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Stochastic investor sentiment, crowdedness and deviation of asset prices from fundamentals

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ABSTRACT

This study constructs a theoretical model to address how stochastic investor sentiment affects investor's crowdedness, and how stochastic investor sentiment and crowdedness affect asset prices. An asset pricing model incorporating stochastic investor sentiment and crowdedness is developed, which can provide efficient explanations for the deviations of asset prices from fundamentals and the maverick risk of investors. This model indicates that the optimistic (pessimistic) investor sentiment and the long (short) crowdedness caused by optimistic (pessimistic) sentimental investors can push asset price above (below) fundamental value. Also, the sentimental investors who are wrong and alone would take the maverick risk. Our results are consistent with the idea that investor sentiment and investor behavior matter for the asset prices and the deviations of asset prices from fundamentals.

1. Introduction

Traditional finance views asset prices reflect all information about future cash flows and argues that rational arbitrageurs eliminate the role of noise traders to drive asset prices close to their fundamental values (Fama, 1965, 1970). However, the abnormal phenomena of deviation of asset prices from fundamentals emerge in financial market. Consequently, some literature explain the deviation of asset prices from fundamentals from the perceptive of investor sentiment or crowdedness in behavioral finance (Delong et al., 1990; Barberis et al., 2001; Stein, 2009; Stambaugh and Yuan, 2016). Furthermore, some literature empirically demonstrate the combined roles of investor sentiment and crowdedness[1] on asset prices (Yang and Zhou, 2016; Ryu et al., 2017; Gao and Yang, 2018). However, rare study theoretically incorporates investor sentiment and crowdedness into asset pricing model. This paper provides an asset pricing model from the perspectives of stochastic investor sentiment (how investors think in asset markets) and crowdedness (how investors buy or sell the asset in concert with each other in asset markets), and further studies the deviations of asset prices from fundamentals from these two perspectives.

We first study a representative sentimental investor model to demonstrate the role of investor sentiment on asset prices. We find that the representative sentimental investor overvalues (undervalues) the perceived value of the risky asset if the representative sentimental investor is optimistic (pessimistic). However, the representative sentimental investor setup cannot address the trading behaviors of different investors in the market and analyze the role of crowdedness.

To address this issue, we then study the heterogeneous sentimental investors' model to analyze the roles of investor sentiment and crowdedness on asset prices. According to this model, when sentimental investors play major roles in the financial market, a large number of optimistic (pessimistic) investors who overvalue (undervalue) and rush into (sell) an asset lead to long (short) crowdedness.[2] Besides, the deviations of asset prices from fundamentals persist over time by the roles of stochastic investor sentiment and crowdedness: positive expected investor sentiment makes the sentimental investor to go long, increasing optimistic sentimental investors leads to long crowdedness and pushes asset price above fundamental value; vice versa, negative expected

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[1] If too many investors are on the same side of the trade, these investors may cause coordination problem and the crowded trade problem, therefore, we employ investors' crowdedness to describe how many investors adopt the same strategy in the financial markets, which is consistent with Stein (2009), Yan (2013) and Hong et al. (2015).

[2] Long crowdedness describes that investors buy the same stocks in concert with each other in asset market; short crowdedness describes that investors sell the same stocks in concert with each other in asset market.

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investor sentiment makes the sentimental investor to go short, increasing pessimistic sentimental investors leads to short crowdedness and pushes asset prices below fundamental values.

In addition, our model indicate that the sentimental investors take great maverick risk[3] which is the greatest peril in financial markets when investors face the risk of wrong and the risk of alone (Arnott, 2003; Allen, 2007). In this case, the sentimental investor has not enough capacity to affect asset price, and the sentimental investor may be compelled to liquidate his positions and suffers losses regardless of whether his perception is right or wrong.

This paper contributes to the literature in the following ways. First, this paper theoretically constructs a model that explores the effect of stochastic investor sentiment on the formation of crowdedness. Specifically, the heterogeneous sentimental investors are divided into optimistic sentimental investors, pessimistic sentimental investors and neutral sentimental investors[4] to measure long crowdedness, short crowdedness and non-crowdedness. Second, this paper presents an asset pricing model with stochastic investor sentiment and crowdedness. In our framework, long crowdedness and optimistic investor sentiment can push asset price above fundamental value; however, short crowdedness and pessimistic investor sentiment can push asset price below fundamental value. Third, this paper offers efficient explanations for the deviations of asset prices from fundamentals and the maverick risk.

The remainder of this paper is organized as follows. Section 2 reviews the literature in this area to highlight our contributions. Section 3 builds a representative sentimental investor model to illustrate the role of stochastic investor sentiment on the deviations of asset prices from fundamentals. Section 4 builds an asset pricing model that shows how the heterogeneity of investor sentiment and crowdedness affect asset prices and describes the equilibrium characterizations. Section 5 presents the relationship between the stochastic investor sentiment, crowdedness and the deviations of prices from fundamentals. Section 6 demonstrates that the sentimental investors who are wrong and alone face the maverick risk, while Section 7 concludes.

2. Literature review

Deviation of asset prices from fundamentals is one of the most important issues in the asset pricing literature. In particular, economists traditionally views asset prices as fully informative about future cash flows, and also argues that rational arbitrageurs eliminate the role of noise traders to drive asset prices close to their fundamental values (Fama, 1965, 1970). However, behavioral finance proves the persistence of the deviations of asset prices from fundamentals from the perceptive of investor sentiment or investor behavior (Delong et al., 1990; Barberis et al., 1998, 2001; Daniel et al., 1998; Hong and Stein, 1999; Abreu and Brunnermeier, 2002, 2003; Stein, 2009; Yan, 2013; Stambaugh and Yuan, 2016). Specially, Shleifer (2000) questions why the deviations of asset prices from fundamentals persist, and describes that investors who buy or sell the same security at the same time will drive prices up without any fundamental news. Shiller (2014) concludes that how people really think and act play important roles on the formation of asset prices. Furthermore, some papers empirically demonstrate that how people think and act will affect asset prices (Yang and Zhou, 2015, 2016; Ryu et al., 2017; Gao and Yang, 2018). Therefore, we should incorporate investor sentiment and investor's crowdedness into asset pricing model to verify their significant roles on asset price.

In the related studies, behavioral finance explains the deviations of prices from fundamental values from two perspectives: investor sentiment and crowdedness. On the one hand, one possible explanation for the deviation of asset prices from fundamentals is that investor sentiment affects asset prices (Lee et al., 2002; Baker and Wurgler, 2006, 2007; Berger and Turtle, 2012; Stambaugh et al., 2012; Fong and Toh, 2014; Kim et al., 2014; Greenwood and Shleifer, 2014; Qian, 2014; Chiarella et al., 2017; Aboody et al., 2018), and the sentimental investors have a biased valuation for the asset (Hong et al., 2012; Yang and Zhang, 2013). Specially, Shleifer and Vishny (1997) and Hong et al. (2012) demonstrate the greater pessimistic or optimistic shocks make the arbitrageurs fail to bring prices back to fundamental values. Barberis and Thaler (2003) prove that investors prefer to trade in the same direction as other investors, thereby exacerbating the mispricing, rather than against them. Moreover, a large number of empirical literature demonstrate that investor sentiment affects trading activities and asset prices in behavior finance (Delong et al., 1990; Kumar and Lee, 2006; Barber et al., 2009; Kumar et al., 2013).

On the other hand, an alternative explanation for the deviations of asset prices from fundamentals is that crowdedness affects asset prices (Stein, 2009; Sias et al., 2015; Hong et al., 2015; Blocher, 2016; Bruno et al., 2018). Specifically, Stein (2009) focuses on how many others using the same strategy and the coordination problem, and further argues that investors overreact to the initial underreaction and push prices beyond fundamental values when an unexpectedly large number of investors adopt the same strategy. Brunnermeier and Pedersen (2009) describe that the commonality across securities affects the asset prices and leads to an evaporation of market liquidity. Pojarliev and Richard (2011) measure carry crowdedness, trend crowdedness and value crowdedness to prove that crowded trades harbor potential risk once sentiment induces liquidation of positions. Yan (2013) uses short interest ratio and the institutional ownership data to measure crowdedness of momentum, further argues that crowdedness of momentum leads to momentum crash and pushes prices away from fundamental values. Sias et al. (2015) verify that hedge funds' demand shocks are, on average, positively related to subsequent raw and risk-adjusted returns when they do buy and sell the same stocks. Furthermore, Bruno et al. (2018) investigate how crowding occurs through the portfolio construction process in an attempt to understand the linkages between portfolio construction and crowding.

In conclusion, extant literature provide strong evidence on crowdedness and investor sentiment in financial markets. However, existing studies neglect the theoretical analysis of investor sentiment on crowdedness so far, and are lack of theoretical models to support the combined roles of investor sentiment and crowdedness on asset prices and the deviation of asset prices from fundamentals. Therefore, this paper sheds new lights on the internal relation between sentiment and crowdedness, and further addresses their roles on deviations of asset prices from fundamentals.

3. A representative sentimental investor model

We first study a representative sentimental investor model to develop our intuition and key insights transparently. Consider a simple economy with an asset (i.e. stock) whose price is denoted by P_t. The asset is in limited supply with a unitary mass, and it is traded in a competitive market at price P_t. Time $t \in [0, \infty)$ is continuous. For simplification, we assumed the fundamental value of the asset is $V_t = V_0 e^{rt}$, where V_0 is the fundamental value at time 0, r is the risk-free rate.

The representative sentimental investor thinks that the value of the risky asset changes with time and has his own beliefs about the value of risky asset. Generally, the representative sentimental investor overvalues the asset value with optimistic investor sentiment, and undervalues the asset value with pessimistic investor sentiment (Brown and Cliff, 2004; Statman et al., 2008). Moreover, the systematic role of investor sentiment on asset price has been proved by some empirical and theoretical literature (Baker and Wurgler, 2006, 2007; Yu and Yuan, 2011; Stambaugh et al., 2012; Yang and Zhang, 2013; Kim et al., 2014). With a more realistic assumption, each sentimental investor's opinion about the

[3] Maverick risk is the greatest peril which is the risk of being wrong and alone, which is in line with the definition of Arnott (2003).

[4] The expected investor sentiment of optimistic (pessimistic) sentimental investor is larger (smaller) than zero, the expected investor sentiment of neutral sentimental investor equals to zero.

perceived value includes both optimistic beliefs and pessimistic beliefs, because he doesn't know the fundamental values and has no information shocks. Each sentimental investor will take actions according to the possibility of the optimistic beliefs and the possibility of pessimistic beliefs. To simplify the calculations, we assume the sentiment of each investor is stochastic and follows a two-dimensional discrete distribution. For the representative sentimental investor in period t, the investor sentiment is stochastic and follows a two-dimensional discrete distribution and the distribution of investor sentiment is given by:

$$S_t = \begin{cases} S_t^+, & q_t \\ S_t^-, & 1-q_t \end{cases}, \tag{1}$$

where $S_t \in [S_{\min}, S_{\max}]$, $S_t^- \in [S_{\min}, 0]$, $S_t^+ \in [0, S_{\max}]$, $S_t^+ > 0 > S_t^-$, $q_t \in [0,1]$ represents the possibility of optimistic sentiment at time t, $S_{\min}$ represents the minimum investor sentiment value, and $S_{\max}$ represents the maximum investor sentiment value.

Thus, the perceived value of the representative sentimental investor is given by

$$V_t^s = V_0 e^{rt} + f\left(S_t^+ q_t + S_t^-(1-q_t)\right), \tag{2}$$

where $f(S_t^+ q_t + S_t^-(1-q_t))$ represents the impact of representative sentimental investor on perceived value, and $S_t^+ q_t + S_t^-(1-q_t)$ is the expected investor sentiment of representative sentimental investor.

Based on Shleifer and Vishny (1997), the demand of the representative sentimental investor for the risky asset at time t is given by

$$D_t^s = \frac{V_0 e^{rt} + f\left(S_t^+ q_t + S_t^-(1-q_t)\right)}{P_t}. \tag{3}$$

By using the market-clearing condition and the representative sentimental investor's demand for the risky asset, we characterize the equilibrium price of the risky asset. The equilibrium condition is that the aggregate demand of the asset must be equal to the unit supply, which is given by

$$D_t^s = 1. \tag{4}$$

Equilibrium price of the representative sentimental investor at time t is

$$P_t^* = V_0 e^{rt} + f\left(S_t^+ q_t + S_t^-(1-q_t)\right), \tag{5}$$

where $f(S_t) = \alpha S_t$, and $\alpha > 0$ is constant. Here, $V_0 e^{rt}$ is a fundamental component and $f(S_t^+ q_t + S_t^-(1-q_t))$ is the second component of the price that is attributed to the role of investor sentiment. The second component of equilibrium price is given by

$$f\left(S_t^+ q_t + S_t^-(1-q_t)\right) = \alpha\left(S_t^+ q_t + S_t^-(1-q_t)\right), \tag{6}$$

where α is constant, and $\alpha > 0$.

The deviation of price from fundamental value at time t can be given by

$$Dev_t = \frac{P_t^* - V_0 e^{rt}}{V_0 e^{rt}} = \frac{f\left(S_t^+ q_t + S_t^-(1-q_t)\right)}{V_0 e^{rt}}. \tag{7}$$

Proposition 1. As the expected investor sentiment $S_t^+ q_t + S_t^-(1-q_t)$ increases, the equilibrium price P_t^* and the deviation of price from fundamental value Dev_t increase at time t.

Intuitively, α is the sensitivity coefficient of investor sentiment, indicating that the second component of equilibrium price monotonically increases with the expected investor sentiment $S_t^+ q_t + S_t^-(1-q_t)$ as follows:

(1) If $S_t^+ q_t + S_t^-(1-q_t) > 0$, $f(S_t^+ q_t + S_t^-(1-q_t)) = \alpha(S_t^+ q_t + S_t^-(1-q_t)) > 0$, then $V_0 e^{rt} + f(S_t^+ q_t + S_t^-(1-q_t)) > V_0 e^{rt}$.
(2) If $S_t^+ q_t + S_t^-(1-q_t) < 0$, $f(S_t^+ q_t + S_t^-(1-q_t)) = \alpha(S_t^+ q_t + S_t^-(1-q_t)) < 0$, then $V_0 e^{rt} + f(S_t^+ q_t + S_t^-(1-q_t)) < V_0 e^{rt}$.
(3) If $S_t^+ q_t + S_t^-(1-q_t) = 0$, $f(S_t^+ q_t + S_t^-(1-q_t)) = \alpha(S_t^+ q_t + S_t^-(1-q_t)) = 0$, then $V_0 e^{rt} + f(S_t^+ q_t + S_t^-(1-q_t)) = V_0 e^{rt}$.

It means that the asset prices and the deviations of asset prices from fundamentals are solely determined by the stochastic investor sentiment. The representative sentimental investor drives the equilibrium price up if the expected investor sentiment is optimistic; vice versa, the representative sentimental investor derives the equilibrium price down if the expected investor sentiment is negative.

However, the heterogeneous beliefs and differences in opinions affect asset prices in financial markets (Miller, 1977; Chen et al., 2002; Fama and French, 2007; Banerjee, 2011; Carlin et al., 2014). Shleifer (2000) points out that different investors using different models of future cash flows can generate different demands and trade with each other. Therefore, there are two limitations in the representative sentimental investor model. First, the representative sentimental investor model cannot address crowdedness of heterogeneous sentimental investors in the asset market. Second, the representative sentimental investor model cannot study the inner connection between stochastic investor sentiment and crowdedness. In the following analysis, we shall overcome these limitations.

4. Heterogeneous sentimental investors model

4.1. Setup

Shleifer (2000) points out that different investors form different models of future cash flows can generate different demands and trade with each other, and they will drive prices up without any fundamental news when they use different models to generate similar predictions and try to invest the same security at the same time. Following Shleifer (2000), we improve our original model (the representative sentimental investor model) by incorporating the heterogeneous beliefs among heterogeneous investors, which considers financial markets as places where different types of investors trade and better explains how stochastic investor sentiment drives sentimental investors to buy or sell the same securities in concert. There are heterogeneous sentimental investors (denoted i) in the economy, and we assumed that $i \in \{1, 2, 3, \cdots\cdots, N\}$. The focus of the analysis is on the deviations of prices from fundamental values from stochastic investor sentiment and the crowdedness.

The sentimental investors think that the value of the risky asset changes with time. For simplification, the mass of sentimental investors is normalized to one. Each sentimental investor has his own beliefs about the value of risky asset. With a more realistic assumption, each sentimental investor's opinion about the perceived value includes both optimistic beliefs and pessimistic beliefs, because they don't know the fundamental values and have no information shocks. Each sentimental investor will take actions according to the possibility of the optimistic beliefs and the possibility of pessimistic beliefs. To simplify the calculations, we make the following assumptions: first, the sentiment of each investor is stochastic and follows a two-dimensional discrete distribution; second, the sentimental investors have the same optimistic beliefs and the same pessimistic beliefs; third, the sentimental investors disagree about the possibility of the optimistic beliefs and the possibility of pessimistic beliefs. We make this assumption of binary investor sentiment for two technical reasons. On the one hand, a binary choice in investor sentiment simplifies an investor's choice, and then we can simply compute the expected investor sentiment to distinguish optimistic sentimental investors, pessimistic sentimental investors and neutral sentimental investors. On the other hand, a binary choice in investor sentiment and optimal decision of sentimental investors combine to simplify to measure the degree of trade and the degree of crowdedness by sentimental investors, so the internal connection of stochastic investor sentiment and crowdedness is clear.

For sentimental investor i, in period t, the stochastic investor sentiment follows a two-dimensional discrete distribution. The distribution of investor sentiment is given by

$$S_{i,t} = \begin{cases} S_t^+, & q_{i,t} \\ S_t^-, & 1 - q_{i,t} \end{cases}, \tag{8}$$

where $S_{i,t} \in [S_{\min}, S_{\max}]$, $S_t^- \in [S_{\min}, 0]$, $S_t^+ \in [0, S_{\max}]$, $i \in \{1, 2, 3, \cdots\cdots, N\}$, $q_{i,t}$ is a continuous random variable with a support of $[0, 1]$, and $q_{i,t}$ is i.i.d across sentimental investors (denoted i) and over time (denoted t).

Based on the characteristics of stochastic investor sentiment, the sentimental investors can be divided into optimistic sentimental investors, pessimistic sentimental investors and neutral sentimental investors. If $|S_t^+ q_{i,t}| > |S_t^-(1 - q_{i,t})|$, then sentimental investor i has more optimistic beliefs to the risky asset. That is, those with sufficiently large optimistic beliefs $q_{i,t}$ will have higher perceived value of the risky asset. In summary, the optimal decision of sentimental investor i at time t is to overvalue the risky asset and go long, that is

$$h_{i,t}(S_{i,t}, q_{i,t}) = I_{\{|S_t^+ q_{i,t}| > |S_t^-(1 - q_{i,t})|\}}, \tag{9}$$

where $I_{\{\}}$ is an indicator function that takes the value one if the condition in parentheses is satisfied and zero if the condition in parentheses is unsatisfied. We employ a variant of Cen et al. (2013) as the basis to calculate the proportion of optimistic sentimental investor at date t. The proportion of optimistic sentimental investors who will overvalue the risky asset and go long at time t is

$$n_{o,t} = \int_0^1 h_{i,t}(S_{i,t}, q_{i,t}) di = E[h_{i,t}(S_{i,t}, q_{i,t}) | S_{i,t}]. \tag{10}$$

Here, $n_{o,t}$ optimistic sentimental investors overvalue the risky asset and go long at time t, The sentiment distribution of optimistic sentimental investors at time t can be given by

$$S_{o,t} = \begin{cases} S_t^+, & q_{o,t} \\ S_t^-, & 1 - q_{o,t} \end{cases}, \tag{11}$$

where $q_{o,t} = \frac{\sum_{i=1}^{n_{o,t}N} q_{i,t}}{n_{o,t}N}$ is the possibility of optimistic sentiment for the optimistic sentimental investors and the expected sentiment of optimistic sentimental investors $\overline{S_{o,t}} = S_t^+ q_{o,t} + S_t^-(1 - q_{o,t})$ is larger than zero. When sentimental investors play major roles in the financial market, more optimistic investors who overvalue and rush into an asset can lead to long crowdedness–investors buy the same asset in concert with each other in asset market.

Specifically, at time t, $n_{o,t}$ optimistic sentimental investors perceive

$$V_t^s = V_0 e^{rt} + f\left(\overline{S_{o,t}}\right). \tag{12}$$

Similarly, if $|S_t^+ q_{i,t}| < |S_t^-(1 - q_{i,t})|$, then sentimental investor i has more pessimistic beliefs to the risky asset. That is, those with sufficiently large pessimistic beliefs $1 - q_{i,t}$ will have lower perceived value of the risky asset. In summary, the optimal decision of sentimental investor i at time t is to undervalue the risky asset and go short, that is

$$g_{i,t}(S_{i,t}, 1 - q_{i,t}) = I_{\{|S_t^+ q_{i,t}| < |S_t^-(1 - q_{i,t})|\}}, \tag{13}$$

where $I_{\{\}}$ is an indicator function that takes the value one if the condition in parentheses is satisfied and zero otherwise. We employ a variant of Cen et al. (2013) as the basis to calculate the proportion of pessimistic sentimental investor at date t. The proportion of pessimistic sentimental investors who will undervalue the risky asset and go short at time t is

$$n_{p,t} = \int_0^1 g_{i,t}(S_{i,t}, 1 - q_{i,t}) di = E[g_{i,t}(S_{i,t}, q_{i,t}) | S_{i,t}]. \tag{14}$$

Here, $n_{p,t}$ pessimistic sentimental investors undervalue the risky asset and go short at time t. The sentiment distribution of pessimistic investors at time t can be given by

$$S_{p,t} = \begin{cases} S_t^+, & q_{p,t} \\ S_t^-, & 1 - q_{p,t} \end{cases}, \tag{15}$$

where $q_{p,t} = \frac{\sum_{i=1}^{n_{p,t}N} q_{i,t}}{n_{p,t}N}$ is the possibility of optimistic sentiment for the pessimistic sentimental investors, and the expected sentiment of pessimistic sentimental investors $\overline{S_{p,t}} = S_t^+ q_{p,t} + S_t^-(1 - q_{p,t})$ is smaller than zero. When sentimental investors play major roles in the financial market, more pessimistic investors who undervalue and sell an asset can lead to short crowdedness– investors sell the same asset in concert with each other in asset market.

Specifically, at time t, $n_{p,t}$ pessimistic sentimental investors perceive

$$V_t^s = V_0 e^{rt} + f\left(\overline{S_{p,t}}\right). \tag{16}$$

If $|S_t^+ q_{i,t}| = |S_t^-(1 - q_{i,t})|$, then the sentimental investor i is neutral and plays waiting strategy, the proportion of the sentimental investors who are neutral at time t is

$$n_{n,t} = 1 - n_{o,t} - n_{p,t}. \tag{17}$$

Specifically, at time t, $n_{n,t}$ neutral sentimental investors perceive

$$V_t^s = V_0 e^{rt}. \tag{18}$$

The whole perceived value of the sentimental investors can be given by

$$V_t^s = \begin{cases} V_0 e^{rt} + f\left(\overline{S_{o,t}}\right), & n_{o,t} \\ V_0 e^{rt} + f\left(\overline{S_{p,t}}\right), & n_{p,t} \\ V_0 e^{rt}, & n_{n,t} \end{cases}, \tag{19}$$

Where $n_{o,t} + n_{p,t} + n_{n,t} = 1$.

The whole perceived value of the sentimental investors can be given by

$$V_t^s = V_0 e^{rt} + n_{o,t} f\left(\overline{S_{o,t}}\right) + n_{p,t} f\left(\overline{S_{p,t}}\right). \tag{20}$$

The whole perceived value of sentimental investors is not always equal to the right valuation with the heterogeneity of investor sentiment and the heterogeneity of investor behavior, even without short-sales constraints. This valuation distinguishes our model from many other valuations in two ways. On the one hand, it differs from the valuation that on average the heterogeneous investors have the rational valuation, the heterogeneity of investors has no effect on price: heterogeneous beliefs can offset each other without short-sales constraints (Chen et al., 2002). On the other hand, it details how the deviations of asset prices from fundamentals are affected by the stochastic investor sentiment and crowdedness of sentimental investors to substitute the noise traders shocks (Shleifer and Vishny, 1997; Hong et al., 2012; Hombert and Thesmar, 2014) and the simple aggregate bias (Cen et al., 2013 and Hong and Sraer, 2013). Our model offers an alternative way to calculate the whole perceived value of sentimental investors.

4.2. Equilibrium characterizations

In this section, we solve for the equilibrium and analyze the deviations of asset prices from fundamental values in all kinds of circumstances. At each date, the aggregate state variables of the economy are $(V_t^s; \overline{S_{o,t}}; \overline{S_{p,t}}; n_{o,t}; n_{p,t})$, and they determine all aggregate market outcomes including the asset prices and crowdedness of sentimental investors. The sentimental investors' individual state variables are $(S_t^+, q_{i,t}; S_t^-, 1-q_{i,t}; V_0e^{rt})$ and they use these variables to determine the beliefs of the sentimental investors, the crowdedness of the sentimental investors and the whole perceived value of the sentimental investors, and hence their investment decisions.

Based on the demand function concept of Shleifer and Vishny (1997), the demand of the sentimental investors for the risky asset at time t are given by

$$D_t^s = \frac{V_0e^{rt} + n_{o,t}f\left(\overline{S_{o,t}}\right) + n_{p,t}f\left(\overline{S_{p,t}}\right)}{P_t}, \tag{21}$$

where $n_{o,t}$ and $n_{p,t}$ are endogenous. Although the heterogeneous sentimental investor model is based on the calculation principles of Shleifer and Vishny (1997), it is formulated somewhat differently. Specifically, this model makes the formation of the perceived value of sentimental investors as detailed as possible, and makes the logic behind the relationship between investor sentiment and crowdedness as transparent as possible.

By using the market-clearing condition and the investors' demand for the risky asset, we characterize the equilibrium prices of the risky asset. The equilibrium condition is that the aggregate demand of the asset must be equal to the unit supply, which is given by

$$D_t^s = 1. \tag{22}$$

Equilibrium price at time t is

$$P_t^* = V_0e^{rt} + n_{o,t}f\left(\overline{S_{o,t}}\right) + n_{p,t}f\left(\overline{S_{p,t}}\right). \tag{23}$$

where $n_{o,t} \in [0,1]$ and $n_{p,t} \in [0,1]$.

Equation (23) shows that the equilibrium price of the risky asset (P_t^*) is composed of three components: V_0e^{rt}, $n_{o,t}f(\overline{S_{o,t}})$, and $n_{p,t}f(\overline{S_{p,t}})$. Specifically, the first component V_0e^{rt} is the fundamental value of the risky asset. The second component $n_{o,t}f(\overline{S_{o,t}})$ captures the role of optimistic investor sentiment and the capacity of optimistic sentimental investors. The third component $n_{p,t}f(\overline{S_{p,t}})$ captures the role of pessimistic investor sentiment and the capacity of pessimistic sentimental investors.

Proposition 2. If there are heterogeneous sentimental investors (denoted i) in the economy, then the equilibrium price P_t^* increase with stochastic investor sentiment, increase with the number of optimistic sentimental investors, and decrease with the number of pessimistic investors. Moreover, four cases arise at time t.

(i) If all the investors are optimistic sentimental investors $n_{o,t} = 1$, the equilibrium price is $P_t^* = V_0e^{rt} + n_{o,t}f(\overline{S_{o,t}})$;
(ii) If all the investors are pessimistic sentimental investors $n_{p,t} = 1$, the equilibrium price is $P_t^* = V_0e^{rt} + n_{p,t}f(\overline{S_{p,t}})$;
(iii) If all the investors are neutral sentimental investors $n_{n,t} = 1$, then the equilibrium price is $P_t^* = V_0e^{rt}$;
(iv) If $0 < n_{o,t} < 1$, $0 < n_{p,t} < 1$, $0 < n_{n,t} < 1$, then the equilibrium price is on average of all investors' beliefs $P_t^* = V_0e^{rt} + n_{o,t}f(\overline{S_{o,t}}) + n_{p,t}f(\overline{S_{p,t}})$.

The second proposition simply describes the equilibrium prices with heterogeneous sentimental investors. This model shows that the equilibrium prices depend on stochastic investor sentiment and crowdedness of sentimental investors. We will further study how stochastic investor sentiment and crowdedness of sentimental investors affect equilibrium asset prices and the deviations of asset prices from fundamentals in the next section.

5. Deviation of asset prices from fundamentals

The objective in this section is to identify the asset that is traded by a large number of sentimental investors and study the deviations of asset prices from fundamental values by stochastic investor sentiment and crowdedness of sentimental investors. Firstly, we describe the benchmark case that we use to identify the deviations of asset prices from fundamental values in section 5.1. Secondly, we describe breadth of investor sentiment to identify the difference of optimistic investor sentiment and pessimistic investor sentiment in section 5.2. Thirdly, we define the degree of trade and the degree of crowdedness by sentimental investors in section 5.3 and study the relationship among breadth of investor sentiment, crowdedness and the deviations of asset prices from fundamental values in section 5.4.

5.1. Benchmark case

To complete the description of the role of the sentimental investors in the deviations of asset prices from fundamental values, we need to explore the impact of the sentimental investors in different circumstances. Before analyzing the pattern of prices in our model, we specify what the benchmark is. The benchmark is efficient market, in which the investors are all rational and have access to the fundamental values. In this case, since the investors are all rational, the price at time t is equal to the fundamental value at time t,

$$P_t = V_t = V_0e^{rt}. \tag{24}$$

5.2. Breadth of investor sentiment

For the purpose of our analysis, we are interested in establishing the connection between the deviations of asset prices from fundamental values and the breadth of investor sentiment. We define breadth of investor sentiment B_t^s as:

$$B_t^s = \overline{S_{o,t}} - \overline{S_{p,t}}, \tag{25}$$

where $\overline{S_{o,t}} \in (0, S_{\max}]$ and $\overline{S_{p,t}} \in [S_{\min}, 0)$. Breadth of investor sentiment is bounded between zero and $S_{\max} - S_{\min}$. It is $S_{\max} - S_{\min}$ when the optimistic investor sentiment and the pessimistic investor sentiment are in extreme values, and it approaches to zero when both the optimistic investor sentiment and pessimistic investor sentiment approach to zero. Therefore, breadth of investor sentiment captures the heterogeneity of investors' beliefs: a higher breadth of investor sentiment means that sentimental investors hold more diverse beliefs about the stock.

5.3. Measuring crowdedness

The objective in this part is to identify the asset that is traded by a large number of sentimental investors. Our measure of crowdedness consists both the degree of trade by sentimental investors and the degree of crowdedness by sentimental investors. Here, most of sentimental investors trade this asset and the effect of sentimental investors can't be offset by the heterogeneous trading directions. To measure crowdedness, we first identify whether the asset is over-trading by the sentimental investors. In this case, the proportion of neutral sentimental investors is small enough—i.e., the proportion of neutral sentimental investors is less than $n_{n,t}^*$. The over-trading of sentimental investors only captures trading by sentimental investors but ignores the degree of crowdedness. We then define long crowdedness, non-crowdedness and short crowdedness according to the degree of trade and the degree of crowdedness by sentimental investors.

The degree of trade by sentimental investors at time t is

$$Deg_t = n_{o,t} + n_{p,t}. \tag{26}$$

The degree of crowdedness by sentimental investors at time t is defined as

$$C_t = Max\left\{\left|\frac{n_{o,t}f\left(\overline{S_{o,t}}\right)}{n_{o,t}f\left(\overline{S_{o,t}}\right) - n_{p,t}f\left(\overline{S_{p,t}}\right)}\right|, \left|\frac{n_{p,t}f\left(\overline{S_{p,t}}\right)}{n_{o,t}f\left(\overline{S_{o,t}}\right) - n_{p,t}f\left(\overline{S_{p,t}}\right)}\right|\right\}. \tag{27}$$

Long crowdedness by the over-trading of optimistic sentimental investors has the following characteristics:

1) Most sentimental investors are trading in this stock—i.e., the proportion of neutral sentimental investors $n_{n,t}$ is less than $n^*_{n,t}$,
2) Optimistic sentimental investors have greater capacity to affect the whole perceived value, which is $\left|\frac{n_{o,t}f(\overline{S_{o,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right| > \left|\frac{n_{p,t}f(\overline{S_{p,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right|$.

Similarly, short crowdedness by the over-trading of pessimistic sentimental investors has the following characteristics:

1) Most sentimental investors are trading in this stock—i.e., the proportion of neutral sentimental investors $n_{n,t}$ is less than $n^*_{n,t}$.
2) Pessimistic sentimental investors have greater capacity to affect the whole perceived value, which is $\left|\frac{n_{o,t}f(\overline{S_{o,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right| < \left|\frac{n_{p,t}f(\overline{S_{p,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right|$.

When $\left|\frac{n_{o,t}f(\overline{S_{o,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right| = \left|\frac{n_{p,t}f(\overline{S_{p,t}})}{n_{o,t}f(\overline{S_{o,t}})-n_{p,t}f(\overline{S_{p,t}})}\right|$, the asset is non-crowdedness no matter how many sentimental investors participate in the asset market.

5.4. The deviations of asset prices from fundamental values

We begin to investigate what kind of relationship among breadth of investor sentiment, crowdedness of sentimental investors and the deviations of asset prices from fundamental values. We are interested in how the deviations of asset prices from fundamentals depend on the following parameters: breadth of investor sentiment and crowdedness of sentimental investors.

To be more specific, we first measure the price-to-value ratio which we take to be the equilibrium price P^*_t divides the fundamental value of risky asset V_0e^{rt}. At time t, the price-to-value ratio is given by

$$\frac{P^*_t}{V_0e^{rt}} = \frac{V_0e^{rt} + n_{o,t}f\left(\overline{S_{o,t}}\right) + n_{p,t}f\left(\overline{S_{p,t}}\right)}{V_0e^{rt}}. \tag{28}$$

Second, we define the deviation of asset price from fundamental value at time t, which we take to be the difference of the equilibrium price P^*_t and the fundamental value of risky asset V_0e^{rt} divides the fundamental value of risky asset V_0e^{rt}. Then at time t, the deviation of price from fundamental value can be decomposed in the following way

$$Dev_t = \frac{P^*_t - V_0e^{rt}}{V_0e^{rt}} = \frac{n_{o,t}f\left(\overline{S_{o,t}}\right) + n_{p,t}f\left(\overline{S_{p,t}}\right)}{V_0e^{rt}}. \tag{29}$$

In this simple model, the deviation of asset prices from fundamentals emerges from two sources: 1) agents are sentimental about the asset value and 2) a large number of sentimental investors adopt the optimistic beliefs or the pessimistic beliefs to drive prices up or down.

Proposition 3. If there are heterogeneous sentiment investors (denoted i) in the economy, then price-to-value ratios and the deviations of asset prices from fundamentals are affected by stochastic investor sentiment, the number of optimistic sentimental investors and the number of pessimistic investors. Moreover, four cases arise at time t.

(i) If all the investors are optimistic $n_{o,t} = 1$, then, $B^s_t = \overline{S_{o,t}}$, $Deg_t = 1$, $C_t = 1$, long crowdedness of optimistic sentimental investors makes that the price-to-value ratio $\frac{P^*_t}{V_0e^{rt}} = \frac{V_0e^{rt}+n_{o,t}f(\overline{S_{o,t}})}{V_0e^{rt}}$ and the deviation of asset price from fundamental $Dev_t = \frac{P^*_t - V_0e^{rt}}{V_0e^{rt}} = \frac{n_{o,t}f(\overline{S_{o,t}})}{V_0e^{rt}}$ only depend on the expected optimistic sentiment.
(ii) If all the investors are pessimistic $n_{p,t} = 1$, then, $B^s_t = -\overline{S_{p,t}}$, $Deg_t = 1$, $C_t = 1$, short crowdedness of pessimistic sentimental investors makes that the price-to-value ratios $\frac{P^*_t}{V_0e^{rt}} = \frac{V_0e^{rt}+n_{p,t}f(\overline{S_{p,t}})}{V_0e^{rt}}$ and the deviation of asset price from fundamental $Dev_t = \frac{P^*_t - V_0e^{rt}}{V_0e^{rt}} = \frac{n_{p,t}f(\overline{S_{p,t}})}{V_0e^{rt}}$ only depends on the expected pessimistic sentiment.
(iii) If all the investors are neutral sentimental investors and play waiting strategy, then, $B^s_t = 0$, $Deg_t = 0$, $C_t = 0$, non-crowdedness of neutral sentimental investors makes that $\frac{P^*_t}{V_0e^{rt}} = 1$, $Dev_t = 0$.
(iv) If $0 < n_{o,t} < 1$, $0 < n_{p,t} < 1$, $0 < n_{n,t} < 1$, then $B^s_t = \overline{S_{o,t}} - \overline{S_{p,t}}$, $0 < Deg_t < 1$, then, the crowded trade, the price-to-value ratio and the deviation of asset price from fundamental Dev_t depend on the degree of trade Deg_t, the capacity of optimistic sentimental investors $n_{o,t}f(\overline{S_{o,t}})$ and the capacity of pessimistic sentimental investors $n_{p,t}f(\overline{S_{p,t}})$.

Our third proposition simply describes the deviations of asset prices from fundamentals depending on the stochastic investor sentiment and crowdedness of heterogeneous sentimental investors. We will verify the proposition from the following aspects from Table 1 to Table 2: 1) the role of the degree of trade Deg_t, 2) the role of breadth of investor sentiment, 3) the role of crowdedness and 4) the role of investor sentiment.

For different degrees of trade in Table 1, I compute the degree of crowdedness by sentimental investors C_t, the price-to-value ratio P^*_t/V_t and the deviation of asset price from fundamental value Dev_t to verify the role of the degree of trade, experimenting with different values of: 1) the proportion of optimistic sentimental investors $n_{o,t}$ and breadth of investor sentiment B^s_t.

Based on the constant expected optimistic investor sentiment and expected pessimistic investor sentiment, the key message from Table 1 can be summarized as follows. First, the deviations of asset prices from fundamental values are small and the price-to-value ratios close to one when there is a low value of Deg_t. Second, the deviations of asset prices from fundamental values and the price-to-value ratios are large when there is a high value of Deg_t. Specifically, the deviations of asset prices from fundamentals range from -0.036 to 0.06 when Deg_t equals to 0.1; the deviations of asset prices from fundamentals range from -0.18 to 0.30 when Deg_t equals to 0.5; the deviations of asset prices from fundamentals range from -0.42 to 0.54 when Deg_t equals to 0.9. Therefore, we can obtain the above two results from Table 1. These properties are intuitive and straightforward conditional on the constant expected optimistic investor sentiment and expected pessimistic investor sentiment: when a large number of investors become optimistic about a stock, more of them will go long, leading to overtrading a stock, generating a long crowdedness and pushing asset prices above fundamental values; when a large number of investors become pessimistic about an asset, more of them will go short, leading to overtrading an asset, generating a short crowdedness and pushing asset prices below fundamental values; in contrast, when rarely investors become sentimental about an asset, less of them will go long or short, leading to little change in asset price, generating non-crowded situations.

In Table 1, we summarize the degree of trade is a precondition for crowdedness of sentimental investors. Assumed that $n^*_{n,t}$ is the largest proportion when the sentimental investors overcrowd in the market. If

Table 1
Solutions to the pricing model for various parameter values.

Panel A: $n_{n,t}=0.1$, $Deg_t=0.9$, $f(S)=\alpha S$, $\alpha=0.1$, $\overline{S_{o,t}}=6$, $V_0e^{rt}=1$						
	$B_t^s=7$	$B_t^s=8$	$B_t^s=9$	$B_t^s=10$	$B_t^s=11$	$B_t^s=12$
Crowdedness						
$n_{o,t}=0.1$	(0.5714)	(0.7273)	(0.8000)	(0.8421)	(0.8696)	(0.8889)
$n_{o,t}=0.3$	0.7500	0.6000	0.5000	(0.5714)	(0.6250)	(0.6667)
$n_{o,t}=0.5$	0.8824	0.7895	0.7143	0.6522	0.6000	0.5556
$n_{o,t}=0.7$	0.9545	0.9130	0.8750	0.8400	0.8077	0.7778
$n_{o,t}=0.9$	1	1	1	1	1	1
Price/Value						
$n_{o,t}=0.1$	0.98	0.90	0.82	0.74	0.66	0.58
$n_{o,t}=0.3$	1.12	1.06	1.00	0.94	0.88	0.82
$n_{o,t}=0.5$	1.26	1.22	1.18	1.14	1.10	1.06
$n_{o,t}=0.7$	1.40	1.38	1.36	1.34	1.32	1.30
$n_{o,t}=0.9$	1.54	1.54	1.54	1.54	1.54	1.54
Deviations of asset prices from fundamentals						
$n_{o,t}=0.1$	(0.02)	(0.10)	(0.18)	(0.26)	(0.34)	(0.42)
$n_{o,t}=0.3$	0.12	0.06	0.00	(0.06)	(0.12)	(0.18)
$n_{o,t}=0.5$	0.26	0.22	0.18	0.14	0.10	0.06
$n_{o,t}=0.7$	0.40	0.38	0.36	0.34	0.32	0.30
$n_{o,t}=0.9$	0.54	0.54	0.54	0.54	0.54	0.54
Panel B: $n_{n,t}=0.5$, $Deg_t=0.5$, $f(S)=\alpha S$, $\alpha=0.1$, $\overline{S_{o,t}}=6$, $V_0e^{rt}=1$						
	$B_t^s=7$	$B_t^s=8$	$B_t^s=9$	$B_t^s=10$	$B_t^s=11$	$B_t^s=12$
Crowdedness						
$n_{o,t}=0.1$	0.6000	(0.5714)	(0.6667)	(0.7273)	(0.7692)	(0.8000)
$n_{o,t}=0.2$	0.8000	0.6667	0.5714	0.5000	(0.5556)	(0.6000)
$n_{o,t}=0.3$	0.9000	0.8182	0.7500	0.6923	0.6429	0.6000
$n_{o,t}=0.4$	0.9600	0.9231	0.8889	0.8571	0.8276	0.8000
$n_{o,t}=0.5$	1	1	1	1	1	1
Price/Value						
$n_{o,t}=0.1$	1.02	0.98	0.94	0.9	0.86	0.82
$n_{o,t}=0.2$	1.09	1.06	1.03	1.00	0.97	0.94
$n_{o,t}=0.3$	1.16	1.14	1.12	1.08	1.06	1.04
$n_{o,t}=0.4$	1.24	1.23	1.22	1.21	1.20	1.19
$n_{o,t}=0.5$	1.30	1.30	1.30	1.30	1.30	1.30
Deviations of asset prices from fundamentals						
$n_{o,t}=0.1$	0.02	(0.02)	(0.06)	(0.10)	(0.14)	(0.18)
$n_{o,t}=0.2$	0.09	0.06	0.03	0.00	(0.03)	(0.06)
$n_{o,t}=0.3$	0.16	0.14	0.12	0.08	0.06	0.04
$n_{o,t}=0.4$	0.24	0.23	0.22	0.21	0.20	0.19
$n_{o,t}=0.5$	0.30	0.30	0.30	0.30	0.30	0.30
Panel C: $n_{n,t}=0.9$, $Deg_t=0.1$, $f(S)=\alpha S$, $\alpha=0.1$, $\overline{S_{o,t}}=6$, $V_0e^{rt}=1$						

Table 1 (*continued*)

Panel A: $n_{n,t}=0.1$, $Deg_t=0.9$, $f(S)=\alpha S$, $\alpha=0.1$, $\overline{S_{o,t}}=6$, $V_0e^{rt}=1$						
	$B_t^s=7$	$B_t^s=8$	$B_t^s=9$	$B_t^s=10$	$B_t^s=11$	$B_t^s=12$
	$B_t^s=7$	$B_t^s=8$	$B_t^s=9$	$B_t^s=10$	$B_t^s=11$	$B_t^s=12$
Crowdedness						
$n_{o,t}=0.02$	0.6000	(0.5714)	(0.6667)	(0.7273)	(0.7692)	(0.8000)
$n_{o,t}=0.04$	0.8000	0.6667	0.5714	0.5000	(0.5556)	(0.6000)
$n_{o,t}=0.06$	0.9000	0.8182	0.7500	0.6923	0.6429	0.6000
$n_{o,t}=0.08$	0.9600	0.9231	0.8889	0.8571	0.8276	0.8000
$n_{o,t}=0.1$	1	1	1	1	1	1
Price/Value						
$n_{o,t}=0.02$	1.004	0.996	0.988	0.98	0.972	0.964
$n_{o,t}=0.04$	1.018	1.012	1.006	1.000	0.994	0.988
$n_{o,t}=0.06$	1.032	1.028	1.024	1.020	1.016	1.012
$n_{o,t}=0.08$	1.046	1.044	1.042	1.040	1.038	1.036
$n_{o,t}=0.1$	1.06	1.06	1.06	1.06	1.06	1.06
Deviations of asset prices from fundamentals						
$n_{o,t}=0.02$	0.004	(0.004)	(0.012)	(0.020)	(0.028)	(0.036)
$n_{o,t}=0.04$	0.018	0.012	0.006	0.000	(0.006)	(0.012)
$n_{o,t}=0.06$	0.032	0.028	0.024	0.020	0.016	0.012
$n_{o,t}=0.08$	0.046	0.044	0.042	0.040	0.038	0.036
$n_{o,t}=0.1$	0.06	0.06	0.06	0.06	0.06	0.06

Table 1 **displays:** 1) the degree of crowdedness by sentimental investors C_t; 2) the price-to-value ratio P_t^*/V_t; and 3) the deviation of asset price from fundamental value Dev_t with the decreasing of degree of trade by sentimental investors from panel A to panel C. For example, $C_t=(0.8889)$ means the degree of short crowdedness, $C_t=0.7778$ means the degree of long crowdedness, $Dev_t=(0.07)$ means the deviation of asset price from fundamental value is -0.07, $Dev_t=0.05$ means the deviation of asset price from fundamental value is 0.05.

$n_{n,t}\le n_{n,t}^*$, most investors are sentimental investors and invest in the stock. Within this region, sentimental investors are optimistic to lead to long crowdedness and drive prices up if $n_{o,t}>n_{p,t}$; sentiment investors are pessimistic to lead to short crowdedness and drive prices down. If $n_{n,t}>n_{n,t}^*$, more investors play waiting strategy, it is hard to generate crowdedness of sentimental investors in the market, so the deviations of asset prices from fundamental values are relatively small. In a word, $n_{n,t}\le n_{n,t}^*$ is the precondition for the crowdedness of sentimental investors.

For different expected sentiment of optimistic sentimental investors in Table 2, We compute the degree of crowdedness by sentimental investors C_t, the price-to-value ratio P_t^*/V_t and the deviation of price from fundamental value Dev_t to verify the role of stochastic investor sentiment, experimenting with different values of: 1) the proportion of optimistic sentimental investors $n_{o,t}$ and 2) breadth of investor sentiment B_t^s.

If we consider most investors invest in the stock, then the crowdedness depends on how many sentimental investors trade on the same direction. Based on $n_{n,t}=0.1$, the sentimental investors are overtrading the asset, Table 2 demonstrates how breadth of sentiment and the proportion of optimistic sentimental investor can affect crowdedness, the price-to-value ratios and the deviations of prices from fundamental values. Firstly, when the optimistic investor sentiment and the proportion of optimistic sentimental investors are constant, increasing breadth of investor sentiment about an asset will decrease long crowdedness, in

Table 2
Solutions to the pricing model in heterogenous investor sentiment.

Panel A: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $\overline{S_{o,t}} = 6$, $V_0e^{rt} = 1$						
	$B_t^s = 7$	$B_t^s = 8$	$B_t^s = 9$	$B_t^s = 10$	$B_t^s = 11$	$B_t^s = 12$
Crowdedness						
$n_{o,t} = 0.1$	(0.5714)	(0.7273)	(0.8000)	(0.8421)	(0.8696)	(0.8889)
$n_{o,t} = 0.3$	0.7500	0.6000	0.5000	(0.5714)	(0.6250)	(0.6667)
$n_{o,t} = 0.5$	0.8824	0.7895	0.7143	0.6522	0.6000	0.5556
$n_{o,t} = 0.7$	0.9545	0.9130	0.8750	0.8400	0.8077	0.7778
$n_{o,t} = 0.9$	1	1	1	1	1	1
Price/Value						
$n_{o,t} = 0.1$	0.98	0.90	0.82	0.74	0.66	0.58
$n_{o,t} = 0.3$	1.12	1.06	1.00	0.94	0.88	0.82
$n_{o,t} = 0.5$	1.26	1.22	1.18	1.14	1.10	1.06
$n_{o,t} = 0.7$	1.40	1.38	1.36	1.34	1.32	1.30
$n_{o,t} = 0.9$	1.54	1.54	1.54	1.54	1.54	1.54
Deviations of asset prices from fundamentals						
$n_{o,t} = 0.1$	(0.02)	(0.10)	(0.18)	(0.26)	(0.34)	(0.42)
$n_{o,t} = 0.3$	0.12	0.06	0.00	(0.06)	(0.12)	(0.18)
$n_{o,t} = 0.5$	0.26	0.22	0.18	0.14	0.10	0.06
$n_{o,t} = 0.7$	0.40	0.38	0.36	0.34	0.32	0.30
$n_{o,t} = 0.9$	0.54	0.54	0.54	0.54	0.54	0.54

Panel B: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $\overline{S_{o,t}} = 3$, $V_0e^{rt} = 1$						
	$B_t^s = 4$	$B_t^s = 5$	$B_t^s = 6$	$B_t^s = 7$	$B_t^s = 8$	$B_t^s = 9$
Crowdedness						
$n_{o,t} = 0.1$	(0.7273)	(0.8421)	(0.8889)	(0.9143)	(0.9302)	(0.9412)
$n_{o,t} = 0.3$	0.6000	(0.5714)	(0.6667)	(0.7273)	(0.7692)	(0.8000)
$n_{o,t} = 0.5$	0.7895	0.6522	0.5556	(0.5161)	(0.5714)	(0.6154)
$n_{o,t} = 0.7$	0.9130	0.8400	0.7778	0.7241	0.6774	0.6364
$n_{o,t} = 0.9$	1	1	1	1	1	1
Price/Value						
$n_{o,t} = 0.1$	0.95	0.87	0.79	0.71	0.63	0.55
$n_{o,t} = 0.3$	1.03	0.97	0.91	0.85	0.79	0.73
$n_{o,t} = 0.5$	1.11	1.07	1.03	0.99	0.95	0.91
$n_{o,t} = 0.7$	1.19	1.17	1.15	1.13	1.11	1.09
$n_{o,t} = 0.9$	1.27	1.27	1.27	1.27	1.27	1.27
Deviations of asset prices from fundamentals						
$n_{o,t} = 0.1$	(0.05)	(0.13)	(0.21)	(0.29)	(0.37)	(0.45)
$n_{o,t} = 0.3$	0.03	(0.03)	(0.09)	(0.15)	(0.21)	(0.27)
$n_{o,t} = 0.5$	0.11	0.07	0.03	(0.01)	(0.05)	(0.09)
$n_{o,t} = 0.7$	0.19	0.17	0.15	0.13	0.11	0.09
$n_{o,t} = 0.9$	0.27	0.27	0.27	0.27	0.27	0.27

Panel C: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $\overline{S_{o,t}} = 1$, $V_0e^{rt} = 1$

Table 2 (*continued*)

Panel A: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $\overline{S_{o,t}} = 6$, $V_0e^{rt} = 1$						
	$B_t^s = 7$	$B_t^s = 8$	$B_t^s = 9$	$B_t^s = 10$	$B_t^s = 11$	$B_t^s = 12$
	$B_t^s = 2$	$B_t^s = 3$	$B_t^s = 4$	$B_t^s = 5$	$B_t^s = 6$	$B_t^s = 7$
Crowdedness						
$n_{o,t} = 0.1$	(0.8889)	(0.9411)	(0.9600)	(0.9697)	(0.9756)	(0.9796)
$n_{o,t} = 0.3$	(0.6667)	(0.8000)	(0.8571)	(0.8889)	(0.9091)	(0.9231)
$n_{o,t} = 0.5$	0.5556	(0.6154)	(0.7059)	(0.7619)	(0.8000)	(0.8276)
$n_{o,t} = 0.7$	0.7778	0.6364	0.5385	(0.5334)	(0.5882)	(0.6316)
$n_{o,t} = 0.9$	1	1	1	1	1	1
Price/Value						
$n_{o,t} = 0.1$	0.93	0.85	0.77	0.69	0.61	0.53
$n_{o,t} = 0.3$	0.97	0.91	0.85	0.79	0.73	0.67
$n_{o,t} = 0.5$	1.01	0.97	0.93	0.89	0.85	0.81
$n_{o,t} = 0.7$	1.05	1.03	1.01	0.99	0.97	0.95
$n_{o,t} = 0.9$	1.09	1.09	1.09	1.09	1.09	1.09
Deviations of asset prices from fundamentals						
$n_{o,t} = 0.1$	(0.07)	(0.15)	(0.23)	(0.31)	(0.39)	(0.47)
$n_{o,t} = 0.3$	(0.03)	(0.09)	(0.15)	(0.21)	(0.27)	(0.33)
$n_{o,t} = 0.5$	0.01	(0.03)	(0.07)	(0.11)	(0.15)	(0.19)
$n_{o,t} = 0.7$	0.05	0.03	0.01	(0.01)	(0.03)	(0.05)
$n_{o,t} = 0.9$	0.09	0.09	0.09	0.09	0.09	0.09

Table 2 **displays:** 1) the degree of crowdedness by sentimental investors C_t; 2) the price-to-value ratio P_t^*/V_t; and 3) the deviation of asset price from fundamental value Dev_t with the decreasing of the expected sentiment of optimistic sentimental investors $\overline{S_{o,t}}$ from Panel A to Panel C in Table 2. For example, $C_t = (0.8889)$ means the degree of short crowdedness, $C_t = 0.7778$ means the degree of long crowdedness, $Dev_t = (0.07)$ means the deviation of asset price from fundamental value is -0.07, $Dev_t = 0.05$ means the deviation of asset price from fundamental value is 0.05.

contrast, increasing breadth of investor sentiment about an asset will increase short crowdedness. That is to say, based on the constant optimistic investor sentiment and the proportion of optimistic sentimental investors, long crowdedness gradually turns into short crowdedness with breadth of investor sentiment. The mechanism is new to the literature, although its intuition is straightforward: when the optimistic investor sentiment is constant, increasing breadth of investor sentiment means to the more pessimistic investor sentiment, which is positive with short crowdedness and negative with long crowdedness.

Secondly, increasing the proportion of optimistic sentimental investors will increase long crowdedness and increase the capacity of optimistic sentimental investors to drive prices up; in contrast, increasing the proportion of pessimistic sentimental investors will increase short crowdedness and increase the capacity of pessimistic sentimental investors to drive prices down. Thirdly, if the only source of variation in the model were difference across optimistic investors in $\overline{S_{o,t}}$, one can obtain a clear-cut prediction: the price-to-value ratios and the deviations of asset prices from fundamental values will increase with increasing $\overline{S_{o,t}}$. These properties are intuitive and realistic: when sentimental investors as a group become optimistic about a stock, more of them will go long, lead to long crowdedness and drive prices up; when sentimental investors as a group become pessimistic about a stock, more of them will go short, lead to short crowdedness and drive prices down.

In Table 2, these comparative static results are intuitive, and they can be summarized as the following. Suppose $n_{n,t} \le n^*_{n,t}$, the sentimental investors generate long crowdedness and push asset prices above fundamental values if $n_{o,t}f(\overline{S_{o,t}}) > n_{p,t}f(\overline{S_{p,t}})$. An increase in $n_{o,t}$ leads to an increase in the degree of long crowdedness, the price-to-value ratios and the deviations of asset prices from fundamental values; an increase in $\overline{S_{o,t}}$ leads to an increase in the degree of long crowdedness, the price-to-value ratios and the deviations of asset prices from fundamental values. Vice versa, the sentimental investors generate short crowdedness and push asset prices below fundamental values if $n_{o,t}f(\overline{S_{o,t}}) < n_{p,t}f(\overline{S_{p,t}})$. An increase in $n_{p,t}$ leads to an increase in the degree of short crowdedness, an increase in the deviations of asset prices from fundamental values and a decrease in price-to-value ratios; an increase in $\overline{S_{p,t}}$ leads to an increase in the degree of short crowdedness, an increase in the deviations of asset prices from fundamental values and a decrease in price-to-value ratios. The heterogeneity of sentimental investors will be offset and can't generate crowdedness of sentimental investors if $n_{o,t}f(\overline{S_{o,t}}) = n_{p,t}f(\overline{S_{p,t}})$.

Overall, this model is too stylized to yield decisive conclusions about the merits of stochastic investor sentiment and crowdedness of sentimental investors on the deviations of asset prices from fundamental values. Nevertheless, it does highlight the key points: asset prices are much closer to fundamental values without the impact of stochastic investor sentiment and crowdedness of sentimental investors; asset prices are much larger than fundamental values with the impact of optimistic investor sentiment and long crowdedness; asset prices are much smaller than fundamental values with the impact of pessimistic investor sentiment and short crowdedness.

6. Maverick risk

The crowdedness of sentimental investors describes that investors buy or sell the same asset in concert with each other, and the same investment direction strengthens the capacity of sentimental investors. An investor would be wrong when he goes out on a limb and take a position based on his unique expectations, which is consistent with the maverick risk in Arnott (2003) and Allen (2007). Arnott (2003) illustrates the danger of maverick risk is the greatest peril which is the risk of being wrong and alone. However, rare study focuses the role of maverick risk on investment decisions and investment errors. Our model can quantify the danger of maverick risk which can be regarded as a product of two components: the risk of being wrong and the risk of being alone.

When investors in a market disagree with each other, an investor takes a position based on his unique expectations could face the risk of being wrong (Carlin et al., 2014), so the risk of being wrong in our model is governed by

$$Dis_{i,t} = V^s_{i,t} - V^s_t. \tag{30}$$

Relative to the market valuation, the valuation of sentimental investor i is wrong, and the first component represents the opposite view of sentimental investor i: when the market valuation is larger than fundamental value V_0e^{rt}, the valuation of sentimental investor i is smaller than fundamental value V_0e^{rt}; in contrast, when the market valuation is smaller than fundamental value V_0e^{rt}, the valuation of sentimental investor i is larger than fundamental value V_0e^{rt}. The second component ensures that the sentimental investor i is alone: when sentimental investor i undervalues (overvalues) the fundamental value and market valuation overvalues (undervalues) the fundamental value, a low $n_{p,t}$ ($n_{o,t}$) means sentimental investor i is alone and faces serious maverick risk.

To capture the maverick risk caused by sentimental investors, we consider the case that sentimental investor i is pessimistic and market sentiment is optimistic. For each stochastic investor sentiment of sentimental investor i in Table 3, we compute the market valuation V^s_t and maverick risk $Dis_{i,t} = V^s_{i,t} - V^s_t$ with different values of: 1) the proportion of pessimistic sentimental investors $n_{p,t}$ and 2) breadth of investor sentiment B^s_t.

Table 3
Maverick risk.

Panel A: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $S_{i,t} = -1$, $\overline{S_{p,t}} = -2$, $V_0e^{rt} = 1$, $V^s_{i,t} = 0.9$

	$B^s_t = 3$	$B^s_t = 4$	$B^s_t = 5$	$B^s_t = 6$	$B^s_t = 7$	$B^s_t = 8$
Market valuation						
$n_{p,t} = 0.05$	1.075	1.160	1.245	1.330	1.415	1.500
$n_{p,t} = 0.10$	1.060	1.140	1.220	1.300	1.380	1.460
$n_{p,t} = 0.15$	1.045	1.120	1.195	1.270	1.345	1.420
$n_{p,t} = 0.20$	1.030	1.100	1.170	1.240	1.310	1.380
$n_{p,t} = 0.25$	1.015	1.080	1.145	1.210	1.275	1.340
Maverick risk						
$n_{p,t} = 0.05$	(0.175)	(0.260)	(0.345)	(0.430)	(0.515)	(0.600)
$n_{p,t} = 0.10$	(0.160)	(0.240)	(0.320)	(0.400)	(0.480)	(0.560)
$n_{p,t} = 0.15$	(0.145)	(0.220)	(0.295)	(0.370)	(0.445)	(0.520)
$n_{p,t} = 0.20$	(0.130)	(0.200)	(0.270)	(0.340)	(0.410)	(0.480)
$n_{p,t} = 0.25$	(0.115)	(0.180)	(0.245)	(0.310)	(0.375)	(0.440)

Panel B: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $S_{i,t} = -3$, $\overline{S_{p,t}} = -2$, $V_0e^{rt} = 1$, $V^s_{i,t} = 0.7$

	$B^s_t = 3$	$B^s_t = 4$	$B^s_t = 5$	$B^s_t = 6$	$B^s_t = 7$	$B^s_t = 8$
Market valuation						
$n_{p,t} = 0.05$	1.075	1.160	1.245	1.330	1.415	1.500
$n_{p,t} = 0.10$	1.060	1.140	1.220	1.300	1.380	1.460
$n_{p,t} = 0.15$	1.045	1.120	1.195	1.270	1.345	1.420
$n_{p,t} = 0.20$	1.030	1.100	1.170	1.240	1.310	1.380
$n_{p,t} = 0.25$	1.015	1.080	1.145	1.210	1.275	1.340
Maverick risk						
$n_{p,t} = 0.05$	(0.375)	(0.460)	(0.545)	(0.630)	(0.715)	(0.800)
$n_{p,t} = 0.10$	(0.360)	(0.440)	(0.520)	(0.600)	(0.680)	(0.760)
$n_{p,t} = 0.15$	(0.345)	(0.420)	(0.495)	(0.570)	(0.645)	(0.720)
$n_{p,t} = 0.20$	(0.330)	(0.400)	(0.470)	(0.540)	(0.610)	(0.680)
$n_{p,t} = 0.25$	(0.315)	(0.380)	(0.445)	(0.510)	(0.575)	(0.640)

Panel C: $n_{n,t} = 0.1$, $Deg_t = 0.9$, $f(S) = \alpha S$, $\alpha = 0.1$, $S_{i,t} = -6$, $\overline{S_{p,t}} = -2$, $V_0e^{rt} = 1$, $V^s_{i,t} = 0.4$

	$B^s_t = 3$	$B^s_t = 4$	$B^s_t = 5$	$B^s_t = 6$	$B^s_t = 7$	$B^s_t = 8$
Market valuation						
$n_{p,t} = 0.05$	1.075	1.160	1.245	1.330	1.415	1.500
$n_{p,t} = 0.10$	1.060	1.140	1.220	1.300	1.380	1.460
$n_{p,t} = 0.15$	1.045	1.120	1.195	1.270	1.345	1.420
$n_{p,t} = 0.20$	1.030	1.100	1.170	1.240	1.310	1.380
$n_{p,t} = 0.25$	1.015	1.080	1.145	1.210	1.275	1.340
Maverick risk						
$n_{p,t} = 0.05$	(0.675)	(0.760)	(0.845)	(0.930)	(1.015)	(1.100)
$n_{p,t} = 0.10$	(0.660)	(0.740)	(0.820)	(0.900)	(1.080)	(1.060)
$n_{p,t} = 0.15$	(0.645)	(0.720)	(0.795)	(0.870)	(0.945)	(1.020)
$n_{p,t} = 0.20$	(0.630)	(0.700)	(0.770)	(0.840)	(0.910)	(0.980)
$n_{p,t} = 0.25$	(0.615)	(0.680)	(0.745)	(0.810)	(0.875)	(0.940)

Table 3 **displays:** 1) the market valuation V^s_t; and 2) the maverick risk sentimental investor i from Panel A to Panel C in Table 3. For example, $Dis_{i,t} = (0.675)$ means the valuation of sentimental investor i is 0.675 smaller than the market valuation.

Table 3 clearly summarizes the maverick risk of sentimental investor. Specially, the proportion of pessimistic sentimental investors $n_{p,t}$, conditional on positive market sentiment, captures how many his peers invest in the same direction to identify whether he is alone. Decreasing $n_{p,t}$ will increase the risk of being alone, the difference between individual perceived value and market valuation, and then the maverick risk. Increasing breadth of investor sentiment B^s_t, conditional on constant proportion of pessimistic sentimental investors and expected pessimistic investor sentiment, will increase the maverick risk. Decreasing the individual investor sentiment $S_{i,t}$, fixed other variables, will increase the maverick risk. Thus, if we consider a cross-section of sentimental investors that only vary in the degree of pessimistic investor sentiment, then those investors with the most pessimistic investor sentiment will also face the largest maverick risk and experience the huge loss.

7. Conclusions

A large number of authors have suggested that asset price is affected by investor sentiment or investor behavior (Baker and Wurgler, 2006, 2007; Stein, 2009; Stambaugh et al., 2012; Yan, 2013; Fong and Toh, 2014; Kim et al., 2014; Greenwood and Shleifer, 2014; Qian, 2014; Yang and Zhou, 2016). However, the related literature have not studied the impact of stochastic investor sentiment on crowdedness and their joint effects on deviation of asset prices from fundamentals.

We present an asset pricing model with stochastic investor sentiment and crowdedness to provide efficient explanations for the deviation of asset price from fundamental value and maverick risk for investors. First, consistent with the idea that investor sentiment affects investor behavior, this paper studies the effect of stochastic investor sentiment on the formation of crowdedness. Based on the characteristics of stochastic investor sentiment, the sentimental investors can be divided into optimistic sentimental investors, pessimistic sentimental investors and neutral sentimental investors to measure long crowdedness, short crowdedness and non-crowdedness. Second, this paper presents an asset pricing model with stochastic investor sentiment and crowdedness, and our model could give efficient explanations for the deviation of asset prices from fundamentals. Optimistic investor sentiment and long crowdedness push asset price above fundamental value; while pessimistic investor sentiment and short crowdedness push asset price below fundamental value. Furthermore, our model measures the maverick risk of sentimental investors who are wrong and alone, showing that the investor would be wrong when he goes out on a limb and take a position based on his unique expectations.

These results are broadly consistent with the idea that investor sentiment affects equilibrium prices and deviations of asset prices from fundamentals (Fong and Toh, 2014; Kim et al., 2014; Greenwood and Shleifer, 2014; Qian, 2014; Yang and Zhou, 2016) and crowdedness will push asset prices further away from fundamentals with (Barberis and Thaler, 2003; Yan, 2013). In this regard, our findings tie in nicely with previous researches and further demonstrate the inner connection between stochastic investor sentiment and crowdedness of sentimental investors and their combined effect on asset prices, showing that changes in investor sentiment will result in changes in investor behavior, and each of these changes will in turn affect crowdedness and the movements of asset prices. The conclusions of our study can reveal useful to participants and regulators of financial market. By proving that the role of crowdedness and the risk of being wrong and alone, participants and regulators of financial markets can incorporate investor sentiment and investor behavior to predict the asset market bubbles, develop investment strategies to amend the decision making.

There are several possible directions for future research. First, we can consider incorporating the coordination problems and crowded trade problems of sentimental investors into asset pricing model to study the deviations of asset prices from fundamentals. Second, we can consider involving the capital surplus and capital shortage of sentimental investors into asset pricing models to demonstrate the role of capital on asset prices. Third, the multi-period model can demonstrate the sentimental multiplier effect on asset prices. In short, future work incorporating stochastic investor sentiment and investor behavior would be very valuable.

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