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# The economic value of VIX ETPs[☆]

Kim Christensen[a,1], Charlotte Christiansen[a,b,1], Anders M. Posselt[c,*,1]

[a] DFI, CREATES, Department of Economics and Business Economics, Aarhus University, Fuglesangs Alle 4, 8210 Aarhus V, Denmark
[b] Lund University, Sweden
[c] CREATES, Department of Economics and Business Economics, Aarhus University, Fuglesangs Alle 4, 8210 Aarhus V, Denmark

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ABSTRACT

The fairly new VIX ETPs have been promoted for providing effective and easily accessible diversification, while at the same time having large negative returns. We examine the economic value of using VIX ETPs for diversification of stock–bond portfolios. Our analysis begins in 2009, when the first VIX ETPs are introduced, and therefore only considers the period after the recent financial crisis. For investors with a constant allocation strategy, the diversification benefits of the VIX ETPs do not offset their negative returns. This implies negative economic value of a constant allocation. For a dynamic allocation strategy, including short VIX ETPs in the investment opportunity set can have substantial positive economic value.

## 1. Introduction

In this paper, we use the framework of Fleming et al. (2001) to quantify the economic value stemming from investing in portfolios that not only consist of the benchmark assets (stocks and bonds) but also relies on the new and increasingly popular VIX exchange-traded products (ETPs) for diversification. We extend the previous usage of economic value from evaluating trading on futures based on intra-daily realized variances to its usage for investing in VIX ETPs.

In 2009 the first exchange traded volatility products, *iPath S&P 500 VIX Short-Term Futures Exchange Traded Note* (VXX) and *S&P 500 VIX Mid-Term Futures Exchange Traded Note* (VXZ) are launched by Barclays Capital. The introduction of VIX ETPs makes volatility exposure available to retail investors who are typically too small or unsophisticated to trade in the futures market and to institutions such as pension and endowment funds who may be restricted from trading derivatives. Today, 13 VIX ETPs are listed, which differ in terms and format, e.g., some provide inverse and leveraged exposure to volatility. As other asset classes like stocks and bonds tend to become near perfectly correlated in times of severe distress, volatility exposure is a desirable portfolio component as it diversifies and protects portfolios when it is needed the most. However, this exposure comes at a cost in terms of negative expected returns of long positions in VIX ETPs during normal times (see e.g., Alexander and Korovilas, 2013, Alexander et al., 2015, and Eraker and Wu, 2017).

We use the concept of economic value to look further into the VIX ETPs. Our study is highly timely as the first VIX ETPs, VXX and VXZ, have expired on January 30, 2019, 10 years after their inception. New versions have been launched subsequently. This prompts the question of whether the economic value from investing in these products offer diversification benefits that are sufficient

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* Corresponding author.
*E-mail addresses:* kim@econ.au.dk (K. Christensen), cchristiansen@econ.au.dk (C. Christiansen), amp@econ.au.dk (A.M. Posselt).

[1] The authors contributed equally to this paper.

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to compensate for the substantial negative returns that they generate. The purpose of this paper is to provide an answer. We refer to VIX ETPs as products with long exposure to VIX futures, which is the main focus of our study. However, we also include results for VIX ETPs with short exposure to VIX futures, which we refer to as inverse VIX ETPs.

Our paper makes two main contributions. First, we make use of the popular concept of economic value from Fleming et al. (2001) to measure the advantage of adding a third asset class to the traditional stock–bond portfolios. This allows us to quantify the portfolio performance in an economic sense. This is an improvement compared to basing the analysis on the classical portfolio performance measures such as the Sharpe ratio. Second, we investigate in detail the new and understudied VIX ETPs. Here, we use an up-to-date sample which is longer than in previous studies. Furthermore, we exploit the findings in Cheng (2019) for estimating the expected returns on VIX ETPs, and we use intra-daily data for estimating the conditional covariance matrix, using the method of Oh and Patton (2016), which gives us much more efficient estimates than using only daily data as in the previous literature. A third minor contribution is that we use the intra-daily quote and trade data to estimate the transaction costs of trading in VIX ETPs.[2]

The prior literature studies the benefits of volatility investing. Both Dash and Moran (2005) and Daigler and Rossi (2006) find diversification benefits of adding variance swaps to portfolios of stocks and portfolios of hedge funds, respectively. Szado (2009) finds positive diversification benefits based on arbitrary allocations between VIX futures and other assets in a sample focused on the period around the recent financial crisis. In a sample running from 2004 to 2008, Brière et al. (2010) find that a long stock investor who is mean-value at risk optimizing increases the risk-adjusted return by adding a combination of long VIX futures and short variance swaps to the portfolio. In a sample ending in 2008, Chen et al. (2011) perform a mean–variance spanning test on four US stock portfolios and find that VIX futures enlarge the investment opportunity set. Whaley (2013) investigates how the indices which the VIX ETPs are tracking perform as an asset class on their own. He shows that from 2005 to 2012, investors in VIX ETPs (excluding inverse products) have lost about $3.89 bn.

A recent study is Alexander et al. (2016), who consider three investor types that allocate capital between stocks, bonds, and VIX ETPs on a monthly basis using different optimization methods. They introduce the concept of the optimal diversification threshold, which is the minimum expected return for VIX ETPs in order to be included in the optimal portfolio. They find that diversification with VIX ETPs is frequently ex-ante optimal, however, the apparent benefits are never realized in the ex-post performance due to the high roll costs for these products. They only find diversification benefits of the VIX ETPs (constructing these synthetic prior to 2009) during the banking crisis of 2008.[3] Caloiero and Guidolin (2017) use the same approach where they back-test different portfolios with exposure to either a short-term VIX ETP or the VIX (not investable) on a sample running from 2010 to 2016. In some cases, depending on the allocation strategy, they find benefits of including the VIX in a portfolio but never the VIX ETP. By means of a simple regression analysis Bordonado et al. (2017) determine the weights in the VIX ETP that fully hedge a position in the S&P 500 index. In an in-sample analysis, the performance is then compared with an unhedged position. On a sample running from 2006 to 2013, they find that inclusion of the VIX ETP would have improved the Sharpe ratio marginally.[4] This result, however, is sensitive to the choice of re-balancing frequency and when the impact of the financial crisis in 2008 is filtered out, the inclusion of the VIX ETP offers no improvement on the Sharpe ratio. One of the newest studies on the subject is Berkowitz and DeLisle (2018) who use a five-factor model to do a performance evaluation of portfolios comprised of a broad stock index and VIX ETPs on a sample beginning at the inception of the first VIX ETPs. They find that the VIX ETPs are too expensive and that they underperform the pure stock portfolio.

In this study, we consider two different allocation strategies. One strategy holds a constant allocation in the respective asset classes, the other strategy dynamically allocates capital to different asset classes on the basis of their expected future returns, variances, and covariances. Hence our study examines two very different approaches to investing in VIX ETPs. One which holds VIX ETPs constantly in the portfolio as an insurance, and one which seeks to time the exposure to VIX ETPs in expectation of increased future returns or changes in the correlation structure. For each allocation strategy, we examine three different portfolios. The first portfolio serves as the benchmark and contains only stocks and bonds. The second and third portfolios are extended with a VIX ETP, either a short- or a medium-term VIX ETP.

For each strategy, the realized out-of-sample performance of the two portfolios containing VIX ETPs is compared to the benchmark portfolio. For the performance evaluation, we apply the concept of economic value introduced in Fleming et al. (2001). In our context, the economic value is interpreted as the performance fee that a mean–variance optimizing investor is willing to pay ex-post to have included a VIX ETP in the portfolio. Our findings depend on the allocation strategy. For the investor with a constant allocation, the value of protection during times of market stress is vaporized due to the roll costs associated with the rebalancing strategy of the VIX ETPs, resulting in an overall ex-post negative economic value of holding VIX ETPs continuously. However, for the investor seeking to time the allocation, performance is enhanced by including a short-term VIX ETP (VXX) in the opportunity set.

Like Alexander et al. (2016) and Caloiero and Guidolin (2017), our approach is back-testing in nature. For the investor who times the portfolio allocation, our results are highly influenced by our estimates of expected future returns and the covariance matrix. Hence, we pay special attention to how we model these. Specifically, we exploit the ability of the VIX premium to forecast VIX futures returns, see Cheng (2019), and we follow the procedure of Oh and Patton (2016) for forecasting the covariance matrix.

The literature described above can be divided into three categories based on the sample period considered. These are (i) before and during the recent financial crisis, (ii) before, during, and after the financial crisis, and (iii) after the financial crisis. Our study

[2] The VIX ETP bid–ask spreads obtainable from data vendors such as Bloomberg and Thomson Reuters are missing or flawed for many observation points.

[3] As VIX ETPs were not available during the 2008 banking crisis they construct synthetic returns for this period using VIX futures.

[4] Prior to the inception of the first VIX ETPs, they use returns of the index that the products track.

belongs to the latter category because we only want to use traded data for the VIX ETPs, and thus the sample period must start when the VIX ETPs actually start trading in 2009. As our analysis leaves out the financial crisis, it partly takes the perspective of investors who have used VIX ETPs since their inception in anticipation that these products will shield portfolio performance during times of market stress. Hence, our study can be seen as a test of whether or not the market volatility in the period after the launch of VIX ETPs has been sufficiently high for these products to offer positive economic value. As we find that this is not the case for the strategy with constant allocations, we make a simple simulation of a new market crash of the same magnitude as the recent financial crisis in 2008. Hereby, we investigate if the simulated crash would enable investors who have held constant positions in VIX ETPs to catch up with the benchmark portfolio of stocks and bonds. Even accounting for the simulated market crash is not enough for the VIX ETPs to add ex-post economic value to the investor with a constant allocation. Thus, our conclusion is that holding VIX ETPs constantly in a portfolio as an insurance is too expensive. To take advantage of the exposure to volatility that these products offer, the allocation must be timed. Our findings indicate that the VIX premium may provide an effective signal of when to increase the allocation since the strategy which times the exposure outperforms the benchmark.

We extend the analysis by investigating the impact of the fees paid to the issuers of the VIX products and also whether investing in a single VIX futures contract provides a more effective exposure to volatility. For the investor with a constant allocation, the fees have a non-negligible impact on long-term performance, however what really matters is the roll-cost associated with the rolling strategy of VIX futures. Investing only in the second-month VIX futures improves the performance, but the exposure is still too expensive to include as a constant portfolio component. For an investor who successfully times the exposure, it is more efficient to use the first-month VIX futures instead of a VIX ETP.

Finally we consider allocating capital to inverse VIX products which give constrained investors access to short VIX futures exposure. Despite several drawdowns in portfolio value, an investor with a constant allocation to an inverse VIX product is able to lever the overall portfolio performance in our sample. An investor with a timing strategy can also enhance performance, substantially, by exposure to inverse products, using the VIX premium as a signal of when to decrease or close the position.

The paper is organized into six additional sections. Section 2 reviews the methodology of our analysis. Section 3 provides a thorough description of the VIX ETPs, and Section 4 describes the data that we use. Section 5 documents the economic value of investing in VIX ETPs, while Section 6 discusses the empirical portfolio allocations for the investment strategies. Finally, Section 7 contains the conclusions. Further results are presented in the online appendix.

## 2. Methodology

The aim of our paper is to consider the economic value of portfolio diversification using the new financial product, VIX ETPs, relative to the benchmark portfolio, which only diversifies between stocks and bonds. We evaluate portfolios consisting of VIX ETPs as well as stocks and bonds by considering their economic value compared to the benchmark portfolios. The economic value calculations follow Fleming et al. (2001). Previously, the concept of economic value has been used in relation to realized volatility of futures contracts based on intra-daily data and in relation to forecasting, so we extend the economic value literature.

### 2.1. Allocation strategies

Initially we consider two different VIX ETPs with long volatility exposure that only differ with respect to their maturity (short- and medium-term). The two VIX ETPs are described in detail in Section 3. It is of course possible to use our framework to assess the economic value of adding any other asset than VIX ETPs to the traditional stock–bond portfolio (as in Fleming et al., 2001).

We consider different portfolios that contain the VIX ETP and measure their performance against the benchmark portfolio. The benchmark portfolio (P-bench) only contains US stocks and bonds. The second portfolio (P-short) is comprised of stocks, bonds, and a short-term VIX ETP, while the third portfolio (P-mid) is comprised of a mid-term VIX ETP instead. Hence, the portfolios that we analyze are:

- P-bench: US stocks and bonds
- P-short: US stocks, bonds, and short-term VIX ETP
- P-mid: US stocks, bonds, and mid-term VIX ETP

P-short and P-mid will also be referred to as the "VIX portfolios".

The investor re-balances her portfolio at a monthly frequency (at month end), assuming 21 trading days in a month. At the re-balance date, the investor allocates her funds across assets according to her asset allocation strategy.

We take the perspective of a short-sales constrained investor (e.g., a pension fund or a retail investor) whose portfolio can only be composed of long positions and who cannot apply leverage. We consider two different allocation strategies which we label constant (constant weights), and CE (certainty equivalent optimizing), respectively.

We fix notation first. Let $\mathbf{R}_{t+1}$ denote an $N\times1$ vector of risky asset returns. $\boldsymbol{\mu}_{t+1|t} \equiv \mathrm{E}_t[\mathbf{R}_{t+1}]$ denotes the conditional expected value of $\mathbf{R}_{t+1}$. $\boldsymbol{\Sigma}_{t+1|t} \equiv \mathrm{E}_t[(\mathbf{R}_{t+1}\text{-}\boldsymbol{\mu}_{t+1|t})(\mathbf{R}_{t+1}\text{-}\boldsymbol{\mu}_{t+1|t})']$ is the conditional covariance matrix of $\mathbf{R}_{t+1}$. $R_f$ is the return on the risk-free asset. $\mathbf{w}_t$ is an $N\times1$ vector of portfolio weights on the risky assets.

The first allocation strategy, *constant*, is simply a static portfolio where the weights to each asset are constant through time at conventional levels using weights similar to Szado (2009). For the benchmark portfolio, $\mathbf{w}_t' = (60\%, 40\%, 0\%)$ where the first, second, and third element is the stock, bond, and VIX ETP weight, respectively. This is commonly referred to as the 60/40 rule by the financial media and has historically been used as a rule of thumb by financial planners and advisers. For the two VIX portfolios, $\mathbf{w}_t' = (60\%, 30\%, 10\%)$.

The second strategy, *CE*, is to maximize the expected utility and the certainty equivalent. For a quadratic utility function the investor's realized utility in period $t+1$ can be written as:

$$U(W_{t+1}) = W_t R_{p,t+1} - \frac{aW_t^2}{2} R_{p,t+1}^2, \tag{1}$$

where $W_{t+1}$ is the investor's wealth at $t+1$, $a$ is her absolute risk aversion, and $R_{p,t+1} = (1 - \mathbf{w}_t'\mathbf{1})R_f + \mathbf{w}_t'\mathbf{R_{t+1}}$ is the period $t+1$ return, where $\mathbf{1}' = (1, \dots, 1)$. The utility function can also be expressed in terms of the certainty equivalent:

$$CE \approx \mathrm{E}[R_{p,t+1}] + \frac{1}{2}\frac{U''(\mathrm{E}[R_{p,t+1}])}{U'(\mathrm{E}[R_{p,t+1}])}\mathrm{Var}[R_{p,t+1}]. \tag{2}$$

This implies that the investor maximizes utility by maximizing the certainty equivalent. Then from Eqs. (1) and (2), we get that the certainty equivalent (CE) maximizing strategy is obtained by solving:

$$\begin{aligned} &\max_{\mathbf{w}_t} \mathbf{w}_t'(\boldsymbol{\mu}_{t+1|t} - R_f\mathbf{1}) - \frac{a}{1 - a\mathbf{w}_t'(\boldsymbol{\mu}_{t+1|t} - R_f\mathbf{1})}\mathbf{w}_t'\Sigma_{t+1|t}\mathbf{w}_t, \\ &\text{s.t. } \mathbf{w}_t'\mathbf{1} \le 1, \\ &\mathbf{w}_t \ge 0 \forall t. \end{aligned} \tag{3}$$

We set the absolute risk aversion parameter, $a$, to 4 as in Alexander et al. (2016).

The two strategies are very different in nature. Contrary to the constant strategy, the amount of capital deployed will be varying through time in the CE strategy, as estimates of expected returns and the covariance matrix are updated. Hence, our analysis takes the perspective of an investor who holds VIX ETPs constantly through time as an insurance against drawdowns, and that of an investor who is timing the allocation based on expected returns, volatility, and correlation over the coming investment horizon. Consequently, we must carefully model these inputs, which is addressed in the following two sub-sections.

In the appendix we provide results for alternative constant weights and for two additional strategies. These are the minimum variance strategy as in Fleming et al. (2001) and the maximum diversification strategy as in Yves and Coignard (2008). The appendix also provides performance results for unconstrained allocations, representing investors (e.g., a hedge fund) who can apply leverage and hold short positions.

### 2.2. Expected returns

For the stock and bond components, in any portfolio, we use a backward-looking average of the daily realized returns over the previous month. This is of course simplistic. However, it will be equally wrong for both the benchmark portfolio and the VIX portfolios, so this will not bias our overall conclusions.

For the VIX ETPs we rely on the findings in Cheng (2019). He defines the VIX premium, denoted $VIXP_{T|t}$, as the difference in the expected value of the VIX, at some future date, $T$, under the risk neutral measure and under the physical measure. That is:

$$VIXP_{T|t} \equiv E_t^Q[VIX_T] - E_t^P[VIX_T]. \tag{4}$$

The premium tends to become negative during periods of increased market risk (e.g. elevated levels of the VIX) and it is a good predictor of VIX futures returns. Since VIX ETPs are based on VIX futures, we estimate expected returns via the VIX premium. Denote $R_t^{VIXETP}$ as the month $t$ return of the VIX product. Then we obtain our estimated expected returns via the following regression:

$$R_{t+1}^{VIXETP} = b_0 + b_1 \times VIXP_{T|t} + \epsilon_{t+1}. \tag{5}$$

In the case of the CE investor, our study in essence becomes a test of the economic value of timing the allocation to VIX products via the VIX premium.

### 2.3. Conditional covariances

For estimating the conditional covariance matrix, we rely on the theory of realized measures. The work of Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) suggest that we can use intra-daily returns to construct volatility estimators that are more efficient than those based on daily returns. By a standard no-arbitrage condition, we assume prices are semimartingales. Then, as shown in Andersen et al. (2001), we can think of the quadratic covariation as an unbiased estimator of the conditional covariance matrix, where the quadratic covariation between asset $j$ and $k$ is defined as:

$$\boldsymbol{\Sigma}_{t+1}(j,k) = \int_t^{t+1} \sigma_{j,k}(s)ds. \tag{6}$$

The quadratic covariation may be approximated directly from high-frequency return data. Suppose we divide the time interval $t$ to $t+1$ into $n$ sub-periods of length $h = 1/n$ and let $\mathbf{r}_{t+ih}$ denote the $d \times 1$ vector of continuously compounded returns that starts at time $t + (i-1)h$ and ends at time $t + ih$, $i = 1, \dots, n$. We then define the realized covariance matrix as:

$$\mathbf{V}_{t+1} = \sum_{i=1}^{n} \mathbf{r}_{t+ih}\mathbf{r}_{t+ih}'. \tag{7}$$

Then, Andersen et al. (2001) show that under weak regularity conditions:

$$\text{p-lim}_{n\to\infty}\mathbf{V}_{t+1} = \boldsymbol{\Sigma}_{t+1}, \tag{8}$$

where $\boldsymbol{\Sigma}_{t+1} = (\boldsymbol{\Sigma}_{t+1}(j,k))_{j,k=1,\dots,d}$. Hence, for a sufficiently large $n$, the realized covariance provides a good approximation to the quadratic covariation, which in turn is an unbiased estimator of the conditional covariance matrix, i.e. $E_t(\boldsymbol{\Sigma}_{t+1}) = \boldsymbol{\Sigma}_{t+1|t}$. So, by using intra-day returns, we can construct non-parametric and unbiased estimates of the conditional covariance matrix.

Several studies (see e.g., Renò, 2003, Griffin and Oomen, 2011, and Barndorff-Nielsen et al., 2011) confirm a bias towards zero for realized covariances computed over a short fixed time period due to non-synchronous trading. This phenomenon is often referred to as the Epps effect. However, as long as the price series is fairly liquid, this effect will be small or even negligible (see Renò, 2003 and Zhang, 2011). Another potential issue with intradaily returns is the lack of observations when markets are closed overnight, which causes a downward bias in the realized covariance matrix. To mitigate this, we include the overnight returns when constructing $\mathbf{V}_{t+1}$.

For obtaining forecasts of the realized covariance matrix we apply the HAR-DCC model developed in Oh and Patton (2016), which combines the ideas of the dynamic conditional correlation (DCC) model by Engle (2002) with the heterogeneous autoregressive (HAR) model of Corsi (2009). The approach relies on the following decomposition of $\mathbf{V}_{t+1}$:

$$\mathbf{V}_{t+1} = \sqrt{\mathbf{RVar}_{t+1}} \times \mathbf{RCorr}_{t+1} \times \sqrt{\mathbf{RVar}_{t+1}}, \tag{9}$$

where $\mathbf{RVar}_{t+1}$ is a diagonal matrix with the realized variances on the diagonal and $\mathbf{RCorr}_{t+1}$ is the correlation matrix. First, we model each component of $\mathbf{RVar}_{t+1}$ separately by specifying a HAR model for each (log) realized variance:

$$\log RVar_{i,t+1} = c + \beta_{i,1}\log RVar_{i,t}^{(d)} + \beta_{i,2}\log RVar_{i,t}^{(w)} + \beta_{i,3}\log RVar_{i,t}^{(m)} + \epsilon_{i,t+1}, \tag{10}$$

where $RVar^{(d)}$, $RVar^{(w)}$ and $RVar^{(m)}$ denote the realized variance of the previous day and the average realized variance over the past week and month, respectively. Then for $\mathbf{RCorr}_{t+1}$ we model all the unique elements in the matrix jointly as a restricted HAR model. Let $\rho_{t+1}$ denote the vectorized strictly lower triangular matrix of $\mathbf{RCorr}_{t+1}$, and let $\bar{\rho} = \frac{1}{T}\sum_{t=1}^{T}\rho_t$ denote the sample average. The $K(K-1)/2$-dimensional vector, $\rho_{t+1}$ is modeled using the following specification:

$$\rho_{t+1} = (1 - a_1 - a_2 - a_3)\bar{\rho} + a_1\rho_t^{(d)} + a_2\rho_t^{(w)} + a_3\rho_t^{(m)} + \eta_{t+1}. \tag{11}$$

The models in Eqs. (10) and (11) are estimated using OLS and with these we can obtain the forecast $\mathbf{V}_{t+1|t}$ via the decomposition in Eq. (9). Oh and Patton (2016) show that the forecast $\mathbf{V}_{t+1|t}$ is positive definite, since the original time series of $\mathbf{V}_t$ are all positive definite by construction.

For the portfolio allocation we need estimates of the covariances over the next month. Conveniently, the HAR framework extends to longer forecast horizons than one day (see Andersen et al., 2007 and Bollerslev et al., 2016). Using the above notation the prediction of the average variance and correlation over the next month is given as:

$$\log RVar_{i,t+m}^{(m)} = c + \beta_{i,1}\log RVar_{i,t}^{(d)} + \beta_{i,2}\log RVar_{i,t}^{(w)} + \beta_{i,3}\log RVar_{i,t}^{(m)} + \epsilon_{i,t+m}, \tag{12}$$

and

$$\rho_{t+m}^{(m)} = (1 - a_1 - a_2 - a_3)\bar{\rho} + a_1\rho_t^{(d)} + a_2\rho_t^{(w)} + a_3\rho_t^{(m)} + \eta_{t+m}, \tag{13}$$

from which we can obtain an estimate of the average covariance matrix over the next month. To get the cumulative covariance matrix we multiply this estimate by $m$ which is the number of days (we assume 21) in the month. This will be our final estimate of the conditional covariance matrix $\boldsymbol{\Sigma}_{t+m|t}$, to be applied in the portfolio optimization.

By estimating the $\beta$ or $a$ parameters using the full dataset, we could potentially be introducing a look-ahead bias into our results. However, as in Fleming et al. (2003), this is not an empirical problem since the estimate implied by the minimum MSE criterion is different from the one which maximizes the economic value. The same argument applies for estimating $b_0$ and $b_1$ in Eq. (5).[5]

### 2.4. Economic value

To assess the economic diversification benefits of adding the VIX ETPs to the benchmark portfolio we follow Fleming et al. (2001) and calculate the economic value of portfolio diversification with the short- and mid-term VIX ETPs.

In Eq. (1) we hold $aW_t$ constant. This is equivalent to setting the investor's relative risk aversion, $\gamma_t = -U''/U'W_t = aW_t/(1-aW_t)$ equal to some fixed value $\gamma$. The average realized utility can then be used to estimate the expected utility generated by a given level of initial wealth $W_0$ as follows:

$$\overline{U}(\cdot) = W_0\left(\sum_{t=0}^{T-1} R_{p,t+1} - \frac{\gamma}{2(1+\gamma)}R_{p,t+1}^2\right). \tag{14}$$

[5] Changing our $\beta_1$-estimate for VXX by e.g. $-.01$ gives better economic results of using this product. Likewise, subtracting 0.001 from our estimate of $b_1$ also results in higher economic value.

We estimate the economic value of VIX products by equating the average utilities for two alternative portfolios P-bench and P-short (or P-mid) as follows:

$$\sum_{t=0}^{T-1}(R_{p,t+1}^{short/mid} - \Delta) - \frac{\gamma}{2(1+\gamma)}\left(R_{p,t+1}^{short/mid} - \Delta\right)^2 = \sum_{t=0}^{T-1} R_{p,t+1}^{bench} - \frac{\gamma}{2(1+\gamma)}\left(R_{p,t+1}^{bench}\right)^2, \tag{15}$$

where $R_{p,t+1}^{short/mid}$ and $R_{p,t+1}^{bench}$ are the portfolio returns from the portfolio holding a VIX ETP and the benchmark portfolio, respectively. We can interpret $\Delta$ as the maximum performance fee that the investor would be willing to pay for switching from the benchmark portfolio to the VIX portfolio. $\Delta$ is thereby the economic value of investing in the VIX portfolio. As enlarging the investment opportunity set can only enhance diversification ex-ante, we stress that economic value in our setting is purely an ex-post performance measure.

We report our estimates of $\Delta$ as annualized fees in basis points (bps) using two different values of $\gamma$, 1 and 10.[6]

## 3. Introduction to VIX ETPs

Although the VIX itself is not a tradeable product, the Chicago Futures Exchange launch futures contracts on the VIX in March 2004. A vital property of VIX ETPs is that they are linked to VIX futures and not the VIX. S&P computes four constant maturity VIX futures indexes and all VIX ETPs track one of these.

The index tracked by most products is the *S&P 500 VIX Short-Term Futures Index* (SPVXSP), which tracks a strategy of holding long positions in the first- and second-month VIX futures contracts in proportions such that the average maturity is kept constant at 30 days at the close of trading. On the following day, a fraction of the first-month contract is sold and the same amount is invested in the second-month contract. This rolling strategy continues until the first-month contract expires at which point the position is fully invested in the second-month contract and the cycle repeats. Hence losses and gains are realized on a daily basis.

Consider the following example: On October 30, 2018, the first- and second-month contracts expire on 11/21/2018 and 12/19/2018, respectively. In order to have an average maturity of 30 days, the VIX futures position is comprised of 75% of the November contract and 25% of the December contract. On the next day, the fraction held in the November contract is reduced to 70% and increased for the December contract to 30%.[7] For hedging the exposure, the VIX ETPs must follow a similar strategy.[8]

As the VIX futures term structure is most often in contango (upward sloping), this strategy incurs a roll cost at each rebalancing, by selling the lower priced first-month contract and buying the higher priced second-month contract. This daily roll cost creates highly negative long-run returns for products with long positions in the VIX futures.

We pay special attention to two VIX ETPs, namely VXX and VXZ. Fig. 1 shows the price development of VXX and VXZ from the inception date until September 2018. VXX is the largest VIX ETP, and it is benchmarked to SPVIXSTR with a leverage factor of one, which means that its performance is benchmarked to one times the daily index return (less management fees and expenses). Because of the roll cost a long position in VXX during the sample period lost 99.9% of the initial investment.

VXZ is a mid-term product, linked to a strategy that rolls between the fourth-, fifth-, sixth-, and seventh-month futures to maintain a constant average maturity of five months. Since the curve is typically not as steep for longer maturities, the price difference between the contracts sold and bought is smaller, yielding a smaller roll cost. As a consequence the price deterioration has been less severe than for VXX.

When volatility is high and the VIX spikes, the futures curve inverts and slopes downward (backwardation). Short-term products will benefit more from this than mid-term products due to the steepness at the short end of the curve. So when volatility spikes, the returns of the short-term products tend to be highest. Hence, an investor who wants to insure against corrections in the equity market faces the trade-off between paying higher premiums in terms of negative returns and then getting larger payouts in times of market distress versus paying lower premiums but also getting lower potential payouts from the mid-term products.[9]

## 4. Data set

### 4.1. Sample period

Our sample period is from January 30, 2009 (the inception date of VXX and VXZ) to June 29, 2018. Although our sample is after the financial crisis, it contains several episodes of turbulence such as May 2010 (flash crash), June 2010 (Greek debt crisis), August 2011 (S&P downgrade of the US credit rating), August 2015 (Renminbi devaluation), and February 2018 (Volmageddon). In our sample, we have 2369 trading days, and, assuming 21 trading days per month, we have 112 monthly re-balancing points.

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[6] According to Gandelman and Hernández-Murillo (2015) the most commonly accepted measures of the coefficient of relative risk aversion lie between 1 and 3 hence our chosen levels should indeed test the most extreme cases.

[7] For further elaboration on how these fractions are calculated please see the index methodology by S&P, which is available at https://us.spindices.com/indices/strategy/sp-500-vix-short-term-index-mcap.

[8] Note that most of the ETPs are ETNs that are not required to hold the underlying futures contracts. Hence the issuer of these products can hedge themselves in other products or simply choose not to hedge.

[9] See also Alexander et al. (2015) and Bollen et al. (2017) for descriptive statistics, trading volume and size on different VIX ETPs.

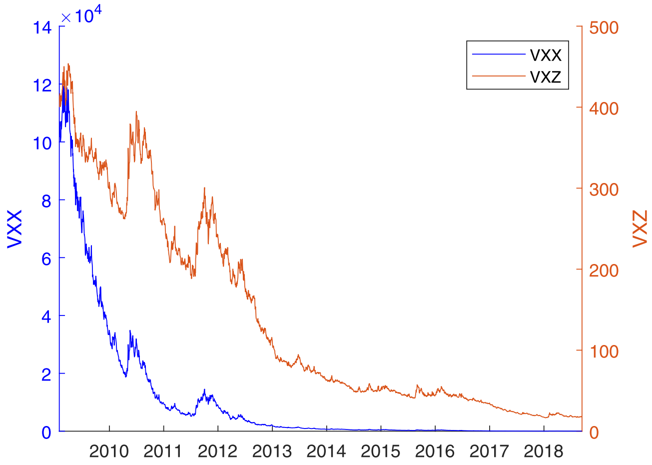



**Fig. 1.** Price development of VXX and VXZ. Over the lifetime of the products, the value of VXX has been severely eroded and the issuer has made no less than five 1-for-4 reverse splits (only one for VXZ). The depicted price development has been adjusted for these hence the magnitude of the left-hand $y$-axis.

### *4.2. Data sources*

For our empirical analysis, we use the products VXX and VXZ for our short-term (P-short) and mid-term (P-mid) VIX portfolios. These are the first VIX ETPs issued, hence using these gives us the longest sample. Furthermore, VXX has the largest market value and is the most liquid of all the VIX ETPs (across both leverage and term).

As a proxy for the US stock and bond components of the portfolios, we use two market-wide indexes traded as exchange-traded funds (ETFs) namely SPY tracking the S&P 500 index and AGG tracking the Bloomberg Barclays U.S. Aggregate Bond Index.[10] For the risk-free interest rate, we use the 1-month US Treasury bill, collected from Kenneth French's webpage.[11]

For calculating the realized covariance matrix and the expected returns, we use intra-daily observations. The data are extracted from the NYSE Trade and Quote (TAQ) database and include both trade and quote data for the official trading hours from 9:30 to 16:00 local New York time. For cleaning the data, we follow the routines proposed in Christensen et al. (2014) and Barndorff-Nielsen et al. (2009). The VIX premium used for estimating expected returns on VIX ETPs are obtained from the web page of Ing-Haw Cheng.[12]

### *4.3. Realized returns*

Very high-frequent data are contaminated with microstructure noise (e.g., bid–ask bounce and price discreteness) which will make our realized covariance estimator diverge. To mitigate this, we use sparse sampling at five-minute intervals, which is the standard frequency in the literature. This is done by constructing a grid of five-minute intervals that spans the trading day. Next, we identify and take the log of the last traded price at each grid point and then take the first differences of these prices. This sampling method gives us 79 prices during the trading day (the first sampling is 9.30) hence 78 log-returns. With the addition of the overnight return we finally get a total of 79 log-returns for each trading day.

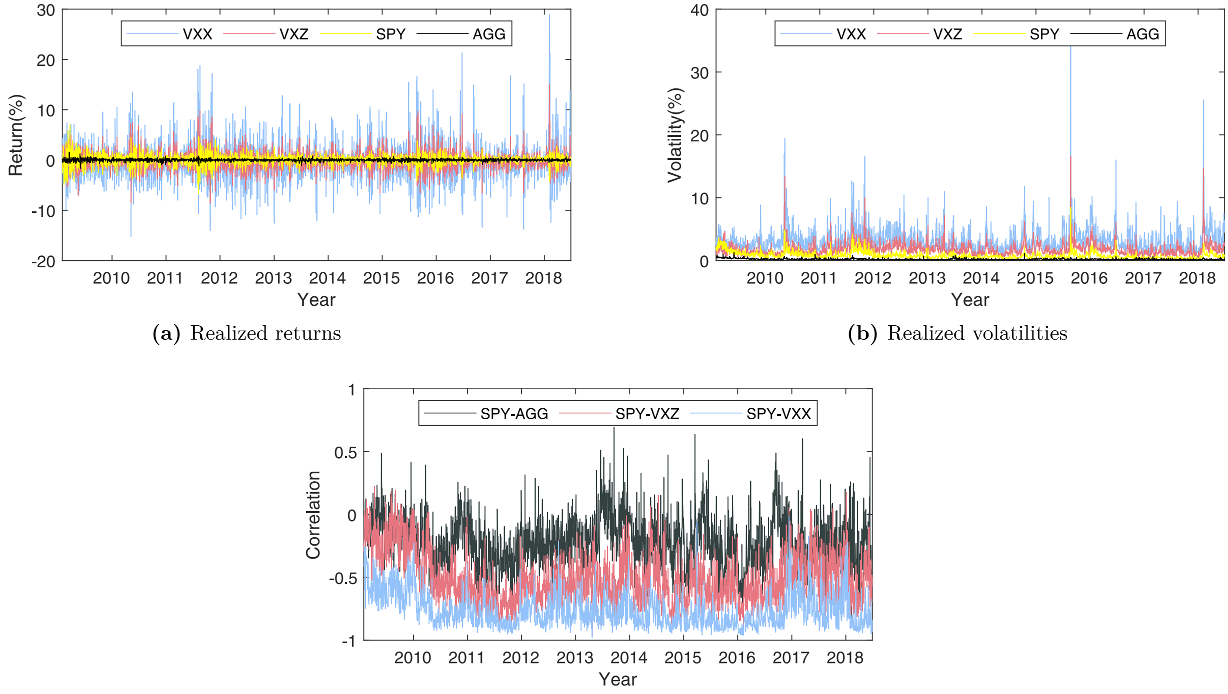
Fig. 2(a) plots the realized returns for each instrument. We note the very severe spikes for both VIX ETPs on February 5, 2018, where the VIX complex blows up (also referred to as Volmageddon). On this day, the VIX make a one-day move from 17.31 to 37.32, and as a consequence, the VIX futures term structure goes into steep backwardation. This yield a very large one-day return for both VXX and VXZ of 28.9% and 15.1%, respectively. We also note severe spikes in June 2016 when stocks tumble due to weakening activity data in the US and China and around August 2011 where the US credit rating is downgraded.

Table 1 provides descriptive statistics for the realized returns. Panel (a) shows that the VIX ETPs have very poor average returns which are more negative than reported in previous studies (e.g., Alexander et al., 2015 and Alexander and Korovilas, 2013). This is, of course, no surprise given the fact that the VIX futures term structure has mainly been in contango during our sample period. The standard deviations indicate that the VIX ETPs are far more volatile than both stocks and bonds. Finally, we note that contrary to SPY and AGG (the stock and bond indexes), the return skewness for both VIX ETPs are positive. This indicates a potential for increasing skewness in a stock–bond portfolio.

[10] The index measures the performance of the total US investment-grade bond market.

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

[12] http://www.dartmouth.edu/~icheng/.




(a) Realized returns

(b) Realized volatilities

(c) Realized correlations

**Fig. 2.** Daily realized returns, realized volatilities and realized correlations. Panel (a) and (b) show the daily realized returns and realized volatilities, respectively, for SPY, AGG, VXX, and VXZ. The realized returns are computed as the sum of intra-day log-returns. The values are not annualized. Panel (c) shows the daily realized cross-market correlations between SPY and the three other tickers used for constructing portfolios. The realized correlations are based on our covariance matrix estimated by the procedure described in Section 2.3.

**Table 1**
Summary statistics for SPY, AGG, VXX and VXZ returns. This table provides summary statistics for daily returns (computed as the sum of intra-day log-returns) on SPY, AGG, VXX and VXZ. Panel (a) reports the mean returns ($\mu$), standard deviations ($\sigma$), and mean realized volatilities ($\overline{\sigma}_t$). These values are annualized using 252 trading days per year. Panel (b) reports the mean realized correlations based on our covariance matrix estimated by the procedure described in 2.3.

Panel (a): Annualized mean return, Standard deviation, Mean realized volatility and Skew

| Ticker | $\mu$ | $\sigma$ | $\overline{\sigma}_t$ | Skew |
|---|---|---|---|---|
| SPY | 12.34 | 16.22 | 13.08 | −0.28 |
| AGG | 0.46 | 3.67 | 3.50 | −0.33 |
| VXX | −84.32 | 62.11 | 52.52 | 0.87 |
| VXZ | −32.34 | 30.27 | 28.81 | 0.66 |

Panel (b): Correlation matrix

| Ticker | SPY | AGG | VXX | VXZ |
|---|---|---|---|---|
| SPY | 1.00 | −0.22 | −0.77 | −0.51 |
| AGG | −0.22 | 1.00 | 0.19 | 0.15 |
| VXX | −0.77 | 0.19 | 1.00 | 0.59 |
| VXZ | −0.51 | 0.15 | 0.59 | 1.00 |

### 4.4. Realized volatilities and correlations

All instruments that we use are fairly liquid, so we do not expect any issues related to the potential Epps effect, cf. the discussion in Section 2.

The average realized volatility estimates are reported as $\overline{\sigma}_t$ in Panel (a) of Table 1 and are generally consistent with the standard deviation of the realized returns. The entire series of realized volatilities are plotted in Fig. 2(b). We see that for both VXX and VXZ, the volatility varies considerably over the entire sample with pronounced spikes in May 2010, August 2015, and February 2018. Furthermore, the estimates for both VIX ETPs are far above those for stocks and bonds for the entire sample.

Fig. 2(c) plots the daily realized correlations. Panel (b) of Table 1 provides the correlation matrix for the different products, where the reported correlations are the average realized correlations over the sample period.

As expected and in line with the results reported by Alexander et al. (2015), both VXX and VXZ correlate very negatively with SPY. The SPY–VXX correlation is between −0.7 and −0.9 with a maximum in absolute terms of −0.98. However on December 13, 2016, SPY, VXX, and VXZ all have a positive realized return and the correlation drops, in absolute terms, to −0.02, and the correlation between SPY and VXZ actually becomes positive. For the entire sample period, VXX is more negatively correlated with SPY than VXZ, which is no surprise given the different structures of the products.

### 4.5. Transaction costs

For assessing the out-of-sample portfolio performance, transaction costs are taken into account. At each portfolio re-balancing point, a cost equal to the product of the bid–ask spread in bps and the absolute change in weights, summed over all assets, is subtracted from the portfolio return.

We estimate the daily bid–ask spread via quote data from TAQ as the size weighted median spread of all the quotes during the trading day. The mid-price is computed as the sum of the size weighted average bid and ask, divided by two.[13]

## 5. Empirical economic value of VIX ETPs

Via the regression in Eq. (5), we estimate the expected returns over the next month for VXX and VXZ as:

$$E_t\left[R_{t+1}^{VXX}\right] = \underset{(-0.21)}{-0.0049} \underset{(-3.55)}{-0.0465} \times VIXP_{T|t} \quad \text{and} \quad E_t\left[R_{t+1}^{VXZ}\right] = \underset{(-0.47)}{-0.0054} \underset{(-2.31)}{-0.0164} \times VIXP_{T|t}, \tag{16}$$

which we input to the CE strategy.[14]

Throughout the sample period, the portfolios are marked to market each day. That is, the ex-post daily return for each portfolio is computed by multiplying the portfolio weights by the observed next-day returns on the components. At each re-balancing day, the weights are changed according to the allocation strategy. We subtract the transaction cost from the portfolio return.

Fig. 3 depicts the evolution of $100 invested in each of the portfolios for both the CE and constant strategy.

The relative performances of the VIX portfolios are dependent on the allocation strategy. Considering the CE strategy, Fig. 3(a), P-short clearly has the best overall performance. Over the entire period, the portfolio has a total return of 83.8% against 41.3% and 37.1% for the benchmark and P-mid respectively. For the first two and a half year of our sample, all the portfolios perform the same, as neither P-short nor P-mid has yet allocated any capital to the respective VIX product (see Section 6). P-short then appreciates sharply in value during August 2011, when the US credit is downgraded. Through this month VXX has an average daily return of 2.2%. The VIX premium becomes negative already by the end of July, which provides a signal of positive expected returns of VXX, resulting in a high allocation to this asset. P-mid does not experience the same increase in value through this period of turmoil, due to a lower allocation and lower daily returns of VXZ. Next, through November 2011 (eurozone debt crisis) P-bench suffers a loss of about 8%. P-short decreases only 3% due to an increased allocation in VXX. Finally there is a sharp upward spike in P-short, occurring December 2012 (concerns about US debt ceiling) where the allocation to VXX has increased. Even though there is a reversal in value as equity markets stabilize, the portfolio still has a positive return of almost 2% through this period, whereas the benchmark is flat.

Panel (a) of Table 2 reports the performance results for the CE strategy. For the entire sample, the two VIX portfolios yield mean returns of 7.01% (P-short) and 3.76% (P-mid). The sample volatilities are 9.86% (P-short) and 8.73% (P-mid). P-short clearly beats the benchmark portfolio which has a mean return of 4.09% and a volatility of 8.71%. This difference in performance translates into an ex-post economic value which is positive. A mean–variance optimizing investor would pay between 282.09 and 286.70 annual bps for swapping the returns of the benchmark portfolio with the returns of P-short. Overall it appears that through tactical allocation to VXX, the CE investor can benefit from periods of market stress.

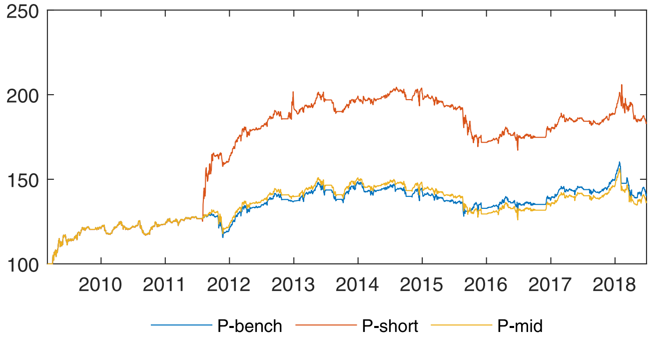
The performance of all three portfolios is also broken down by two-year sub-periods. For only two sub-periods, (2009–2010) and (2015–2016), is the economic value of the returns of P-short negative. As the performance of P-mid is overall worse than the benchmark, the returns generated by P-mid have negative economic value. Hence the VIX premium is not working well as a signal for timing the allocation to VXZ. Since this product is linked to VIX futures contracts on the longer end of the term structure, our results indicate that the VIX premium is a better predictor of short-term futures returns.

For the constant strategy, the results reported in Panel (b) of Table 2, indicate that VIX ETPs are unsuitable as a portfolio component to be held constantly through time like stocks and bonds. For the entire sample and all sub-periods, the economic value for both products is negative. Hence the negative returns that VIX ETPs generate during calm markets is a too large drag on the portfolio returns and as such the insurance that they offer is way too expensive to hold for long periods. Intuitively, we would expect P-short and P-mid to outperform the benchmark during times of market stress. From Fig. 4(a) we see that this is also the case. Here we have plotted the three-month rolling economic values for P-short and P-mid together with movements in the VIX. The overall correlation between the rolling economic values and the VIX is 0.50 for P-short and 0.56 for P-mid.

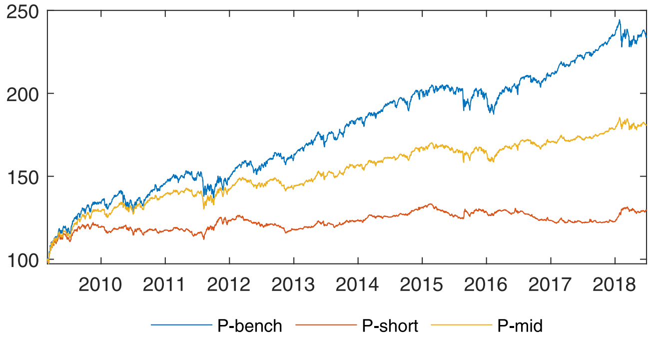
Fig. 4(b) plots the 3-month rolling Sharpe ratios against the VIX. The correlations between the Sharpe ratios and VIX are 0.17 and 0.25 for P-short and P-mid, respectively. As expected, there is a clear connection between changes in the VIX and the value of holding VIX products. This is clearly visible during the pronounced VIX spikes in 2010, 2011, and 2018 where the economic values of both VIX portfolios increase sharply. However, as the VIX then reverts, so does the economic value of both portfolios, and

[13] The bid–ask spread in bps is the spread divided by mid price.

[14] $t$-statistics calculated with Newey–West standard errors are given in parenthesis.




(a) CE strategy




(b) Constant strategy

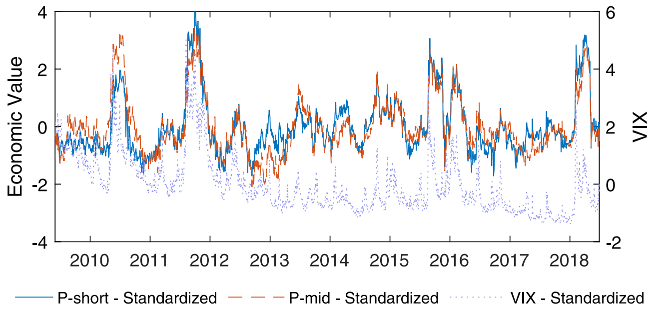
**Fig. 3.** Ex-post performance. Panel (a) and (b) show the ex-post performance for the CE and constant strategy, respectively.

**Table 2**

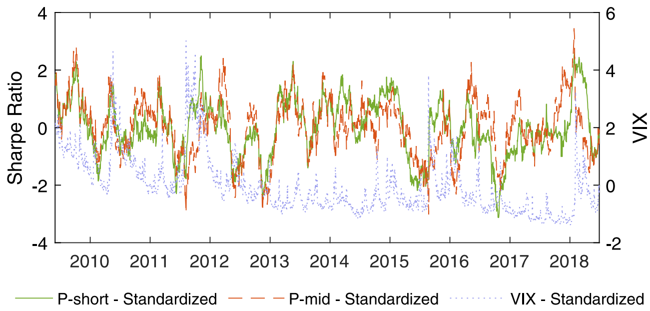
Ex-post performance of constrained portfolios. The table summarizes the ex-post performance for the benchmark portfolio P-bench and the two portfolios with access to VIX products, P-short and P-mid. Panel (a) reports the results for the CE investor with an absolute risk aversion, $a$, equal to 4 and Panel (b) for the investor using a constant allocation. For each portfolio, we report the annualized mean return ($\mu$), the annualized volatility ($\sigma$), the Sharpe Ratio (SR), and skewness of the returns (Skew). Furthermore, for P-short and P-mid, we report the economic value ($\Delta$) over the sample period in annual bps. We report the economic value for a level of relative risk aversion ($\gamma$) of 1 and 10. The sample period is January 30, 2009, through June 2018. The first-month of data is withheld for estimating the expected return and conditional covariance matrix for the first re-balancing date. We also report results for each two-year subsample.

| | P-bench | | | | P-short | | | | | | P-mid | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period | $\mu$ | $\sigma$ | SR | Skew | $\mu$ | $\sigma$ | SR | Skew | $\Delta_1$ | $\Delta_{10}$ | $\mu$ | $\sigma$ | SR | Skew | $\Delta_1$ | $\Delta_{10}$ |
| Panel (a): CE | | | | | | | | | | | | | | | | |
| Entire sample | 4.09 | 8.71 | 0.444 | −0.54 | 7.01 | 9.86 | 0.688 | 1.44 | 286.70 | 282.09 | 3.76 | 8.73 | 0.405 | −0.52 | −34.45 | −33.22 |
| 2009–2010 | 11.96 | 10.79 | 1.095 | 0.15 | 11.94 | 10.79 | 1.093 | 0.15 | −4.92 | −4.92 | 11.98 | 10.80 | 1.096 | 0.15 | 1.85 | 1.23 |
| 2011–2012 | 5.51 | 8.17 | 0.672 | −0.90 | 23.32 | 12.99 | 1.794 | 2.42 | 1755.90 | 1735.0 | 6.47 | 8.28 | 0.779 | −0.86 | 98.44 | 95.98 |
| 2013–2014 | 1.85 | 7.67 | 0.240 | −0.33 | 4.07 | 7.76 | 0.523 | 0.623 | 226.41 | 221.48 | 2.06 | 7.48 | 0.274 | −0.27 | 22.15 | 22.15 |
| 2015–2016 | −1.78 | 8.13 | −0.231 | −0.51 | −5.78 | 9.24 | −0.636 | −0.42 | −404.82 | −408.52 | −4.14 | 8.45 | −0.503 | −0.51 | −237.48 | −238.71 |
| 2017–2018 | 3.26 | 8.90 | 0.253 | −1.58 | 3.58 | 8.79 | 0.293 | 2.00 | 32.0 | 33.22 | 2.90 | 8.80 | 0.215 | −1.55 | −35.68 | −36.91 |
| Panel (b): Constant | | | | | | | | | | | | | | | | |
| Entire sample | 9.58 | 9.22 | 1.015 | −0.07 | 2.93 | 5.67 | 0.477 | 1.05 | −652.15 | −641.07 | 6.63 | 7.14 | 0.897 | 0.37 | −286.70 | −279.32 |
| 2009–2010 | 21.90 | 12.31 | 1.709 | 0.40 | 9.08 | 8.73 | 1.023 | 1.11 | −1191.7 | −1181.3 | 18.43 | 10.36 | 1.764 | 0.65 | −264.55 | −236.25 |
| 2011–2012 | 5.55 | 10.48 | 0.528 | −0.42 | 0.24 | 5.80 | 0.038 | 0.28 | −511.88 | −492.19 | 1.79 | 7.85 | 0.226 | −0.12 | −374.06 | −393.75 |
| 2013–2014 | 10.65 | 6.75 | 1.577 | −0.34 | 5.72 | 3.76 | 1.519 | −0.04 | −486.04 | −478.65 | 7.41 | 5.03 | 1.471 | −0.36 | −318.69 | −315 |
| 2015–2016 | 2.41 | 8.38 | 0.276 | −0.26 | −2.29 | 4.53 | −0.528 | 0.76 | −457.73 | −442.97 | 1.19 | 5.93 | 0.184 | 0.04 | −108.28 | −107.64 |
| 2017–2018 | 8.00 | 6.46 | 1.082 | −1.29 | 3.01 | 4.16 | 0.481 | 0.91 | −493.42 | −488.50 | 4.57 | 4.75 | 0.751 | −0.58 | −338.38 | −333.46 |

it becomes negative for lower levels of the VIX. This suggests that throughout our sample, spikes in volatility have been too rare and too short-lived for these products to have any sustainable long-term value as a constant component in a portfolio. Thus, in the volatility regime we have experienced since the inception of the products, an investor would need to tactically time the allocations for the products to add any value.




(a) Economic value




(b) Sharpe ratio

**Fig. 4.** 3-month rolling performance and the VIX level. Panel (a) shows the three-month rolling economic value of portfolios P-short and P-mid against the level of the VIX. The economic value is for a relative risk aversion ($\gamma$) equal to 1. Panel (b) shows the three-month rolling Sharpe ratios of portfolios P-short and P-mid against the level of the VIX. To facilitate a comparison, the time series are standardized to have a 0 mean and unit standard deviation. These values are for the constant strategy. The correlation between economic value and the VIX is 0.50 (P-short) and 0.56 (P-mid). The correlation between Sharpe ratios and the VIX is 0.17 (P-short) and 0.25 (P-mid).

A final interesting observation from Table 2 is that the P-short and P-mid tend to have a higher skewness than the benchmark portfolio. So by including VIX products, an investor is able to counteract the typical negative skewness of stock and bond portfolios. This could suggest that the negative returns investors with a constant allocation in general pay for holding long positions in VIX ETPs, can be viewed as a skewness premium, in order to reduce negative skewness of common portfolio components.

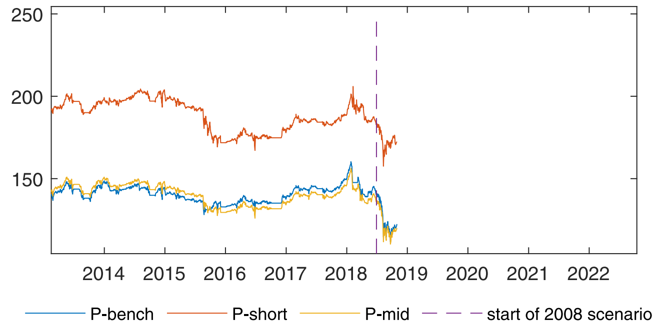
### *5.1. Financial crisis scenario*

A potential weakness of our analysis is that our sample period does not contain the recent financial crisis of 2008 (the 2008 crash in the following), an event during which the previous literature has reported benefits of holding some kind of volatility instrument (variance swaps, VIX futures, or VIX ETPs). This could potentially bias our conclusion about holding constant allocations in VIX ETPs.

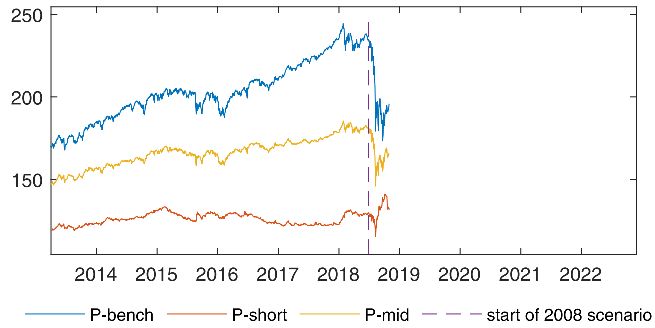
For this reason, we extend our results with a simple what-if analysis. What would have happened to the value of the portfolios if a scenario like the 2008-crash had occurred at the end of the sample period? Would the distress in asset prices during such a crash be sufficient for the VIX portfolios to have caught up with the benchmark portfolio? For the CE investor, an occurrence of such a crash could lead to an even larger out-performance of P-short.

We take the perspective of an investor who has held a position in VIX ETPs in expectation of protection during market crashes. Now, after having endured a long period of suppressed volatility, we simulate that a 2008 crash occurs. The question is whether this gives sufficient reason for having held VIX ETPs constantly in the portfolio, and thereby having suffered long periods of inferior returns compared to the benchmark.

We consider the height of the 2008 crash. We assume that during the simulated new crash, our assets follow the same path as during the actual 2008 crash. We use the period from September 2, 2008, to December 31, 2008, which encapsulates the 2008 crash (e.g. it contains the two historic highs of the VIX at 80.06 and 80.86 in October and November). As VXX and VXZ do not exist in 2008, we proxy these by the returns of their underlying futures indexes (SPVIXSTR and SPVIXMTR). This is also done in other studies (see e.g., Whaley, 2013 and Bordonado et al., 2017). For each portfolio, we use the average weights from our previous analysis, together with the asset returns during the 2008 crash to calculate the portfolio returns during the crash scenario.




(a) CE strategy




(b) Constant strategy

**Fig. 5.** Financial crisis scenario — portfolio performance. Panel (a) and (b) show the ex-post performance for the CE, and constant strategy, respectively, throughout the crash scenario.

**Table 3**
Financial crisis scenario — portfolio performance. The table summarizes the performance of each portfolio throughout the crash scenario. For each portfolio, we report the annualized mean return ($\mu$), and the annualized volatility ($\sigma$). For P-short and P-mid, we report the economic value ($\Delta$) in annual bps for a level of relative risk aversion ($\gamma$) of 1 and 10.

| Portfolio | $\mu$ | $\sigma$ | $\Delta_1$ | $\Delta_{10}$ |
|---|---|---|---|---|
| Panel (a): CE | | | | |
| P-bench | −47.5 | 37.49 | | |
| P-short | −20.9 | 25.34 | 2844.80 | 3002.30 |
| P-mid | −43.33 | 35.53 | 452.81 | 477.42 |
| Panel (b): Constant | | | | |
| P-bench | −46.96 | 37.17 | | |
| P-short | 6.67 | 19.51 | 5591.3 | 5807.8 |
| P-mid | −23.9 | 27.79 | 2460.9 | 2598.8 |

Fig. 5 depicts the value development of each portfolio in the crash scenario. First, for the CE investor the gap in value between P-short and P-bench has increased at the end of the crash scenario. Second, for the constant strategy, in terms of portfolio value, none of the VIX portfolios have caught up with their respective benchmark at the end of the crash period.

Consulting Table 3 for the performance statistics, it is clear that the allocations to the VIX ETPs have to some extent cushioned the portfolio returns through the crash period. All the VIX portfolios have performed much better than the benchmark, which translates into large economic values.

The question is then what the overall picture looks like if we extend the actual portfolio returns with the returns from the crash scenario. Table 4 reports the performance results. Compared with the results based on the actual sample period (cf. Table 2), we see that adding the crash scenario period has increased the ex-post value of holding VIX ETPs in the portfolios. However, for the constant strategy the economic values are still negative, thus the overall conclusion stays the same.

**Table 4**
Portfolio performance: Entire sample extended with financial crisis scenario. The table summarizes the performance of each portfolio for the entire sample with the addition of the crash scenario. For each portfolio, we report the annualized mean return ($\mu$), the annualized volatility ($\sigma$), the Sharpe Ratio (SR), and the skewness of returns (Skew). For P-short and P-mid, we report the economic value ($\Delta$) in annual bps for a level of relative risk aversion ($\gamma$) of 1 and 10.

| Portfolio | $\mu$ | $\sigma$ | SR | Skew | $\Delta_1$ | $\Delta_{10}$ |
|---|---|---|---|---|---|---|
| Panel (a): CE | | | | | | |
| P-bench | 2.60 | 10.08 | 0.234 | 0.05 | | |
| P-short | 6.16 | 10.39 | 0.569 | 1.43 | 354.38 | 352.53 |
| P-mid | 2.39 | 9.99 | 0.215 | 0.22 | −20.61 | −20.00 |
| Panel (b): Constant | | | | | | |
| P-bench | 7.61 | 11.41 | 0.646 | 0.61 | | |
| P-short | 3.06 | 6.65 | 0.424 | 1.56 | −433.43 | −415.60 |
| P-mid | 5.57 | 8.72 | 0.611 | 0.61 | −191.95 | −177.19 |

For investors who hold VIX ETPs constantly as a portfolio component, not even the occurrence of a market crash of the same magnitude as the 2008 crash, is enough for the products to add positive ex-post economic value. For this to happen either the crash should have been even more severe or the VIX portfolios should have larger weights in VIX ETPs. The latter premise would imply a correct timing of the allocation. So for investors who consider holding VIX ETPs at rather constant levels, the conclusion is clear. It has been way too expensive to hold these products over a long calm period and not even a crash of similar magnitude as the 2008 crash would have been enough to break even. For successful application, the allocation to these products must be timed in a tactical manner as in the case of the CE investor's allocations to VXX. Our results indicate that the VIX premium as defined in Cheng (2019) provides a rather good signal of when to increase or decrease the exposure.

In the appendix, we consider various alternative allocation weights for the constant portfolio. None of these yield positive economic value for either of the VIX portfolios.

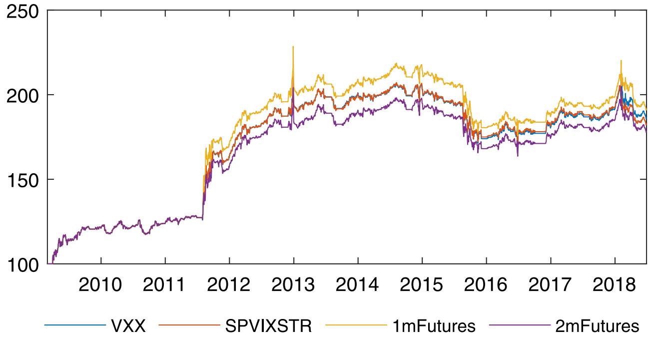
### 5.2. VIX futures

In this subsection we investigate how much performance will be impacted if the VIX ETPs are substituted with their underlying assets, the VIX futures. Both VXX and VXZ have an expense ratio of 0.89%, which is relatively high compared to SPY and AGG for which they are 0.09% and 0.05%. The costs will compound over time and might have a non-negligent impact on long-term returns. Furthermore, previous studies (e.g. Bollen et al., 2017 and Dong, 2016) suggest that VIX ETPs are the "tail wagging the dog" in the sense that much of the issuers hedging end up in the VIX futures market, steepening the term structure and making the roll-strategy, described in Section 3, even more expensive to follow. Thus, it might be more efficient to hold a position in a single VIX futures contract. We evaluate the performance using the returns of SPVIXSTR instead of VXX. Furthermore, we consider two different strategies where only one VIX futures contract is bought. The first strategy is simply to invest in the first-month contract and hold it through the re-balancing period. If the contract has less than three days to expiry, the position will be rolled over to the second-month contract, which is then held for the remaining days until the next re-balancing. The second strategy is to buy the second-month contract. If this contract at some point within our re-balance window becomes the first-month contract we will not roll it over. We will use the same allocations as those applied in our first empirical analysis (see Section 6). As we do not have any bid–ask spreads on the VIX futures, our comparison with VXX will be on a zero transaction cost basis.[15] The portfolio values are displayed in Fig. 6 and Table 5 reports the performance in terms of economic value. In the following we only comment on economic value for $\gamma = 1$.

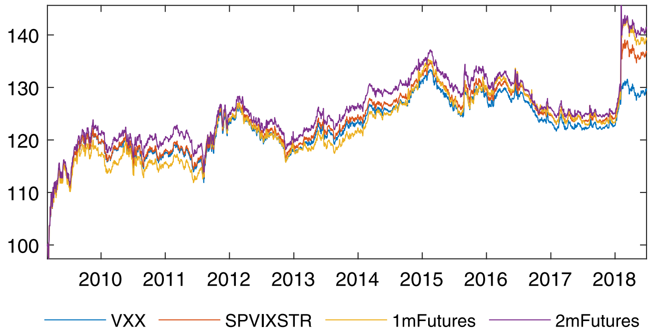
First of all, note that the economic value of VXX has increased for both strategies, compared to our previous analysis (cf. Table 2), by 3 and 2 bps approximately, due to the removal of transaction costs. Consider then the impact of substituting VXX with SPVIXSTR for the CE strategy. The performance of using SPVIXSTR instead of VXX is 16 annual bps worse (difference in economic values) which seems counterintuitive given that SPVIXSTR is not subject to an expense ratio. However, this difference is very much driven by the very volatile period around the Volmagedon event in the beginning of February 2018. On February 6, SPVIXSTR falls by 25%, whereas VXX only falls by 2.4%. A reason for this discrepancy could be that VXX is an ETN and not an ETF. Thus, new shares are solely created by the issuer, which might provide obstacles for market makers to arbitrage away prices above net asset value. Disregarding this period, from Fig. 6(a), we see that the portfolio paths are almost identical, so for an investor who is timing the exposure and only hold VXX for brief periods the expense ratio seems to matter very little for the overall performance. If we consider the constant strategy, however, there is an increase in performance by 59 bps (from −650 to −592) by swapping VXX with SPVIXSTR. Hence, for constant allocations the compounding of expenses paid to the issuer has a substantial impact on the long term.

Next, consider the impact of applying just a single futures contract instead VXX. If the first-month strategy is applied there is a significant performance increase of 55 bps, for the CE strategy. When the futures term structure inverts during periods of market stress, the first-month contract will typically be the one that increases most in value. As the strategy of VXX and SPVIXSTR holds

[15] It is not unrealistic that large institutional investors would be able to trade at prices near the mid-quote.




(a) CE strategy




(b) Constant strategy

**Fig. 6.** VIX futures comparison. This figure compares the performances using VXX against SPVIXSTR, the first- and, the second-month futures contract only. Panel (a) is for the CE strategy and Panel (b) is for the constant strategy.

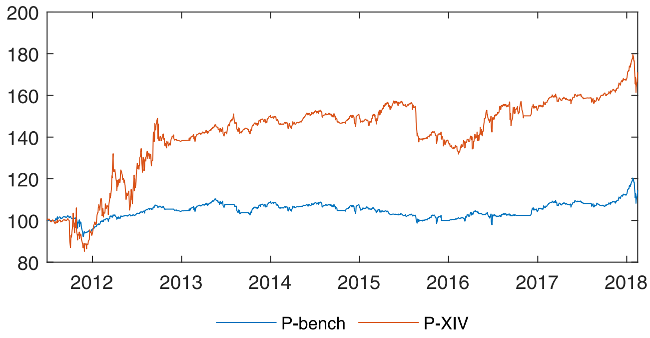
**Table 5**
VIX futures comparison. The table summarizes the performance comparisons using VXX, SPVIXSTR, first-month VIX futures and second-month VIX futures as the exposure to volatility. We report the economic value ($\Delta$) in annual bps for a level of relative risk aversion ($\gamma$) of 1 and 10.

| Panel (a): CE | $\Delta_1$ | $\Delta_{10}$ |
|---|---|---|
| VXX | 289.78 | 285.62 |
| SPVIXSTR | 273.78 | 265.70 |
| 1mFutures | 329.00 | 317.15 |
| 2mFutures | 247.78 | 239.63 |
| **Panel (b): Constant** | | |
| VXX | −650.30 | −639.69 |
| SPVIXSTR | −591.55 | −583.08 |
| 1mFutures | −570.78 | −564.20 |
| 2mFutures | −554.94 | −548.17 |

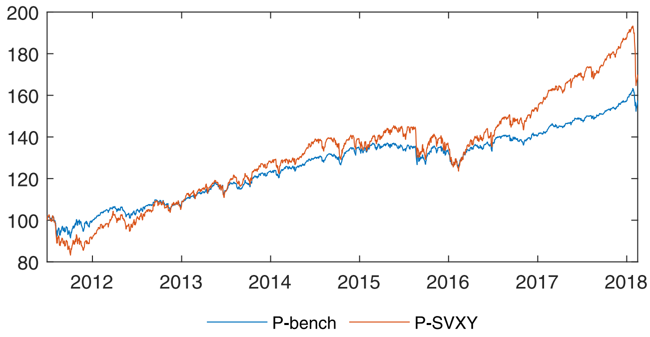
positions in both the first-month and second-month contract, these will not increase as much. The CE strategy proves to be relatively successful at timing these periods, so the optimal strategy is simply to invest only in the first-month contract. Conversely, for the constant strategy what yields the best performance (in terms minimizing underperformance relative to the benchmark), is to apply the second-month contract. This is due to the fact that the further you go on the term structure the lower is the "roll-down" of the futures price. Hence the drag on portfolio performance will be lower by just holding a single position in the second-month contract.

It is clear from the above that the design of volatility exposure has an impact on the overall performance, however the changes in performance that we find here do not change the conclusion that a constant allocation to a VIX futures or assets linked to VIX futures is far too expensive. In order for VIX futures exposure to add economic value the timing aspect is still crucial.

Variance swaps is another important volatility asset. However, applying returns of synthetic variance swaps made available by Johnson (2017) for the period of March 2009–December 2017 yield much worse performance for both strategies. These results




(a) CE strategy




(b) Constant strategy

**Fig. 7.** XIV performance. Panel (a) and (b) show the ex-post performance applying XIV. For the constant strategy, the weights are $\mathbf{w}'_t = (60\%, 40\%, 0\%)$ for the benchmark, and $\mathbf{w}'_t = (50\%, 40\%, 10\%)$ for the VIX portfolio.

are available upon request. Our optimal allocation weights for the CE strategy have been estimated on the basis of assets linked to VIX futures and not variance swaps, which could very well be the reason for this. Proper allocation to variance swaps would in our case imply the calculation of intradaily synthetic variance swaps, which is beyond the scope of this study.

### *5.3. Inverse VIX ETPs*

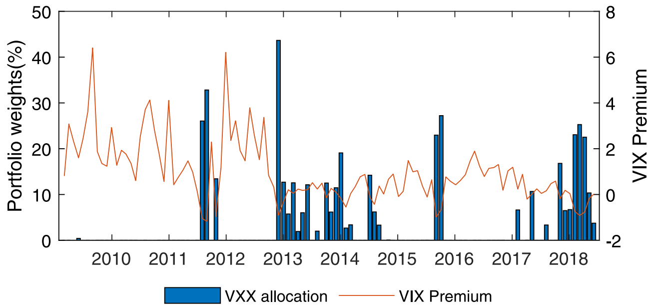
A final important aspect to consider, is the value of including inverse VIX ETPs. These products promise minus the leverage factor times the return of the underlying benchmark, SPVXSP. Hence, for a constrained investor these products provide access to short exposure to VIX futures and will in general earn the roll cost incurred by long VIX ETPs. As in our previous analyzes we take the perspective of an investor who applies a tactical allocation to a VIX ETP and an investor who holds the instrument as a constant portfolio component. However, VXX is now substituted with an inverse VIX ETP. We consider the product XIV, which has a leverage factor of minus one. This is the largest and most liquid inverse product through our sample period. The inception date is November 29, 2010. However, during the first-months after inception the liquidity is very poor. Hence, for this analysis, we start our sample period on June 1, 2011. Due to a 93% loss of value during the Volmageddon event, XIV is closed by the issuer on February 15, 2018. In the appendix we include the results of the second largest inverse product, SVXY.

Fig. 7(a) plots the portfolio values for the CE strategy. Disregarding the beginning of the sample, P-XIV is consistently above P-bench. In particular, there is a large surge in value over the period from December 2011 to October 2012, where on average 43% of capital is allocated to XIV. In this period XIV has an average daily return of 0.65% against 0.078% in SPY. During the last half of August 2015 (Renminbi devaluation) there is a large drawdown in the value of P-XIV. XIV has an average daily return of −5.9% against −0.61% in SPY, which is why P-XIV decreases much more than the benchmark. For the constant investor, we see from Fig. 7(b) that P-XIV also has several drawdowns, particularly at the end of the sample, where XIV is closed. Despite these, the value of P-XIV is above P-bench for the majority of our sample.

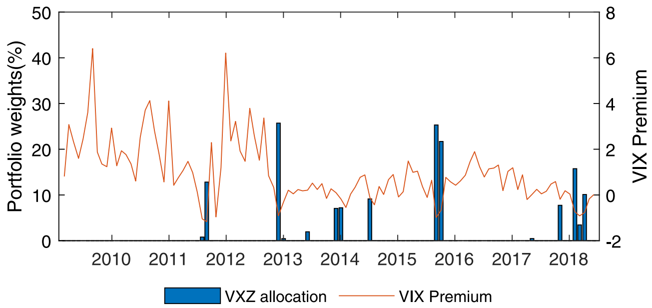
The performance results for XIV are reported in Table 6. For the CE investor the allocations to XIV result in much higher mean returns than for the benchmark (9.50% against 2.31%) but the returns are also more volatile as the standard deviation is 16.63% for P-XIV and only 6.93% for P-bench. These numbers translate into ex-post economic values of 662 and 615 bps for $\gamma = 1$ and 10, respectively. For the constant strategy, P-XIV also outperforms the benchmark, with economic value of 137 and 119 bps. The

**Table 6**
Portfolio performance: XIV. The table summarizes the ex-post performance, applying the inverse VIX ETP, XIV. For each portfolio, we report the annualized mean return ($\mu$), the annualized volatility ($\sigma$), the Sharpe Ratio (SR), and the skewness of returns (Skew). For P-short and P-mid, we report the economic value ($\Delta$) in annual bps for a level of relative risk aversion ($\gamma$) of 1 and 10. For the constant strategy, the weights are $\mathbf{w}_t' = (60\%, 40\%, 0\%)$ for the benchmark, and $\mathbf{w}_t' = (50\%, 40\%, 10\%)$ for the VIX portfolio.

| Portfolio | $\mu$ | $\sigma$ | SR | Skew | $\Delta_1$ | $\Delta_{10}$ |
|---|---|---|---|---|---|---|
| Panel (a): CE | | | | | | |
| P-bench | 2.31 | 6.93 | 0.307 | −1.51 | | |
| P-XIV | 9.50 | 16.63 | 0.560 | −0.93 | 661.99 | 615.23 |
| Panel (b): Constant | | | | | | |
| P-bench | 7.21 | 8.33 | 0.843 | −0.52 | | |
| P-XIV | 8.80 | 12.61 | 0.684 | −0.96 | 136.58 | 118.74 |




(a) Allocation to VXX




(b) Allocation to VXZ

**Fig. 8.** CE allocations to VXX and VXZ. Panel (a) and (b) show the optimal allocation to VXX and VXZ, respectively. Weights are shown for the CE strategy with an absolute risk aversion, $a$, equal to 4.

inclusion of XIV in the opportunity set has levered the performance for both strategies. In our previous analyzes we find that the rareness of large volatility in our sample make it too expensive to hold long VIX ETPs (VXX and VXZ) constantly as an insurance. Since inverse products are basically on the other side of the trade, selling insurance, it is not surprising that a constant allocation of capital to XIV has added value, despite several drawdowns. However as with any leveraged position, timing the allocation is crucial for avoiding drawdowns, and our results for the CE strategy suggest that the VIX premium is a useful signal as the added value is much higher for this strategy.

## 6. Empirical portfolio allocations

Here we examine when and how often the optimal portfolio for the CE strategy contains either VXX or VXZ. First, the CE strategy allocates capital to VXX and VXZ, 30% and 13% respectively, of all the re-balancing points.

Considering the question of how much capital is allocated to the VIX ETPs, Fig. 8 shows the weights held in VXX and VXZ, together with the level of the VIX premium.

Conditional on the weights being greater than zero, the average allocation is 13% for VXX and 10% for VXZ. The allocation is very much dependent on the level of the VIX premium and no capital is invested in either VXX or VXZ when the VIX premium is

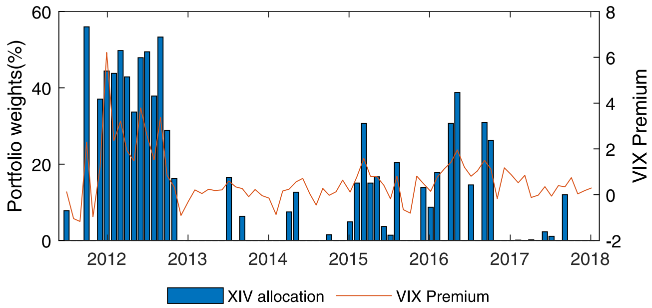



**Fig. 9.** CE allocations to XIV. This figure shows the optimal allocation to XIV. Weights are shown for the CE strategy with an absolute risk aversion, $a$, equal to 4.

large. This is clearly evident in the period from 2009 to mid 2011 and from 2012 to 2013. We note the high allocations in VXX at the end of July, 2011 (US credit downgrade), November, 2012 (concerns about US debt ceiling) and September 2015 (aftermath of Renminbi devaluation). These are all periods where the VIX premium drops below zero. The correlation between allocations to VXX and the VIX premium is −0.52. For VXZ the corresponding correlation is −0.36.

Fig. 9 shows the allocations to the inverse product, XIV, along with the level of the VIX premium. The allocation frequency is 49%. Conditional on a positive allocation, 23% of the capital is on average invested in XIV. In comparison, 70% of capital is invested into the stock component. As depicted, the amount of capital allocated to XIV is closely related to the level of the VIX premium. The correlation between the two series is 0.80. We see that the VIX premium turns negative and remains close to zero from the end of 2012 to mid 2013, and no position is opened in XIV.

## 7. Conclusion

In this paper, we use the concept of economic value of Fleming et al. (2001) to evaluate the portfolio performance of allocation strategies that include VIX ETPs in addition to stocks and bonds.

With the proliferation of VIX ETPs more than ten years ago, retail and restricted institutional investors have gained access to volatility trading and the potential of improving portfolio diversification in periods of severe market turmoil. The VIX ETPs have become quite popular, as the number of listed products and the combined market value has increased significantly over the decades. The purpose of this paper has been to quantify the diversification benefits that these products offer in terms of economic value.

Our empirical study considers two different allocation strategies, representing two different approaches to investing in VIX ETPs. The first strategy is to hold a constant fraction (10%) of the portfolio value in the VIX ETP, while the certainty equivalent strategy (CE) changes the allocation dynamically based on changes in expected returns and the covariance matrix over the investment horizon.

For the CE strategy we estimate the expected returns on the VIX ETPs via the VIX premium, defined in Cheng (2019), and we employ high-frequency data along with the methodology suggested in Oh and Patton (2016) for forecasting the covariance matrix. This gives us more sophisticated and forward-looking estimates than in the prior literature. Our analysis covers the period after the recent financial crisis, which implies that it is a period where VIX ETPs have been less valuable to investors than during crisis periods.

Evaluating the ex-post performance of the CE strategy, the short-term VIX portfolio, outperforms the benchmark equity-bond portfolio. For the constant strategy, both VIX portfolios perform worse than the benchmark equity-bond portfolio. In economic terms, the value added of these VIX products is negative, implying that an investor would be willing to pay for not having included these in the portfolio. A simple what-if simulation analysis shows that not even the occurrence of a market crash of similar magnitude as the recent financial crisis in 2008, will imply a positive economic value of the VIX ETPs. Hence, the costs of holding these products constantly in a portfolio as an insurance, clearly outweigh the diversification benefits in periods of market turmoil. To successfully take advantage of the long exposure to volatility, the allocation to VIX ETPs must be timed in a tactical manner, and our results suggest that the VIX premium is a useful tool for this.

We evaluate the impact of the expense ratio by substituting the VIX ETPs with their underlying indexes. For the CE strategy, the expense ratio does not have an impact on performance, but it would more efficient to only invest in the first-month VIX future contract. For the constant strategy, the expense ratio has a non-negligent impact on the long-term performance, but not enough to make up for the under-performance relative to the benchmark portfolio.

Finally, we consider the application of the inverse VIX ETP, XIV, which gives constrained investors access to short VIX futures exposure. By using the VIX premium for tactical allocation to XIV, the CE investor can lever performance substantially compared to the benchmark. For the constant strategy, the portfolios with XIV beats the benchmark, despite several large drawdowns in portfolio value during the sample period.

## Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jempfin.2020.05.009.

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