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Measuring liquidity commonality in financial markets

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This paper contributes to the literature by developing a new methodology, termed the beta index, for measuring liquidity commonality in financial markets which is derived from the dynamics of liquidity co-movements. We show that computing the beta index is a straightforward process. In addition, not only is the proposed beta index more efficient in controlling for confounding factors and addressing the associated statistical inference issues, but it will also enhance the accuracy of estimation. We apply the beta index to track liquidity commonality in the foreign exchange markets over the study period and to identify important financial and economic events that caused liquidity commonality. We detect periods of high and low liquidity commonalities that would especially benefit active market traders who frequently rebalance portfolios and require knowledge of liquidity commonality as an important early signal and indication of diversification benefit.

Keywords: Commonality; Financial market; Foreign exchange; Liquidity

JEL Classification: C58, F31, G10

1. Introduction

Comovement of liquidity is a phenomenon regularly observed across asset classes whose liquidities are driven by common influences such as monetary shocks (Chordia *et al.* 2000). This phenomenon in the financial market has recently attracted considerable attention (Chordia *et al.* 2005; Hameed *et al.* 2010; Karolyi *et al.* 2012; Mancini *et al.* 2013; Karnaukh *et al.* 2015; Koch *et al.* 2016; Anthony *et al.* 2017; among many others). Commonality in liquidity has important implications for investors. The liquidity-adjusted capital asset pricing model developed by Acharya and Pedersen (2005) shows that investors want to be compensated for holding a security that becomes illiquid when the market in general becomes illiquid. Asset prices are consequently affected by liquidity risk and commonality in liquidity.

This paper develops a new measure for commonality in liquidity, termed the beta index, which is derived from the dynamics of liquidity comovements. To help explain the construction of the index, we apply this new method to the foreign exchange (FX) markets where the time-varying beta index is tracked over the study period to examine the changes in liquidity commonality following the various market events and crises that took place in the FX markets.

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Measuring liquidity commonality is challenging in practice due to its complexity. Inspired by Roll (1988), Morck *et al.* (2000) propose using an aggregate index over a specified time period, *R*-squared, to measure the extent to which the stock prices of individual firms within a country move together. *R*-squared is obtained by regressing the individual stock returns on the market return over the specified time period. Using this, Hameed *et al.* (2010) and Karolyi *et al.* (2012) investigate the commonality in liquidity in the U.S. stock market and in the global stock markets respectively. On the other hand, Karnaukh *et al.* (2015) assess liquidity commonality in the FX markets using *R*-squared as a liquidity commonality measure. Recently, Anthony *et al*. (2017) have investigated liquidity commonality in the secondary corporate loan market.

As a measure of liquidity commonality, however, *R*-squared has several limitations. First, when some assets move in a direction different to that of the market, *R*-squared fails to characterize the overall comovement because it does not consider the directions (positive and negative) of the changes. In addition, from a financial perspective, when measuring liquidity commonality, confounding factors such as volatility and market return must be considered to avoid spurious dependence (Chordia *et al.* 2000). Due to its intrinsic nature, *R*-squared of a regression for measuring liquidity commonality reflects not only the explanatory power of the market-wide liquidity, but also the other relevant control variables in the regression. This results in R-squared being an inaccurate assessment of liquidity commonality.

To address these issues, we propose an alternative approach to measuring commonality in liquidity, termed the beta index, which is time-varying in nature and describes the extent to which the rates of changes in individual assets' liquidities are related to systematic movements of the market liquidity. The magnitude of the beta index therefore measures liquidity commonality with the movements of the market liquidity as a benchmark: a large (or small) beta index indicates that the liquidities of the individual assets do (or do not) co-move together with the entire market. We illustrate the time-varying beta index with an application to the FX markets, where the beta index is tracked to examine the changes in liquidity commonality in the FX market following various market events and crises that took place during the study period.

The remainder of this paper proceeds as follows. In Section 2, we develop the methodology of constructing the beta index to measure commonality in liquidity and compare it with the existing R-squared measure of liquidity commonality. In Section 3, we demonstrate construction of the proposed liquidity commonality measure, i.e. the monthly beta index for the FX markets. Then we track liquidity commonality in the FX markets in Section 4. Section 5 summarizes and concludes the paper. An Appendix provides proofs of the propositions derived.

2. The beta index measure for liquidity commonality

Consider N assets in a financial market (e.g. a basket of currencies in the FX market or a portfolio of stocks in the equity market) that is representative of the market over T periods. We measure the liquidity of each asset i ($i = 1, \ldots, N$) by a liquidity variable $L_{i,t}$, e.g. quoted bid-ask spread or Amihud measure (Amihud 2002), in each period t ($t = 1, \ldots, T$). Let $L^i_{M,t}$ represent the corresponding market-level liquidity variable in time period t defined to be the average of the liquidity variable $L_{j,t}$ across all assets j except for i ($j = 1, \ldots, N$ and $j \neq i$) under investigation, i.e. $L^i_{M,t} = \sum_{j\neq i,j=1}^{N} L_{j,t}/(N-1)$; the same method for calculating market-level liquidity is also used in many extant studies (e.g. Hameed *et al.* 2010; Karolyi *et al.* 2012).

Liquidity may potentially be affected by a number of confounding factors, denoted by $C_{h,t}$ ($h = 1, \ldots, H$), such as volatility, market return, day-of-the-week, and holiday effect (see, e.g. Chordia *et al.* 2000, 2005; Anthony *et al.* 2017). To measure liquidity commonality, the effects of the confounding factors must be taken into consideration. For each confounding factor, $C_{h,t}$, we define the corresponding control variable $C^i_{h,t}$ to be the value of $C_{h,t}$ averaged across all assets j except for i under investigation, i.e. $j = 1, \ldots, N$ and $j \neq i$.

We characterize the comovements of the individual assets' liquidities in the financial market by the rates of changes that measure both the direction and magnitude of the liquidity movements. We consider the following regression of the liquidity variable $L_{i,t}$ on the contemporaneous, lead and lag of the market liquidity variable $L^i_{M,t}$, controlling for variables $C^i_{h,t}$ ($h = 1, \ldots, H$):

$$\Delta L_{i,t} = \gamma_{0,i,t} + \sum_{k=-1}^{1} \beta^{(M)}_{k,i,t} \Delta L^i_{M,t+k} + \sum_{h=1}^{H} \gamma_{h,i,t} C^i_{h,t} + e_{i,t}, \quad i = 1, \ldots, N \tag{1}$$

where the error terms $e_{i,t} \sim N(0, \rho_i^2)$ are assumed to be mutually independent. $\gamma_{h,i,t}$ and $\beta^{(M)}_{k,i,t}$ are the regression coefficients. To reflect the comovements (rather than the overall levels) of the assets' liquidities, we investigate the changes, $\Delta L_{i,t} = L_{i,t} - L_{i,t-1}$, in the liquidity variable in equation (1). $\Delta L^i_{M,t+k}$ is defined similarly: $\Delta L^i_{M,t} = L^i_{M,t} - L^i_{M,t-1}$. Similar approach is also used in the literature (Chordia *et al.* 2000; Koch *et al.* 2016; Anthony *et al.* 2017). We follow Chordia *et al.* (2000) and consider only one lead and one lag in equation (1). Theoretically speaking, the one lag/lead in equation (1) can be further extended to include higher-order leads and lags into the analysis. However, conditional on the values in the current trading period and immediately before/after the current periods, their relevance/importance is usually marginal.

The regression coefficients $\gamma_{h,i,t}$ and $\beta^{(M)}_{k,i,t}$ in equation (1) usually do not hold a constant value over the entire time period under investigation; this is particularly the case when the time period is considerably long, as documented in the literature (see, e.g. Kamara *et al.* 2008). To account for the time-varying nature of the regression coefficients, we assume that the regression coefficients in equation (1) follow a state space (random walk) model:

$$\begin{aligned}
\gamma_{h,i,t} &= \gamma_{h,i,t-1} + \varepsilon_{\gamma,h,i} \\
&\qquad i = 1, \ldots, N;\ h = 0, 1, \ldots, H;\ t = 1, \ldots, T, \\
\beta^{(M)}_{k,i,t} &= \beta^{(M)}_{k,i,t-1} + \varepsilon_{\beta,k,i}, \\
&\qquad i = 1, \ldots, N;\ k = -1, 0, 1;\ t = 1, \ldots, T,
\end{aligned} \tag{2}$$

where $\varepsilon_{\gamma,h,i}$ and $\varepsilon_{\beta,k,i}$ are mutually independent error terms, each having a normal distribution with zero mean, i.e. $\varepsilon_{\gamma,h,i} \sim N(0, \sigma^2_{h,i})$ and $\varepsilon_{\beta,k,i} \sim N(0, \tau^2_{k,i})$, with $\sigma^2_{h,i}$ and $\tau^2_{k,i}$ the corresponding variances that characterize the variability of the coefficients in (1). For regression analysis with time-varying coefficients, the Kalman filter is a widely used approach for statistical inference. This follows that the time-varying regression coefficients $\beta^{(M)}_{k,i,t}$ can efficiently be estimated at each time point t for each asset i. See Kim and Nelson (1999), and Tsay (2010) for a general discussion on time-varying regression coefficients and see Kessler and Scherer (2011) for an application to liquidity analysis.

Following Chordia *et al.* (2000), we add up the market betas in the concurrent, next and previous trading periods to generate the sum-beta:

$$\beta^{(M)}_{i,t} = \sum_{k=-1}^{1} \beta^{(M)}_{k,i,t},$$

which characterizes the time-varying relationship in the movement rates between the i-th individual asset's liquidity and the systematic liquidity. Hence, a negative value of the sum-beta indicates that the liquidity of the i-th individual asset moves towards the opposite direction of the overall market. Such information is very helpful for traders who invest in the i-th individual asset.

Now we define the beta index in time period t to be the average sum-beta $\beta_{i,t}^{(M)}$ across all the assets:

$$B_t := \frac{1}{N}\sum_{i=1}^{N}\beta_{i,t}^{(M)} = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=-1}^{1}\beta_{k,i,t}^{(M)}, \quad t = 1, \ldots, T. \quad (3)$$

The beta index B_t in equation (3) describes to what extent the rates of changes $\Delta L_{i,t}$ $(i = 1, \ldots, N)$ in all the individual assets' liquidities are related to the rate of change $\Delta L_{M,t}^{i}$ in the systematic market liquidity at each time point t. The magnitude of the time-varying beta index B_t hence measures the comovements of the assets' liquidities at each time point t, with the rate of change in the market liquidity as a benchmark. Note the difference between the beta index B_t and individual sum-beta $\beta_{i,t}^{(M)}$: the former characterizes liquidity commonality of the entire market, whereas the latter describes the movement of an individual asset against the whole market. In other words, a small (or large) sum-beta $\beta_{i,t}^{(M)}$ indicates that the liquidity of the i-th individual asset does not (or does) co-move with the entire market. The beta index B_t, calculated by aggregating over all the assets, measures the overall comovements across the whole market at time t.

Chordia *et al.* (2000) consider two summary statistics, i.e. the R-squared and sum of betas, to gauge commonality in liquidity when running regression of the changes in a liquidity variable for individual stocks against the changes in the market liquidity in the U.S. equity market. We note that these two commonality measures in Chordia *et al.* (2000) are summary statistics, each providing a *single* number to summarize the market liquidity over a given period (e.g. a year). Due to the limitations of the OLS regression method, this period must be sufficiently long to ensure that the estimated beta and/or R-squared statistic are measured reliably. For example, Table 3 in Chordia *et al.* (2000) summarizes the market-wide commonality in liquidity with a *single* number (the beta or R-squared) in year 1992 using 253 daily observations. With the same dataset, however, the proposed time-varying beta index method in this paper can efficiently measure commonality in liquidity to provide a time series of commonality measure, even on a daily basis.

Clearly when a substantial number of assets move along different directions, liquidity commonality is very low, and occasionally the beta index B_t in equation (3) can even be a negative value. The other extreme end is the situation when the liquidities of all the individual assets have a strong positive association with the market liquidity, indicating that they move towards the same direction, and hence showing high liquidity commonality. The following proposition characterizes the above discussion; see the Appendix for proof.

PROPOSITION 1 *Consider N random variables y_i $(i = 1, \ldots, N)$ with the observations y_{it} $(t = 1, \ldots, T)$ measured on these variables. Let $x_{it} = \sum_{j=1, j\neq i}^{N} y_{jt}/(N-1)$ be the average across all random variables except for y_i. Consider the following N regression models for each of the indexes $i = 1, \ldots, N$:*

$$y_{it} = \alpha_i + \beta_i x_{it} + e_{it} \quad \text{for} \quad t = 1, \ldots, T.$$

Define $B = (1/N)\sum_{i=1}^{N}\beta_i$ to be the average least square estimate of the slope parameter and let r_{ij} denote the correlation coefficient for variables y_i and y_j. Then we have

(i) $B = 0$ when $r_{ij} = 0$ for all $i, j = 1, \ldots, N$;
(ii) $B \geq 1$ when $r_{ij} = 1$ for all $i, j = 1, \ldots, N$.

Proposition 1 indicates that the beta index is equal to zero when all the rates of changes $y_i = \Delta L_{i,t}$ $(i = 1, \ldots, N)$ are not correlated with each other. On the other hand, the beta index is greater than unity when all the rates of changes $y_i = \Delta L_{i,t}$ $(i = 1, \ldots, N)$ are perfectly correlated.

Following Proposition 1, we suggest two thresholds for the beta index, i.e. 1 and 0, as displayed in Table 1. The former represents the scenario where all the assets have an extremely high correlation with the market, hence indicating a very high degree of liquidity comovement; the latter is where there are a substantially large number of assets moving towards the opposite direction of the market. Note that Koch *et al.* (2016) also investigate the relationship between commonality in stocks' liquidities and the correlations of liquidity demands of the investors' stocks.

In the literature, liquidity commonality is usually measured by the average of the individual regressions' R-squared. Specifically, let R_i^2 denote the R-squared of the time-series regression for each asset i. Then liquidity commonality across the entire market is measured by $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2$ (Roll 1988; Morck *et al.* 2000; Hameed *et al.* 2010; Karolyi *et al.* 2012; Karnaukh *et al.* 2015). It is of interest to investigate in which way the R-squared commonality measure is related to the proposed beta index. The following two propositions shed light on this issue.

PROPOSITION 2 *For N pairs of the random variables y_i and x_i $(i = 1, \ldots, N)$ with the observations y_{it} and x_{it} measured on variables y_i and x_i respectively $(t = 1, \ldots, T)$, consider the following N time-series regressions for index $i = 1, \ldots, N$:*

$$y_{it} = \alpha_i + \beta_i x_{it} + e_{it} \quad \text{for } t = 1, \ldots, T.$$

Define the beta index $B = (1/N)\sum_{i=1}^{N}\beta_i$ to be the average least square estimate of the slope parameter. Then $B = (1/N)\sum_{i=1}^{N}\beta_i$ can be expressed as $B = (1/N)\sum_{i=1}^{N} w_i R_i$,

Table 1. Scale of the beta index and interpretation.

Range of B_t	Strength of the commonality	Interpretation
$B_t < 0$	Very low	Overall the changes in individual assets' liquidities have no substantial association with the changes in the market liquidity.
$B_t \geq 1$	Very high	The changes in individual asset liquidities on average have a very high association with the changes in the market liquidity.

where $R_i = \sqrt{R_i^2}$ is the multiple R for each regression of variable y_i on variable x_i and $w_i = \pm(s_{y_i}/s_{x_i})$ with s_{y_i} and s_{x_i} the standard deviations of y_i and x_i respectively. In addition, each w_i has a positive (negative) sign if y_i and x_i have a positive (negative) association.

See the Appendix for proof. Proposition 2 shows that, under some conditions, the beta index is a weighted average of the multiple Rs of the individual regressions so it captures the information that the average R-squared $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2$ contains. When variables y_i and x_i are both standardized to have a unit variance, the beta index reduces to $B = (1/N)\sum_{i=1}^{N} \pm R_i$. Hence, we have the following result.

Proposition 3 *Under the assumptions of Proposition 2, if variables y_i and x_i are both standardized to have a unit variance, and they have a positive relationship for all $i = 1, \ldots, N$, then*

(i) $B = (1/N)\sum_{i=1}^{N} R_i = 1$ *if and only if* $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2 = 1$;
(ii) $B = (1/N)\sum_{i=1}^{N} R_i = 0$ *if and only if* $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2 = 0$.

Let us consider the scenario where variables y_i and x_i are the rate of changes $\Delta L_{i,t}$ for asset i and the rate of changes for the market $\Delta L^i_{M,t}$ respectively. Proposition 3 shows that, when all the individual liquidities move along the same direction as the market-wide liquidity does (i.e. $\Delta L_{i,t}$ and $\Delta L^i_{M,t}$ are positively correlated), then the beta index B and average $\bar{R}^2$ provide consistent measures on commonality.

However, the conclusion in Proposition 3 is not true in the general situation. The following proposition considers an extreme scenario where the average R-squared is unable to describe the comovement of the market.

Proposition 4 *Under the assumptions of Proposition 2, suppose that variables y_i and x_i are both standardized to have a unit variance ($i = 1, \ldots, N$). If a half of the paired variables (y_i, x_i) have a correlation coefficient of r, and the other half have a correlation of $-r$, then $B = (1/N)\sum_{i=1}^{N} R_i = 0$ but $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2 = r^2$.*

For the market condition characterized in Proposition 4 where a half of the asset liquidities move along one direction whereas the other half along the opposite direction, liquidity commonality, by definition, is low. In such a market condition, Proposition 4 shows that the beta index B and average $\bar{R}^2$ do *not* provide consistent measures on the commonality, and it makes much more sense to use the beta index B. Specifically, it indicates that for such a market condition, the positive and negative multiple Rs are canceled out, leading to a small beta index $B = (1/N)\sum_{i=1}^{N} \pm R_i = 0$ that correctly reflects the low level of commonality. In contrast, the individual R_i^2 $(i = 1, \ldots, N)$ are always positive; hence the average R-squared value $\bar{R}^2 = (1/N)\sum_{i=1}^{N} R_i^2 = r^2$ tends to be much higher since both the positive and negative changes are mistakenly measured as 'comovements'. Hence $\bar{R}^2$ as a commonality measure may not be as informative as it should be in this market condition. In Section 3.4, we will examine empirical evidence to illustrate this important point.

There are some other advantages of applying the beta index when compared with the R-squared. First, we note the R-squared is susceptible to confounding factors, such as market return and volatility. To avoid any spurious dependence, confounding factors must be taken as control variables in the regression analysis (Chordia *et al.* 2000; Anthony *et al.* 2017). Consequently, the R-squared reflects the explanatory powers of the market-wide liquidity and the control variables in the multiple regressions. In theory, it can happen that the changes in all the individual liquidities have little correlation with the market-wide changes (and hence the liquidity commonality is around 0) but the R-squared would still remain high due to the contributions from the control variables. Consider equation (1), for example: when the correlations between $\Delta L_{i,t}$ and $\Delta L^i_{M,t+k}$ are all zero, the beta index by definition is 0. However, owing to the control variables $C^i_{h,t}$, the R-squared of the regression could still be high.

We note some existing studies use a two-stage method to circumvent this difficulty: the effects of the control variables are firstly removed via a filtering regression, and then the residuals of the filtering regression for the individual assets under investigation are subsequently regressed on the counterpart for the market-wide liquidity. The average R-squared for the second-stage regression is then used as a measure of liquidity commonality (see, e.g. Karolyi *et al.* 2012). Econometrically, the efficiency of this two-stage approach for statistical inference is generally lower. In contrast, the beta index method, as a one-stage procedure, allows statistical inference to be drawn together with the confounding factors being controlled (Kim and Nelson 1999).

Finally, to compute R-squared at a certain time frequency (e.g. monthly), the R-squared approach requires the data on individual assets to be available at a higher sampling frequency (e.g. daily or higher); such higher-frequency data may not be available in some applications. Here we use the computation of monthly R-squared as an example. In order to obtain the monthly R-squared measure, daily liquidities are required to run a number of time-series regression, one for each monthly subsample. Note that the raw data on a weekly basis would not suffice to derive a monthly R-squared since there are only 4–5 weeks per month. The problem with using daily data is that daily data within each monthly subsample consists of no more than 23 trading days per month and hence the sample size for each regression is at most 23. Econometrically, when many control variables are included, a sample with up to 23 observations is barely the optimal size to draw reliable statistical inference, as it requires at least 10–20 observations per parameter to detect reasonable size effects with reasonable power (see, e.g. Harrell 2015).

In summary, compared with the R-squared, using the beta index enhances the statistical efficiency and accuracy in the process of computation. More importantly, it acts as a handy tool and benefits active market traders who rebalance portfolios frequently, where knowledge of liquidity commonality gives important early signs and implications on diversification benefit.

3. Monthly beta index for measuring liquidity commonality in the FX market

This section illustrates the methodology in section 2 by empirically constructing a monthly beta index to measure the liquidity commonality in the FX market. It discusses the data and control variables used for the construction of the index, as well as robustness checks carried out to support the obtained beta index. It also discusses liquidity comovement/divergence situations as observed on some currencies in the study period.

3.1. Data

The time period of the undertaken study is chosen from January 1999, the year the euro was launched, to December 2015. This time span covers the 2001 U.S. recession, the recent global financial crisis (GFC) in 2007–2009 and the period of the European sovereign debt crisis after 2009. All the FX data used in this paper are sourced from Thomson Datastream.

Liquidity is a complex concept and can be measured in several different ways, including price impact and return reversal, trading cost, and price dispersion (Goyenko *et al.* 2009; Kessler and Scherer 2011; Mancini *et al.* 2013). For example, Chordia *et al.* (2000) use the quoted spread to measure liquidity, whereas Karolyi *et al.* (2012) and Koch *et al.* (2016) use the Amihud measure (Amihud 2002) in their studies. Although these liquidity measures capture different facets of liquidity, evidence shows that they are highly correlated in the FX market (Mancini *et al.* 2013; Karnaukh *et al.* 2015). For illustration purposes, we follow Hameed *et al.* (2010) and Karnaukh *et al.* (2015), and measure currency liquidity using the proportional quoted bid-ask spread: $S = (P_A - P_B)/P_M$, where P_A and P_B are the quoted ask price and bid price, P_M is the quote midpoint of the bid and ask prices, and where a low proportional quoted bid-ask spread S indicates that the market is liquid (Mancini *et al.* 2013).

We consider a basket of six major currencies against the United States dollar (USD), i.e. the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), euro (EUR), United Kingdom pound (GBP), and Japanese yen (JPY), and measure the values of these currencies. These were the seven most traded currencies based on the turnover list of the FX market in April 2016; in total, they accounted for 85% of the average daily turnover (Bank for International Settlements 2016). To preserve a sufficiently large cross-section of currencies spanning the foreign exchange market, we also include Danish krone (DKK), Norway krone (NOK), New Zealand dollar (NZD), Swedish krona (SEK), and Singapore dollar (SGD) in the analysis. Taking together, these eleven currencies, plus the USD, as a currency basket (CB) represent the most economically important currencies across the foreign exchange market. Similar currency baskets are also considered in the literature (e.g. Kessler and Scherer 2011; Menkhoff *et al.* 2012; Mancini *et al.* 2013; Jurek 2014; Karnaukh *et al.* 2015; Luo *et al.* 2017).

Bid and ask prices of the currencies sourced from Thomson Datastream are based on actual trades executed with bid and offer order rates directly from market transactions. They are pre-screened to remove a few erroneous measurements.

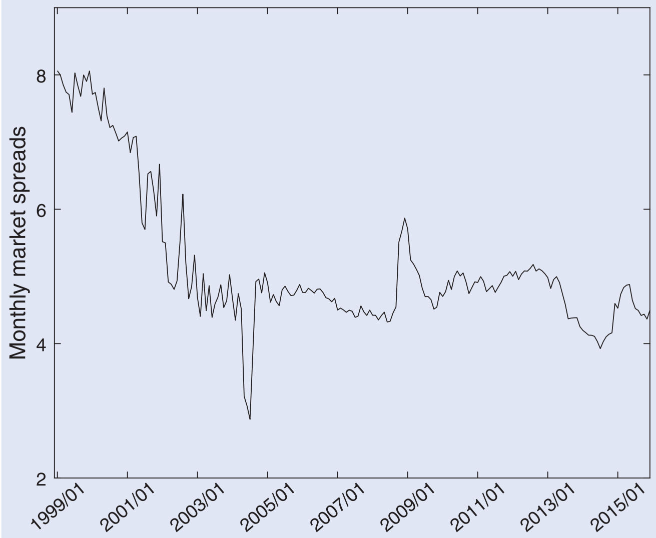

Figure 1. Market liquidity calculated as average proportional quoted spreads in bps of the eleven currencies against the USD.

Figure 1 displays the monthly time series of the market liquidity, $L_{M,t}$, calculated as average proportional quoted bid-ask spreads across the eleven currencies in the basket against the USD. Figure 1 reveals a strong downward trend during 1999–2004; this was primarily due to the introduction of electronic trading systems that substantially reduced the spreads of bid and ask prices. Since then, it has been stable around a level between 4 and 5 bps, except for the recent GFC period in 2007–2009. Table 2, on the other hand, displays summary statistics of the individual liquidities as measured by the daily proportional quoted spreads for the exchange rates of the above eleven currencies against the USD.

The EUR/USD, JPY/USD and GBP/USD were the three most important currency pairs in the FX market; in total they had nearly 50% of market shares in terms of turnover (Bank for International Settlements 2016). Table 2 shows the exchanges rates of the EUR/USD, JPY/USD and GBP/USD across the time period of January 1999 to December 2015 to be more liquid as compared with the other currencies under investigation, whereas the NZD/USD, NOK/USD, and SEK/USD were three of the least liquid.

3.2. Construction of the beta index

We use the monthly average of proportional quoted spreads, denoted by $S_{i,t}$, for currency i in each month t to measure liquidity of this currency ($i = 1, 2, \dots, N$ and $t = 1, \dots, T$), where $N = 11$ is the number of currencies in the basket and $T = 204$ is the number of months over the study period. The following analysis treats the liquidity variable $L_{i,t}$ discussed in Section 2 as the average of the proportional quoted spreads $S_{i,t}$.

Let $\Delta S_{i,t}$ denote the first-order difference of $S_{i,t}$. We use $\Delta S^{i}_{M,t+k} (k = 0, 1, -1)$ to represent the contemporaneous, one lead and one lag of the changes in the market-wide liquidity calculated as the average of $\Delta S_{j,t}$ across all the currency spreads j except for i $(i, j = 1, 2, \dots, N)$, respectively. Regarding the control variables, we follow Chordia *et al.* (2000) and

Table 2. Summary statistics of proportional quoted spreads in bps of the exchange rates of eleven currencies against the USD during Jan. 1999 to Dec. 2015.

Currency	Minimum	Maximum	Mean	S.D.	Skewness	Kurtosis
AUD	1.24	19.26	5.77	3.10	1.84	3.95
CAD	0.62	7.94	4.34	1.59	− 0.39	− 0.67
CHF	0.82	17.78	6.01	1.55	1.30	4.98
DKK	0.66	9.53	3.34	1.07	1.47	3.23
EUR	0.73	10.54	2.93	1.12	1.50	3.12
GBP	0.48	6.88	2.75	1.07	1.09	2.64
JPY	0.84	9.65	3.79	1.31	1.16	2.07
NOK	1.89	28.11	7.65	1.88	2.30	13.15
NZD	1.48	43.32	8.39	4.72	1.94	4.45
SEK	0.96	21.73	6.95	2.35	1.43	2.04
SGD	0.58	14.70	4.86	1.11	2.11	12.15

include the contemporaneous, one lag and one lead of the equally-weighted market returns $R^i_{M,t+k}(k=0,-1,1)$ respectively, as well as the contemporaneous change in the volatility $\Delta V_{i,t}$ of currency i as additional regressors. Based on equation (1), we construct the monthly beta index using the following time-varying coefficients regression:

$$\Delta S_{i,t} = \gamma_{0,i,t} + \sum_{k=-1}^{1} \beta^{(M)}_{k,i,t}\Delta S^i_{M,t+k} + \sum_{h=-1}^{1} \gamma_{h+2,i,t} R^i_{M,t+h} + \gamma_{4,i,t}\Delta V_{i,t} + e_{i,t}, \quad (4)$$

where $e_{i,t} \sim N(0,\rho_i^2)$ are mutually independent error terms, and $\gamma_{h,i,t}$ and $\beta^{(M)}_{k,i,t}$ are the regression coefficients with $\gamma_{0,i,t}$ the intercept term. The lead and lag in (4) are designed to capture any lagged adjustment in commonality (Chordia *et al*. 2000).

Following equation (2), we stipulate the following state-space model for the time-varying regression coefficients:

$$\begin{aligned}\gamma_{k,i,t} &= \gamma_{k,i,t-1} + \varepsilon_{\gamma,k,i},\\ &\quad i=1,\ldots,N;\ k=0,1,\ldots,4;\ t=1,\ldots,T,\\ \beta^{(M)}_{k,i,t} &= \beta^{(M)}_{k,i,t-1} + \varepsilon_{\beta,k,i},\\ &\quad i=1,\ldots,N;\ k=-1,0,1;\ t=1,\ldots,T,\end{aligned} \quad (5)$$

where $\varepsilon_{\gamma,k,i} \sim N(0,\sigma^2_{k,i})$ and $\varepsilon_{\beta,k,i} \sim N(0,\tau^2_{k,i})$ are mutually independent. $\sigma^2_{k,i}$ and $\tau^2_{k,i}$ are the parameters that characterize the variability in these regression coefficients over time.

Following Kim and Nelson (1999), we use the maximum likelihood method to draw statistical inference, where the time-varying beta coefficients $\beta^{(M)}_{k,i,t}$ in equation (4) are estimated with the Kalman filter algorithm (see, e.g. Kim and Nelson 1999; Tsay 2010; Kessler and Scherer 2011). The monthly beta index time series $\{B_t\}$ is calculated based on the beta coefficients using equation (3). The estimated variance parameters are displayed in Table 3.

We first focus on the estimates of the standard deviations $\tau_{k,i}$ $(k=-1,0,1; i=1,\ldots,N)$ for one lag, contemporaneous, and one lead of the changes in the market-wide liquidity $\Delta S^i_{M,t+k}(k=-1,0,1;i=1,\ldots,N)$. We can see from Table 3 that, except for the AUD/USD pair, there is at least one significant standard deviation, either $\tau_{-1,i}$, $\tau_{0,i}$ or $\tau_{1,i}$, at the 10% significance level for all the other currency pairs. This suggests that for these currency pairs, the sum-beta $\beta^{(M)}_{i,t} = \sum_{k=-1}^{1}\beta^{(M)}_{k,i,t}$ is time-varying over the study period, hence justifying the use of state-space model (5) to reflect their time-varying nature. It is hence inappropriate to impose a restriction to hold the betas being constant values over the time span.

Next, we consider the regression coefficients of the control variables. The corresponding variability parameters, i.e. standard deviations $\sigma_{k,i}$ $(k=1,\ldots,4; i=1,\ldots,N)$ for the control variables, are displayed in the remaining four columns of Table 3. First, we consider $\sigma_{1,i}$, $\sigma_{2,i}$ and $\sigma_{3,i}$ for the market return. Table 3 shows that there is at least one significant standard deviation at the 10% significance level, either for one lag, contemporaneous, or one lead, for the currency pairs CAD/USD, DKK/USD, GBP/USD, NOK/USD, NZD/USD and SEK/USD. On the other hand, for the volatility, the standard deviation $\sigma_{4,i}$ is significant at the 10% level for the currency pairs AUD/USD, CHF/USD, DKK/USD, EUR/USD, GBP/USD and NZD/USD. This suggests that the overall confounding effects, i.e. the volatility or market return or both, are in general time-varying over the time span under investigation.

On the basis of the estimates of $\beta^{(M)}_{k,i,t}$ obtained using the Kalman filter algorithm, we calculate the sum-beta $\beta^{(M)}_{i,t}$ for each currency and hence apply equation (3) to obtain the monthly beta index time series $\{B_t\}$ as a measure of the currency liquidity commonality, as depicted in Figure 2.

3.3. Robustness analysis

The results obtained in the previous subsection are based on a basket of 11 currencies against the USD, denoted as CB(11). This subsection investigates the robustness of the obtained beta index by adding/removing a few currencies from the basket. Specifically, we compare the currency basket CB(11) with the following currency baskets against the USD:

- Currency basket CB(6) consists of the AUD, CAD, CHF, EUR, GBP, and JPY against the USD;
- Currency basket CB(7) consists of CB(6) and the NZD/USD;
- Currency basket CB(8) consists of CB(7) and the SEK/USD;

Table 3. Parameter estimation for model (4)–(5) based on the monthly averages of proportional quoted spreads.

Currency against USD	$\tau_{-1,i}$	$\tau_{0,i}$	$\tau_{1,i}$	$\sigma_{1,i}$	$\sigma_{2,i}$	$\sigma_{3,i}$	$\sigma_{4,i}$
AUD	0.022	0.024	0.043	0.000	0.000	0.001	0.002
	(0.368)	(0.238)	(0.273)	(0.004)	(0.002)	(1.434)	(2.164)
CAD	0.269	0.183	0.443	0.000	0.001	0.000	0.000
	(4.913)	(1.840)	(5.011)	(0.005)	(1.770)	(0.727)	(0.000)
CHF	0.342	0.285	0.000	0.000	0.000	0.000	0.004
	(3.626)	(2.771)	(0.001)	(0.000)	(0.000)	(0.000)	(3.087)
DKK	0.035	0.385	0.002	0.000	0.001	0.000	0.002
	(0.912)	(4.574)	(0.110)	(0.022)	(1.847)	(0.025)	(2.323)
EUR	0.000	0.527	0.250	0.001	0.000	0.000	0.003
	(0.000)	(5.826)	(3.566)	(1.515)	(0.000)	(0.000)	(5.059)
GBP	0.000	0.253	0.000	0.002	0.000	0.000	0.001
	(0.002)	(2.469)	(0.001)	(2.579)	(0.000)	(0.000)	(1.767)
JPY	0.000	0.244	0.060	0.001	0.000	0.000	0.000
	(0.001)	(3.390)	(1.585)	(1.097)	(0.608)	(0.000)	(0.964)
NOK	0.143	0.835	0.796	0.018	0.000	0.002	0.000
	(0.721)	(4.917)	(53.259)	(2.778)	(0.011)	(1.616)	(1.003)
NZD	0.079	1.855	2.144	0.002	0.009	0.003	0.004
	(1.281)	(5.804)	(6.817)	(1.588)	(2.480)	(1.787)	(2.754)
SEK	0.268	0.214	0.384	0.003	0.001	0.000	0.001
	(1.080)	(1.759)	(1.310)	(1.786)	(1.605)	(0.000)	(0.825)
SGD	0.055	0.257	0.009	0.000	0.000	0.000	0.000
	(1.481)	(3.474)	(0.177)	(0.000)	(0.000)	(0.000)	(0.000)

Monthly change in the spread for each currency ($\Delta S_{i,t}$) is regressed on the contemporaneous, one lead and one lag of changes in market liquidity $\Delta S^i_{M,t}$, as well as contemporaneous, one lead and one lag of the market return $R^i_{M,t}$, and contemporaneous change in the volatility $\Delta V_{i,t}$, using the specification:

$$\Delta S_{i,t} = \gamma_{0,i,t} + \sum_{k=-1}^{1} \beta^{(M)}_{k,i,t} \Delta S^i_{M,t+k} + \sum_{h=-1}^{1} \gamma_{h+2,i,t} R^i_{M,t+h} + \gamma_{4,i,t} \Delta V_{i,t} + e_{i,t}$$

with $e_{i,t} \sim i.i.d.N(0, \rho_i^2)$, where all regression coefficients are time-varying and follow random walks, i.e. $\gamma_{k,i,t} = \gamma_{k,i,t-1} + \varepsilon_{\gamma,k,i}$ with $\varepsilon_{\gamma,k,i} \sim N(0, \sigma^2_{k,i})$ for $k = 0, 1, \ldots, 4$ and $\beta^{(M)}_{k,i,t} = \beta^{(M)}_{k,i,t-1} + \varepsilon_{\beta,k,i}$ with $\varepsilon_{\beta,k,i} \sim N(0, \tau^2_{k,i})$ for $k = -1, 0, 1$. The maximum likelihood estimates of the parameters of interest are reported with the t-values in parentheses. The results for the intercepts and parameters ρ_i are not tabulated.

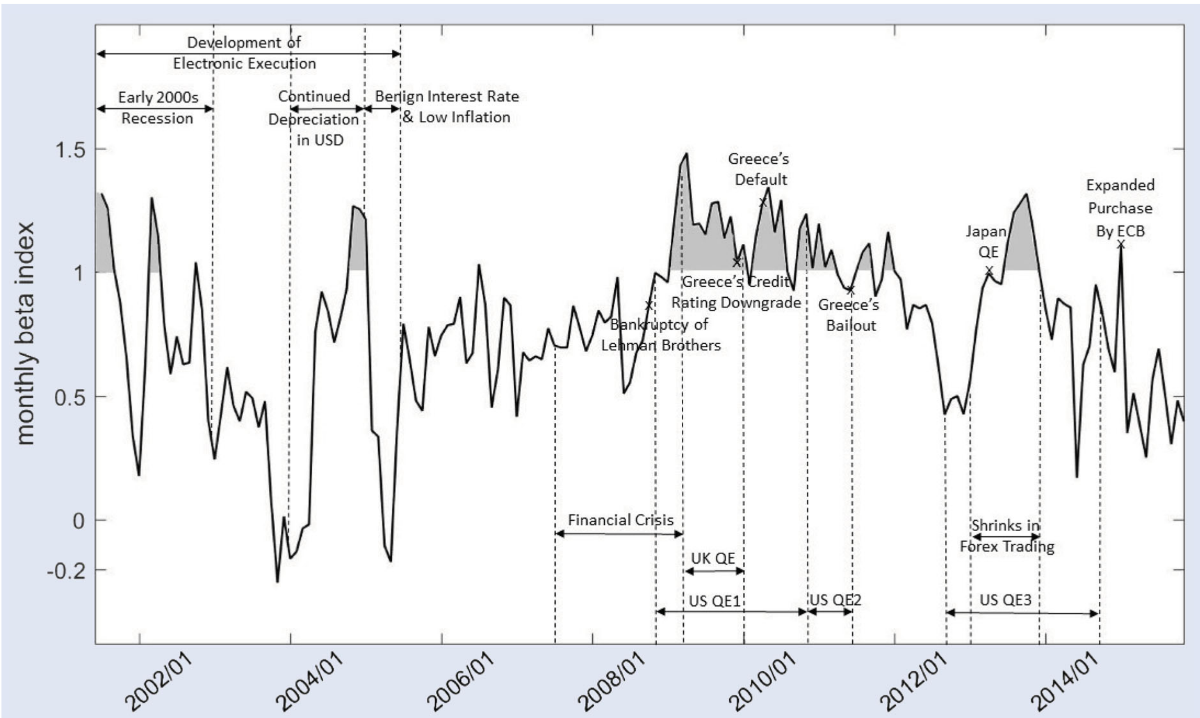

Figure 2. Time-series plot of liquidity commonality in the FX market measured by monthly beta index from Jul. 2001 to Nov. 2015.

Table 4. The correlation coefficients of CB(11)-based beta index with other beta indexes based on different CBs.

Currency basket	CB(6)	CB(7)	CB(8)	CB(9)	CB(15)
Correlation	0.572	0.831	0.902	0.831	0.819

- Currency basket CB(9) consists of CB(8) and the SGD/USD;
- Currency basket CB(15) consists of CB(11), plus the CNY/USD, KRW/USD, MXN/USD, and RUB/USD.†

We construct beta indexes using the above listed CBs. They are compared with the CB(11)-based beta index to find out their correlation coefficients. The results are reported in Table 4. Table 4 reveals that when adding/removing a few currencies, the different CBs-based beta indexes are highly correlated with the CB(11)-based beta index. Hence, it is robust against small changes in the currency basket.

However, when the currency basket consists of only liquid currencies, i.e. the CB(6), the discrepancy of the beta indexes constructed with the CB(6) and CB(11) respectively becomes relatively large. This implies the importance of defining how ‘wide’ the market-wide liquidity covers. One could argue that the currency basket CB(6) represents only a portion of the FX market, i.e. the liquid currencies only, and hence the liquidity commonality based on the CB(6) cannot characterize the comovement of the entire FX market. On the other hand, one could also question the representativeness of the CB(11) for the FX market: should the currency basket be further expanded to include more currencies with lower turnovers? This investigation/debate should be left for future research.

When aggregating the individual currencies, this paper follows the existing literature (e.g. Hameed *et al.* 2010; Karolyi *et al.* 2012) and uses an equally weighted average to construct the beta index. This can straightforwardly be extended to a weighted average. Market-wide indexes in financial markets are often value-weighted averages for the chosen assets, e.g. the Standard & Poor’s 500. Future research is required to investigate if a weighted-average approach to determine weighting would improve the beta index method as a measure of liquidity commonality.

3.4. *Liquidity comovement versus divergence of individual currencies*

This section discusses the comovements of liquidities of individual currencies. In particular, we focus on the divergence of currencies’ liquidities from commonality movement. Figure 2 tracks the liquidity commonality pattern in the FX market in the study period using the constructed monthly beta index time series $\{B_t\}$. The shaded areas as highlighted in Figure 2 are associated with the range of B_t greater than 1, which implies very high commonality in the FX market as listed in Table 1. Interestingly, such phenomenon was rare prior to the 2007–2009 periods. Instead, a couple of episodes were observed for the FX liquidity commonality at very low levels with $B_t < 0$ in the periods 2003–2004 and around early 2005, due to various idiosyncratic events associated with a couple of currency pairs, in particular, the SGD/USD (2003–2004) and NZD/USD (early 2005).

We observe the liquidities of these currency pairs did not comove with the others during these two time periods. We now discuss their respective liquidity situations as compared to the FX market liquidity, and illustrate the use of the beta index for measuring liquidity commonality.

Figure 3 depicts the time series plots of the liquidity for the SGD, $L_{i,t}$, and the FX market liquidity $L_{M,t}$, while Figure 4 shows the time series plots of their monthly changes, i.e. $\Delta L_{i,t}$

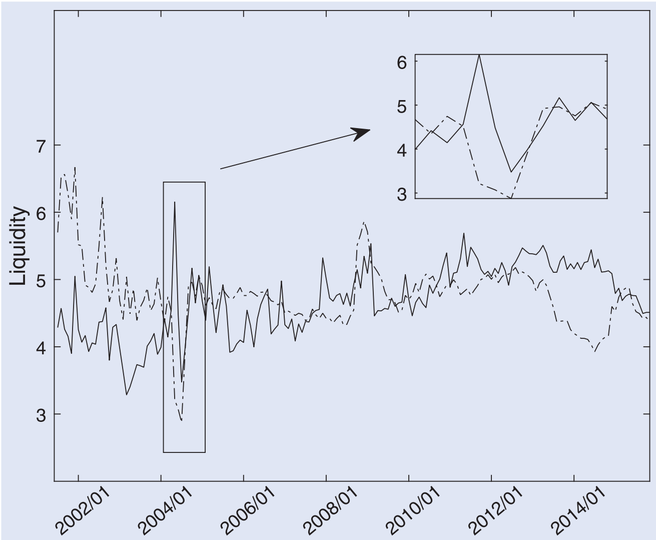

Figure 3. Time series plots of the liquidity for the SGD (the solid line) and the FX market liquidity (the broken line): It shows that during the period immediately after 2004/01, the former moved towards the opposite direction of the latter.

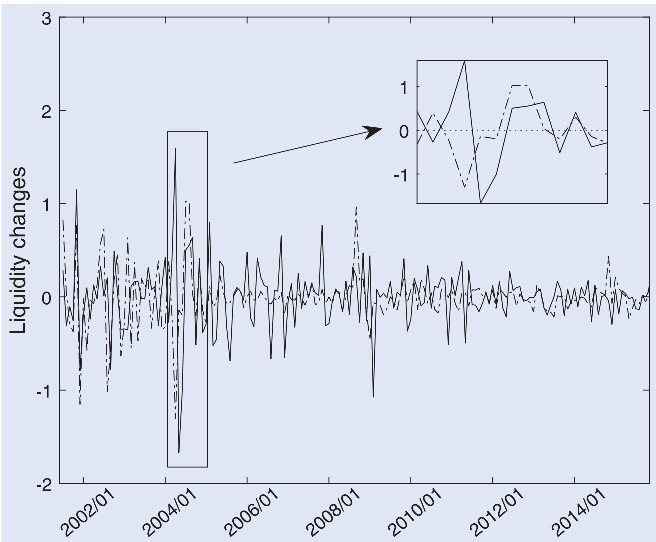

Figure 4. Time series plots of the monthly changes in liquidity for the SGD (the solid line) and the monthly changes in the FX market liquidity (the broken line): It shows that during the period immediately after 2004/01, the former was positive (or negative) whereas the latter was negative (or positive), implying a negative bivariate relationship.

† Note that the currency basket CB(15) consists of all 13 currencies with more than 2% share of turnover in 2016 (Bank for International Settlements 2016), plus the Chinese Yuan (CNY) and Russian Ruble (RUB).

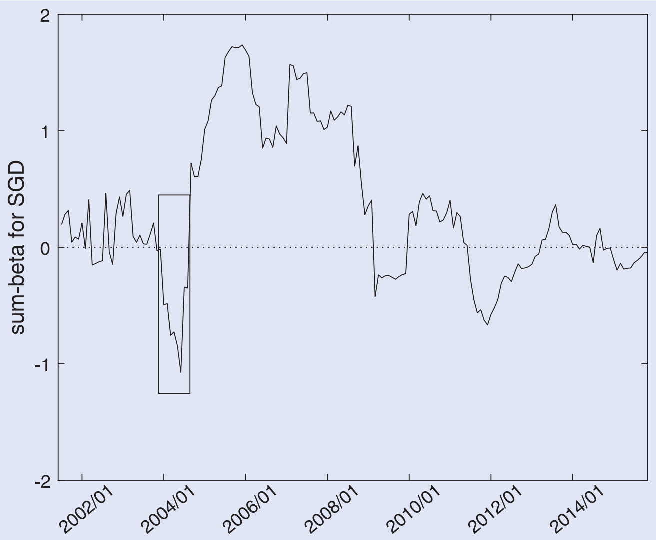

Figure 5. Time series plot of the sum-beta for the SGD (the solid line): It shows that it is negative during the period immediately after 2004/01, indicating that the liquidity movements of the SGD deviated from the overall market liquidity.

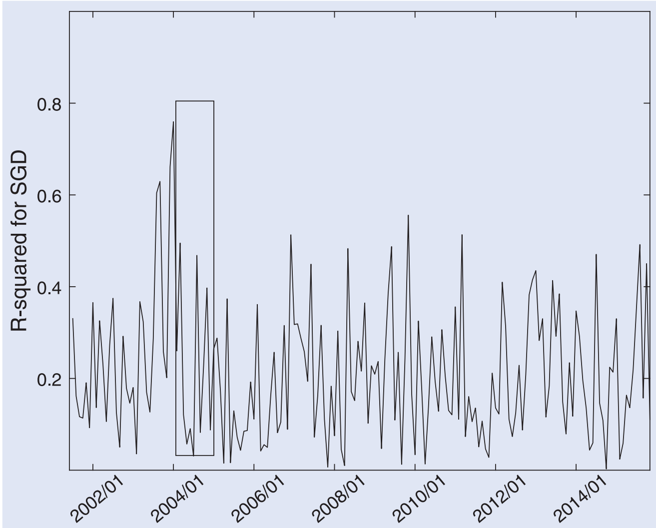

Figure 6. Time series plot of the R-squared for the liquidity of the SGD: It shows that the deviation of the SGD from the market liquidity for the time period immediately after 2004/01 is not flagged up by the R-squared measure.

and $\Delta L_{M,t}$. From Figure 3, we see that during the period immediately after year 2004/01, the liquidity of the SGD moved towards the opposite direction of the FX market: when the market liquidity went down (or up), the SGD liquidity went up (or down). Correspondingly, we see from Figure 4 that during this time period, the monthly changes $\Delta L_{i,t}$ for the SGD are positive (or negative) whereas the monthly changes $\Delta L_{M,t}$ of the overall market are negative (or positive), i.e. they have a negative bivariate relationship during this time period.

The negative relationship shown in Figure 4 leads to a negative sum-beta for the SGD, i.e. $\beta_{i,t}^{(M)} < 0$, for this period, as displayed in Figure 5, since the betas are the coefficients of $\Delta L_{M,t}$ and its one lead/lag when $\Delta L_{i,t}$ is regressed against these changes in the market liquidity, after controlling for the other variables. Apparently, the negative values of the sum-beta for the SGD contributes to the negative region of the beta index around 2004/01 in Figure 2. Hence, this shows that the sum-beta $\beta_{i,t}^{(M)}$ for an individual currency is able to reflect whether this currency comoves with the entire market or deviates from it, and the beta index as a whole is a useful indicator of liquidity commonality of the entire market.

Figure 6 examines the R-squared for the SGD/USD during this time period. The subtle and important deviations of the SGD liquidity movements in this time period are not captured by the R-squared. This supports Proposition 4 in Section 2 which states that the R-squared measure does not account for the direction of movements as it is always positive. The R-squared, as shown in Figure 6, therefore cannot differentiate if the liquidity of an individual currency comoves with the entire market or deviates from it, and may give misleading liquidity commonality signals in the financial markets.

We find supportive empirical evidence to explain the above deviations of the SGD/USD's liquidity from those of the overall FX market. From the perspective of the economy and monetary policies of Singapore around 2003–2004, Chow (2010) reports that Singapore took a position to ease on its monetary policy with reduction in the 3-month S$ interbank rate in 2003. As a small economy exposed to external demand shock and considering the downside risk from the global environment, it also prompted the Monetary authority of Singapore (MAS) to maintain a zero-appreciation path for Singapore dollar as measured by its trade-weighted exchange rate index. The slow economic growth of 2003, which was also evidenced by the relatively smaller FDI in Singapore (Chow 2010), affected Singapore dollar demand, impacted its bid-ask spread and its relationship with the market liquidity spread, and hence contributed to a negative beta index value towards the end of 2003. For the case of the beta index increasing and entering the positive region in late 2004, it could be explained by the announcement of the Singapore's Monetary Authority MAS in April 2004 shifting towards a gradual and modest appreciation of the trade-weighted index underlying the Singapore dollar value. This shifted the demand towards Singapore dollar and was also evident by the increase in the country's FDI in 2004 (Chow 2010) supporting the currency's strength. This, in turn, reduced the value of holding other currencies and contributed to the effects on the US dollars values, alongside other reasons.

Next, we discuss the NZD/USD and examine its movements around early 2005. Figure 7 depicts the time series plots of the liquidity for the NZD, $L_{i,t}$, and the FX market liquidity $L_{M,t}$, while Figure 8 shows the time series plots of their monthly changes, i.e. $\Delta L_{i,t}$ and $\Delta L_{M,t}$. Figure 7 shows that around early 2005, the liquidity of the NZD did not closely follow the movements of the FX market: the former fluctuates dramatically, whereas the latter remains stable. In terms of the monthly changes, Figure 8 shows that there is very little change for the overall FX market during this time period. The $\Delta L_{i,t}$ for the NZD, on the other hand, is initially negative, and then becomes positive and negative again, until it coincides with the overall market changes.

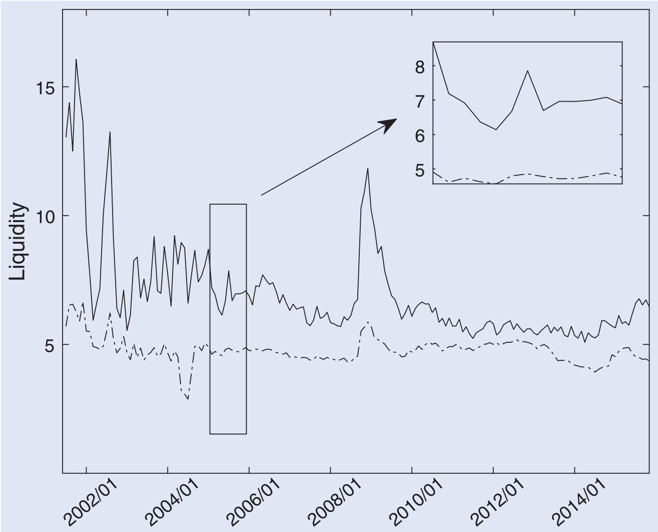

Figure 7. Time series plots of the liquidity for the NZD (the solid line) and the FX market liquidity (the broken line): It shows the divergence of the NZD from the overall FX market around early 2005.

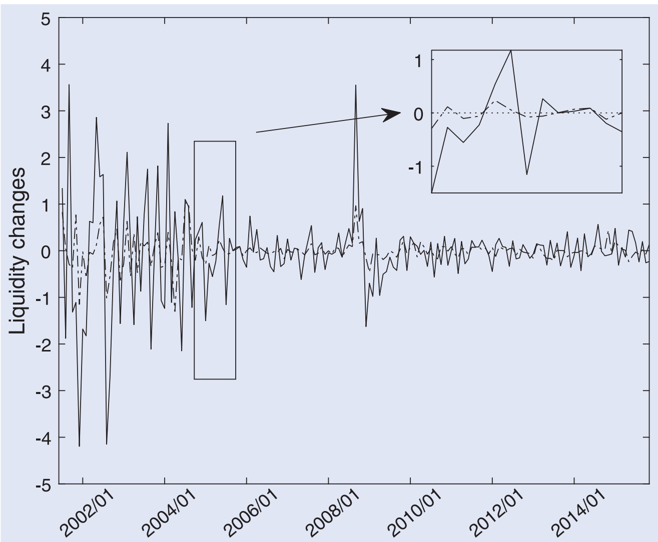

Figure 8. Time series plots of the monthly changes in liquidity for the NZD (the solid line) and the monthly changes in the FX market liquidity (the broken line): It shows that the former did not comove with the latter around early 2005.

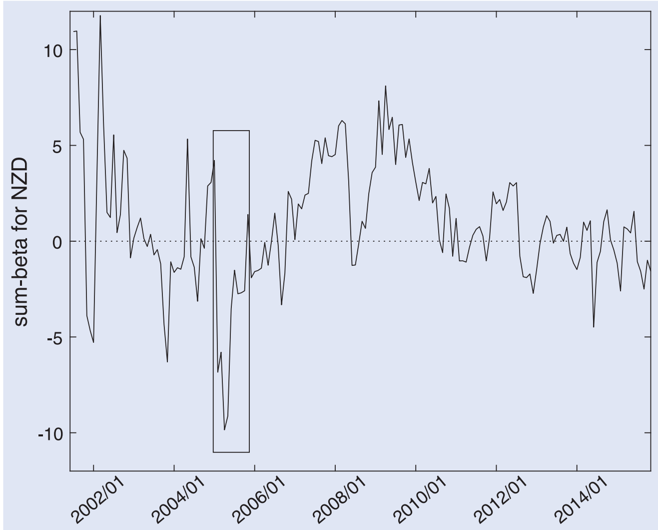

Figure 9. Time series plot of the sum-beta for the NZD (the solid line): It shows that it is negative around early 2005, indicating that the liquidity movements of the NZD deviated from the overall market liquidity.

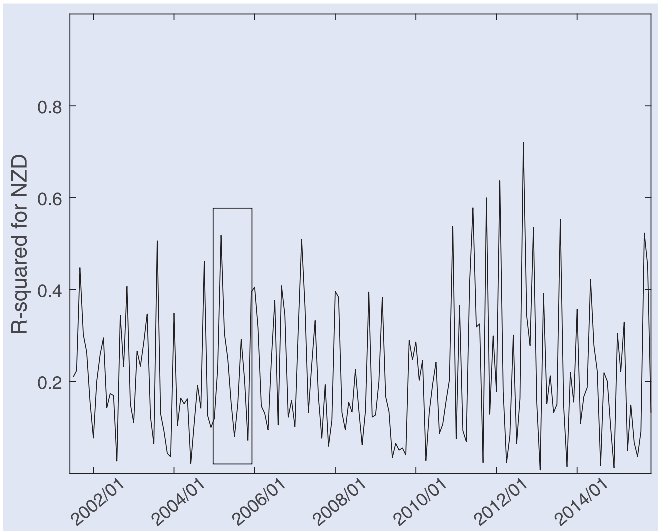

Figure 10. Time series plot of the R-squared for the liquidity of the NZD: It shows that the deviation of the NZD from the market liquidity around early 2005 is not flagged up by the R-squared measure.

The divergence relationship between the overall market and the NZD in Figure 8 is highlighted by the negative values of the sum-beta $\beta_{i,t}^{(M)}$ for the NZD (as displayed in Figure 9), after adjusted for the confounding factors in equation (4). Figure 10 also shows the R-squared for NZD/USD cannot capture the subtle and important deviations of the NZD liquidity movements from the overall market in this time period. This empirical observation again supports Proposition 4 in Section 2, which is in favor of using the beta index to measure liquidity commonality, instead of using the R-squared.

Regarding the explanations for such deviations, Smyth (2009) observe that NZD/USD is heavily involved in carry trade, where it is used as a 'high yielding' (investment) currency between 2001–2006. Though in the time period around early 2005, US dollar's interest rate rose to around 4.25% and created opportunities for profitable carry trade with other currencies considering changes in interest rate differentials, the interest rate of the New Zealand dollar reached at about 7% at the time. This may have affected the demand and supply of the New Zealand dollar and impacted on the bid-ask spread and its relationship with the market-wide liquidity and contributed to a short period of negative sum-beta values in 2005. Section 4 in the following further tracks the financial and economic events that contributed to the high/low liquidity commonality levels in the full study period as shown in Figure 2.

4. Tracking liquidity commonality in the FX market

This section discusses the patterns of liquidity commonality as exhibited in Figure 2. Comovement of liquidity is a phenomenon that individual assets' liquidities are driven by common influences (Chordia *et al.* 2000). This discussion therefore focuses on likely events/influences that caused the liquidity commonality over the study period.

We interpret liquidity commonality based on the description in Table 1, with those shaded areas in Figure 2 associated to the periods of 'very high' level of liquidity commonality, implying the level of the individual currency liquidities' movement to be highly correlated with that of the overall market, brought about by some common causes. The constructed beta index is based on eleven currencies against the US dollar. The US is being impactful as one of the world's largest economy; we therefore focus our discussion based on market events or news from the US or globally when explaining causes of liquidity commonality. In the following analysis, we divide our discussions into the GFC in 2007–2009, as well as the time periods prior and after the GFC.

4.1. *Financial and economic events from 2001 to 2005*

Figure 2 shows that liquidity commonality fell throughout 2001 as it was the year where US dollar was strong with the US dollar index rising to 110.74 from 102.98 as the year closed. The September 11 crisis in 2001 didn't immediately affect the FX market. There were fluctuations within a narrow range, but quickly regained back to pre-crisis level (Makinen 2002). This caused sustained US dollar demand and supported the liquidity commonality at the low level. The year 2002 began with a pessimistic outlook of the US economy. With downside risks intensified (Blecker 2003), the attractiveness of holding US dollar as an investment currency reduced and resulted in a strong sell off leading to a depreciation of the value against most major currencies (Blecker 2003). This caused an increase of the liquidity commonality to the 'very high' level by the first quarter of 2002.

The US dollar remained weak in the year 2003. Crawford *et al.* (2004) commented that this benefited the US economy with exports hitting the highest level in 3 years as it reduced trade gap to $38 billion from $41.6 billion. Crawford *et al.* (2004) also reported that some Asian central banks were big buyers of the US treasury securities in 2003, such as China, increased its dollar foreign exchange reserves by 41 percent, contributing to the demand of holding US dollars and its value and caused a reduced commonality as seen in Figure 2.

The US dollar in 2004, however, turned weaker. Brook *et al.* (2004) believe the continued depreciation of the US dollar in 2004 is due to the increase in the U.S. public and current-account deficits which were the main determinants of 2004 exchange-rate movements worldwide. Brook *et al.* (2004) note the shortfall between funding and investment may pressure on exchange rate adjustment. Banti and Phylaktis (2015) also note capital flows' effects among countries may impact on FX market liquidity. Most industrialized countries' currencies appreciated against the US dollar on trade-weighted terms in 2004, and contributed to liquidity commonality level reaching the 'very high' level.

The year 2005 saw the US dollar demonstrated considerable strength with the first half of the year appreciating against its trading partners, and appreciated by 3.5% in July (Elwell 2008), due partly to the rise in the U.S. interest rates causing a wider interest rate differential with other currencies. This increased demand to hold dollar- rather than euro-based assets and motivated carry trades activities as funding constraints were expected to be less stringent and stimulated FX trading volume and contributed to reduction in liquidity commonality. Figure 2 also shows a short period of the commonality falling below zero in 2005,† before becomes positive as it enters 2006.

4.2. *Global financial crisis 2007–2009*

Turning to the more recent GFC from 2007 to 2009, we observe that the event of Leman Brothers‡ failure did cause an increase in liquidity commonality. However, contagion from one market to another took time, and the crisis in the FX market came relatively late in the recent GFC (Melvin and Taylor 2009). It was in the 4th stage of the financial crisis (Filardo *et al.* 2010), i.e. late 2008 to Q1 2009, that liquidity commonality rose to the 'very high' level according to Figure 2. At this stage, financial markets were roiled by increasingly dire macroeconomic data releases and earnings reports, punctuated by short-lived periods of optimism, especially in January 2009, where it became clear of evidence of a global economic downturn, with prices for financial assets dragged further down, alongside weaker US Q4 2008 economic growth. This led to a huge depreciation of US dollar against most currencies towards the Q1 of 2009 (Filardo *et al.* 2010), arising from reduced demand of the dollar, and resulted in a 'very high' liquidity commonality level as seen in Figure 2.

4.3. *Quantitative easing implementations after the GFC*

The years following 2009 were characterized by a few episodes of Quantitative Easing (QE) programs. The QE program aimed at stimulating the economy via open market operation to increase the money supply of the economy, which influenced the downward pressure of the currency. The UK and the US QE implementations can reasonably explain the movement of the liquidity commonality during the period 2009–2014. The decision and the announcement to implement the QE program had a negative effect on the value of the US

† Detailed discussions can be found in Section 3.4.
‡ Our paper focused on time periods extended beyond the whole GFC period and need to discuss main events throughout the whole period. For the GFC period, we focused only on the failure of Lehman brothers as this failure is one of the main discussion points during the crisis. According to Melvin and Taylor (2009), the failure of Lehman added an entirely new dimension to perceptions of risk, as this is a sign showing that the US government had demonstrated that the market's belief in major institutions being 'too big to fail' was misplaced.

dollar and prompted strong selling off pressure and lowered the demand for holding US dollar. This reduced FX market liquidity and explains why most of the periods the liquidity commonality remained at the 'very high' level. Even at the very high level, the commonality level was observed to fluctuate lower at times, which was also observed at the various points of the QE implementations, such as the US QE2 on the 3rd of November 2010, where the Fed announced that it would buy $600 billion in long-term Treasuries over the subsequent eight months ending June 2011.

With the QE implementation driving further value of US dollar to depreciate, this caused unwinding of carry trades, which according to Mancini *et al.* (2013), further enhanced selling pressure on the dollar that reduced FX liquidity in the market. On the other hand, the liquidity injection underlying the QE implementation from the central bank eliminated the liquidity strains on the dollar or other investment currencies. This helped sustain the effect of the depreciation of the investment currencies. Figure 2 shows that the liquidity injection did impact on reducing liquidity commonality but remained within the 'very high' level. With future US QE implementation to be expected and that US dollar to be depreciated further, it is not surprising that the selling pressure and motivation remained strong and hence led to the 'very high' commonality level.

The U.S. QE3 in 2012 led to some market speculations in 2013 following further plans to purchase securities later the year as announced in June 2013. This occurred as the market started to speculate the timing of the 'tapering' of the asset purchased to be made by the U.S. Fed. News sources (Strauss 2014) revealed that the interpretation of the timing, gathered from information released by the Fed, had led to losses due to wrongly betting the direction of the FX market, resulting in the collapse of some major currency hedge funds in the third quarter of 2013 and caused sudden FX liquidity shock and this impacted on the liquidity commonality to rise and remained in the 'very high' level as showed in Figure 2. News report also revealed that the FX liquidity condition eased beginning of 2014 and that explains the reduction of liquidity commonality out of the 'very high' level to slightly lower level in the same period as indicated in Figure 2.

5. Conclusions and discussion

Recent academic research use R-squared as a liquidity commonality measure (Chordia *et al*. 2005; Hameed *et al*. 2010; Karolyi *et al*. 2012; Mancini *et al*. 2013; Karnaukh *et al*. 2015). This paper, however, re-introduces the beta investigated in Chordia *et al*. (2000) as a measure of liquidity commonality which has been overlooked in the recent literature. Using an econometric approach, we aim to methodologically contribute to the literature by developing a time-varying beta index to closely measure liquidity commonality at every time point of interest. Using the time-varying beta index, this paper tracks the liquidity commonality of the FX markets over the study period.

Liquidity commonality is an early sign indicating diversification benefits which is expected to drive asset price direction throughout an investment period (say, a 1-day period). Commonality level in a financial market, specifically for FX traders in the FX markets, also implies the extent of funding constraints and impacts successful execution of a carry trade strategy. In our analysis, we have discussed and explained market events that caused common movements. We show that the 'very high' level of liquidity commonalities was observed more often in the period after the GFC, and less often prior to that. This is in relation to the implementation of QE programs by major economies such as the US and the UK.

Taking a wider perspective, Acharya and Pedersen (2005) investigate liquidity and liquidity commonality within a liquidity-adjusted capital asset pricing model, where an asset is subject to three liquidity risks: (i) covariance between the asset's illiquidity and the market illiquidity, (ii) covariance between a security's return and the market liquidity, and (iii) covariance between a security's illiquidity and the market return. This paper focuses on the first type of liquidity risk that concerns more about the 'covariance between the asset's illiquidity and the market illiquidity', i.e. liquidity commonality.

The approach to measuring liquidity commonality proposed in this paper can be extended to incorporate it within the framework of Acharya and Pedersen (2005) and to further assess how much an FX investor would gain if his/her investment is based on beta-commonality information. By constructing several portfolios, with and without using the information on liquidity commonality, one can compare the gains/losses of these portfolios.

This research can also be extended to relatively illiquid markets such as the equity market, which is equally important for industry practitioners. As a general methodology for measuring commonality of financial markets, the method of beta-index construction can be further extended to analyse market characteristics other than liquidity, for example, commonality in volatility. These should remain potential areas for future research.

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Appendix. Proofs of the propositions

We offer the proofs for the propositions in the appendix. We first consider a lemma.

LEMMA *For the optimization problem: min $\sum_{i=1}^{N} 1/a_i$, subject to $\sum_{i=1}^{N} a_i = 1$ and $a_i > 0$, the global minimum is attained at $a_i^* = 1/N$ for $i = 1, \ldots, N$. In addition, $\sum_{i=1}^{N} 1/a_i^* = N^2$.*

Proof Define the Lagrange function of the problem as $L = \sum_{i=1}^{N} 1/a_i + \lambda\left(\sum_{i=1}^{N} a_i - 1\right)$. We can obtain $\partial L/\partial a_i = -(1/a_i^2) + \lambda$. Thus the first-order condition gives the stationary point: $a_i^* = \lambda^{-1/2}$ for all $i = 1, \ldots, N$. From the constraint $\sum_{i=1}^{N} a_i = 1$, we obtain $a_i^* = 1/N$. It can be easily verified that $\partial^2 L/\partial a_i^2 = (2/a_i^3) > 0$ and $\partial^2 L/\partial a_i \partial a_j = 0$ for all $i, j = 1, \ldots, N$. Hence the second-order condition indicates the global minimum at the stationary point $a_i^* = 1/N$. Finally, submitting the stationary point $a_i^* = 1/N$ into the criterion function $\sum_{i=1}^{N} 1/a_i$ yields N^2. This completes the proof. ■

Proof of Proposition 1 Let $\bar{x}_i = (1/T)\sum_{t=1}^{T} x_{it}$ and $\bar{y}_i = (1/T)\sum_{t=1}^{T} y_{it}$ denote the means, and let $s_{x_i}^2 = \sum_{t=1}^{T} (x_{it} - \bar{x}_i)^2/(T-1)$ and $s_{y_i}^2 = \sum_{t=1}^{T} (y_{it} - \bar{y}_i)^2/(T-1)$ denote the variances of variables x_i and y_i, respectively. In addition, let $s_{x_i y_i} = \sum_{t=1}^{T}(x_{it} - \bar{x}_i)(y_{it} - \bar{y}_i)/(T-1)$ denote the covariance for variables x_i and y_i. The least square estimate of β_i is given by $\beta_i = s_{x_i y_i}/s_{x_i}^2$. Hence we obtain $B = (1/N)\sum_{i=1}^{N} \beta_i = (1/N)\sum_{i=1}^{N} s_{x_i y_i}/s_{x_i}^2$ which can be rewritten by some algebra as:

$$B = [(N-1)/N]\sum_{i=1}^{N} \frac{\sum_{j=1, j\neq i}^{N} r_{ij} s_{y_i} s_{y_j}}{\sum_{j=1, j\neq i}^{N} s_{y_i}^2 + \sum_{j=1, j\neq i}^{N}\sum_{k=1, k\neq i}^{N} r_{jk} s_{y_j} s_{y_k}}.$$

When $r_{ij} = 0$ for all $i, j = 1, \ldots, N$, we immediately obtain $B = 0$. On the other hand, when $r_{ij} = 1$ for all $i, j = 1, \ldots, N$, the above equation reduces to

$$B = [(N-1)/N]\sum_{i=1}^{N} \frac{s_{y_i}\sum_{j=1, j\neq i}^{N} s_{y_j}}{\sum_{j=1, j\neq i}^{N} s_{y_i}^2 + \sum_{j=1, j\neq i}^{N}\sum_{k=1, k\neq i}^{N} s_{y_j} s_{y_k}}.$$

Note that the denominator can be written as $\sum_{j=1, j\neq i}^{N} s_{y_i}^2 + \sum_{j=1, j\neq i}^{N}\sum_{k=1, k\neq i}^{N} s_{y_j} s_{y_k} = \left(\sum_{j=1, j\neq i}^{N} s_{y_j}\right)^2$. Therefore, we obtain $B = [(N-1)/N]\sum_{i=1}^{N}\left(s_{y_i}/\sum_{j=1, j\neq i}^{N} s_{y_j}\right)$. Finally, let $A = \sum_{i=1}^{N} s_{y_i}$ and $a_i = (A - s_{y_i})/(A(N-1))$. Then we can verify that $\sum_{i=1}^{N} a_i = 1$. The beta index can be written as $B = [(N-1)/N]\sum_{i=1}^{N}(1/(N-1) - a_i)/a_i = [(N-1)/N]\{(N-1)^{-1}\sum_{i=1}^{N}(1/a_i) - N\}$. Hence, from the lemma, we obtain $B \geq 1$. This completes the proof. ■

Proof of Proposition 2 Following the notation used in Proposition 1, the least square estimates of α_i and β_i are given by $\alpha_i = \bar{y}_i - \beta_i \bar{x}_i$ and $\beta_i = s_{x_i y_i}/s_{x_i}^2$ for the ith ($i = 1, \ldots, N$) regression:

$$y_{it} = \alpha_i + \beta_i x_{it} + e_{it} \quad \text{for } t = 1, \ldots, T.$$

On the other hand, the coefficient of determination, the R-squared, for the above regression is defined as $R_i^2 = 1 - \left(\sum_{t=1}^{T}(y_{it} - \hat{y}_{it})^2/\sum_{t=1}^{T}(y_{it} - \bar{y}_t)^2\right)$, where $\hat{y}_{it} = \alpha_i + \beta_i x_{it}$ is the predicted y-value when the independent variable takes the value of x_{it}. Substituting $\hat{y}_{it} = \alpha_i + \beta_i x_{it}$ into R_i^2, we can show, by some algebra, that $R_i^2 = (s_{x_i}^2/s_{y_i}^2)\beta_i^2$. Hence we obtain $B = (1/N)\sum_{i=1}^N \beta_i = (1/N)\sum_{i=1}^N w_i R_i$, with the weight $w_i = \pm(s_{y_i}/s_{x_i})$. ■

Proof of Proposition 3 Since the variables y_i and x_i are both standardized and they have a positive relationship, we have $w_i = 1$ for all $i = 1, \ldots, N$. Hence, we can write $B = (1/N)\sum_{i=1}^N R_i$ and $\bar{R}^2 = (1/N)\sum_{i=1}^N R_i^2$. Noting that $0 \leq R_i \leq 1$, we obtain that $B = 1$ (or 0) if and only if $R_i = 1$ (or 0) for all $i = 1, \ldots, N$; the latter condition is equivalent to $\bar{R}^2 = 1$ (or 0). This completes the proof. ■

The proof for Proposition 4 is immediate from Proposition 2.