Murphy, 2015).

Several authors have attempted to increase investor utility by incorporating non-Gaussian characteristics into their portfolio construction approach. Patton (2004) finds that accounting for asymmetry and skewness using skewed-t marginals and a rotated Gumbel copula leads to a small improvement in realised utility relative to a passive 1/N strategy. Jondeau and Rockinger (2005) combine a Taylor series approach with DCC-GARCH and find that the approach significantly outperforms mean–variance for investors with high levels of risk aversion (see also Dark, 2018). Adler and Kritzman (2007) show that full-scale optimisation in conjunction with S-shaped and bi-linear utility enhances investor utility. Aït-Sahalia and Brandt (2001) model portfolio weights as a function of asset characteristics under CRRA utility, obviating the need to model joint asset returns. Their approach leads to substantial uplifts in utility that persist out of sample (Brandt et al., 2009). This conclusion however is challenged by DeMiguel et al. (2009).

Alcock and Hatherley (2009) employ Gaussian marginals in combination with the Clayton copula. They find that accounting for asymmetric dependence leads to a significant uplift in average returns and higher returns in down-markets. Viebig and Poddig (2010) develop models that account for all four stylised facts[3]; however, generalising the Archimedean copula approach used in these studies to higher dimensions ($n > 2$) requires highly restrictive assumptions. Martellini and Ziemann (2010) use a Taylor Series approach to investigate the effect of incorporating investor preferences for moments higher than two. The authors find that incorporating higher moments does not improve investor welfare unless sophisticated shrinkage estimators of higher moments are employed. Xiong and Idzorek (2011) use the truncated Lévy-flight distribution to capture skew and heavy-tails and find that the approach outperforms mean–variance marginally during the 2007–8 financial crisis (see also Blanchett and Ratner, 2015, for a similar approach for income-based portfolios and Bernardi et al., 2017 for a banking application).

It is notable that of the out-of-sample studies discussed above, it is rare to find one that uses more than one data set to test models. Because financial data are inherently noisy, and since the extreme market conditions that provide stress tests of an approach are by definition rare, we argue for employing multiple long history data sets. A further limitation in the majority of studies is the absence of formal tests to determine whether the uplifts in investor welfare are statistically significant.

[2] The results of McAleer and Da Veiga (2008) however are mixed .

[3] Patton (2004) uses time varying copulas and moments up to the fourth order that are functions of exogenous variables. Viebig and Poddig (2010) use GARCH, extreme-value theory and Archimedean copulas.

3. A scalable multivariate non-Gaussian approach

In this section, we outline an approach that can accommodate heavy-tails, skew, stochastic volatility, and asymmetric dependence into the risk management process, using VaR methods to illustrate the benefits. Our approach is an extension of an existing body of work that uses GARCH models and extreme-value theory in conjunction with copulas (Nystrom and Skoglund, 2002; Ghorbel and Trabelsi, 2009; Viebig and Poddig, 2010). Copulas are a valuable addition to the econometrician's toolbox because they enable the researcher to model the dependence structure separately from the univariate densities using the inference function for margins (IFM) method of Joe and Xu (1996). A parallel to the IFM method can be seen in the estimation of constant and dynamic conditional correlation (CCC and DCC) multivariate GARCH models (Bollerslev, 1990; Engle, 2002).

In our approach, we use a univariate exponential GARCH (EGARCH) procedure to capture stochastic volatility and the leverage effect, the generalised Pareto distribution (GPD) to capture asymmetry and heavy-tails in the GARCH residuals, and the skewed-t copula to allow for asymmetric tail dependence. Hereinafter, we refer to this as the GSEV approach.[4] We break the estimation problem down into three parts: the fitting of the GARCH processes, the estimation of the univariate marginals, and the modelling of dependence structure. There are benefits from this sequential approach. Filtering with a GARCH process yields a series that is iid which facilitates fitting a parametric distribution. The estimation procedure is also simplified and accelerated with a minimal loss in efficiency (Joe, 2005; Patton, 2006).

We use the exponentially-weighted GARCH developed by Nelson (1991) with Student-t innovations. Rather than imposing a distribution on the data, we use extreme-value theory (EVT). Extreme-value theorem provides the theoretical basis for how the tails of all iid distributions behave asymptotically. EVT has been used to model rare events in a variety of fields. We follow McNeil and Frey (2000), and Nystrom and Skoglund (2002), and fit piecewise distributions to the univariate GARCH residuals. Specifically, we fit GPDs to the upper and lower deciles of the data, and a Gaussian kernel to the inner 80% of the density using maximum likelihood estimation. We model the dependence structure of the GARCH residuals using the skewed-t copula. The skewed-t copula is particularly attractive because it can accommodate tail dependence, asymmetric dependence between the upper and lower tails, and heterogeneous dependence across asset pairs. Further it is scalable in high dimensions. We employ the skewed-t distribution of Demarta and McNeil (2005) that follows from the Generalised Hyperbolic (GH) distribution proposed by Barndorff-Nielsen (1976).[5] We simulate the skewed-t copula from the multivariate skewed-t distribution through simulation (see Sun et al., 2008).

We estimate and simulate the model using the following steps:

Algorithm 1. GSEV approach: estimation and simulation

I. Estimate an AR(1)-EGARCH(1,1) model for each asset i, and obtain parameter estimates, θ^i_{EG}, residuals, $\varepsilon_{i,t}$ and conditional sigmas, $\sigma_{i,t}$.
II. Calculate the standardised residuals, $\epsilon_{i,t} = \varepsilon_{i,t}/\sigma_{i,t}$.
III. Fit the univariate piecewise distributions, $\hat{f}_{pw,i}$ to $\epsilon_{i,t}$:
   i. Fit univariate GPDs to the upper and lower 10% of the densities of $\epsilon_{i,t}$, yielding parameters θ^i_{GP}
   ii. Fit the interior 80% of the distribution with a Gaussian smoothing kernel.
IV. Fit univariate skewed-t distributions to each vector of residuals from step II by MLE.
V. Estimate the covariance matrix $\hat{\Sigma}$ given by:

$$\hat{\Sigma} = \frac{\upsilon-2}{\upsilon}\left(cov\left(X\right) - \hat{\beta}\hat{\beta}'\frac{2\upsilon^2}{(\upsilon-2)^2\left(\upsilon-4\right)}\right)$$

See Demarta and McNeil (2005)

where υ and β are parameters of the skewed-t distributions estimated in IV. *Cov(X)* is the sample covariance matrix.
VI. Draw N independent d-dimensional vectors from the multivariate Gaussian distribution defined by:

$$Z \sim N\left(0, \hat{\Sigma}\right)$$

VII. Draw N independent random numbers from the inverse gamma distribution defined by:

$$W \sim IG\left(\frac{\upsilon}{2}, \frac{\upsilon}{2}\right)$$

VIII. Substitute Z and W into the following function to yield multivariate skewed-t variables, $X \sim mskt\left(\hat{\upsilon}, \hat{\mu}, \hat{\Sigma}, \hat{\beta}\right)$:

$$X = \mu + \beta W + \sqrt{W}Z$$

See Demarta and McNeil (2005)

[4] In place of the rather cumbersome EGARCH/skewed-t/extreme-value approach.

[5] There is a bewildering array of variants of the skewed-t distribution including Hansen (1994), Fernandez and Steel (1998), Branco and Dey (2001), Bauwens and Laurent (2005), Azzalini and Capitanio (2003), Jones and Faddy (2003), Sahu et al. (2003), and Patton (2004). Each of these distributions have polynomial upper and lower tails, so while they can fit heavy-tailed data well, they cannot accommodate significant asymmetry (Aas and Haff, 2006).

IX. Approximate the cumulative distribution functions $\hat{F}_{sk,j,i}(x)$, for the univariate marginals $\hat{f}_{sk,j,i}$ by numerical integration.[6]
X. Transform X, based on VIII, into uniformly distributed variables using the univariate cumulative distribution functions:

$$U_{sk,j,i} = \hat{F}_{sk,j,i}(X_{j,i})$$

XI. Convert U_{sk} to innovations, I_{sk}, using the inverse cumulative distribution functions for the univariate piecewise distributions developed in step III:

$$I_{sk} = \hat{F}^{-1}_{pw,i}(U_{sk})$$

XII. Simulate from an AR(1)-EGARCH(1,1) model for each asset i using innovations, I, and parameters θ^{i}_{EG}

This algorithm is rapid, and can be applied to problems with large dimensions.

4. Data and methodology

We compare our proposed approach to portfolio construction models commonly used by practitioners or that are prominent in the literature across eight data sets including five asset classes.

4.1. Risk model forecast evaluation

We re-estimate the non-elliptical models each day in the sample and forecast the daily VaR as the 0.5% quantile of the equally weighted series of the asset returns. We follow McNeil et al. (2005) and use 1000-day estimation windows. To estimate the VaR of the elliptical models, we use the closed-form solutions. We evaluate the VaR estimate over a one-day horizon, as is common in the VaR validation literature (Skoglund et al., 2010). Specifically, we evaluate the quality of each VaR forecast using the unconditional coverage (UC), serial independence (SI), and conditional coverage (CC) tests proposed by Christoffersen (1998) and used throughout the VaR model validation literature (for example, McNeil et al., 2005; Ghorbel and Trabelsi, 2009).

4.2. Out-of-sample portfolio rebalancing

We quantify the economic benefits of the portfolio construction approaches in an out-of-sample portfolio rebalancing framework (Solnik, 1993; Fletcher, 1997). We employ conditional value-at-risk (CVaR), also known as expected shortfall, as our primary risk measure. The advantages of CVaR are numerous (see Acerbi and Tasche, 2002). CVaR is a coherent measure of risk (see Artzner et al., 1997, 1999) and is amenable to optimisation. The Basel Committee on Banking Supervision have recently argued for the adoption of conditional value at risk (CVaR), over VaR.

We follow Lucas and Klaasen (1998), Campbell et al. (2001), and Chekhlov et al. (2005) and maximise the expected return of the portfolio, subject to a constraint on risk, in our case CVaR.

$$\arg max\ w'\mu \quad s.t. \quad w \geq 0, w'i = 1 \quad \text{and} \quad CVaR(w) = CVaR_{targ} \tag{1}$$

where w is a vector of portfolio weights, μ is a vector of expected returns, i is a vector of ones, and $CVaR(.)$ is a function that estimates the conditional value-at-risk, with $\text{CVaR}_{\text{targ}}$ being a (potentially varying) target level of risk. Specifically, we use the technique pioneered by Rockafellar and Uryasev (2000) which calculates VaR and optimises CVaR simultaneously. To ensure our results are not confounded by our return forecasts, we use identical expected return forecasts for each asset.

We set the one-day 99% CVaR target to 2% and prohibit shorting. This corresponds to the historical 99% one-day CVaR of the average pension fund allocation invested 50% in equities and 50% in fixed-income and cash.[7,8] Each model is re-estimated each day for each data set using rolling 1000-day estimation windows. An optimal portfolio is then produced for each model for each data set. We then examine the daily returns of the respective portfolio construction models through time.

Our primary performance measure is the ratio of the mean return in excess of the risk free rate divided by the realised $CVaR_{\beta}$. In the literature, there is a near absence of studies evaluating the statistical significance of the difference in performance between non-Gaussian and benchmark portfolio construction techniques. For Gaussian performance metrics such as the Sharpe ratio, it is trivial to evaluate the statistical significance of the difference between two strategies using the Jobson and Korkie (1981) t-statistic with Memmel's (2003) correction and we do so in the current work. However, a similar developed literature to evaluate the statistical significance of non-Gaussian measures is lacking. We employ a block bootstrap technique to estimate the standard-errors of our performance measures following Politis and White (2004).

Performance ratios allow for many insights. They do not however, allow us to readily gauge the uplift in investor welfare of a portfolio construction technique. We follow West et al. (1993), and Fleming et al. (2001) and estimate the value-added, ϕ, as

[6] Since there is no closed form solution for the cdf of this family of skew-t distributions we resort to numerical integration. Specifically, we incrementally compute 10,000 points in the internal −100 to 100 using the pdf (where there is a closed form solution). This produces a discrete cdf which we can use to estimate specific points based on interpolation.

[7] This is estimated based on a 50% weight in the S&P 500 index and a 50% weight in the Barclays Aggregate Government Bond index for the period 5/1990 to 10/2013.

[8] See Ibbotson and Kaplan (2000).

Table 1
Benchmark methodologies. Table 1 provides an overview of the seven benchmark methodologies.

Methodology	Type	Components	Key references
Gaussian	Unconditional, Parametric	Gaussian distribution	Bloomfield et al. (1977) Jobson and Korkie (1981)
Student-t	Unconditional, Parametric	Student-t distribution	Lauprete et al. (2002) Hu and Kercheval (2007)
Historical Simulation	Unconditional, non-Parametric	Bootstrapped from historical distribution	Efron and Tibshirani (1993), Sortino (2010)
Gaussian Marginals/ Clayton-Copula	Unconditional parametric	Gaussian marginals and the Clayton copula	Alcock and Hatherley (2009), Clayton (1978)
EWMA	Conditional, Parametric	Exponentially-weighted covariance matrix	*RiskMetrics Technical Document, 1996*
Filtered Historical Simulation	Conditional, non-Parametric	EGARCH simulation applied to bootstrapped returns	Barone-Adesi et al. (1998), Hull and White (1998)
GGEV	Conditional, Parametric	EGARCH Generalised Pareto distribution to fit tails Gaussian Copula	Nelson (1991), Nystrom and Skoglund (2002)

the value that equates the expected utility of the proposed approach with a benchmark approach, in our case the Gaussian (mean–variance) model. We assume investors' true utility is power utility which displays constant relative risk aversion, rewards positive skew, and penalises excess kurtosis. We use three levels of risk aversion, $\gamma = 5$, $\gamma = 10$, and $\gamma = 15$, consistent with estimates given in the literature (for example, Jondeau and Rockinger, 2005).

Definition 1 (*Value-added: Power Utility*)**.** The value-added refers to the value, ϕ_P, that solves the following equation, where γ refers to the coefficient of risk aversion, and $x_{m,t}$ and $x_{b,t}$ refer to the return of the model and benchmark portfolios in period t respectively.

$$\frac{1}{T}\sum_{t=1}^{T}\frac{\left(1+x_{m,t}-\phi_P\right)^{\gamma}}{1-\gamma}=\frac{1}{T}\sum_{t=1}^{T}\frac{\left(1+x_{b,t}\right)^{\gamma}}{1-\gamma}$$

4.3. Benchmark models

Table 1 provides an overview of the benchmark portfolio construction models.

4.4. Data

Our data sets encompass multiple periods of market turbulence, including the 1987 stock market crash, the 1997 Asian financial crisis, the Russian debt default and the demise of Long-Term Capital Management in 1998, the dot-com crash in 2000, the 2001 terrorist attacks in the US, and the 2007–8 financial crisis. We limit ourselves to value-weighted indices to help ensure that the strategies are investible in practice. The first data set replicates the asset allocation problem of an institutional investor, and is comprised of the following asset classes: US equities (S&P 500), international equities (MSCI EAFE), US corporate bonds (Barclays Corporate Bond index), a broad-based commodities index (S&P/GSCI), and US REITs (FTSE NAREIT US). The second, third, and fourth data sets include US sectors at three levels of granularity and have been selected to replicate the investment universe of an equity portfolio manager. The fifth data set includes the ten largest US equities that have been continuously listed since 1985. The sixth and seventh data sets have been chosen to replicate the investment universes of two funds of funds managers that allocate between large-cap and small-capitalisation funds, and value and growth funds, respectively. Our eighth data set includes the three Fama–French factors (Fama and French, 1993), the momentum factor used in the four factor market of Carhart (1997) and the short-term reversal factor (Jegadeesh, 1990) used by statistical arbitrage hedge funds. This data set captures the opportunity set of a fund of funds hedge fund manager allocating between different hedge fund styles. All our return series are daily, include dividends where relevant, and are in USD.

5. Results

5.1. The data sets

Table 3 provides the average summary statistics for the assets in each of the eight data sets. We divide the data sets into overlapping 1000-day sub-periods and conduct Jarque–Bera tests for normality. In roughly 80% of sub-periods, normality is rejected for each asset class. There is pronounced autocorrelation in the absolute value of returns consistent with volatility clustering.

We quantify the degree of asymmetric dependence (AD) using exceedance correlations. To determine whether or not the observed asymmetry is statistically significant, we use the signed J-test of Alcock and Hatherley (2009) with exceedance thresholds of $\delta = [0.2, 0.4, 0.6, 0.8, 1.0]$. We calculate the signed J-statistic for overlapping 260 day sub-periods. We estimate the tail dependence of

Table 2
Data set descriptions.

Data set	Description	Source	Period
1. 5 Asset classes	US equities (S&P 500), EAFE equities (MSCI), corporate bonds index (Barclays), commodities (GSCI), US REITs (FTSE NAREIT)	DataStream, Bloomberg	12/89 – 5/13
2. 5 US industries	Value-weighted	K.R. French	12/89 – 5/13
3. 10 US industries	Value-weighted	K.R. French	12/89 – 5/13
4. 30 US industries	Value-weighted	K.R. French	12/89 – 5/13
5. 10 equities	Largest 10 US equities continuously listed for the period 31/12/1985 to 31/5/2013	Factset, Ex-share database	12/84 – 5/13
6. 5 size portfolios	Value-weighted quintiles sorted by market capitalisation	K.R. French	12/89 – 5/13
7. 5 book-to-marketportfolios	Value-weighted quintiles sorted by book-to-market ratio	K.R. French	12/89 – 5/13
8. Fama–French factors	Market, size, value, momentum, short-term reversal	K.R. French	12/89 – 5/13

Table 3
Average summary statistics: All data sets.

	Asset allocat.	5 Indust.	10 Indust.	30 Indust.	10 Stocks	Size factors	Value factors	Fama–French factors
Annualised ret.	5.9	11.6	11.5	11.0	10.8	11.1	11.9	9.0
Standard-dev.	20.5	19.5	20.3	22.9	30.6	18.2	18.1	12.4
Skewness	−0.6	−0.5	−0.4	−0.3	−0.1	−0.6	−0.7	−0.2
Kurtosis	16.3	15.5	14.4	12.5	20.3	12.1	17.6	18.0
Maximum	9.6	12.2	12.8	13.8	22.0	9.8	11.1	8.0
Minimum	−10.0	−17.5	−17.4	−17.7	−25.6	−13.9	−17.4	−9.8
P(JB = 0)	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
% JB failures	80	79	81	80	85	82	80	79
% sig. skew.	43	46	46	46	50	57	46	57
% sig. kurt.	82	79	81	80	82	82	79	78
$\rho(r_t, r_{t-1})$	0.04	0.01	0.02	0.03	−0.04	0.04	−0.00	0.10
$\rho(\lvert r_t\rvert, \lvert r_{t-1}\rvert)$	0.21	0.17	0.18	0.20	0.23	0.28	0.18	0.25

Notes: Table 3 provides the median summary statistics for the eight data sets listed in Table 2. The time period is 1/1983 to 12/2012 for all data sets except for the asset allocation problem (12/1989 to 5/2013) and the stock selection problem (12/1983 to 3/2013). The annualised total returns are calculated geometrically. The percentage of Jarque–Bera failures refers to the proportion of 1000-day sub-periods where normality is rejected at the 5% level. The mean positive (negative) exceedance ρ refers to the average exceedance correlation above (below) 1.5 standard-deviations. $\rho(r_t, r_{t-1})$ refers to the autocorrelation in daily returns at one lag. $\rho\left(\lvert r_t\rvert, \lvert r_{t-1}\rvert\right)$ refers to the autocorrelation in the absolute value of daily returns at one lag.

Table 4
Asymmetric dependence: All data sets.

	Asset allocation	5 industries	10 industries	30 industries	10 stocks	Size portfolios	Value portfolios	Fama–French factors
Mean ‘bear’ ρ	0.13	0.67	0.57	0.48	0.30	0.81	0.79	0.16
Mean ‘bull’ ρ	0.17	0.55	0.44	0.36	0.18	0.70	0.72	0.21
% sign. −AD	8.33	1.72	2.15	4.30	7.33	0.34	0.34	11.03
% sign. + AD	5.00	0.34	1.38	2.43	4.86	0.00	0.00	12.07

Notes: Table 4 provides the mean exceedance correlations for each data set with respect to the S&P 500. The upside and downside exceedance correlations are estimated using returns in excess of one standard deviation from the mean using overlapping 260-day sub-periods. Statistical significance is determined using the signed J-test at the 5% level.

each asset with the S&P 500. The mean ‘bear’ (downside) and ‘bull’ (upside) correlations are shown in Table 4 for absolute returns in excess of one-standard-deviation. In six out of eight of the data sets, the mean bear correlation exceeds the mean bull correlation. In seven out of the eight data sets, statistically significant downside asymmetric dependence is more common than upside asymmetric dependence. The Fama–French data exhibit asymmetric dependence most frequently, approximately twice as often as would be expected by chance.

5.2. Value at risk model performance

If the models are well calibrated, we would expect 0.5% VaR violations, or one every 200 days. It is apparent from Table 5 that the elliptical models systematically underestimate tail risk. For the Gaussian model there are, on average, 260% too many violations across all data sets. Similarly, the Student-t and EWMA models yield 160% and 180% too many violations respectively. The FHS and GGEV approaches produce 50% and 70% too many violations whereas the GSEV approach produces only 20% too many. We can reject the hypothesis of serial independence for all of the unconditional models for all of the data sets, and only for one of the data sets for the FHS, GGEV, and GSEV approaches. The GSEV approach outperforms the GGEV approach consistently. The GGEV approach produces 66% too many violations, while the GSEV approach only produces 20% too many. Further, the conditional coverage p-values of the GGEV approach exceed 5% in six out of eight of the data sets and only in one data set for the GSEV approach. The only difference between the GGEV and GSEV approaches is the use of the Gaussian copula instead of the skewed-t copula.

Consistent with prior literature we have shown that conditional approaches outperform unconditional approaches. We have also demonstrated the benefits of using extreme-value theory to produce more accurate VaR predictions. Finally, we have highlighted the importance of accounting for asymmetric dependence patterns between assets where the skewed-t copula outperforms the Gaussian copula. The GSEV approach incorporates all of these elements to yield reliable VaR forecasts at 99.5% confidence level. The finding that the GARCH-EVT-copula approach dominates other methods is consistent with Ghorbel and Trabelsi (2009).

5.3. Out-of-sample portfolio rebalancing

Tables 6 and 7 summarise the portfolio performance of the models. For every data set, the out-of-sample CVaRs of the Gaussian, Student-t, HS, EWMA models are markedly higher than the 2% target. In contrast, the FHS, GGEV, and GSEV approaches generate CVaRs that are quite close to the target level, with the GSEV approach producing the smallest risk forecast error on average. The GSEV approach produces the smallest risk forecast error for seven out of eight of the data sets. This mirrors our findings in Section 5.2. The conditional models also provide a consistently lower maximum drawdown,[9] defined as the peak to trough return, a metric commonly used by practitioners. The GSEV approach produces the maximum drawdown with the smallest absolute value for all eight investment problems.

The unconditional approaches, Gaussian, Student-t, historical simulation, and the Clayton copula approach generate similar Sharpe ratios to each other. The relatively unimpressive performance of the Clayton copula model runs counter to Alcock and Hatherley (2009) who show a substantial performance uplift relative to the Gaussian case. The Alcock and Hatherley (2009) work considers triplets of industry indices whereas the current work considers larger investment sets. Given that the standard Clayton copula imposes an identical dependence structure across asset pairs, it is probable that the performance of the approach degrades as the number of assets and complexity increases. The GSEV approach generates the highest average Sharpe ratio of 1.13 across all data sets. The Sharpe ratio of the GSEV approach is statistically greater than for the Gaussian model for each data set at the 1% level. On average the uplift in the Sharpe ratio is 125%.

The GSEV approach generates higher mean/CVaR ratios than the Gaussian model for every data set. In addition, the improvement is statistically significant at the 99% level in all cases. The 182% average increase in mean/CVaR ratio is even larger than the increase in the Sharpe ratio. The GSEV approach also dominates the other benchmark models. In six out of the eight data sets the GSEV approach produces the highest mean/CVaR ratio, whilst the GGEV approach produces the highest mean/CVaR ratio in two of the data sets. This provides further evidence of the importance of accounting for asymmetric dependence. As we noted in Section 5.1, the two investment problems with the most frequent statistically significant asymmetric dependence are the asset allocation and the Fama–French problems. It is perhaps no surprise that these are the two data sets where the GSEV approach adds the most value relative to the GGEV approach. For the Fama–French data set, the increase in the mean/CVaR ratio relative to the GGEV approach is 13% and statistically significant. This is consistent with the Hong et al. (2007) conclusion that incorporating correlation asymmetries leads to substantial uplifts in welfare.

In Table 8 we quantify the economic significance of the uplift in performance using Definition 1. The 1/N rule and the Clayton copula model subtract value relative to the Gaussian approach. The t-distribution and historical simulation approaches generate meaningful amounts of incremental economic value. This makes sense given that these are heavy-tailed models and the power utility function rewards distributions with less tail risk. The filtered historical simulation, GGEV, and GSEV approaches again add the most value. Finally, the GSEV approach, which accounts for asymmetric dependence, outperforms the GGEV approach, which does not.

The value-added of the FHS, GGEV, and GSEV approaches appears to be larger than the gains in welfare that have been reported in much of the literature to date (for example, Patton, 2004; Xiong and Idzorek, 2011; Martellini and Ziemann, 2010). Brandt et al. (2009) report similar gains in welfare to those shown in Table 8. These authors, however, use forecasts of expected returns which means that their results are not strictly comparable. Overall, our results are closest to Jondeau and Rockinger (2005) who document an uplift of over 10% p.a. relative to mean–variance from accounting for departures from normality. Jondeau and Rockinger (2005) employ DCC-GARCH with the skewed-t copula, and thus account for all four stylised facts. Also, consistent with these authors, we find that the gains from accounting for non-normality are increasing in the level of risk aversion.

[9] The largest peak to trough decline.

Table 5
VaR forecast evaluation: 99.5% confidence level.

β = 99.5%	Gaussian	Student-t	Historical Simulation	Clayton Copula/ Gaussian Marginals	Exp. Weighted Moving Average	Filtered Historical Simulation	GGEV	GSEV
Asset Allocation								
Violations	2.11%	1.32%	1.18%		1.45%	0.92%	1.01%	0.48%
UC: p-value	0.00	0.00	0.00		0.00	0.01	0.00	0.90
Consecutive viol	14.6%	13.3%	7.4%		2.9%	4.5%	4.2%	8.3%
SI: p-value	0.00	0.00	0.04		0.53	0.21	0.25	0.05
CC: p-value	0.00	0.00	0.00		0.00	0.02	0.01	0.15
5 Industries								
Violations	1.79%	1.20%	0.95%	1.43%	1.43%	0.81%	0.75%	0.61%
UC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.21
Consecutive viol	9.4%	10.3%	9.7%	11.8%	4.3%	1.9%	2.0%	2.5%
SI: p-value	0.00	0.00	0.00	0.00	0.06	0.46	0.39	0.25
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.23
10 Industries								
Violations	1.81%	1.46%	0.98%	1.17%	1.49%	0.81%	0.81%	0.64%
UC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11
Consecutive viol	11.0%	10.5%	6.3%	11.8%	4.1%	1.9%	1.9%	2.4%
SI: p-value	0.00	0.00	0.00	0.00	0.07	0.46	0.46	0.28
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16
30 Industries								
Violations	1.86%	1.69%	0.97%	0.98%	1.46%	0.83%	0.86%	0.58%
UC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36
Consecutive viol	7.4%	8.2%	7.9%	9.4%	3.2%	1.9%	1.8%	2.6%
SI: p-value	0.00	0.00	0.00	0.00	0.23	0.47	0.51	0.22
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31
10 Stocks								
Violations	1.47%	1.05%	0.77%	0.36%	1.05%	0.54%	0.75%	0.59%
UC: p-value	0.00	0.00	0.01	0.10	0.00	0.67	0.01	0.34
Consecutive viol	6.7%	7.8%	4.3%	9.1%	1.5%	2.9%	2.1%	2.7%
SI: p-value	0.00	0.00	0.05	0.00	0.72	0.19	0.38	0.22
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.38	0.02	0.30
Size Portfolios								
Violations	1.89%	1.37%	0.86%	1.83%	1.52%	0.69%	0.66%	0.58%
UC: p-value	0.00	0.00	0.00	0.00	0.00	0.04	0.08	0.36
Consecutive viol	11.4%	9.0%	7.1%	10.9%	4.0%	2.2%	2.3%	2.6%
SI: p-value	0.00	0.00	0.00	0.00	0.09	0.32	0.29	0.22
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.07	0.13	0.31
Value Portfolios								
Violations	1.86%	1.45%	0.92%	1.85%	1.43%	0.80%	0.65%	0.63%
UC: p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.15
Consecutive viol	10.7%	9.6%	6.7%	10.8%	3.2%	1.9%	2.4%	2.4%
SI: p-value	0.00	0.00	0.00	0.00	0.21	0.44	0.28	0.26
CC: p-value	0.00	0.00	0.00	0.00	0.00	0.01	0.15	0.19
Fama–French Factors								
Violations	1.60%	0.92%	0.72%		1.22%	0.66%	1.18%	0.69%
UC: p-value	0.00	0.00	0.02		0.00	0.08	0.00	0.04
Consecutive viol	17.3%	21.7%	23.4%		7.6%	2.3%	7.8%	2.2%
SI: p-value	0.00	0.00	0.00		0.00	0.29	0.00	0.32
CC: p-value	0.00	0.00	0.00		0.00	0.12	0.00	0.07
Averages								
Violations	1.80%	1.30%	0.92%	1.27%	1.38%	0.76%	0.83%	0.60%
UC: p-value	0.00	0.00	0.00	0.02	0.00	0.10	0.03	0.31
Cons. viol	11.07%	11.29%	9.09%	10.65%	3.87%	2.45%	3.06%	3.23%
SI: p-value	0.00	0.00	0.01	0.00	0.24	0.35	0.32	0.23
CC: p-value	–	0.00	0.00	0.00	0.00	0.08	0.04	0.22

Notes: Table 5 provides the summary statistics for the VaR forecasting models for the equally-weighted portfolio for each data set at the 99% confidence level. The table provides the proportion of violations, the proportion of consecutive violations, and the p-values of the unconditional, serial-independence, and conditional coverage tests. The tests are run from the 1000th day of each data set to the end of each data set.

6. Conclusions

In line with previous work, we show that Gaussian approaches systematically underestimate tail risk. This conclusion holds even if we are using a conditional volatility approach such as EWMA. Any approach that uses the Gaussian distribution to quantify tail

Table 6
Out-of-sample portfolio results.

	Gaussian	Student-t	Historical Simulation	Clayton Copula/ Gaussian Marginals	Exp. Weighted Moving Average	Filtered Historical Simulation	GGEV	GSEV
Asset Allocation								
Annualised return	8.9	2.9	4.7		17.0	25.1	24.9	23.9
Standard-dev.	14.8	11.7	9.7		14.3	12.2	12.3	11.2
Kurtosis	23.0	32.4	13.3		3.0	2.2	2.2	2.6
CVaR 99%	−4.8	−4.0	−2.9		−3.3	−2.5	−2.5	−2.3
Risk forecast error (%)	141%	98%	45%		67%	23%	26%	16%
Max. DD	−71.6	−64.1	−53.2		−50.0	−38.6	−38.7	−36.6
Sharpe ratio	0.65	0.30**	0.52		1.16*	1.89**	1.86**	1.96**
Mean/CVaR 99%	2.00	0.89	1.75		5.00**	9.38**	9.12**	9.53**
5 Industries								
Annualised return	5.4	4.6	4.2	5.4	7.1	8.8	9.1	8.8
Standard-dev.	13.6	11.2	10.2	14.3	13.6	11.2	11.3	11.0
Kurtosis	45.7	65.1	30.6	60.8	5.2	6.9	7.7	8.3
CVaR 99%	−3.5	−2.9	−2.6	−3.6	−3.1	−2.5	−2.6	−2.5
Risk forecast error (%)	74%	45%	30%	82%	57%	27%	29%	25%
Max. DD	−33.9	−29.0	−28.0	−37.6	−33.2	−27.0	−26.1	−25.6
Sharpe ratio	0.45	0.45	0.45	0.44	0.57	0.80**	0.82**	0.81**
Mean/CVaR 99%	1.78	1.76	1.77	1.73	2.48**	3.56**	3.63**	3.60**
10 Industries								
Annualised return	5.9	5.0	4.9	5.8	9.2	10.4	10.8	10.3
Standard-dev.	13.7	11.8	10.8	14.1	14.1	11.8	11.9	11.4
Kurtosis	31.1	33.4	26.9	29.2	4.3	5.3	6.6	7.2
CVaR 99%	−3.4	−3.0	−2.7	−3.5	−3.1	−2.6	−2.6	−2.5
Risk forecast error (%)	70%	50%	37%	77%	57%	29%	31%	24%
Max. DD	−35.4	−31.6	−27.7	−35.7	−32.9	−26.7	−27.8	−23.3
Sharpe ratio	0.48	0.46	0.49	0.47	0.69	0.89**	0.92**	0.91**
Mean/CVaR 99%	1.96	1.84	1.94	1.88	3.13**	4.11**	4.16**	4.19**
30 Industries								
Annualised return	3.9	3.6	3.5	4.1	7.7	8.5	9.1	8.2
Standard-dev.	13.8	12.7	11.3	14.0	15.9	12.5	12.8	12.3
Kurtosis	36.7	49.1	30.8	29.4	4.4	4.0	6.3	7.0
CVaR 99%	−3.5	−3.3	−2.9	−3.5	−3.6	−2.8	−2.9	−2.8
Risk forecast error (%)	75%	63%	44%	77%	80%	38%	45%	41%
Max. DD	−41.2	−36.2	−35.9	−34.3	−39.7	−30.2	−31.2	−29.8
Sharpe ratio	0.34	0.34	0.36	0.35	0.54	0.71**	0.74**	0.70**
Mean/CVaR 99%	1.37	1.33	1.41	1.40	2.40**	3.25**	3.27**	3.07**

Notes: Table 6 shows the summary statistics for the seven portfolio construction methodologies for the Asset allocation, 5 Industries, 10 industries, and 30 Industries data sets for the period 12/1986 to 12/2012. The annualised return is calculated geometrically. The standard deviation is annualised. Kurtosis refers to excess kurtosis. CVaR 99% refers to the conditional value-at-risk at the 99% confidence level. Risk forecast error (%) refers to the percentage difference in the realised CVaR 99% and the CVaR 99% target of 2%. The Sharpe ratio is calculated arithmetically and is annualised. Mean/CVaR 99% refers to the ratio of the annualised arithmetic excess return divided by the realised CVaR 99%.

*Denotes statistical significance at the 95% level.

**Denotes statistical significance at the 99% level.

risk courts danger. We have also shown that the most widely used approach in practice, Historical Simulation, leads to too many consecutive VaR violations out-of-sample. This is problematic, as in times of market stress it may not be feasible to increase capital buffers. Conditional volatility approaches lead to a significant reduction in the proportion of consecutive violations and appear to be an essential component of a sensible VaR estimation model. The GSEV approach generates more accurate VaR forecasts than the GGEV approach which uses a symmetrical dependence structure, which may be indicative of the importance of accounting for asymmetric dependence. We are of the view that these results would hold for other risk management procedures, but leave that for further research.

From our out-of-sample portfolio rebalancing analysis, we conclude that several approaches outperform mean–variance. Historical simulation generates a modest uplift in economic value versus the mean–variance approach. The value-added of the FHS, GGEV, and GSEV approaches is substantial. The common thread uniting these approaches is the use of a conditional volatility approach in conjunction with a heavy-tailed distribution.

The FHS, GGEV, and GSEV approaches consistently outperform the Gaussian and EWMA approaches. In line with our analysis of the VaR models, it would appear desirable to employ a conditional non-Gaussian approach to estimate tail-risk. The FHS, GGEV, and GSEV approaches all perform similarly well, generating almost 7.5 to 10% in annual economic value-added depending on the level of risk aversion. Considering mutual fund management fees are in the region of 1% (Malkiel, 2013), this is a significant result.

Table 7
Out-of-sample portfolio results.

	Gaussian	Student-t	Historical Simulation	Clayton Copula/ Gaussian Marginals	Exp. Weighted Moving Average	Filtered Historical Simulation	GGEV	GSEV
10 Stocks								
Annualised return	9.5	7.5	8.0	6.2	13.1	9.0	10.0	9.7
Standard-dev.	12.6	10.6	11.3	11.9	15.6	12.4	12.0	11.6
Kurtosis	7.0	7.6	4.1	6.8	3.8	2.1	2.4	2.4
CVaR 99%	−2.9	−2.5	−2.6	−2.7	−3.3	−2.6	−2.5	−2.4
Risk forecast error (%)	45%	26%	30%	34%	63%	29%	24%	19%
Max. DD	−31.8	−25.9	−34.1	−27.0	−31.6	−32.4	−27.2	−25.6
Sharpe ratio	0.78	0.72	0.73	0.56	0.86	0.75	0.84	0.85
Mean/CVaR 99%	1.30	1.18	1.23	0.96	1.59*	1.39	1.58**	1.60**
Size Portfolios								
Annualised return	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1
Standard-dev.	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.1
Kurtosis	33.9	45.5	17.5	49.4	7.3	6.1	6.9	7.2
CVaR 99%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Risk forecast error (%)	104%	91%	35%	124%	76%	25%	24%	22%
Max. DD	−0.5	−0.5	−0.4	−0.5	−0.4	−0.3	−0.3	−0.3
Sharpe ratio	0.38	0.35	0.36	0.31	0.63*	1.10**	1.12**	1.12**
Mean/CVaR 99%	1.32	1.18	1.37	1.06	2.50**	4.80**	4.85**	4.86**
Value Portfolios								
Annualised return	0.1	0.1	0.0	0.1	0.1	0.1	0.1	0.1
Standard-dev.	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Kurtosis	41.4	47.9	39.8	47.3	5.7	5.7	6.3	5.9
CVaR 99%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Risk forecast error (%)	79%	56%	33%	88%	54%	29%	28%	26%
Max. DD	−0.4	−0.3	−0.3	−0.4	−0.3	−0.3	−0.3	−0.3
Sharpe ratio	0.53	0.53	0.48	0.51	0.67	0.68	0.68	0.69
Mean/CVaR 99%	2.05	2.03	1.83	1.97	2.98**	2.94**	2.97**	3.00**
Fama–French Factors								
Annualised return	0.1	0.1	0.1		0.2	0.3	0.3	0.3
Standard-dev.	0.1	0.1	0.1		0.1	0.1	0.1	0.1
Kurtosis	25.6	74.3	8.5		4.4	2.6	4.3	3.5
CVaR 99%	0.0	0.0	0.0		0.0	0.0	0.0	0.0
Risk forecast error (%)	80%	37%	49%		48%	25%	36%	16%
Max. DD	−0.4	−0.3	−0.4		−0.3	−0.4	−0.4	−0.4
Sharpe ratio	0.50	0.53	0.93**		1.38**	1.76**	1.80**	1.98**
Mean/CVaR 99%	2.05	2.07	4.13**		6.98**	9.40**	9.16**	10.3**

Notes: Table 7 shows the summary statistics for the seven portfolio construction methodologies for the 10 Stocks, 5 Size portfolios, 5 Value portfolios, and Fama–French factors for the period 12/1986 to 12/2012. The annualised return is calculated geometrically. The standard deviation is annualised. Kurtosis refers to excess kurtosis. CVaR 99% refers to the conditional value-at-risk at the 99% confidence level. Risk forecast error (%) refers to the percentage difference in the realised CVaR 99% and the CVaR 99% target of 2%. The Sharpe ratio is calculated arithmetically and is annualised. Mean/CVaR 99% refers to the ratio of the annualised arithmetic excess return divided by the realised CVaR 99%.

*Denotes statistical significance at the 95% level.

**Denotes statistical significance at the 99% level.

The GSEV approach has the highest average value-added across all eight data sets and levels of risk aversion. The GSEV approach also has the lowest average out of sample maximum drawdown and the most accurate out-of sample portfolio VaR estimates. This is indicative of the importance of accounting for asymmetric dependence.

There are several implications of our research for practitioners. First, if asset return volatility is stochastic, practitioners should use a conditional volatility model to estimate tail risk and to optimise portfolios. Potential choices include exponentially-weighted covariance estimators, GARCH, stochastic volatility models, or realised volatility using high-frequency data. In the case of high-frequency data, care must be taken to deal with asynchronicity and market microstructure noise (see for example Ait-Sahalia et al., 2010). Second, if conditional returns are non-Gaussian, investors should use a heavy-tailed distribution. We have shown that the generalised Pareto distribution performs well in this context.

Finally, investors should consider accounting for asymmetric dependence when estimating VaR and determining portfolio weights. The skewed-t copula appears to be a sensible and underutilised choice. A potential area for future work is to investigate the dispersion of the risk estimates of our multivariate approach. Recent work from Kellner et al. (2016) find that EVT-based risk estimates tend to have high standard errors which may warrant additional capital buffers. This problem may be more acute in the multivariate approach we have presented. Our approach is computationally intensive requiring the estimation of a large number of parameters. It would be interesting to compare our approach to the more parsimonious portfolio choice approach developed by Aït-Sahalia and Brandt (2001).

Table 8
Economic value-added: power utility.

	Student-t	Historical Simulation	Clayton Copula/ Gaussian Marginals	Exp. Weighted Moving Average	Filtered Historical Simulation	GGEV	GSEV
$\gamma = 5$ (Aggressive)							
Asset Allocation	−3.9%	−1.2%		7.6%	15.5%	15.3%	15.0%
5 Industries	0.5%	0.6%	−0.4%	1.8%	4.5%	4.8%	4.6%
10 Industries	0.1%	0.5%	−0.3%	2.9%	5.3%	5.6%	5.3%
30 Industries	0.4%	1.0%	0.1%	2.4%	5.2%	5.5%	5.0%
10 Stocks	−1.0%	−0.8%	−2.7%	1.5%	−0.3%	0.7%	0.6%
Size Portfolios	0.0%	1.0%	−1.5%	3.8%	8.9%	9.0%	9.0%
Value Portfolios	0.3%	−0.1%	−0.4%	2.2%	2.1%	2.1%	2.2%
Fama–French	0.9%	6.1%		13.3%	17.2%	18.3%	18.5%
Average	−0.3%	0.9%	−0.9%	4.4%	7.3%	7.7%	7.5%
$\gamma = 10$ (Moderate)							
Asset Allocation	−1.5%	2.7%		8.6%	18.0%	17.8%	18.2%
5 Industries	2.3%	3.3%	−1.4%	2.5%	6.8%	7.1%	7.0%
10 Industries	1.6%	2.7%	−0.4%	3.1%	7.0%	7.2%	7.3%
30 Industries	1.2%	3.0%	0.2%	1.3%	6.7%	6.8%	6.6%
10 Stocks	0.2%	0.0%	−2.2%	−0.7%	−0.1%	1.1%	1.2%
Size Portfolios	1.0%	4.3%	−2.8%	4.7%	11.8%	11.9%	12.0%
Value Portfolios	1.7%	2.7%	−1.1%	3.0%	4.5%	4.6%	4.7%
Fama–French	3.7%	7.6%		13.7%	18.7%	19.5%	20.8%
Average	1.3%	3.3%	−1.3%	4.5%	9.2%	9.5%	9.7%
$\gamma = 15$ (Conservative)							
Asset Allocation	1.3%	7.4%		10.3%	21.4%	21.1%	22.2%
5 Industries	4.8%	7.3%	−3.6%	4.5%	10.5%	10.7%	10.9%
10 Industries	3.5%	5.4%	−0.6%	3.9%	9.5%	9.6%	10.0%
30 Industries	2.3%	5.6%	0.7%	1.1%	9.2%	9.0%	9.1%
10 Stocks	1.4%	0.8%	−1.7%	−3.1%	0.1%	1.5%	1.9%
Size Portfolios	2.1%	8.5%	−5.0%	6.4%	15.7%	15.9%	16.0%
Value Portfolios	3.8%	6.5%	−2.3%	5.0%	8.2%	8.3%	8.5%
Fama–French	6.8%	9.7%		14.7%	20.9%	21.2%	23.7%
Average	3.2%	6.4%	−2.1%	5.4%	11.9%	12.2%	12.8%

Notes: Table 8 provides the value-added of the respective models relative to the Gaussian model for a power utility investor. The value-added is defined as the annual return that equates the expected power utility of the given model and the Gaussian model.

CRediT authorship contribution statement

David Allen: Conceptualization, Methodology, Data curation, Formal analysis, Writing - original draft. **Colin Lizieri:** Validation, Supervision, Writing - review & editing, Visualization. **Stephen Satchell:** Conceptualization, Methodology, Supervision, Validation, Writing - review & editing.

References

Aas, K., Haff, I., 2006. The generalized hyperbolic skew student's t-distribution. J. Financ. Econ. 4, 275–309.
Acerbi, C., Tasche, D., 2002. Expected shortfall: A natural coherent alternative to value at risk. Econ. Notes 31, 379–388.
Adler, T., Kritzman, M., 2007. Mean–variance versus full-scale optimisation: In and out of sample. J. Asset Manage. 7, 302–311.
Aït-Sahalia, Y., Brandt, M.W., 2001. Variable selection for portfolio choice. J. Finance 56, 1297–1351.
Ait-Sahalia, Y., Fan, J., Xiu, D., 2010. High-frequency covariance estimates with noisy and asynchronous financial data. J. Amer. Statist. Assoc. 492, 1504–1517.
Aït-Sahalia, Y., Lo, A., 2000. Nonparametric risk management and implied risk aversion. J. Econometrics 94, 9–51.
Alcock, J., Hatherley, A., 2009. Asymmetric dependence between domestic equity indices and its effect on portfolio construction. Aust. Actuar. J. 15, 143–180.
Alcock, J., Satchell, S. (Eds.), 2018. Asymmetric Dependence in Finance: Diversification, Correlation and Portfolio Management in Market Downturns. John Wiley and Sons, Hoboken.
Artzner, P., Delbaen, J.M., Eber, F., Heath, D., 1997. Thinking coherently. Risk 10, 68–71.
Artzner, P., Delbaen, J.M., Eber, F., Heath, D., 1999. Coherent measures of risk. Math. Finance 9, 203–228.
Azzalini, A., Capitanio, A., 2003. Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution. J. R. Stat. Soc. Ser. B Stat. Methodol. 65, 367–389.
Barndorff-Nielsen, O., 1976. Exponentially decreasing distributions for the logarithm of particle size. Proc. R. Soc. A 353, 401–419.
Barone-Adesi, G., Bourgoin, F., Giannopoulos, K., 1998. Don't look back. Risk 11, 31–58.
Bauwens, L., Laurent, S., 2005. A new class of multivariate skew densities, with application to generalized autoregressive conditional heteroscedasticity models. J. Bus. Econom. Statist. 23, 346–354.
Bauwens, L., Laurent, S., Rombouts, J.V.K., 2006. Multivariate GARCH models: A survey. J. Appl. Econometrics 21, 79–109.
Berkowitz, J., O'Brien, J., 2002. How accurate are value-at-risk models at commercial banks? J. Finance 57, 1093–1111.
Bernardi, M., Maruotti, L., Petrella, A., 2017. Multiple risk measures for multivariate dynamic heavy–tailed models. J. Empir. Financ. 43, 1–32.

Blanchett, D., Ratner, H., 2015. Building efficient income portfolios. J. Portf. Manag. 41, 117–125.
Bloomfield, T., Leftwich, R., Long, J.B., 1977. Portfolio strategies and performance. J. Financ. Econ. 5, 201–218.
Bollerslev, T., 1990. Modeling the coherence in short-run nominal exchange-rates - A multivariate generalized ARCH model. Rev. Econ. Stat. 72, 498–505.
Brandt, M.W., Santa-Clara, P., Valkanov, R., 2009. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. Rev. Financ. Stud. 22, 3411–3447.
Brooks, C., Persand, G., 2003. Volatility forecasting for risk management. J. Forecast. 22, 1–22.
Campbell, R., Huisman, R., Koedijk, K., 2001. Optimal portfolio selection in a Value-at-Risk framework. J. Banking Finance 25, 1789–1804.
Carhart, M.M., 1997. On persistence in mutual fund performance. J. Finance 52, 57–82.
Chekhlov, A., Uryasev, S., Zabarankin, M., 2005. Drawdown measure in portfolio optimization. Int. J. Theor. Appl. Finance 8, 13–58.
Chen, X., Fan, Y., 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. J. Econometrics 135, 125–154.
Christoffersen, P.F., 1998. Evaluating interval forecasts. Internat. Econom. Rev. 39, 841–862.
Christoffersen, P.F., 2009. Value-At-Risk Models. Springer, Heidelberg, Berlin.
Clayton, D.G., 1978. Model for association in bivariate life tables and its application in epidemiological-studies of familial tendency in chronic disease incidence. Biometrika 65, 141–151.
Dark, J., 2018. Multivariate models with long memory dependence in conditional correlation and volatility. J. Empir. Financ. 48, 162–180.
Demarta, S., McNeil, A.J., 2005. The t copula and related copulas. Internat. Statist. Rev. 73, 111–129.
DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Rev. Financ. Stud. 22, 1915–1953.
Dufour, J., Kurz-Kim, J.-R., Palm, F., 2010. Heavy tails and stable paretian distributions in empirical finance. J. Empir. Financ. 17, 177–179.
Efron, B., Tibshirani, R., 1993. An Introduction To the Bootstrap. Chapman and Hall.
Engle, R., 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. J. Bus. Econom. Statist. 20, 339–350.
Erb, C., Harvey, C., Viskanta, T., 1994. Forecasting international equity correlations. Financ. Anal. J.
Fabozzi, F.J., Focardi, S., Jonas, C., 2007. Trends in quantitative equity management: Survey results. Quant. Finance 7, 115–122.
Fama, E.F., 1965. The behavior of stock market prices. J. Bus. 38, 34–105.
Fama, E.F., French, K.R., 1993. Common risk-factors in the returns on stocks and bonds. J. Financ. Econ. 33, 3–56.
Fernandez, C., Steel, M.F.J., 1998. On Bayesian modelling of fat tails and skewness. J. Amer. Statist. Assoc. 93, 359–371.
Fleming, J., Kirby, C., Ostdiek, B., 2001. The economic value of volatility timing. J. Finance 56, 329–352.
Fletcher, J., 1997. An examination of alternative estimators of expected returns in mean–variance analysis. J. Financ. Res. 20, 129–143.
Ghorbel, Trabelsi, A.A., 2009. Measure of financial risk using conditional extreme value copulas with EVT margins. J. Risk 11, 51–85.
Gurrola-Perez, P., Murphy, D., 2015. Filtered historical simulation value-at-risk models and their competitors. Bank of England Working Paper 525. London. Bank of England.
Hansen, B.E., 1994. Autoregressive conditional density-estimation. Internat. Econom. Rev. 35, 705–730.
Hong, Y., Tu, J., Zhou, G., 2007. Asymmetries in stock returns: Statistical tests and economic evaluation. Rev. Financ. Stud. 20, 1547–1581.
Hu, W., Kercheval, A.N., 2007. Risk management with generalized hyperbolic distributions. In: Proceedings of the Fourth IASTED International Conference on Financial Engineering and Applications. pp. 19–24.
Hull, J., White, A., 1998. Incorporating volatility updating into the historical simulation method for value at risk. J. Risk 1, 5–19.
Ibbotson, R., Kaplan, P., 2000. Does asset allocation explain 40, 90, or 100 percent of performance? Financ. Anal. J.
Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. J. Finance 45, 881–898.
Jobson, J.D., Korkie, B., 1981. Estimation for Markowitz efficient portfolios. J. Am. Stat. Assoc. 75, 544–554.
Joe, H., 2005. Asymptotic efficiency of the two-stage estimation method for copula-based models. J. Multivariate Anal. 94, 401–419.
Joe, H., Xu, J., 1996. The Estimation Method of Inference Functions for Margins for Multivariate Models. Technical Report No, 166, Department of Statistics, University of British Columbia.
Jondeau, E., Rockinger, M., 2005. Conditional asset allocation under non-normality: how costly is the mean–variance criterion? FAME International Centre for Financial Asset Management and Engineering Research Paper, Vol. 132. pp. 1–43.
Jones, M.C., Faddy, M.J., 2003. A skew extension of the t-distribution, with applications. J. R. Stat. Soc. Ser. B Stat. Methodol. 65, 159–174.
Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263–291.
Karolyi, G.A., Stulz, R.M., 1996. Why do markets move together? An investigation of US-Japan stock return comovements. J. Finance 51, 951–986.
Kellner, R., Rösch, D., Scheule, H., 2016. The role of model risk in extreme value theory for capital adequacy. J. Risk 18 (6), 39–70.
Kraus, A., Litzenberger, R.H., 1976. Skewness preference and valuation of risk assets. J. Finance 31, 1085–1100.
Lauprete, G.J., Samarov, A.M., Welsch, R.E., 2002. Robust portfolio optimization. Metrika 55, 139–149.
Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. J. Finance 56, 649–676.
Lucas, A., Klaasen, P., 1998. Extreme returns, downside-risk, and optimal asset allocation. J. Portf. Manag. 25, 71–79.
Malkiel, B.G., 2013. Asset management fees and the growth of finance. J. Econ. Perspect. 27, 97–108.
Mandelbrot, B., 1963. The variation of certain speculative prices. J. Bus. 36, 394–419.
Markowitz, H.M., 1952. Portfolio selection. J. Finance 7, 77–91.
Markowitz, H.M., 1959. Portfolio Selection: Efficient Diversification of Investments. John Wiley and Sons, Inc.
Martellini, L., Ziemann, V., 2010. Improved estimates of higher-order comoments and implications for portfolio selection. Rev. Financ. Stud. 23, 1467–1502.
McAleer, M., 2009. The ten commandments for optimizing value-at-risk and daily capital charges. J. Econ. Surv. 23, 831–849.
McAleer, M., Da Veiga, B., 2008. Single-index and portfolio models for forecasting value-at-risk thresholds. J. Forecasting 27, 217-23.
McNeil, A., Frey, R., 2000. Estimation of tail related risk measures for heteroskedastic time series: An extreme value approach. J. Empir. Financ. 7, 271–300.
McNeil, A., Frey, R., Embrechts, P., 2005. Quantitative Risk Management: Concepts, Techniques and Tools. Princeton University Press.
Memmel, C., 2003. Performance hypothesis testing with the sharpe ratio. Finance Lett. 1, 21–23.
Moreira, A., Muir, T., 2017. Volatility managed portfolios. J. Finance 72, 1611–1643.
Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica 59, 347–370.
Nystrom, K., Skoglund, J., 2002. A framework for scenario based risk management. SAS Institute and the University of Umea Working Paper.
Patton, A., 2004. On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. J. Financ. Econ. 2, 130–168.
Patton, A., 2006. Estimation of multivariate models for time series of possibly different lengths. J. Appl. Econometrics 21, 147–173.
Pérignon, C., Smith, D.R., 2008. A new approach to comparing VaR estimation methods. J. Deriv. 16, 54–66.
Politis, D., White, H., 2004. Automatic block length selection for the dependent bootstrap. Econometric Rev. 23, 53–70.
Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. J. Risk 2, 493–517.
Sahu, S.K., Dey, D.K., Branco, M.D., 2003. A new class of multivariate skew distributions with applications to Bayesian regression models. Can. J. Stat.-Revue Can. Stat. 31, 129–150.
Skoglund, J., Erdman, D., Chen, W., 2010. The performance of value-at-risk models during the crisis. J. Risk Model Valid. 4, 3–21.
Solnik, B., 1993. The performance of international asset allocation strategies using conditioning information. J. Empir. Financ. 1, 33–55.

Sortino, F.A., 2010. The Sortino Framework for Constructing Portfolios. Elsevier Science.
Sortino, F.A., van der Meer, R., 1991. Downside risk. J. Portf. Manag. 17, 27–31.
Sun, W., Rachev, S., Fabozzi, F.J., 2008. Multivariate skewed student's t copula in the analysis of nonlinear and asymmetric dependence in the German equity market. Stud. Nonlinear Dyn. Econom. 12, 1–37.
Viebig, J., Poddig, T., 2010. Modelling extreme returns and asymmetric dependence structures of hedge fund strategies using extreme value theory and copula theory. J. Risk 13, 23–55.
West, K.D., Edison, H.J., Cho, D., 1993. A utility-based comparison of some models of exchange-rate volatility. J. Int. Econ. 35, 23–45.
Xiong, J.X., Idzorek, T.M., 2011. The impact of skewness and fat tails on the asset allocation decision. Financ. Anal. J. 67, 23–35.