Fine-Tuning Private Equity Replication Using Textual Analysis

Ananth Madhavan and Aleksander Sobczyk

Ananth Madhavan
is managing director at BlackRock, Inc., in San Francisco, CA.
ananth.madhavan@blackrock.com

Aleksander Sobczyk
is director at BlackRock, Inc., in San Francisco, CA.
aleksander.sobczyk@blackrock.com

Private equity is an attractive asset class to investors seeking superior returns and low correlation to public equity markets. By *private equity*, we refer to buyouts of mature companies and growth equity—that is, established and expanding firms.[1] Private equity investors make money through management fees and performance-based fees. The illiquid nature of private equity and asymmetric information may drive risk premiums. In comparison to private markets, public markets are more transparent, and asymmetric information risk is mitigated by accounting that abides by generally accepted accounting principles, company regulation, and the attention of financial analysts and short sellers. Illiquidity is a major consideration. Private equity investments typically require a long-term commitment of 10 years or more, with the first 2 to 4 years being investment years (capital calls or periodic draw) and subsequent years the harvest period. Although the investment period is known up front to investors, capital calls are unpredictable and can vary significantly in size. The schedule of capital calls may be estimated by managers at launch (and through annual updates) but for many private equity investors in the early years the experience is quite variable.

These considerations imply that private equity investors and venture capitalists have a need for an interim beta solution to mitigate cash drag and the risk of underperformance arising from large and unpredictable capital calls. The need for interim exposure applies even to investors who have a full private equity allocation (beyond the early years) who face reinvestment risk with distributions (e.g., return of capital, realized gains, interest income) or liquidity needs for various expenses. Note the interim solution is not intended to permanently replace private equity, given the unique risk premiums associated with this asset class, but rather to supplement an actual private equity position within a portfolio.

What would such an interim portfolio solution look like? First, it should be liquid because the funds could be needed at short notice, and it should be long-only because there may be investment constraints on short positions. The solution should also be unlevered; any desired leverage can be added on top of the portfolio. Finally, and perhaps most importantly, the interim beta solution should provide dynamic economic exposure to the asset class. A dynamic portfolio is important because private equity opportunities vary over time with changes in regulation,

[1] Buyouts are often structured as leveraged buyouts but can also refer to so-called turnaround approaches through earnings growth, often accompanied by cost-reduction strategies. By contrast, we think of venture capital as more early-stage investment in firms that have yet to go public.

technology, and the business cycle. In this article, we develop a *holdings-based* methodology using modern data science to create a liquid investable portfolio to mimic dynamically the factor characteristics of private equity over time.

The alternative to a holdings-based approach is to use reported returns. Indeed, a substantial literature on hedge fund replication looks to create an investable liquid proxy for this asset class by regressing fund returns on factors associated with public equities. The returns-based regression approach has the distinct advantage of requiring only return data, but it faces some particularly difficult challenges with private equity, as we describe in detail later, because of return smoothing and other distortions.

It is generally recognized that holdings-based data provide a more accurate way to measure performance and gauge a fund's exposure to factors.[2] But how do we measure at a point in time holdings that are, by definition, not public? The approach taken here involves several steps using modern data science techniques. First, using textual analysis, we create a dictionary of private equity firms from a variety of sources. We then identify firms taken private by those private equity firms in the 10-year period ending June 2018. This step is needed because there is no explicit flag for private equity transactions in the data. Previous analyses (see, e.g., Stafford 2017) used a combination of methods and heuristics. Our approach allows us to create a *dynamic* portfolio that resembles that of private equity at a point in time.

Next, using a multifactor risk model, we measure on a quarterly basis the cross-sectional factor exposures of firms immediately prior to the *announcement* (not *effective*) date when the firms were being acquired by a private equity firm.[3] This analysis is of interest in itself because it complements the growing literature (see, e.g., Kinlaw, Kritzman, and Mao 2014) on the factor characteristics of private equity. We show that the private equity deal portfolio looks quite different from other transactions. Finally, we use holdings-based optimization to build a liquid, investable, unlevered, long-only portfolio that mimics the factor characteristics of the stocks taken private. This portfolio evolves dynamically on a quarter-by-quarter basis and, overall, has risk–return characteristics that are similar to those of reported private equity returns. Interestingly, the mimicking portfolio does not load heavily on small size or the broader market, and the factor loadings vary substantially over time. Value and minimum volatility are important attributes overall, indicative of a preference for cheaper, more stable firms, but traditional quality metrics such as profitability are not preferred, perhaps because private equity firms seek turnaround companies.

PREVIOUS LITERATURE

The article is related to several distinct areas of the literature. First, we complement previous empirical studies of the characteristics of private equity. In particular, Stafford (2017) noted that private equity funds tend to select small firms with low multiples of price to earnings before interest, tax, depreciation, and amortization (EBITDA) (i.e., smaller, value firms). He found that a passive portfolio of small, low-EBITDA-multiple stocks with modest leverage and hold-to-maturity accounting produces an unconditional return distribution that is highly consistent with that of the pre-fee aggregate private equity index.[4] This passive replicating strategy represents an economically large improvement in risk- and liquidity-adjusted returns over direct allocations to private equity funds. Franzoni, Nowak, and Phalippou (2012) estimated a four-factor model to private equity returns and reported significant exposure to factors for the market, liquidity, and value, but not size. The four-factor alpha is zero, and the liquidity risk premium is about 3% annually.

[2] Use of stock-level information in return analysis dates back to at least Brinson and Fachler (1985) and Brinson, Hood, and Beebower (1995) in holdings-based return attribution. Chen, Forsberg, and Gallagher (2016) used institutional holdings data and concluded that hedge funds are superior to other institutional investors at security selection, and hedge funds, mutual funds, and pension funds are able to successfully time the market. Lo (2008) and Hsu, Kalesnik, and Myers (2010) showed how to identify the factor and nonfactor components of active returns using security-level holdings. Grinold (2006) proposed a holdings-based attribution method using characteristic portfolios. When managers dynamically change factor loadings in response to changing economic environments, regression-based approaches may result in excessively smoothed coefficients.

[3] This approach is well established in asset pricing; an early approach was used by Ferson and Harvey (1991), who assumed that betas are a linear function of characteristics.

[4] Indexes are unmanaged, and it is not possible to invest directly in an index.

Our work is also related to efforts to capture the returns of private equity using liquid assets. Kinlaw, Kritzman, and Mao (2014) used a proprietary database of private equity returns to measure the excess return of private equity over public equity and to partition it into two components: an asset class alpha and compensation for illiquidity. They found that private equity managers generate alpha by anticipating the relative performance of economic sectors, consistent with our notion that capturing the dynamics of the opportunity set is important. They interpreted the balance of excess return as a premium for illiquidity. They also noted that their results suggest that investors can capture the asset class alpha of private equity by using liquid assets such as exchange-traded funds (ETFs) to match the sector weights of private equity investors.

The approach is also related to a large literature on hedge fund replication that seeks to create an investable liquid proxy for an asset class by regressing returns of that asset class on factors associated with public equities. Private equity investment returns are reported only quarterly and appear to provide high levels of return with only modest amounts of volatility, as we show later. Return series available to researchers are typically smoothed, the result of appraisal-based valuations.[5] As a result of artificial smoothing, which we investigate empirically in the following, investment returns exhibit high levels of autocorrelation and understate true volatility. Leverage further enhances returns, but transaction costs are understated because of the lack of liquidity. These factors tend to produce high Sharpe ratios that are difficult to proxy with public companies.

By working with unlevered, whitened returns, we can potentially approximate the true unobserved returns to private equity. One approach was illustrated by Pedersen, Page, and He (2014), who employed a lagged factor model to describe the performance of a variety of alternative and illiquid asset classes. The authors described how to estimate risk factor exposures when the available asset return series may be smoothed (owing to the difficulty of obtaining market-based valuations). They showed that private equity has exposure to beta, size, value, and liquidity factors.

An alternative source of return data is use of the returns of publicly traded firms that have private equity portfolios. Although approximately 60 global companies that invest in private equity are publicly traded (including well-known firms such as Apollo Global Management, Blackstone Group, and KKR), many private equity companies (such as Bain Capital) are structured as private partnerships. It is not clear that the returns of public companies are necessarily a representative proxy for the returns to the asset class in general.[6] However, even if we have accurate return information for private equity, the challenge of regression-based time-series coefficients being the (weighted) average over the particular sample period remains, and any attempt to replicate their exposures is inherently static.

Finally, it is worth noting that the holdings-based approach taken here is consistent with the literature showing that factor loadings vary over time. Indeed, conditional factor models, beginning with the conditional capital asset pricing model, predict that betas are a function of the economic environment, time-varying company characteristics, or the changing risk aversion of economic agents.[7] The time variation of factor loadings also significantly affects the interpretation and estimation of econometric and statistical models, including those for private equity. For example, Jagannathan and Wang (1996) showed that conditional betas are an omitted state variable, and failing to take this into account causes other coefficients, including factor loadings and alphas, to be biased. These private equity factor benchmarks we construct at each point in time are dynamic, investable, and without look-ahead bias.

EMPIRICAL ANALYSIS OF RETURNS

Before we turn to our holdings-based approach, it is useful to provide some evidence on returns to motivate the analysis to follow. We gathered quarterly total return data for private equity (Cambridge Associates US Private Equity) and two small-capitalization public equity proxies (see, e.g., Stafford 2017), namely the

[5] Returns reflect management fees, which can range widely (Stafford 2017 estimated fees of 3.5% to 5% annually), and performance fees on the profits (up to 20%). Returns may reflect unrealized and realized gains from the investments, as well as income from the investment in credit instruments. See also Ang et al. (2018).

[6] As an aside, several ETFs hold public companies that invest in private equity, an indicator of interest in this asset class.

[7] See Ang (2014) for a comprehensive review of the major literature in this area.

EXHIBIT 1
Summary Statistics on Quarterly Returns, January 1999 to April 2018

Statistics	Private Equity Returns	Russell 2000 Index	S&P 600 Index
Mean	**3.18**	**2.61**	**2.98**
Std. Dev.	5.19	10.07	9.36
Min.	–15.44	–26.12	–25.17
First Quartile	0.61	–3.78	–1.25
Median	3.88	3.21	3.66
Third Quartile	5.45	8.89	8.73
Max.	18.26	23.42	21.06

Note: Statistics are returns, in percent, observed on a quarterly basis.

Source: Based on data from Bloomberg, FactSet, and Cambridge Associates.

Russell 2000 Index and the S&P Small Cap 600 Index, for the period of January 1999 to April 2018, for a total of 78 quarters. Summary statistics on all three return series are presented in Exhibit 1. Consistent with the previous literature, we find the following:

- Private equity average returns are higher than both public indexes.
- Private equity returns are less variable in terms of measures such as the interquartile range (third quartile less first quartile) or the (quarterly) standard deviation of returns. There is little difference between the two small-cap public equity return series in terms of the measures of central tendency and dispersion, and the correlation in the two public return series is approximately 0.98.
- The risk–return trade-off is seemingly quite favorable to private equity. Approximate annual returns are 12.7% for private equity, and the annualized standard deviation is 10.4%, a ratio of return to risk of 1.22. By contrast, for the Russell 2000 Index, the approximate corresponding figures are 10.4% and 20.1%, respectively, for a return to risk ratio of 0.52.

Private equity reported returns are strongly statistically related to contemporaneous US small-cap equity returns. How well can we model the time-series pattern of returns to private equity in Exhibit 1 using a public small-cap equity index? We regress the quarterly reported private equity returns on the Russell 2000 quarterly return. The coefficient on small-cap returns is only 0.36, significantly less than 1, and the intercept (or Jensen's alpha)[8] is 2.23% per *quarter*; both are highly significant (t-values of 5.17 and 8.72, respectively).[9] Although the R^2 is relatively high at 0.49, this simple model illustrates that small-cap returns alone are unable to explain the risk and return characteristics of reported private equity returns.

Unlike the two public equity indexes of Exhibit 1, neither of which have autocorrelations at any lag that are statistically significantly different from zero, the reported returns of private equity show complicated dynamics possibly reflective of interpolation, smoothing, and appraisal-based valuations. We fit an autoregressive moving-average model to the private equity returns to model these dynamics parsimoniously. The model for reported private equity returns is given by:

$$r_t = \mu + \sum_{i=1}^{p} \rho_i r_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \tag{1}$$

Here r_t is the reported private equity return in quarter t, μ is a constant term, and $\{\epsilon_t\}$ is a weak white noise process with expectation zero and constant variance. In the $ARMA(p, q)$ model of Equation 1, returns are a function of p quarters of past returns through the autoregressive coefficients $\{\rho_i\}$ and q quarters of moving average terms via the coefficients $\{\theta_i\}$. The estimated autoregressive $\{\rho_i\}$ and moving average coefficients $\{\theta_i\}$ are shown in Exhibit 2 for $(p, q) = (5, 5)$. Not only are the autoregressive elements important and significant, it is also clear that all the moving-average terms up to quarterly lag 5 are highly statistically significant.

Recall that the autocorrelation function of an $ARMA(p, q)$ process exhibits exponential decay toward zero, but with possibly damped oscillations. The conclusion that emerges from our analysis of the time series of reported private equity returns is that even with 78 quarters of data, return dynamics are very complex. Although it is certainly possible to try to correct for the impact of smoothing, staleness, interpolation, and

[8] See, for example, Jensen (1968). We get very similar results when using the S&P 600 series instead of the Russell 2000.

[9] Note also that the low beta coefficient on small-cap returns is to be expected if the proxy (i.e., the Russell 2000) is very noisy because of a well-known errors-in-variables problem.

EXHIBIT 2
ARMA(p, q) Model of Reported Private Equity Returns

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	θ_1	θ_2	θ_3	θ_4	θ_5
Coefficients	**0.98**	0.11	**0.69**	**–1.14**	0.27	**–1.79**	**0.50**	**–0.56**	**1.79**	**–0.94**
Std. Errors	0.15	0.12	0.07	0.11	0.14	0.11	0.19	0.18	0.14	0.09

Notes: Coefficients that are statistically significant are marked in bold. The log likelihood is −222.48, and the Akaike information criterion is 466.95. Technically, we estimate an ARIMA(5, 1, 5) model to handle nonstationarity.

Source: Based on return data from Cambridge Associates, in percent, observed on 78 quarters.

appraisal-based valuations (e.g., whitening), this is not an easy task given the complexity shown in Exhibit 2. Accordingly, we turn to a different approach, based on holdings, that allows us to dynamically model the factor attributes of private equity.

DYNAMIC HOLDINGS-BASED LIQUID ALTERNATIVES MODELING

Private Equity Acquisitions of Public Companies

Our holdings-based approach to liquid alternatives modeling consists of three elements:

- Use public equity markets, identify companies that were acquired by private equity firms;
- Use a multifactor model to measure cross-sectional characteristics of those firms prior to the announcement; and
- Use holdings-based factor characteristic mapping to build liquid, investable, long-only portfolios that vary over time to mimic the dynamic target private equity portfolio.

Our sample covers the 10-year period between June 2008 and June 2018. We use the FactSet M&A database and the Bloomberg CAX database to identify 1,107 mergers and acquisitions (M&A) events that resulted in a public US company being delisted (approximately 110 instances per calendar year). Unfortunately, there is no easy way to detect which of these events involved private equity. Although it is possible to make such a determination by hand, this is not a scalable approach—certainly not globally or on a going-forward basis for a possible product or a client portfolio. To develop a systematic approach to identify private equity deals, we used textual identification against custom word dictionaries (based on relevant textual sources for broad industry designations and including firm-specific words) to identify acquirers that are private equity firms or part of private equity–led consortia or private groups. This exercise resulted in 159 events. Exhibit 3 provides a sample textual analysis of an event; Panel A shows the dictionary, and Panel B shows how we distinguish between two M&A events, one of which involves private equity whereas the other is a within-industry biopharma acquisition.

Not surprisingly, the great majority of private equity deals (*deal portfolio*) involve cash (97%), as shown in Exhibit 4. Furthermore, the majority of private equity acquirers are private (86%), with some notable exceptions (e.g., Blackstone, Apollo). Private equity firms target smaller deals (average of $2.5 billion compared to $3.8 billion for non–private equity acquirers), but there are some notable exceptions. This finding suggests that using the returns of public companies that invest in private equity as proxies for private equity returns may not be representative. We note in passing that the sector and industry composition of the deal portfolio changes over time, as one would expect given a time-varying opportunity set, again motivating the need for a dynamic approach.

Mapping Style Factor Exposures

The next step is to determine the factor exposures of the deal portfolio at each point in time, as a prelude to mapping them to investable, long-only factors. For individual stocks, we collect risk characteristics produced by an industry multifactor risk model, BlackRock's Fundamental US Equity Risk Model (BFRE USAM), that includes style characteristics such as momentum, volatility, size, value, and trading, as well as individual sector

EXHIBIT 3
Sample Textual Analysis

Panel A: Custom Dictionary

Dictionary for broad industry designations →	{'private group', 'private equity', 'venture capital'}
Firm-specific dictionary →	{'silver lake', '3g capital', 'apollo', 'fortis', 'longview', 'capital partners', 'blackstone', 'lone star', 'bain capital', 'golden gate', 'goldman sachs', 'vista equity', 'kkr & co', 'pamplona', 'tpg capital', 'kohlberg kravis roberts', 'california public employees retirement system', 'carlyle', 'thoma bravo', 'apq asset', 'cvc capital', 'lqp management', 'advent', 'ares capital', 'ares partner', 'ares management', 'citic capital', 'siris capital', 'elliott', 'francisco partners', 'omers private equity', 'apax partners', 'warburg pincus', 'pictet', 'de rothschild', 'cerberus', 'fortress', 'h.i.g.', 'marlin', 'veritas capital', 'accel partners', 'accelkkr', 'hellman & friedman'}

Panel B: Distinguishing between M&A Events

Target	Announcement Date	Completion Date	Method of Pay	Acquirer	Notes
Kite Pharma, Inc.	8/28/2017	10/3/2017	Cash	Gilead Sciences, Inc.	Gilead Sciences Inc, acquired Kite Pharma Inc for US$10.3 billion in cash, via a tender offer. Under the terms of the agreement, Gilead Sciences Inc paid US$180 in cash for every Kite Pharma Inc share. The consideration represents a 29% premium to Kite's closing stock on August 25, 2017, and a 50% premium to the its 30 day volume weighted average. The transaction was funded from a combination of Gilead Sciences Inc's cash on hand, bank debt and senior unsecured notes.
Strategic Hotels & Resorts, Inc.	9/8/2015	12/11/2015	Cash	The Blackstone Group LP	The Blackstone Group LP, through its Blackstone Real Estate Partners VIII LP fund, acquired Strategic Hotels & Resorts Inc for approximately US$4 billion in cash. Under the terms of the agreement, Blackstone Group offered to pay US$14.25 cash per Strategic Hotels & Resorts Inc share. The offer represents a premium of approximately 13% over the unaffected price on the intraday price July 23, 2015 when an article was published reporting a potential transaction for Strategic Hotels & Resorts Inc. The acquisition is part of The Blackstone Group LP's long term investments in the lodging industry.

Source: BlackRock, based on FactSet M&A and Bloomberg CAX data.

EXHIBIT 4
M&A-Driven Delistings of US Public Companies, 2008 to 2018

Acquirer	Number of Deals	Public Acquirer (%)	Cash Deal (%)	Deal Size ($Millions) Average	Deal Size ($Millions) Median
Private Equity	159	14	97	2,514	795
Other	948	83	61	3,794	1,318
All	1,107	73	66	3,610	1,259

Source: BlackRock, based on FactSet M&A and Bloomberg CAX data.

exposures. The choice of risk model has little impact on the results, given the similarity in models across different providers. Our set of tradable factors are MSCI single factor indexes, which offer a variety of targeted factor exposures and can be traded by an investor at low cost through ETFs. The individual factors, beyond the broad market (Russell 3000), are size, value, momentum, minimum volatility, and quality.

When to Map Factors?

We have a choice in the date of factor mapping. One approach is to use the last calendar month-end date before delisting (i.e., the effective date). Thus, if a company was taken private and delisted from an exchange on June 13, 2001, we could use factor exposures on May 31, 2001. An alternative is to use the style factor characteristics prior to the deal announcement date—that is, if a company was delisted from an exchange on June 13, 2001, but the deal was announced on March 15, 2001, we could use factor exposures on February 28, 2001. The advantage of using the deal announcement date is that this analysis controls for any post-announcement price movement (i.e., from the market price before announcement to the target price). Indeed, we find that the factor

EXHIBIT 5

Comparison of US Public Companies Delisted, 2008 to 2018

	Private Equity Deals			
	Announcement	**Effective**	**Other Deals**	***t*-Statistic**
Momentum	–0.57	0.20	0.63	**–12.26**
Volatility	0.69	0.69	0.79	–1.15
Earnings Yield	–0.15	–0.24	–0.56	**4.22**
Size	–2.00	–1.96	–1.80	**–2.46**
Growth	–0.16	–0.19	–0.06	–1.12
Leverage	0.01	–0.02	–0.06	0.74
Reversal	0.22	0.16	0.25	–0.33
Value	0.53	0.28	–0.16	**7.77**
Dividend Yield	–0.46	–0.55	–0.41	–0.65
Small Cap	2.04	1.93	1.61	**3.80**
Liquidity	–1.27	–1.99	–1.45	1.64
Profitability	–0.25	–0.25	–0.57	**3.27**

Notes: The column "Other" represents all 948 nonprivate deals out of a total of 1,107 deals. The t-statistic refers to a two-tailed test of pre-announcement date factor loadings for private versus other deals, with significant differences marked in bold.

Source: BlackRock, based on FactSet M&A and Bloomberg CAX data.

exposures of private equity targets are significantly more differentiated when mapping their style exposures on the announcement date than on the effective date. Not surprisingly, the effects are strongest for the momentum factor. Just prior to the announcement, the momentum Z-score of the deal portfolio averages −0.57 versus 0.20 on the effective date. Price appreciation to the new target price leads to momentum and a reversal toward the effective date. There is also an evident effect on the liquidity factor and, through price appreciation, on the value factor. In what follows, we will use the (pre) announcement date for factor matching.

Comparison of Deals

We assume a standard multifactor model in which the returns of stock i at time t, $r_{i,t}$, are a linear function of K factors with betas that vary over time:

$$r_{i,t} = \alpha_i + \sum_{k=1}^{K} \beta_{i,k,t-1} F_{k,t} + \varepsilon_{i,t}, \tag{2}$$

where $\beta_{i,k,t-1}$ denotes the exposure of stock i to factor k at time t. Note that timing is explicit in the subscripts. We instrument the beta $\beta_{i,k,t-1}$ for returns at time t to emphasize that it is measurable with respect to information at time $t-1$. We summarize this information in a characteristics vector, denoted by $z_{i,t-1}$. The constant or alpha is not time subscripted, meaning it does not itself vary with time, but returns in excess of the time-varying factor exposures are subject to stochastic shocks, $\varepsilon_{i,t}$.

We estimate factor loadings, $\beta_{i,k,t-1}$, using cross-sectional information, $z_{i,t-1}$, by assuming that various sets of factors are functions of security-level risk characteristics. Exhibit 5 shows the mean Z-score by factor model (using BFRE USAM) for the sample of companies identified as private equity deals based on two dates, announcement and effective. The analysis in Exhibit 5 also shows the changes in average style factor exposure between the deal announcement date and the effective (delisting) date for private equity targets. The column "Other" represents other deals.

Of special interest is the comparison of the deal portfolio to other M&A transactions. Exhibit 6 shows the t-statistic for a two-tailed test of a difference between the mean (pre-announcement date) of the private deal sample versus the "other" deal category. There are marked differences in the factor characteristics of the deal portfolio compared with other public M&A targets. Consistent with the previous literature, we find that compared to all other public M&A targets, targets of private equity firms are

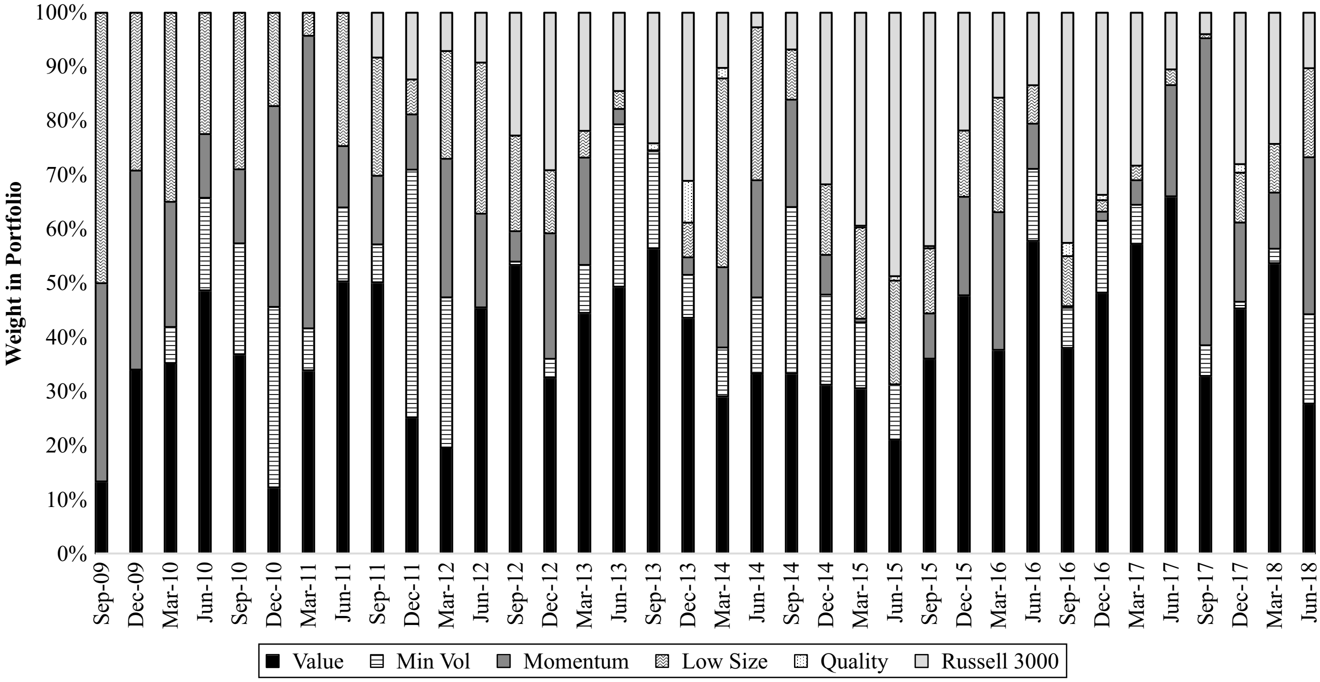
- smaller (size and small cap; also less liquid)
- cheaper (value and earnings yield)
- higher quality (profitability)

Important differences between the deal and other portfolios relate to factors such as momentum (private equity deals have significantly negative momentum relative to other deals when estimated pre-announcement). There are also important and statistically significant differences for value, quality, yield, and profitability.

Factor Index Portfolios—Intuition

The next step is to translate the private equity portfolio's cross-sectional risk characteristics into investable

E X H I B I T 6
Investable Private Equity Mimicking Portfolio



Source: BlackRock, based on FactSet M&A and Bloomberg CAX data from September 30, 2009, to June 30, 2018.

factor index portfolios at each point in time. We follow the approach of Ang, Madhavan, and Sobczyk (2017):

- We start with risk characteristics: variables such as beta or book-to-price for stocks, but also sectors, countries, and currencies.
- The risk model maps securities onto risk characteristics such as value and momentum, as described by Equation 2 earlier.
- At each point in time, optimally match the risk characteristics of a given company to the risk characteristics of a set of third-party long-only style indexes using optimization.
- The resulting portfolio is dynamic, investable, and without any look-ahead bias.

We assume that traded securities have security-level risk attributes. Some of these characteristics, such as valuation ratios or past returns, are sometimes directly used to form style (or smart beta) factors, following Fama and French (1993). We compute factor loadings for our proxy private equity fund at a given time by finding the combination of factors with the closest match, in terms of characteristics, to that fund's holdings. The formal optimization problem is laid out in the next section, but the intuition is quite simple. Suppose that the private equity portfolio at a particular point in time has a value Z-score (e.g., using metrics such as earnings/price or book/price) of 0.40 and a momentum (e.g., trailing-12-month returns omitting the most recent month) Z-score of 0.25. Suppose a long-only, investable value index has Z-scores to value and momentum of 0.90 and −0.10, respectively. Furthermore, suppose a long-only momentum index has Z-scores to value and momentum of −0.10 and 0.70, respectively. It is easy to see then that the investable factor-mimicking portfolio is composed of 50% value index and 50% momentum index. A formal exposition follows.

Formal Optimization Objective

The formal objective is to translate cross-sectional risk characteristics (exposures) into investable factor index exposures. At the start of period t, for any given fund, define an *index factor portfolio* comprising weights $w_{j,t-1}^{IND}$ in an investable index factor $j = 1 \ldots M$, where the number of investable funds (e.g., ETFs) does not exceed the number of possible risk factors in Equation 1 (i.e., $M \le K$). We require the weights in the index portfolio to satisfy $0 \le w_{j,t-1}^{IND} \le 1$ and $\Sigma_{j=1}^{M} w_{j,t-1}^{IND} = 1$ (i.e., the portfolio is long-only and fully invested). Denote by $\hat{\beta}_{j,k,t}^{IND}$ the exposure of investable fund j to risk factor k in period t. It follows that the expected return of the private equity *factor portfolio* with weights $w_{j,t-1}^{IND}$ (where $j = 1\ldots M$) in t is:

$$\mathrm{E}[R_t^{IND}] = \sum_{j=1}^{M} w_{j,t-1}^{IND}\left(\sum_{k=1}^{K}\mathrm{E}(\hat{\beta}_{j,k,t}^{IND})\mathrm{E}(F_{k,t})\right) \tag{3}$$

The difference between the fund's expected total return attributable to static exposures to the K risk factors (from Equation 2) and the expected return of the index factor portfolio (from Equation 3) is denoted by $\hat{\eta}_t$, where

$$\hat{\eta}_t = \sum_{k=1}^{K}\mathrm{E}(\hat{\beta}_{k,t})\mathrm{E}(F_{k,t}) - \sum_{j=1}^{M} w_{j,t-1}^{IND}\left(\sum_{k=1}^{K}\mathrm{E}(\hat{\beta}_{j,k,t}^{IND})\mathrm{E}(F_{k,t})\right) \tag{4}$$

The ordinary least squares estimate for the index factor portfolio at time t is the set of M weights $w_{j,t-1}^{IND}$ that minimizes the squared residual in Equation 4, subject to the following constraints:

$$\sum_{j=1}^{M} w_{j,t-1}^{IND} = 1,$$
$$\text{and}\quad 0 \le w_{j,t-1}^{IND} \le 1,\ \text{for each } j = 1\ldots M. \tag{5}$$

In other words, we require full investment and long-only positions in the factor indexes. This approach could be used more broadly for alpha capture with other return drivers.

Investable Liquid Portfolios

Following the methodology described earlier, we constructed an investable factor-mimicking portfolio for public companies that were targets of private equity acquisitions. The mimicking portfolio is rebalanced quarterly (although we could use an alternative frequency such as monthly, albeit with fewer constituents) and created using the following liquid public instruments:

- MSCI USA Enhanced Value Index (Value)
- MSCI USA Minimum Volatility Index (Min Vol)
- MSCI USA Momentum Index (Momentum)
- MSCI USA Risk Weighted Index (Low Size)
- MSCI USA Sector Neutral Quality Index (Quality)
- Russell 3000 Index (Market)

Exhibit 6 shows the composition of the investable mimicking portfolio based on private deals over the sample period of 10 years, from Q3 2009 through Q2 2018, with no look-forward bias. The factor-mimicking portfolio is dynamic, changing with the latest quarter's private deal portfolio.

Exhibit 6 shows that the mimicking portfolio is not simply composed of the broader market (e.g., the Russell 3000 Index has a median weight of only 12.9% over the whole period) but reflects time-varying factor attributes. Nor is the mimicking portfolio completely dominated by small-cap proxies. Consistent with our earlier regression results of private equity returns on small-cap indexes, low size is an important (median of 15.5%; range of 0.2%–50.0%), but by no means dominant, factor. Rather, value is the largest component, with a median allocation of 37.3% over the entire period, but it also exhibits significant time-series variation. The quality factor generally has a very low or zero weight in most quarters (the median weight is zero, the lowest possible given the long-only constraint), possibly because private equity firms seek out companies they can turn around that score low on quality metrics such as return on equity and profitability. By contrast, minimum volatility has a median weight of 8.4%, consistent with the notion that private equity firms prefer companies with stable cash flows over the business cycle that have the capacity to carry additional debt.

Recall that the private equity mimicking portfolio represented in Exhibit 6 has no look-ahead bias. It is worth emphasizing that we do not use reported private equity returns (which could reflect smoothing and leverage) to construct this portfolio. Assuming quarterly reconstitution and no leverage, the annualized return for the mimicking portfolio in the period beginning September 30, 2009, and ending June 30, 2018, based on the returns to the factors, is 15.1%, with a volatility of 11.6%.[10] By contrast, from Exhibit 1, the reported

[10] The computation takes the sum of factor weights times the returns to the associated factors. Indexes are unmanaged, and it is not possible to invest directly in an index. In the last three years, there are ETFs that offer low-cost proxies (15 basis points) for the factor indexes considered here.

(approximate) annual private equity returns are 12.7% with an annualized standard deviation of 10.4%. It is important to note, though, that the investable mimicking portfolio is not intended to be a substitute for private equity (which offers liquidity and other risk premiums), but rather is a way to deploy excess cash in anticipation of possible future capital calls.

CONCLUSIONS

Private equity is of considerable interest as an asset class. In the early years, private equity investors are committed, but they face challenges because capital calls are unpredictable and can vary significantly in size, risking a performance shortfall. Consequently, private equity investors and venture capitalists have a need for liquid solutions that provide economic exposure to the asset class (so-called *interim beta*) to deploy excess cash, mitigate the risk of underfunding, and manage large and unpredictable capital calls.

In this article, we develop a *holdings-based* methodology using modern data science to create an investable portfolio to dynamically replicate the factor characteristics of private equity. The alternative to a holdings-based approach is to use reported returns, following a large literature on hedge fund replication that seeks to create an investable liquid proxy by regressing hedge fund returns on factors associated with public equities. The returns-based regression approach has the distinct advantage of requiring only return data, but it faces some particularly difficult challenges in private equity, as we showed. Specifically, using 78 quarters of data from January 1999 to April 2018, we find significant evidence of autocorrelation and moving average terms at up to five quarterly lags. These complex dynamics reflect effects such as smoothing, interpolation, and appraisal-based valuations, but perhaps also cyclical factors or error correction. Further research into the dynamics of reported returns is clearly important to investors seeking to understand the diversification benefits of private equity across the business cycle and as part of a public portfolio.

As an alternative to using reported private equity returns, we explore a holdings-based approach to mimic the factor characteristics of a private equity portfolio dynamically. Using textual analysis, we create a dictionary of private equity firms and then identify firms taken private by those firms in the 10-year period ending June 2018. We measure the cross-sectional factor exposures of firms immediately prior to the announcement that they were being acquired by a private equity firm using a risk model. We show the importance of measuring factor exposures of the private equity deal portfolio prior to the announcement date and demonstrate significant changes in momentum and value between the announcement and effective dates. Private equity portfolios look different from other deals: They are smaller (size and small cap; also, less liquid), cheaper (value and earnings yield), and higher quality (profitability).

Finally, we use holdings-based robust optimization to build a portfolio of factor indexes that replicate the factor characteristics of the stocks taken private. This exercise can be repeated at any interval. It would be interesting to understand better how the dynamic holdings-based approach described here compares and contrasts with a returns-based approach and whether both could be integrated in some fashion. In recent years, the mimicking portfolio loads not only on small size but on value and other factors. From a practical perspective, the ability to create a factor-mimicking portfolio that is liquid, investable, and long-only offers a valuable way for private equity investors to maintain exposure and control the risks associated with unpredictable capital calls.

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