



Contents lists available at ScienceDirect

## Journal of Econometrics

journal homepage: [www.elsevier.com/locate/jeconom](http://www.elsevier.com/locate/jeconom)

## Editorial

## Quantile regression



This special issue of the *Journal of Econometrics* is based upon an econometrics conference held in 2017 in Urbana, Illinois, co-organized by Victor Chernozhukov, Antonio F. Galvao, Xuming He, and Zhijie Xiao, and sponsored by the Department of Economics of the University of Illinois at Urbana–Champaign. The theme of the conference was “Quantile Regression”.

The conference was also an occasion to celebrate Professor Roger Koenker’s 70th birthday, his achievements, and to honor his scholarly contributions in econometrics and statistics, in particular, the quantile regression methodology. Since the seminal work of Koenker and Bassett (1978), quantile regression (QR) has attracted considerable interest in a variety of fields and provided a valuable tool of statistical analyses. QR models the conditional quantile functions that provide important insight into heterogeneous effects of policy variables. QR has become a very active field in econometrics over the past forty years, with Professor Roger Koenker being one of the most influential contributors in the field. Moreover, empirical research has benefited significantly from QR. Early developments on QR in the literature include, among others, inference procedures (Koenker and Bassett, 1982; Koenker and Xiao, 2002), censored models (Powell, 1986), computational methods (Koenker and D’Orey, 1987), and nonparametric models (Koenker et al., 1994) and He et al. (1998). More recently, panel data (Koenker, 2004; Wei and He, 2006; Kato et al., 2012), autoregressive models (Koenker and Xiao, 2006), treatment effects (Chernozhukov and Hansen, 2005), and high dimensional models (Belloni and Chernozhukov, 2011). There are several more recent developments in QR, as for example, errors in variables, missing data, sample selection, multivariate, and functional data. Professor Roger Koenker published a book “Quantile Regression” in 2005, and it has become a standard reference for the subject.

The papers for this volume can be put into five categories: (i) nonparametric QR; (ii) panel data; (iii) moment conditions; (iv) topics on inference and treatment effects; and (v) studies on prediction and time-series.

## 1. Nonparametric QR

The first category consists of two papers.

The paper by Alexandre Belloni, Victor Chernozhukov, Denis Chetverikov, and Ivan Fernández-Val, titled “Conditional Quantile Processes Based on Series or Many Regressors” develops a nonparametric quantile regression (QR)-series framework, covering many regressors as a special case, for performing inference on the entire conditional quantile function and its linear functionals. In this framework, they approximate the entire conditional quantile function by a linear combination of series terms with quantile-specific coefficients and estimate the function-valued coefficients from the data. They also develop large sample theory for the QR-series coefficient process, and propose four resampling methods (pivotal, gradient bootstrap, Gaussian, and weighted bootstrap) that can be used for inference on the entire QR-series coefficient function.

In “Penalized Sieve GEL for Weighted Average Derivatives of Nonparametric Quantile IV Regression”, Xiaohong Chen, Demian Pouzo and James L. Powell consider estimation and inference on a weighted average derivative (WAD) of a nonparametric quantile instrumental variables regression (NPQIV) model. They first characterize the semi-parametric efficiency bound for a WAD of the NPQIV, and propose a penalized sieve generalized empirical likelihood (GEL) estimation and inference procedure, which is based on the unconditional WAD moment restriction and an increasing number of unconditional moments that are implied by the conditional NPQIV restriction, where the unknown quantile function is approximated by a penalized sieve. Under some regularity conditions, they show that the self-normalized penalized sieve GEL estimator of the WAD of a NPQIV is an asymptotically standard normal.

## 2. Panel data

There are three papers in the second category.

The paper by Yingying Zhang, Huixia Judy Wang, and Zhongyi Zhu, “Quantile-Regression-Based Clustering for Panel Data”, proposes a quantile-regression-based clustering method for panel data. They develop an iterative algorithm using a similar idea of k-means clustering to identify subgroups with heterogeneous slopes at a single quantile level or across multiple quantiles. The asymptotic properties of the group membership estimator and corresponding group-specific slope estimator are established. The finite sample performance of the proposed method is assessed through simulation and the analysis of an economic growth data.

In “Panel Data Quantile Regression with Grouped Fixed Effects” Jiaying Gu and Stanislav Volgushev introduce estimation methods for grouped latent heterogeneity in panel data quantile regression. They assume that the observed individuals come from a heterogeneous population with a finite number of types. The number of types and group membership is not assumed to be known in advance and is estimated by means of a convex optimization problem. They provide conditions under which group membership is estimated consistently and establish asymptotic normality of the resulting estimators. Simulations show that the method works well in finite samples when time-series is reasonably large.

Zhijie Xiao and Lan Xu’s paper, “What Do Mean Impacts Miss? Distributional Effects of Corporate Diversification”, argues that the existing empirical analyses ignore some important data features, especially cross sectional heterogeneity, predicted by both theories and casual observations on corporate diversification, and thus cannot provide a complete picture of the diversification discount. Using a quantile regression analysis on U.S. public firms, they investigate the importance of heterogeneity of diversification as well as other firm characteristics. Estimated quantile treatment effects exhibit substantial heterogeneity as predicted. Thus mean impacts miss a great deal. They also tie back differences in the effect of diversification in high-valued and low-valued firms to observable agency characteristics; the most interesting finding is that CEOs seem to play vastly different roles in high-valued and low-valued firms.

## 3. Moment conditions

The third category consists of two papers.

The paper by Luciano de Castro, Antonio F. Galvao, David M. Kaplan, and Xin Liu, “Smoothed GMM for Quantile Models”, considers estimation of finite-dimensional parameters identified by general conditional quantile restrictions, including instrumental variables quantile regression. Within a generalized method of moments framework, moment functions are smoothed to aid both computation and precision. Consistency and asymptotic normality are established under weaker assumptions than previously seen in the literature, allowing dependent data and nonlinear structural models. Simulations illustrate the finite-sample properties. An in-depth empirical application estimates the consumption Euler equation derived from quantile utility maximization.

In “Quantiles via Moments” Jose A. F. Machado J. M. C. Santos Silva study the conditions under which it is possible to estimate regression quantiles by estimating conditional means. The advantage of this approach is that it allows the use of methods that are only valid in the estimation of conditional means, while still providing information on how the regressors affect the entire conditional distribution. They consider two settings in which our approach can be particularly useful: panel data models with individual effects and models with endogenous explanatory variables. Besides presenting the estimator and establishing the regularity conditions needed for valid inference, they perform a small simulation experiment, present two simple illustrative applications, and discuss possible extensions.

## 4. Topics on inference and treatment effects

The fourth category consists of three papers.

The paper by Thomas Parker, “Asymptotic Inference for the Constrained Quantile Regression Process”, investigates the asymptotic distribution of linear quantile regression coefficient estimates when the parameter lies on the boundary of the parameter space. In order to allow for inferences made across many conditional quantiles, a uniform characterization of constrained quantile regression estimates as a stochastic process over an interval of quantile levels is provided. To accomplish this, he poses the process of estimates as solutions to a parameterized family of constrained optimization problems, parameterized by quantile level. A uniform characterization of the dual solution to these problems – the so-called regression rankscore process – is also derived, which can be used for score-type inference in quantile regression.

Andreas Hagemann’s paper, “Placebo Inference on Treatment Effects When the Number of Clusters Is Small”, introduces a general, Fisher-style randomization testing framework to conduct nearly exact inference about the lack of effect of a binary treatment in the presence of very few, large clusters when the treatment effect is identified across clusters. The proposed randomization test formalizes and extends the intuitive notion of generating null distributions by assigning placebo treatments to untreated clusters. It is shown that under simple and easily verifiable conditions, the placebo test leads to asymptotically valid inference in a very large class of empirically relevant models.

In “Partial Identification of the Treatment Effect Distribution and its Functionals”, Sergio Firpo and Geert Ridder show that bounds on functionals of the quantile process that use Makarov bounds are not sharp, because the Makarov bounds are pointwise, but not uniformly sharp. This allows them to propose improved bounds on functionals of the cumulative distribution function (c.d.f.). As an intermediate result, they find that the Makarov bounds on the region that contains the c.d.f. of the treatment effect distribution in a finite number of points can be improved. They also provide numerical illustrations throughout the paper permitting a clear visualization of how the method works.

## 5. Studies on prediction and time-series

The final category comprises of four papers.

In “On the Predictive Risk in Misspecified Quantile Regression” Alexander Giessing and Xuming He investigate the predictive risk of possibly misspecified quantile regression functions. The in-sample risk is well-known to be an overly optimistic estimate of the predictive risk and we provide two relatively simple (asymptotic) characterizations of the associated bias, also called expected optimism. They propose estimates for the expected optimism and the predictive risk, and establish their uniform consistency under mild conditions. Their results hold for models of moderately growing size and allow the quantile function to be incorrectly specified. Empirical evidence from their estimates is encouraging as it compares favorably with cross-validation.

Rui Fan and Ji Hyung Lee’s paper, “Predictive Quantile Regressions under Persistence and Conditional Heteroskedasticity”, provides inference for predictive quantile regressions with persistent predictors and conditionally heteroskedastic errors. They propose a size-corrected bootstrap inference thereby avoiding the nuisance parameter estimation. The bootstrap consistency is shown even with the nonstationary predictors and conditionally heteroskedastic innovations. Monte Carlo simulation confirms the significantly better test size performances of the new methods. The empirical exercises on stock return quantile predictability are revisited.

Stephen Portnoy’s paper, “Edgeworth’s Time Series Model: Not AR(1) But Same Covariance Structure”, shows that though the AR(1) process cannot be distinguished from the Edgeworth Process by second order properties, inferences based on an AR(1) assumption can fail under the Edgeworth model. This model has many additional surprising features, among which is that it has Markov structure, but is not generated by a one-step transition operator.

Finally, Gib Bassett’s paper, “Review of Median Stable Distributions and Schröder’s Equation”, reviews median stable distributions in light of its connection to Schröder’s functional equation. Median stable distributions are an extension of traditional (mean) stable distributions. The traditional definition of stability (in terms of sums of iid random variables) is recast as a condition on the sampling distribution of an estimator. For the traditional (mean) stable distribution, the sample mean’s (rescaled) sampling distribution is identical to the distribution of the iid data. Median stable distributions are defined similarly by replacing the sample mean with the sample median. Since the sampling distribution of the median is a functional its stable distribution is the solution to a functional equation. It turns out that this defining functional equation is an instance of a famous equation due to Schröder from 1870.

## References

- Belloni, A., Chernozhukov, V., 2011.  $\ell_1$ -Penalized quantile regression in high-dimensional sparse models. *Ann. Statist.* 39, 84–130.  
 Chernozhukov, V., Hansen, C., 2005. An IV model of quantile treatment effects. *Econometrica* 73, 245–261.  
 He, X., Ng, P., Portnoy, S., 1998. Bivariate quantile smoothing splines. *J. R. Stat. Soc. Ser. B Stat. Methodol.* 60, 537–550.  
 Kato, K., Galvao, A.F., Montes-Rojas, G., 2012. Asymptotics for panel quantile regression models with individual effects. *J. Econometrics* 170, 76–91.  
 Koenker, R., 2004. Quantile regression for longitudinal data. *J. Multivariate Anal.* 91, 74–89.  
 Koenker, R., Bassett, G.W., 1978. Regression quantiles. *Econometrica* 46, 33–49.  
 Koenker, R., Bassett, G.W., 1982. Robust tests for heteroscedasticity based on regression quantile. *Econometrica* 50, 43–61.  
 Koenker, R., D’Orey, V., 1987. Algorithm AS 229: Computing regression quantiles. *J. R. Stat. Soc. Ser. C. Appl. Stat.* 36, 383–393.  
 Koenker, R., Ng, P., Portnoy, S., 1994. Quantile smoothing splines. *Biometrika* 81, 673–680.  
 Koenker, R., Xiao, Z., 2002. Inference on the quantile regression process. *Econometrica* 70, 1583–1612.  
 Koenker, R., Xiao, Z., 2006. Quantile autoregression. *J. Amer. Statist. Assoc.* 101, 980–990.  
 Powell, J.L., 1986. Censored regression quantiles. *J. Econometrics* 32, 143–155.  
 Wei, Y., He, X., 2006. Conditional growth charts. *Ann. Statist.* 34, 2069–2097.

Victor Chernozhukov\*

*Department of Economics, MIT, United States*

*E-mail address: [vchern@mit.edu](mailto:vchern@mit.edu).*

Antonio F. Galvao

*Department of Economics, University of Arizona, United States*

*E-mail address: [agalvao@email.arizona.edu](mailto:agalvao@email.arizona.edu).*

Xuming He

*Department of Statistics, University of Michigan, United States*

*E-mail address: [xmhe@umich.edu](mailto:xmhe@umich.edu).*

Zhijie Xiao

*Department of Economics, Boston College, United States*

*E-mail address: [zhijie.xiao@bc.edu](mailto:zhijie.xiao@bc.edu).*

Available online 10 April 2019

\* Corresponding editor.