Deep Learning, Predictability, and Optimal Portfolio Returns*

Mykola Babiak† Jozef Baruník‡

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Abstract

We study optimal dynamic portfolio choice of a long-horizon investor who uses deep learning methods to predict equity returns when forming optimal portfolios. The results show statistically and economically significant out-of-sample portfolio benefits of deep learning as measured by high certainty equivalent returns and Sharpe ratios. Return predictability via deep learning generates substantially improved portfolio performance across different subsamples, particularly the recession periods. These gains are robust to including transaction costs, short-selling and borrowing constraints.

Keywords: Return Predictability, Portfolio Allocation, Machine Learning, Neural Networks, Empirical Asset Pricing

JEL codes: C45, C53, E37, G11, G17

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†Department of Accounting and Finance, Lancaster University Management School, Lancaster, LA1 4YX, UK. E-mail: m.babiak@lancaster.ac.uk Web: mykolababiak.com

‡Institute of Economic Studies, Charles University, Opletalova 26, 110 00, Prague, CR and Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Pod Vodarenskou Vezi 4, 18200, Prague, Czech Republic. E-mail: barunik@utia.cas.cz Web: barunik.github.io

1 Introduction

Large empirical asset pricing literature has documented supportive evidence for equity return predictability.[1] Abundance of proposed predictive variables has further spurred interest in applying machine learning methods for making the best prediction using a large set of available variables.[2] This new growing literature has demonstrated superior performance of machine learning methods compared to linear regressions that are commonly used by researchers.[3] Yet it is unclear how a sound statistical performance of these complex econometric methods transmits to portfolio benefits of an investor making optimal portfolio decisions. Indeed, the existing evidence for linear models shows that an ensemble of additional features is required to improve portfolio performance from using linear predictive regressions, despite their fine statistical accuracy.[4]

In this paper, we examine the economic value of non-linear machine learning methods, such as neural networks (NNs), for an investor forming optimal portfolios. Specifically, we study an asset allocation of a long-horizon investor with a power utility choosing between a market portfolio and a risk-free asset. The optimal portfolio design follows the structure outlined in Johannes et al. (2014), whereas a comparison of various econometric models to forecast excess market returns is based on their out-of-sample performance as in Gu et al. (2020). Methodologically, we consider univariate and multivariate linear regressions and a variety of machine learning architectures such as shallow and deep NNs as well as a long-

[1] See, for example, Campbell (1987); Campbell and Shiller (1988); Fama and French (1988, 1989); Ferson and Harvey (1991); Pesaran and Timmermann (1995); Lettau and Ludvigson (2001); Lewellen (2004) and Ang and Bekaert (2007) among many others.

[2] See, for example, Rapach et al. (2010); Kelly and Pruitt (2013, 2015); Sirignano et al. (2016); Giannone et al. (2017); Giglio and Xiu (2017); Heaton et al. (2017); Messmer (2017); Feng et al. (2018); Fuster et al. (2018); Chen et al. (2019); Feng et al. (2019); Kelly et al. (2019); Bianchi et al. (2020); Freyberger et al. (2020); Gu et al. (2020); Kozak et al. (2020).

[3] Goyal and Welch (2008) use around 20 financial and macroeconomic variables for the aggregate market returns. Green et al. (2013) list more than 330 return predictive signals used by the existing literature over the 1970-2010 period. Harvey et al. (2016) report 316 "factors" useful for predicting stock returns.

[4] Additional ingredients include learning about predictability with informative priors (Wachter and Warusawitharana, 2009) or an ensemble of estimation risk and time-varying volatility (Johannes et al., 2014).

short-term-memory (LSTM) recurrent NNs. An LSTM is a specialized form of a neural network, which is capable of learning extremely complex long-term temporal dynamics that a vanilla NN is unable to learn. Our focus on the NNs is motivated by the fact that these machine learning methods proved to be the most useful in detecting predictable variations in a variety of the financial markets, particular the equity. Therefore, they are the natural suspects for portfolio managers.

Our contribution to the portfolio literature is threefold. First, we show that non-linear machine learning methods are useful for the construction of optimal portfolios, as indicated by economically significant gains. Specifically, we document that deviating from the Expectations Hypothesis and using NNs to forecast excess returns lead to more than three and two times higher Sharpe ratios (SRs) and certainty equivalent returns (CERs), respectively. This evidence contributes to the debate on the economic value of equity return predictability (Goyal and Welch, 2008; Johannes et al., 2014; Rossi, 2018). Furthermore, our evidence on the benefits of NNs is robust to alternative measures of portfolio performance measures (cumulative return, maximum drawdown, and maximum one-month loss), to the inclusion of transaction costs, short-selling and borrowing constraints.

Moreover, dissecting economic gains of NNs across subsamples, we find that historically machine learning methods would have generated the highest CERs during each of the seven decades in the post-war period. Interestingly, NNs generate on average a two-fold increase in SRs during the NBER recessions compared to the expansion periods. In particular, we find that all NNs manage to generate significant gains during the 2007-2008 Financial Crisis. Finally, the investor benefits from NNs more by rebalancing her portfolio more frequently as opposed to a passive strategy. Despite more frequent trading, we show that these gains are not eliminated by the increased turnover.

Second, compared to the existing evidence for linear models, deep learning methods provide a single "silver bullet" by generating out-of-sample gains without relying on additional ingredients. We demonstrate that the portfolio performance from using NNs dominates those strategies using the linear predictive models even when omitting time-varying return volatility. Our evidence is consistent with Goyal and Welch (2008) and Johannes et al. (2014), as we also do not find any benefits from using linear models without estimation risk and time-varying volatility. We contribute to the literature by showing that the empirical evidence for equity return predictability is economically significant even in the absence of these additional ingredients provided the investor uses non-linear machine learning methods for detecting this predictive variation.

Our third contribution is related to the properties of economic gains implied by NNs. We find that increasing the complexity of deep learning architectures does not necessarily need to translate into improved portfolio performance. We document that moving from shallow settings with one hidden layer to deeper specifications does not bring additional gains. This seems to be surprising result, but finance and especially return prediction faces a challenging data environment that differ substantially from other domains where deep learning brings large improvements. Specifically, return prediction with the goal of optimal portfolio construction is a small data problem with data facing very low signal-to-noise ratio (Israel et al., 2020) and increased complexity of the network does not need to help necessarily. Importantly, we document that inclusion of deep recurrent LSTM networks that capture important temporal dynamics improves the performance according to all portfolio performance measures considered. In this respect, our paper contributes to the evidence on the economic information captured by NNs. Specifically, we extend the evidence presented by Rossi (2018) for boosted regression trees and show that, apart from the important non-linear relationship, long-term memory effects are particularly beneficial in short samples.

The remainder of this paper is organized as follows. Section 2 provides a discussion of standard approach for assessing return predictability, introduces non-linear machine learning methods we consider, describes a portfolio choice problem of an investor, and outlines a variety of performance measures. Section 3 describes the data and summarizes the results.

Section 4 dissects the economic gains obtained from using NNs across subperiods and provides the robustness checks to using alternative performance measures or including transaction costs, borrowing and short-selling constraints. Section 5 concludes.

2 Evaluating Predictability via Portfolio Performance

2.1 Simple Linear

The standard approach used to forecast excess equity returns is a linear model of the form

$$r_{t+1} = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{x}_t + \varepsilon^r_{t+1}, \tag{1}$$

where r_{t+1} are monthly log excess returns, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are coefficients to be estimated, $\boldsymbol{x}_t = (\boldsymbol{x}^1_t, ..., \boldsymbol{x}^n_t)$ is a set of predictor variables, and ε^r_{t+1} is a normal error term. A large strand of the empirical literature has examined the linear regression models with multiple predictors including prominent variables such as the dividend yield, valuation ratios, various interest rates and spreads among others.[5] Although researchers have proposed numerous candidates for predicting stock market returns, the empirical evidence on the degree of predictability is mixed at best. Goyal and Welch (2008) find that most linear specifications with multiple predictors have performed poorly and remain insignificant even in-sample. Furthermore, the authors show that an investor using linear models to forecast equity returns would not be able to improve portfolio performance compared to no predictability benchmark.

There are several reasons for the lack of robust evidence on the equity return predictability and its benefits for portfolio construction. The specification defined by Eq.(1) assumes linear and time-invariant relationship between log excess returns and predictors, which is at odds

[5]See, for example, Shiller (1981); Hodrick (1992); Stambaugh (1999); Avramov (2002); Cremers (2002); Ferson et al. (2003); Lewellen (2004); Torous et al. (2004); Campbell and Yogo (2006); Ang and Bekaert (2007); Campbell and Thompson (2008); Cochrane (2008); Lettau and Van Nieuwerburgh (2008); Pástor and Stambaugh (2009).

with the theoretical and empirical evidence.[6] Bayesian learning about uncertain parameters in the linear regression has been proposed as one of the ways to introduce time-varying relationship between the returns and predictor variables. However, sequential parameter learning leads to significant portfolio benefits only in the presence of a highly informative prior (Wachter and Warusawitharana, 2009) or an ensemble of estimation risk and time-varying volatility (Johannes et al., 2014). Thus, prior knowledge about the nature of return predictability or careful modeling of its conditional features, especially time variation in return volatility, are critical for generating the economic gains.

This paper follows an alternative approach inspired by the recent development of machine learning in the empirical asset pricing.[7] Specifically, we apply neural networks to approximate the functional association between the set of predictors and returns for the optimal portfolio construction. In doing so, we do not impose a priori known form of this relationship and instead allow for a flexible identification of potentially nonlinear interactions from the data. Our choice of neural networks as opposed to other machine learning methods (for instance, tree-based approaches) is motivated by the fact that they deliver the most accurate statistical performance as documented by the existing literature. The aim of this paper is to revisit the evidence documented by Goyal and Welch (2008) and to show that, unlike linear predictive regressions, a sound statistical performance of neural networks indeed translates into substantial portfolio improvements for an investor using these novel methods when dynamically forming an optimal portfolio.

2.2 From Linear Regression Towards Deep Learning

Machine learning have a long history in economics and finance (Hutchinson et al., 1994; Kuan and White, 1994; Racine, 2001; Baillie and Kapetanios, 2007). At its core, machine

[6]Leading examples of this literature include Menzly et al. (2004); Paye and Timmermann (2006); Santos and Veronesi (2006); Lettau and Van Nieuwerburgh (2008); Henkel et al. (2011); Dangl and Halling (2012).

[7]Leading studies include Giglio and Xiu (2017); Heaton et al. (2017); Feng et al. (2018, 2019); Chen et al. (2019); Kelly et al. (2019); Freyberger et al. (2020); Gu et al. (2020); Kozak et al. (2020).

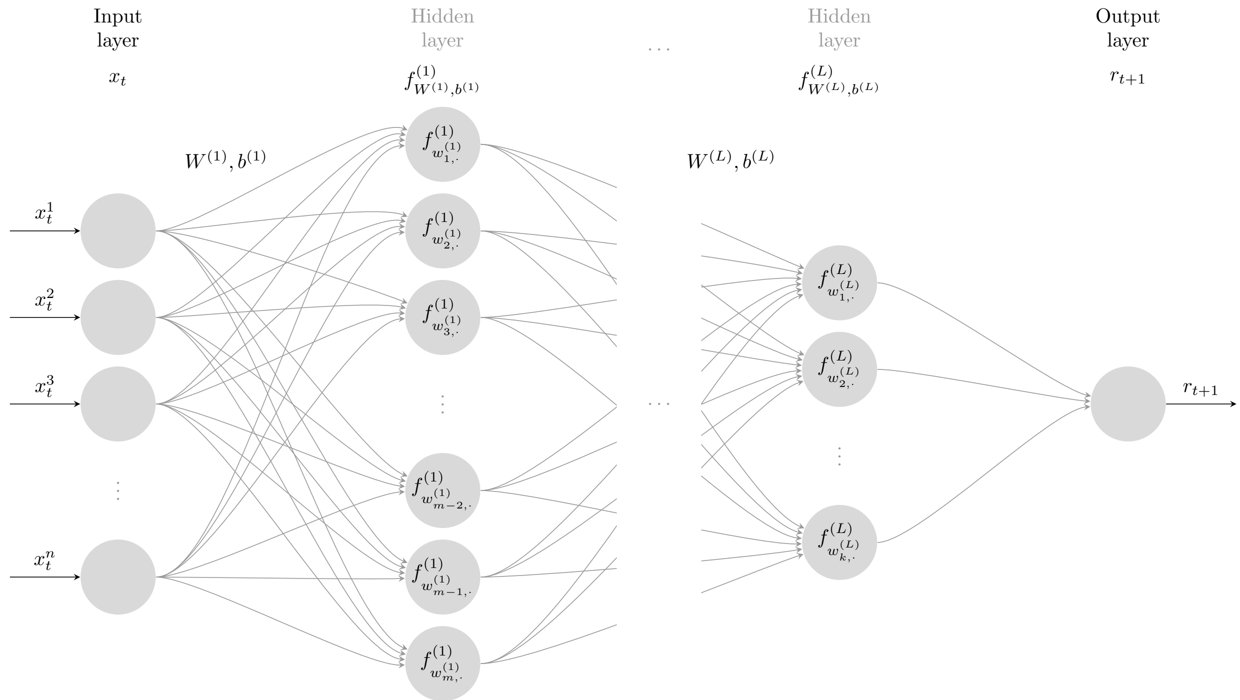
learning can be viewed as a general statistical analysis that can be used by economists to capture complex relationships hidden to observer when using simple linear methods. As emphasized by Breiman et al. (2001), maximizing prediction accuracy in the face of an unknown data model is a key differentiating the machine learning methods from traditional statistical objective of estimating a known data generating the model. More specifically, machine learning seeks to choose a preferred model from the unknown pool of models using sophisticated and innovative optimization techniques. Instead of traditional fit metrics, machine learning is focused on out-of-sample forecasting and understanding the bias-variance tradeoff; that is the tradeoff between a more complex model versus over-fitted one. In addition, deep learning is data-driven and focuses on finding structure in large datasets.

While finance is focused on return prediction, machine learning techniques being able to find the relationship in data no matter how complex and nonlinear it is seem perfectly suited for financial application. Furthermore, dismissing the “black-box” view of machine learning as a misconception (Lopez de Prado, 2019) it seems nothing should stop a researcher in exploring the power of these methods in financial data. Yet, problems in finance differ from a typical machine learning applications in many aspects. In order to enjoy benefits of the machine learning, user needs to understand key challenges brought by financial data.

As noted by Israel et al. (2020), machine learning applied to finance is challenged by small sample sizes, natural low signal-to-noise ratios making market behavior difficult to predict and dynamic character of markets. Because of these critical issues, benefits of machine learning are not so obvious as in other fields and research understanding how impactful machine learning will be for asset management is only emerging. Especially with the surge in deep learning literature machine learning applications in finance started to emerge too (Heaton et al., 2017; Feng et al., 2018; Bryzgalova et al., 2019; Bianchi et al., 2020; Chen et al., 2020; Gu et al., 2020; Tobek and Hronec, 2020; Zhang et al., 2020). Here we describe the core ideas we use for building a deep learning models to predict the returns.

Figure 1. (Deep) Feedforward Network

The figure illustrates a deep neural network model $r_{t+1} = \mathfrak{f}_{W,b}(x_t) + \varepsilon^r_{t+1}$ that predicts output return r_{t+1} with set of predictor variables $x_t = (x^1_t, ..., x^n_t)$. The network is deep with large number of hidden layers L.



2.2.1 (Deep) Feedforward Networks. Deep feedforward networks, also often called feedforward neural networks, or multilayer perceptrons lie at hearth of deep learning models and are universal approximators that can learn any functional relationship between input and output variable with sufficient data.

A feedforward network is a form of supervised machine learning that use hierarchical layers to represent high-dimensional non-linear predictors with the goal to predict output variable. Figure 1 illustrates how $\ell \in \{1, \ldots, L\}$ hidden layers transform input data $x_t = (x^1_t, ..., x^n_t)$ in a chain using collection of non-linear activation functions $f^{(1)}, \ldots, f^{(L)}$. More formally, we can define our prediction problem by characterizing excess equity returns as:

$$r_{t+1} = \mathfrak{f}_{W,b}(x_t) + \varepsilon^r_{t+1}, \tag{2}$$

where $x_t = (x_t^1, ..., x_t^n)$ is a set of predictor variables that enter input layer, and ε_{t+1}^r is a i.i.d. error term, $\mathfrak{f}_{W,b}$ is a neural network with L hidden layers such as

$$\widehat{r}_{t+1} := \mathfrak{f}_{W,b}(x_t) = f^{(L)}_{W^{(L)},b^{(L)}} \circ \ldots \circ f^{(1)}_{W^{(1)},b^{(1)}}(x_t), \tag{3}$$

and $W = \left(W^{(1)}, \ldots, W^{(L)}\right)$ and $b = \left(b^{(1)}, \ldots, b^{(L)}\right)$ are weight matrices and bias vector. Any weight matrix $W^{(\ell)} \in \mathbb{R}^{m \times n}$ contain m neurons as n column vectors $W^{(\ell)} = [w^{(\ell)}_{\cdot,1}, \ldots, w^{(\ell)}_{\cdot,n}]$, and $b^{(\ell)}$ are threshold or activation level which contribute to the output of a hidden layer allowing the function to be shifted. A commonly used activation functions $f^{(\ell)}_{W^{(\ell)},b^{(\ell)}}$

$$f^{(\ell)}_{W^{(\ell)},b^{(\ell)}} := f_\ell\left(W^{(\ell)}x_t + b^{(\ell)}\right) = f_\ell\left(\sum_{i=1}^{m} W_i^{(\ell)}x_t + b_i^{(\ell)}\right) \tag{4}$$

are sigmoidal (e.g. $f_\ell(z) = 1/(1+\exp(-z))$) or $f_\ell(z) = \tanh(z)$, or rectified linear units (ReLU) ($f_\ell(z) = \max\{z, 0\}$). Note that in case functions $\mathfrak{f}$ are linear, $\mathfrak{f}_{W,b}(x_t)$ is simple linear regression, regardless of number of layers L and hidden layers are redundant. For example with $L = 2$, model becomes reparametrized simple linear regression as $\widehat{r}_{t+1} = W^{(2)}(W^{(1)}x_t + b^{(1)}) + b^{(2)} = \beta x_t + \alpha$. In case $\mathfrak{f}_{W,b}(x_t)$ is non-linear, neural network complexity grows with increasing m, and with increasing number of hidden layers L, or deepness of the network, we have a deep neural network.

2.2.2 (Deep) Recurrent Networks. Many predictors used in finance are usually non-i.i.d., dynamically evolve in time, and hence traditional neural networks assuming independence of data may not approximate the relationships sufficiently well. Instead, a Recurrent Neural Network (RNN) that takes into account time series behavior may help in the prediction task. In addition, Long-Short-Term-Memory (LSTM) is designed to find hidden state processes allowing for lags of unknown and potentially long time dynamics in the time series. Figure 2 illustrates how the network structure additionally uses lagged information.

More formally, RNNs are a family of neural networks for processing sequences of data. They transform a sequence of input predictors to another output sequence introducing lagged hidden states as

$$h_t = f(W_h h_{t-1} + W_x x_t + b_0). \tag{5}$$

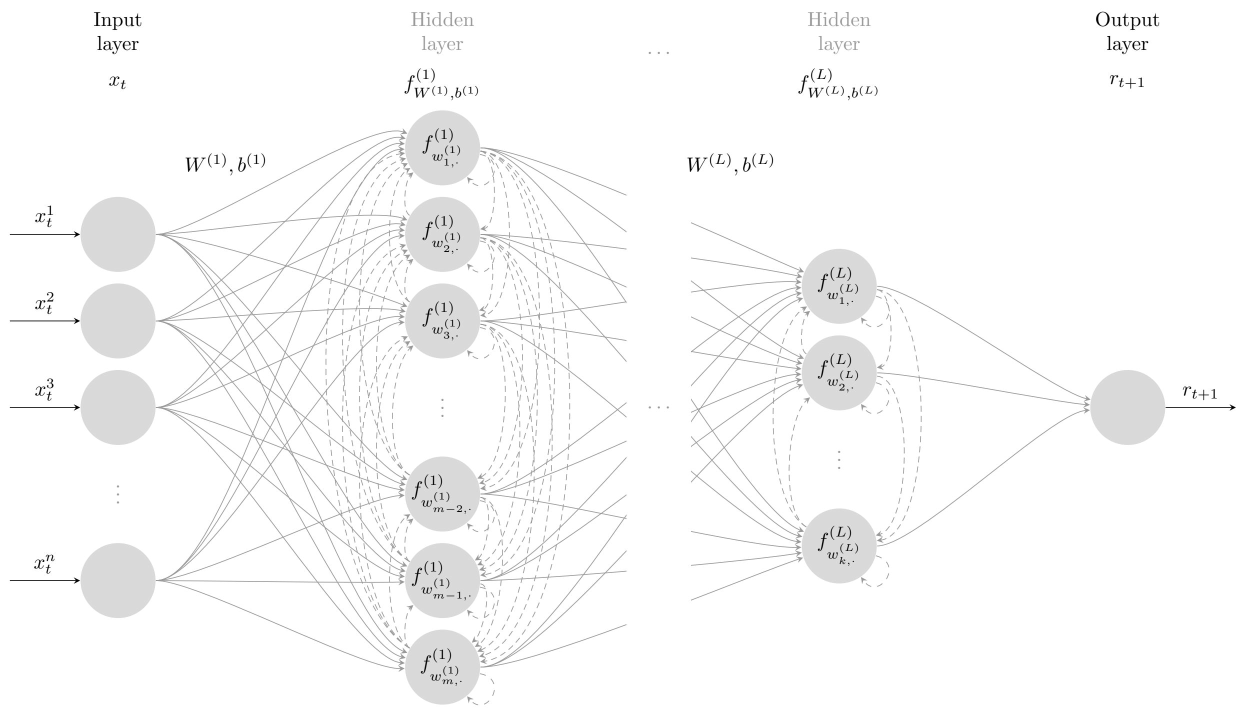
Intuitively, RNN is a non-linear generalization of an autoregressive process where lagged variables are transformations of the lagged observed variables. Figure 2 depicts W_h by dashed lines and W_x by solid lines. Nevertheless, this structure is useful in case only immediate past is relevant. In case the the time series dynamics are driven by events that are further back in the past, addition of complex LSTM is further required.

2.2.3 Long-Short-Term-Memory (LSTM). An LSTM are a particular form of recurrent network which provide a solution to short memory problem by incorporating memory units Hochreiter and Schmidhuber (1997). Memory units allow the network to learn when to forget previous hidden states and when to update hidden states given new information. Specifically, in addition to a hidden state, LSTM includes an input gate, a forget gate, an input modulation gate, and a memory cell. The memory cell unit combines the previous memory cell unit which is modulated by the forget and input modulation gate together with the previous hidden state, modulated by the input gate. These additional cells enable an LSTM to learn extremely complex long-term temporal dynamics that a vanilla RNN is not capable of. Such structures can be viewed as a flexible hidden state space model for a large dimensional system. Additional depth can be added to LSTM by stacking them on top of each other, using the hidden state of the LSTM as the input to the next layer.

More formally, at each step a new memory cell c_t is created with current input x_t and previous hidden state h_{t-1} and it is then combined with forget gate controlling amount of

Figure 2. (Deep) Recurrent Network

The figure illustrates a deep recurrent neural network model.



information stored in the hidden state as

$$h_t = \sigma\left(\underbrace{W_h^{(o)} h_{t-1} + W_x^{(o)} x_t + b_0^{(o)}}_{\text{output gate}}\right) \circ \tanh(c_t) \tag{6}$$

$$c_t = \sigma\left(\underbrace{W_h^{(g)} h_{t-1} + W_x^{(g)} x_t + b_0^{(g)}}_{\text{forget gate}}\right) \circ c_{t-1} + \sigma\left(\underbrace{W_h^{(i)} h_{t-1} + W_x^{(i)} + b_0^{(i)}}_{\text{input gate}}\right) \circ \tanh(k_t). \tag{7}$$

The term $\sigma(\cdot) \circ c_{t-1}$ introduces the long-range dependence, k_t is new information flow to the current cell. The forget gate and input gate states control weights of past memory and new information. In the Figure 2, c_t is the memory pass through multiple hidden states in the recurrent network.

2.2.4 Estimation, Hyperparameters, Details.

Due to the high dimensionality and non-linearity of the problem, estimation of a deep neural network is a complex task. Here, we provide a detailed summary of the model architectures and their estimation considered. We work with variety of deep learning structures and compare them with recurrent LSTM network and regularized OLS. Namely, we consider an NN1, NN2 and NN3 models that contain 16, 32–16 and 32–16–8 neurons in the 1,2, and 3 hidden layer structures respectively, and LSTM model which is a NN with 3 recurrent layers with 32-16-8 neurons in each and LSTM cells introduced to the last layer.

To prevent the model from over-fitting and reduce large number of parameters, we use dropout, which is a common form of regularization that has generally better performance in comparison to traditional l_1 or l_2 regularization. The term dropout refers to dropping out units in neural networks and can be shown to be a form of ridge regularization. To fit the networks, we adopt popular and robust adaptive moment estimation algorithm (Adam) with weight decay regularization introduced by Kingma and Ba (2014) and we used the Huber loss function in the estimation.

Further, we follow the most common approach in the literature and select tuning parameters adaptively from the data in a validation sample. We split the data into training and validation sample that maintain temporal ordering of the data and tune hyperparameters with respect to the statistical as well as economic criteria. We search the optimal models in the following grid of 100 randomly chosen combinations of the following hyperparameters: learning rate $\in [0.001, 0.02]$, decay regularization $\in [0, 0.001]$, dropout $\in [0\%, 60\%]$ of weights and activation function $\in \{\text{sigmoid}, \text{ReLU}\}$ with 1000 epochs with early stopping. Since the sample at each window is rather small, and final models can depend on initial values in the optimization, we use ensemble averaging of five models with randomly chosen initial values.[8]

[8] We have estimated our models on the two servers with 48 core Intel® Xeon® Gold 6126 CPU@ 2.60GHz and 24 core Intel® Xeon® CPU E5-2643 v4 @ 3.40GHz, 768GB memory and two NVIDIA GeForce RTX 2080 Ti GPUs. We have used `Flux.jl` with JULIA 1.4.0. for the model fitting. A complete rolling window estimation with hyperparameter tuning takes around two days. We have confirmed that our estimation results are robust to using a larger hyperparameter space. As a full hyperparameter search

2.3 Optimal Portfolios

We consider a portfolio choice problem of an agent with the investment horizon of T periods in the future who maximizes her expected utility over the cumulative portfolio return. There are two assets: a one-period Treasury bill and a stock index.[9] If $\omega_{t+\tau}$ is the allocation to the stock index at time $t+\tau$, the investor solves the following optimization problem at time t

$$\max_{\omega} \mathbb{E}_t \left[U(r_{p,t+T}) \right] \tag{8}$$

in which the end-of-horizon portfolio return $r_{p,t+T}$ is defined as

$$r_{p,t+T} = \prod_{\tau=1}^{T} \left[(1-\omega_{t+\tau-1}) \exp(r^f_{t+\tau}) + \omega_{t+\tau-1} \exp(r^f_{t+\tau} + r_{t+\tau}) \right], \tag{9}$$

and $r^f_{t+\tau}$ denotes a zero-coupon default-free log bond yield between $t+\tau-1$ and $t+\tau$. Following Johannes et al. (2014), we consider various choices of horizons T to assess the impact of the length of the investment period. Specifically, we report the results for the two cases of six months ($T=6$) and two years ($T=24$). Furthermore, we allow the investor to rebalance portfolio weights with different frequencies. The allocations between a Treasury bill and a stock index are updated every three months or once per year for the shorter or longer investment horizons, respectively. These choices of horizons and rebalancing periods allow us to compare two investment strategies. The former reflects more actively managed portfolio with frequent changes in the allocations, whereas the latter corresponds to relatively passive investment portfolio with less frequent rebalancing. We further winsorize the weights for the stock index to $-1 \leq \omega_{t+\tau} \leq 2$ to prevent extreme investments. In the sensitivity analysis, we check the robustness of our results to alternative assumptions about the portfolio

on a larger hyperparameter space can easily take weeks or months even on our fast GPU cluster, we have selectively tested further hyperparameters.

[9]Extending our analysis to multiple assets is straightforward; however, we consider a portfolio choice problem with two assets as in Barberis (2000) and more recently Johannes et al. (2014) and Rossi (2018) to make our results directly comparable to other studies.

weights, particularly incorporating the borrowing and short-selling constraints.

We also assume a power utility investor

$$U(r_{p,t+\tau}) = \frac{r_{p,t+\tau}^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of risk aversion. The expected utility is defined by the predictive distribution of cumulative portfolio returns $r_{p,t+\tau}$ given by Eq.(9), which in turn depends on the corresponding model used to predict future excess returns $r_{t+\tau}$ and the law of motion of predictor variables $\boldsymbol{x}_t$. For $\boldsymbol{x}_t$, we adopt a parsimonious AR(1) framework, that is, each variable $\boldsymbol{x}_t^i$ satisfies

$$\boldsymbol{x}_t^i = \boldsymbol{\alpha}^{x^i} + \boldsymbol{\beta}^{x^i}\boldsymbol{x}_{t-1}^i + \varepsilon_t^{x^i}.$$

where $\boldsymbol{\alpha}^{x^i}$ and $\boldsymbol{\beta}^{x^i}$ are coefficients, $\varepsilon_t^{x^i}$ are normal error terms. To proxy for the joint variance-covariance matrix of the error terms $\varepsilon_t = (\varepsilon_t^r, \varepsilon_t^x)$, we employ a sample variance estimator $\hat{\Sigma}_t = \hat{\varepsilon}_t\hat{\varepsilon}_t'$, where ε_t are forecast errors. Finally, we set the risk aversion parameter $\gamma = 4$ to compare our results to the existing literature (Johannes et al., 2014; Rossi, 2018).

In sum, the investor maximizes her expected utility and optimally rebalances portfolio weights quarterly or annually for investment horizons of six months and two years, respectively. To compute her expected utility, she uses the distribution of returns predicted by the linear regressions or neural networks. To evaluate the impact of the investor's conditioning information, we consider different assumptions about the set of predictors and sample periods used to estimate the models. In particular, we consider the following specifications:

1. The no-predictability expectations hypothesis (EH) framework assumes a constant mean and constant variance framework with no predictors in Eq.(1), that is, $\boldsymbol{\beta} = 0$.
2. A simple linear regression of excess log returns with the dividend yield as a single predictor and a "kitchen sink" linear regression with all available variables. For each of the two cases, we further implement OLS regressions using all data up to time t or over

a 10-year rolling window as in Johannes et al. (2014). The univariate models with the expanding and rolling windows are denoted OLS1 and OLS2, whereas the multivariate versions are OLS3 and OLS4, respectively.

3. A set of machine learning architectures including neural networks with 1 layer 16 neurons (NN1), 2 layers and 32-16 neurons (NN2), and 3 layers 32-16-8 (NN3) as well as LSTM model with 3 recurrent layers and 32-16-8 neurons and LSTM cells introduced to the last layer. All NNs use a "kitchen sink" approach by utilizing all available data to predict log excess returns and are trained on the 10-year rolling window to account for time-varying relationship between the predictors and returns.

There are many dimensions to generalize our modelling approach. More general specifications could add additional predictor variables (McCracken and Ng, 2016), parameter uncertainty (Wachter and Warusawitharana, 2009; Johannes et al., 2014; Bianchi and Tamoni, 2020), economic restrictions (Van Binsbergen and Koijen, 2010), or consider a larger set of investable assests and alternative preferences (Dangl and Weissensteiner, 2020) among other extensions. Most notably, modelling stochastic volatility via a parsimonious mean-reverting process (Johannes et al., 2014) or more complex GARCH- and MIDAS-type volatility estimators (Rossi, 2018) would certainly improve the performance of our strategies. Instead, we consider all specifications with a constant volatility setting to evaluate the sole impact of neural networks on the performance of dynamic allocation strategies. The aim is to demonstrate out-of-sample portfolio gains from using deep learning in the most restrictive setting.

2.4 Performance Evaluation

In our analysis, we employ a number of metrics measuring the statistical accuracy of the methods considered and their economic gains for the investor. With respect to the statistical performance, we first consider a common measure of mean squared prediction error (MSPE) defined as

$$MSPE = \frac{1}{T_0 - t_0 + 1} \sum_{t=t_0}^{T_0} \left(r_t - \hat{r}_t^{\mathcal{M}_s} \right)^2, \tag{10}$$

where r_t denotes the observed excess log return, $\hat{r}_t^{\mathcal{M}_s}$ is the return predicted by a particular framework $\mathcal{M}_s$, whereas t_0 and T_0 are the months of the first and last predictions, respectively. Notice that the investor rebalances her allocations with different frequencies. Thus, we compute the prediction errors only in those periods when she reoptimizes her portfolio.

As in Campbell and Thompson (2008), we also compute the out-of-sample predictive R^2_{oos}

$$R^2_{oos} = 1 - \frac{\sum_{t=t_0}^{T_0} \left(r_t - \hat{r}_t^{\mathcal{M}_s} \right)^2}{\sum_{t=t_0}^{T_0} \left(r_t - \bar{r}_t \right)^2},$$

where $\bar{r}_t$ is the historical mean of returns. By construction, the R^2_{oos} statistics compares the out-of-sample performance of the chosen model $\mathcal{M}_s$ relative to the historical average forecast. Notice that we compute the historical mean over the same sample used to estimate $\mathcal{M}_s$ that corresponds to either an expanding sample or a 10-year rolling window. The positive value of R^2_{oos} indicate that the model-implied forecast has smaller mean squared predictive error compared to the one generated by the historical average forecast. Thus, we perform a formal test of the null hypothesis $R^2_{oos} \leq 0$ against the alternative hypothesis $R^2_{oos} > 0$ by implementing the MSPE-adjusted Clark and West (2007) test. Note that we calculate the Clark and West (2007) only if R^2_{oos} is positive.

Once we compare different models in terms of the statistical accuracy of their predictions, we assess whether superior statistical fit translates into economic gains. It is worth noting that this relationship is non-trivial. Indeed, Campbell and Thompson (2008) and Rapach et al. (2010) note that seemingly small improvements in R^2_{oos} could generate large benefits in practice. We start our investigation of the size of the improvement by calculating the average Sharpe ratio of portfolio returns as a common measure of portfolio performance used in finance. The drawback of this metric is that it does not take into account tail behaviour.

Consequently, we follow Fleming et al. (2001) and compute the certainty equivalent return (CER) by equating the utility from CER to the average utility implied by an alternative model. Finally, we visualize the performance of all specifications by plotting the cumulative log portfolio returns over the sample period considered. This allows us to clearly see the time intervals when the investor benefits the most from using different frameworks.

To evaluate the statistical significance of portfolio gains, we follow Bianchi et al. (2020) and implement the test á la Diebold and Mariano (2002) . Specifically, we perform a pairwise comparison between the CERs generated by each framework under consideration and those yielded by the EH specification.[10] For each model $\mathcal{M}_s$, we estimate the regression

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$.

3 Empirical Results

3.1 Data and preliminary results

The empirical analysis of the S&P 500 excess return predictability is based on the applications of a variety of linear models and non-linear machine learning methods as discussed in Section 2.3. We use a set of economic predictor variables considered by Goyal and Welch (2008) to make our results directly comparable to the literature. Specifically, we focus on the monthly historical data of twelve predictors such as dividend yield, log earning price ratio, dividend

[10]For the significance of SRs, we first need to simulate artificial returns under a null model of no predictability, that is, a model with the constant mean and constant volatility. For each simulation, we need to obtain the forecasts for all models considered and construct optimal portfolios. Since a complete exercise of hyperparameter tuning takes around 2 days on the supercomputer cluster, repeating it, say, 500 times will increase cluster computing time proportionally. This makes the task computationally infeasible given the current computing capacity, unless more resources for parallel computing are available.

Table 1. Statistical Accuracy of Excess Return Forecasts

The table reports the mean squared prediction error and out-of-sample R^2_{oos} obtained from using different methodologies used to predict future S&P 500 excess returns as outlined in Section 2.3. We compute the out-of-sample R^2_{oos} in comparison to the expectations hypothesis using the historical mean to predict returns. Panel A shows the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B demonstrates the results for a 2-year horizon and annual rebalancing. We compute statistical accuracy measures in those periods when the investor reevaluates her allocations, meaning with quarterly or annual frequency. We also report a p-value (in parentheses) of the null hypothesis $R^2_{oos} \leq 0$ following Clark and West (2007). We report a statistical significance only if R^2_{oos} is positive. The forecast starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Panel A: 6-month horizon and quarterly rebalancing									
MSPE$\times 10^4$	17.4	18.0	18.2	18.9	18.0	16.3	16.6	16.6	17.3
R^2_{oos}	0.5%	-2.5%	-3.6%	-8.0%	-2.5%	7.1%	5.1%	5.6%	1.6%
p-value	(0.152)					(0.006)	(0.007)	(0.002)	(0.002)
Panel B: 2-year horizon and annual rebalancing									
MSPE$\times 10^4$	14.2	15.0	14.7	16.7	19.9	12.2	11.1	11.9	12.1
R^2_{oos}	2.2%	-2.8%	-0.8%	-15.0%	-36.9%	16.0%	23.8%	17.6%	17.2%
p-value	(0.171)					(0.001)	(0.014)	(0.007)	(0.008)

payout ratio, book to market ratio, net equity expansion, treasury bills rate, term spread, default yield spread, default return spread, cross-sectional premium, inflation growth, and monthly stock variance.[11]

Table 1 reports statistical accuracy of the considered models. Panel A and B show the MSPEs and R^2_{oos} based on those periods when quarterly and annual rebalancing is happening, respectively. As show in Panel A, all linear regressions generate larger MSPEs compared to the constant mean and constant volatility case, while neural networks provide the best fit with the data.

A multivariate linear model does not necessarily outperform a univariate case. Indeed, a linear regression estimated on the rolling window (OLS3) is more noisy and generates a larger MSPE compared to the regressions using dividend yield only (OLS1), whereas the "kitchen sink" linear regression with an expanding window estimation (OLS4) slightly outperforms a single predictor case (OLS2). Furthermore, consistent with Goyal and Welch (2008), none

[11]The data are retrieved from Amit Goyal's website and are available via the following link http://www.hec.unil.ch/agoyal/docs/PredictorData2019.xlsx as of 26th August 2020.

of the linear regressions can beat simple historical mean as indicated by the negative R^2_{oos}. In contrast, we find that deep learning methods achieve the positive R^2_{oos}, indicating the statistical benefits of accounting for the nonlinear relationship between stock market returns and predictors similarly to Feng et al. (2018) and Rossi (2018). A formal test confirms that equity return predictability generated by NNs is statistically different from a naive historical mean forecast. In the unreported results, we verify that, among machine learning methods, the performance is statistically the same. Panel B also shows the results in favor of NNs in a setting with less frequent rebalancing.

3.2 Portfolio Results

Table 2 provides the summary of annualized CERs and monthly SRs of portfolio returns for each model assuming a 6-month (Panel A) and 2-year (Panel B) investment horizon. The summary statistics in each panel are computed for the whole sample as well as recession and expansion periods as defined by the NBER recession indicator. The risk aversion parameter is $\gamma = 4$.

For traditional methods, we recover a standard result: linear regressions do not generate out-of-sample improvements as measured by the CERs compared to the constant mean and constant volatility model. In terms of model-generated SRs, linear models perform slightly better than the expectations hypothesis model, with higher Sharpe ratios in case of more predictor variables. The rolling-window estimation introduces time-varying slope coefficients and leads to a modest improvement. However, ignoring the estimation risk and stochastic volatility of returns results in lower CERs relative to a constant mean and volatility specification, which is consistent with Johannes et al. (2014).

Turning to NNs, we observe that the improved R^2_{oos} obtained with machine learning methods directly translate into economic gains for an investor. Specifically, the best performing NN – the LSTM model – generates more than two- and three-fold increases in the annual

Table 2. Certainty Equivalent Returns and Sharpe Ratios

The table reports the annualized certainty equivalent returns and monthly Sharpe ratios for different models outlined in Section 2.3. Panel A shows the results when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B shows the results for a 2-year horizon and annual rebalancing. Each panel computes the statistics for the whole sample, expansion and recession periods as defined by the NBER. For the statistical significance of CERs, we report a one-sided p-value (in parentheses) of the test á la Diebold and Mariano (2002). In particular, we regress the difference in utilities for each model $\mathcal{M}_s$ and EH

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$. We flag in **bold font** those CER values that are significant at the 10% confidence level. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Panel A: 6-month horizon and quarterly rebalancing									
1955-2018									
CER	4.737	2.643	-0.030	2.781	2.491	**7.295**	**6.984**	5.491	**10.007**
p-value		(1.000)	(1.000)	(0.935)	(0.954)	(0.027)	(0.032)	(0.292)	(0.000)
SR	0.049	0.046	0.062	0.088	0.095	0.166	0.157	0.144	0.175
Expansions									
CER	4.948	3.073	-0.173	4.598	2.045	5.873	5.280	5.304	**7.998**
p-value		(0.998)	(1.000)	(0.654)	(0.982)	(0.258)	(0.398)	(0.403)	(0.000)
SR	0.100	0.077	0.048	0.092	0.108	0.149	0.143	0.135	0.149
Recessions									
CER	3.311	-0.274	1.401	-9.079	6.752	**19.024**	**20.806**	6.936	**26.770**
p-value		(0.995)	(0.648)	(0.944)	(0.221)	(0.000)	(0.000)	(0.200)	(0.000)
SR	-0.193	-0.182	0.154	0.091	0.036	0.284	0.255	0.204	0.358
Panel B: 2-year horizon and annual rebalancing									
1955-2018									
CER	4.542	1.068	0.040	0.923	-0.067	**6.342**	**6.879**	**6.437**	**5.622**
p-value		(1.000)	(1.000)	(0.999)	(0.997)	(0.000)	(0.000)	(0.000)	(0.012)
SR	0.048	0.044	0.046	0.083	0.081	0.138	0.136	0.129	0.118
Expansions									
CER	4.448	0.826	0.514	0.321	0.231	**6.051**	**6.390**	**5.913**	**5.537**
p-value		(1.000)	(1.000)	(0.999)	(0.987)	(0.002)	(0.000)	(0.000)	(0.011)
SR	0.100	0.076	0.037	0.097	0.137	0.136	0.149	0.132	0.112
Recessions									
CER	5.235	2.866	-2.924	5.930	-2.138	**8.611**	**10.975**	**10.836**	6.353
p-value		(0.997)	(1.000)	(0.262)	(1.000)	(0.006)	(0.000)	(0.000)	(0.284)
SR	-0.190	-0.170	0.102	0.017	-0.104	0.160	0.111	0.149	0.154

CER (around 10% vs 4.7%) and monthly SR (0.175 vs 0.049) relative to the model ignoring return predictability. The LSTM model, which is a three-layer network, is directly comparable to NN3 in terms of the structure complexity. Nevertheless, LSTM dominates a standard network, emphasizing the importance of learning complex long-term temporal dynamics in addition to non-linear predictive relationships. In general, comparing NN1 through NN3, we observe that increasing the complexity of NNs does not need to improve the portfolio performance, while all machine learning structures remain statistically equivalent between each other. A formal one-sided test confirms that, except for NN3, the portfolio performance of NNs is significantly better than that generated by the EH model. Further, the comparison between the results in Panels A and B demonstrates that the investor benefits more from using NNs when she manages her portfolio more actively. Overall, these results indicate that return predictability generated by applying nonlinear methods provides a valuable information for the portfolio construction.

We dissect this superior performance by looking at portfolio return statistics in expansion and recession periods. Table 2 shows that economic gains generated by NNs are large during both regimes and are especially pronounced in recessions. For instance, the annualized CER generated by the LSTM is on average around 8% in good times, which is more than 5% predicted by the EH model. In bad times, the difference in the performance is extremely large, with around 26% and 3% CERs in the two cases, respectively. A pairwise test confirms that the improvement of LSTM over EH is statistically significant during both expansions and recessions. In contrast, the portfolio returns of NN1 through NN3 are indistinguishable from EH in expansions, while shallower networks exhibit significantly better performance in recessions.

The investor ignoring return predictability experiences on average around -19% Sharpe ratios in recessions. In contrast, the LSTM model helps generate significant portfolio gains around 36% SRs, with other NNs generating at least 20% SRs on the monthly basis. Further, all NNs outperform linear regressions across good and bad times. The existing evidence for

Table 3. Portfolio Return Statistics

The table reports mean, standard deviation, skewness, and kurtosis of optimal portfolio returns for different models used outlined in Section 2.3. All statistics are expressed in monthly terms. Panel A shows the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B shows the results for a 2-year horizon and annual rebalancing. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Panel A: 6-month horizon and quarterly rebalancing									
Mean	0.937	2.213	4.138	4.676	6.762	10.728	9.605	9.533	11.715
St.dev.	5.504	13.871	19.122	15.353	20.502	18.601	17.641	19.121	19.343
Skew	-0.472	-0.615	-0.893	-0.332	-0.881	-0.844	-0.816	-0.786	-0.046
Kurt	4.353	8.609	10.400	7.631	9.182	11.707	12.237	11.172	4.860
Panel B: 2-year horizon and annual rebalancing									
Mean	0.978	2.184	3.058	5.093	4.908	7.333	5.634	4.445	7.722
St.dev.	5.849	14.443	19.189	17.655	17.512	15.331	11.937	9.916	18.87
Skew	-0.452	-0.492	-1.058	-0.989	-0.787	-0.275	0.469	0.799	-0.013
Kurt	4.386	7.104	10.566	13.550	10.737	9.113	10.416	14.775	6.433

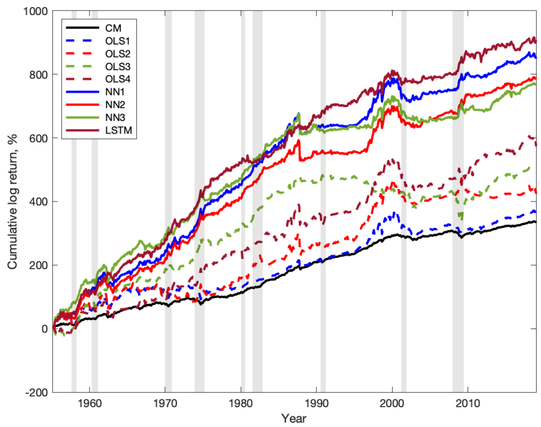
equities (Rapach et al., 2010; Dangl and Halling, 2012) indicates that return predictability is concentrated in bad times.[12] Our findings extend the existing literature by showing that, unlike linear models, NNs help the investor to effectively convert predictive variation in stock market returns into substantial economic gains across different business cycle conditions.

Table 3 presents additional statistics of portfolio returns for different methodologies. The models using NNs generate out-of-sample returns with significantly larger mean. Intuitively, this occurs because machine learning methods specifically excel in risk premium prediction, that is, the conditional expectation of returns. The linear regressions and vanilla NNs do not take into account the time-varying volatility of returns and hence these models predict negative skewness and excess kurtosis (since they ignore a fat-tailed return distribution). Interestingly, although an LSTM network does not consider time variation in return volatility, it is able to identify the periods of high return variance using the long-term memory of its cells (including realized return variance as one of the predictors also helps). This results in better skewness and lower excess kurtosis. The statistics for the longer horizon portfolio

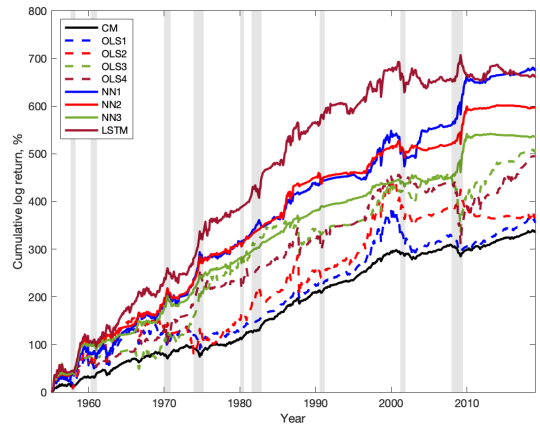
[12] Gargano et al. (2019) report a similar result for bond returns. Recently, Bianchi et al. (2020) show that bond return predictability is also present in expansions when machine learning methods are employed.

Figure 3. Cumulative Returns

The figure illustrates the cumulative log returns of optimal portfolio strategies from different models outlined in Section 2.3. The left panel shows the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. The right panel shows the results for a 2-year horizon and annual rebalancing. The shaded areas denote the recession periods as defined by the NBER. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.



(a) 6-month horizon and quarterly rebalancing



(b) 2-year horizon and annual rebalancing

are improved for the standard neural networks, where properties remain largely the same or slightly deteriorate for other models.

We visually summarize the previous results in Figure 3, which shows the cumulative sum of log portfolio returns. The left panel shows that NNs outperform other models by a large margin. Among NNs considered, the LSTM dominates remaining networks by the end of the considered period, with a particularly pronounced difference in the second half of the sample considered. In relation to specific historical events, all NNs produce a steady positive portfolio performance during the 2007-2008 Financial Crisis. Interestingly, the LSTM network additionally avoids largely unexpected stock market crash, Black Monday, on October 19, 1987. Figure 3 also shows that weaker statistical performances for the case of a passive strategy with annual rebalancing leads to lower cumulative returns across all models.

4 Further Analysis

This section dissects the performance of portfolio returns across seven decades in the post-war period considered. Also, it provides robustness of our conclusions to alternative measures of portfolio performance, transaction costs, borrowing and short-selling constraints.

4.1 Subsample Analysis

We start by examining whether superior portfolio performance implied by NNs varies over subsamples other than expansions and recessions. Table 4 shows the certainty equivalent yields and Sharpe ratios computed separately for each decade in our sample. For the CERs, we extend the main finding of the paper: NNs, particularly LSTM, outperform the expectations hypothesis model in most cases. Specifically, the table shows that, except for the last decase, the LSTM network generates the certainty equivalent values above those implied by no predictability framework. Interestingly, the formal test indicates that the improvement of LSTM over EH is significant during the first three decades, while higher CERs in later periods are statistically equivalent to those from the EH model.

The linear models perform well during the 1990s and 2010s, when the stock market has grown steadily over the two-decade period considered. Also, the rolling-window linear regressions tend to perform better compared to those using the expanding-window estimation, emphasizing the role of time-varying betas and changing information set. For instance, Goyal and Welch (2008) show that dividend-yield exhibited a strong predictive power for stock market returns from 1970 to mid-1990, with a weaker but mostly positive out-of-sample performance during the first two decades after World War II. In contrast, it had large prediction errors during the 1995-2000 and 2000s periods. As a result, Table 4 shows that the OLS3 model generates high CERs from 1955 to 1989, exhibiting a statistically better performance than EH in some case, but it falls in later years when the forecast based on dividend yield had strong underperformance.

Turning to the SRs, NNs provide the investor with substantially higher Sharpe ratios with an exception of the 1990s and 2010s when they perform slightly worse. These results are consistent with our previous findings. Indeed, the U.S. stock market has been strongly bullish during the last decades of the previous and current centuries. The two episodes are marked by prolonged stock market expansions. In contrast, the Black Monday crash happened in 1987 and the S&P 500 index recovered slowly only by the end of the 1980s. Further, the beginning of the new millennium experienced two major crashes driven by the burst of the dot-com bubble and the the subprime mortgage crisis. Table 4 shows that NNs perform significantly better compared to other specifications during the decades with major stock market bears and provide statistically equal results during stock market bulls, which is consistent with our previous results across expansions and recessions.

4.2 Alternative Measures of Performance

Although certainty equivalent yields and Sharpe ratios represent common measures of the portfolio performance considered by the literature, the investor may use alternative statistics to evaluate their investment strategies including maximum drawdown, maximum one-month loss, and average monthly turnover. For each model $\mathcal{M}_s$, we define maximum drawdown

$$\text{Max DD} = \max_{t_0 \leq t_1 \leq t_2 \leq T_0} \left[\hat{r}_{t_0}^{t_1,\mathcal{M}_s} - \hat{r}_{t_0}^{t_2,\mathcal{M}_s} \right], \tag{11}$$

in which $\hat{r}_{t_0}^{t,\mathcal{M}_s}$ denote the cumulative portfolio return from time t_0 through t, while t_0 and T_0 are the months of the first and last predictions, respectively. The maximum one-month loss measures the largest portfolio decline during the period considered. The average monthly

Table 4. Portfolio Performance across Subsamples

The table reports the annualized certainty equivalent returns and monthly Sharpe ratios for different models outlined in Section 2.3. The table shows the results when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. The table computes the statistics for each of the last seven decades. For the statistical significance of CERs, we report a one-sided p-value (in parentheses) of the test á la Diebold and Mariano (2002). In particular, we regress the difference in utilities for each model $\mathcal{M}_s$ and EH

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$. We flag in **bold font** those CER values that are significant at the 10% confidence level. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
1955-1959									
CER	5.467	4.376	3.219	9.495	5.545	**15.611**	**12.220**	**23.120**	**15.455**
p-value		(0.631)	(0.707)	(0.149)	(0.490)	(0.000)	(0.017)	(0.000)	(0.001)
SR	0.225	0.188	0.209	0.258	0.154	0.319	0.278	0.431	0.341
1960-1969									
CER	4.197	0.580	-4.193	**8.354**	0.619	**7.498**	**7.608**	5.627	**14.015**
p-value		(0.959)	(0.991)	(0.024)	(0.931)	(0.094)	(0.042)	(0.360)	(0.000)
SR	0.062	0.064	0.030	0.157	0.067	0.164	0.148	0.181	0.241
1970-1979									
CER	3.312	0.599	0.223	**9.275**	**8.580**	**17.750**	**15.690**	**13.618**	**20.658**
p-value		(1.000)	(0.847)	(0.015)	(0.026)	(0.000)	(0.000)	(0.000)	(0.000)
SR	-0.107	-0.097	0.005	0.149	0.180	0.274	0.248	0.224	0.309
1980-1989									
CER	9.215	7.243	-3.130	11.276	-1.450	3.315	1.216	2.603	10.241
p-value		(0.992)	(0.983)	(0.148)	(0.969)	(0.820)	(0.906)	(0.864)	(0.282)
SR	0.048	-0.005	0.072	0.139	0.055	0.166	0.123	0.142	0.112
1990-1999									
CER	8.101	**11.808**	**13.704**	-4.393	**12.085**	**10.817**	**10.301**	6.429	9.815
p-value		(0.002)	(0.002)	(1.000)	(0.039)	(0.052)	(0.099)	(0.842)	(0.202)
SR	0.222	0.185	0.219	-0.168	0.218	0.168	0.172	0.100	0.150
2000-2009									
CER	0.000	-7.445	-7.091	-15.834	-10.520	-3.889	-0.205	-7.991	1.726
p-value		(1.000)	(0.990)	(0.998)	(0.998)	(0.869)	(0.531)	(0.995)	(0.238)
SR	-0.091	-0.125	-0.055	-0.039	-0.040	0.028	0.060	-0.062	0.084
2010-2018									
CER	4.230	**6.268**	0.997	**9.757**	**9.508**	**6.642**	**5.690**	**7.241**	2.074
p-value		(0.000)	(0.999)	(0.000)	(0.000)	(0.004)	(0.049)	(0.023)	(0.842)
SR	0.237	0.220	-0.020	0.255	0.161	0.150	0.175	0.213	0.096

Table 5. Drawdowns, Maximum Loss, and Turnover

The table reports the alternative out-of-sample performance measures — maximum drawdown, maximum 1-month loss, and turnover — of optimal portfolio returns for different methodologies used to predict future S&P 500 excess returns as outlined in Section 2.3. All statistics are expressed in percentages. Panel A shows the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B demonstrates the results for a 2-year horizon and annual rebalancing. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Panel A: 6-month horizon and quarterly rebalancing									
Max DD	22.795	76.236	74.251	144.760	100.956	74.572	68.248	82.72	45.995
Max 1M Loss	7.795	33.325	57.974	31.756	57.974	57.974	57.974	57.974	35.011
Turnover	0.506	4.286	10.968	17.890	34.531	23.008	23.407	29.616	32.814
Panel B: 2-year horizon and annual rebalancing									
Max DD	25.002	90.896	83.370	123.279	145.663	74.972	34.562	25.136	64.433
Max 1M Loss	8.036	29.431	57.974	57.974	38.862	34.375	24.419	25.136	35.011
Turnover	0.584	4.289	8.804	10.773	11.329	11.352	8.725	6.771	17.459

turnover is defined as

$$\text{Turnover} = \frac{1}{T_0 - t_0} \sum_{t=t_0+1}^{T_0} \left| \omega_t - \omega_{t-1} \cdot \hat{r}_{t-1}^{\mathcal{M}_s} \right|, \tag{12}$$

where ω_{t-1} is the weight of the stock index.

Table 5 shows the results for alternative performance statistics. We first focus on actively managed portfolios with quarterly rebalancing and then move to more passive investment strategies with annual rebalancing. The maximum drawdown experienced by NN1 through NN3 is between 68% and 83% on the monthly basis. The linear models predict comparable or even larger drawdowns, whereas the constant mean and constant volatility model delivers a mild loss of around 23%. In contrast, the maximum drawdown for LSTM is around 46%, the most mild decline among the predictive models. Panel A further shows a similar picture for the maximum one-month loss of the portfolio: linear models and NNs tend to generate the worst one-period performance, while the LSTM strategy experiences a milder loss. Thus, the LSTM specification is the most successful in avoiding big losses over short- and long-term periods, even though it comes in the expense of the higher turnover.

Panel B in Table 5 shows that the investor using less frequent portfolio rebalancing is generally less efficient in forming the optimal portfolio if he relies on the linear regressions. Interestingly, the benefits of deep learning methods remain similar or even improve in some cases. For instance, the maximum one-month and drawdown losses tend to increase between 83% to more than 140% for the linear models, while NNs produce the largest declines from 25% to 35% per month. Furthermore, as the portfolio weights are kept unchanged for longer investment periods, the turnover is reduced. Thus, the passive investor who is mainly interested in reducing his short- and long-term tail risks would still find NNs useful, whereas she does not benefit from linear predictive models.

In sum, exploiting return predictability for the portfolio construction leads to more risky investments. It also generates the increased turnover, especially for the best performing model using the LSTM network. A natural question arises if these benefits are offset by large transaction costs implied by more aggressive buying or selling stocks

4.3 Portfolio Performance with Transaction Costs

This subsection extends the main analysis by accounting for the effect of transaction costs. Specifically, we consider low and high transaction costs that are equal to the percentage paid by the investor for the change in the value traded. Let τ denote a transaction costs parameter. Then the transaction costs adjusted returns are defined as

$$\hat{r}_t^{\tau,\mathcal{M}_s} = \hat{r}_t^{\mathcal{M}_s} - \tau \left| \omega_t - \omega_{t-1} \cdot \hat{r}_{t-1}^{\mathcal{M}_s} \right|,$$

where τ can attains one of the two possible values $\tau_l = 0.1\%$ or $\tau_h = 0.5\%$.

Table 6 presents summary statistics of the out-of-sample portfolio returns with low (Panels A and B) and high (Panels C and D) transaction costs. The results show that: (1) portfolio performance is monotonically decreasing in the percentage paid in transaction costs

Table 6. Portfolio Performance with Transaction Costs

The table reports the annualized certainty equivalent returns and Sharpe ratios for different models outlined in Section 2.3. The top and bottom parts of the table compute the optimal returns with low ($\tau = 0.1\%$) and high ($\tau = 0.5\%$) transaction costs, respectively. Panels A and C show the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B and D show the results for a 2-year horizon and annual rebalancing. Each panel computes the statistics for the whole sample. For the statistical significance of CERs, we report a one-sided p-value (in parentheses) of the test á la Diebold and Mariano (2002). In particular, we regress the difference in utilities for each model $\mathcal{M}_s$ and EH

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$. We flag in **bold font** those CER values that are significant at the 10% confidence level. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Low Transaction Costs									
Panel A: 6-month horizon and quarterly rebalancing									
CER	4.731	2.589	-0.171	2.548	2.036	**6.996**	**6.689**	5.115	**9.592**
p-value		(1.000)	(1.000)	(0.953)	(0.978)	(0.045)	(0.054)	(0.391)	(0.000)
SR	0.049	0.045	0.060	0.084	0.089	0.162	0.153	0.139	0.169
Panel B: 2-year horizon and annual rebalancing									
CER	4.535	0.998	-0.085	0.765	-0.241	**6.191**	**6.783**	6.366	**5.413**
p-value		(1.000)	(1.000)	(0.999)	(0.998)	(0.001)	(0.000)	(0.000)	(0.033)
SR	0.048	0.043	0.044	0.081	0.079	0.135	0.134	0.127	0.115
High Transaction Costs									
Panel C: 6-month horizon and quarterly rebalancing									
CE	4.706	2.370	-0.736	1.609	0.193	5.791	5.501	3.592	**7.910**
p-value		(1.000)	(1.000)	(0.990)	(0.999)	(0.214)	(0.263)	(0.784)	(0.000)
SR	0.048	0.041	0.053	0.068	0.066	0.145	0.134	0.117	0.145
Panel D: 2-year horizon and annual rebalancing									
CE	4.506	0.717	-0.586	0.129	-0.943	**5.579**	**6.396**	**6.080**	4.563
p-value		(1.000)	(1.000)	(1.000)	(0.999)	(0.022)	(0.000)	(0.000)	(0.453)
SR	0.047	0.039	0.038	0.073	0.070	0.125	0.123	0.117	0.102

(2) the key findings reported in the main analysis remain the same, that is, the NNs consistently outperform the traditional linear predictive regressions and the expectations hypothesis framework by generating substantially higher CER and SR values; and (3) among the NNs considered, the LSTM architecture remains a dominant specification. Quantitatively, the annualized CERs for all NNs decline by less than 0.5% and 2.1% for the low and high

transaction cost parameters, respectively. In terms of SRs, the decline in the performance never exceeds 2% and 3% on the monthly basis for the plain vanilla NNs and LSTM, respectively. However, despite a slightly detrimental effect of transaction costs, the best performing models (NN1 and LSTM) with an actively managed portfolio generate more than two- and three-fold increases in the CERs and SRs compared to the scenario ignoring equity return predictability. The formal test shows that the CER gains are also statistically significant.

4.4 Borrowing and Short-selling Constraints

We consider an additional robustness check to alternative assumptions about the portfolio weights. The main analysis allows the investor to borrow the money or to short-sell the stock by considering the weights in the interval $-1 \leq \omega_t \leq 2$. In this subsection, we perform a two-step analysis: we first impose borrowing constraints by restricting the optimal weight on the risk-free investment to be non-negative and then additionally imposing short-selling constrains with the weights $0 \leq \omega_t \leq 1$.

Table 7 reports the results for the two scenarios. We focus on the quarterly rebalancing case reported in Panels A and C. The corresponding results for the passive portfolios, which are shown in Panels B and D, remain qualitatively similar. Several observations are noteworthy. First, winsorizing the weights to narrower intervals leads to ambiguous conclusions about the performance of linear predictive models. On the one hand, the constraints prevent optimal investments and hence lead to smaller out-of-sample Sharpe ratios. On the other hand, using the certainty equivalent as a measure of portfolio performance, the linear specifications consistently generate improved results, with the CERs above 3.5% in all cases. Thus, constraints on the optimal weights result in higher CERs. The reason for this seemingly counterintuitive result is that such restrictions prevent the expected utility from achieving unbounded large values (Johannes et al., 2014) and, therefore, avoid extreme investments based on unstable predictions of linear regressions (Goyal and Welch, 2008). Since

Table 7. Portfolio Performance with Borrowing and Short-Selling Constraints

The table reports the annualized certainty equivalent returns and Sharpe ratios for different models outlined in Section 2.3. The top part of the table imposes borrowing constrains, while the bottom part additionally assumes short-selling constraints. Panels A and C show the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panels B and D show the results for a 2-year horizon and annual rebalancing. For the statistical significance of CERs, we report a one-sided p-value (in parentheses) of the test á la Diebold and Mariano (2002). In particular, we regress the difference in utilities for each model $\mathcal{M}_s$ and EH

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$. We flag in **bold font** those CER values that are significant at the 10% confidence level. The portfolio construction starts in February 1955. The sample period spans from January 1945 to December 2018.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Borrowing Constraint									
Panel A: 6-month horizon and quarterly rebalancing									
CER	4.737	3.662	3.371	4.149	4.560	**9.122**	**7.632**	**6.900**	**8.560**
p-value		(0.999)	(0.958)	(0.770)	(0.591)	(0.000)	(0.000)	(0.003)	(0.000)
SR	0.049	0.046	0.061	0.074	0.080	0.176	0.146	0.135	0.157
Panel B: 2-year horizon and annual rebalancing									
CER	4.542	2.780	2.936	2.974	1.560	4.964	**5.336**	**5.275**	**5.321**
p-value		(1.000)	(1.000)	(1.000)	(1.000)	(0.147)	(0.007)	(0.008)	(0.033)
SR	0.048	0.044	0.051	0.049	0.013	0.101	0.094	0.087	0.100
Borrowing and Short-Selling Constraints									
Panel C: 6-month horizon and quarterly rebalancing									
CER	4.737	3.704	4.707	5.353	5.708	**7.500**	**6.758**	**7.128**	**7.775**
p-value		(0.998)	(0.528)	(0.080)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)
SR	0.049	0.047	0.066	0.093	0.093	0.146	0.129	0.138	0.150
Panel D: 2-year horizon and annual rebalancing									
CER	4.542	2.780	3.757	4.261	**5.010**	**5.809**	**5.320**	**5.408**	**6.117**
p-value		(1.000)	(1.000)	(0.859)	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)
SR	0.048	0.044	0.054	0.054	0.071	0.107	0.109	0.098	0.107

the certainty equivalent measure takes into account tail behaviour of returns, less extreme investments ultimately yield the improved results.

Second, unlike the linear regressions, we document a negative impact of imposing borrowing and short-selling constraints on the portfolio performance implied by NNs. For instance, Panels A and B in Table 7 demonstrate a decline in both CERs and SRs for all NNs, with a larger drop in performance measures in response to more stringent assumptions about the weights. Nevertheless, despite weaker performance of machine learning methods, the table confirms the key results of the main analysis. Specifically, traditional predictive models hardly generate a positive value for the investor, whereas there is a robust statistical evidence on the substantial improvement from using NNs.

4.5 Different Rolling Window Sizes

The subperiod analysis presented in Table 4 reveals a slightly declining performance of NNs by the end of the sample. In particular, the LSTM generates higher CERs than the EH model, however, the difference turns out to be statistically indistinguishable over the last four decades. This raises the question whether the evidence of this paper holds for more recent data. This subsection demonstrates that the main conclusions of this paper indeed remain intact.

Table 8 reports summary statistics of the out-of-sample portfolio returns, which are obtained for the subperiod from February 1969 to December 2018 as in Rossi (2018). In relation to the models using the rolling-window estimation, we assume a 20-year horizon to assess the impact of longer history on the performance of different methodologies, particularly machine learning methods that are supposed to work better in larger samples. Notice that the quantitative predictions of this exercise are not directly comparable to the previous results due to the difference in the historical data. In particular, the period from February 1969 to December 2018 is characterized by slightly weaker market performance that ultimately translates into a less favorable opportunity set of the investor. The return statistics in Table 8 are consistent with this intuition. The average Sharpe ratio implied by the model with no predictability becomes twice smaller compared to the benchmark analysis. The linear models experience a comparable deterioration in the results.

For NNs with quarterly rebalancing, we document several interesting observations. First,

Table 8. Portfolio Performance from Feb 1969:02 to Dec 2018: 20-year rolling window

The table reports the annualized certainty equivalent returns and Sharpe ratios for different models outlined in Section 2.3. The rolling window estimation uses 20 years of recent data. Panel A shows the results for the case when the investor maximizes a 6-month portfolio return and changes the allocations quarterly. Panel B shows the results for a 2-year horizon and annual rebalancing. Each panel computes the statistics for the whole sample, expansion and recession periods as defined by the NBER. For the statistical significance of CERs, we report a one-sided p-value (in parentheses) of the test á la Diebold and Mariano (2002). In particular, we regress the difference in utilities for each model $\mathcal{M}_s$ and EH

$$\mathcal{U}_{t+T}^{\mathcal{M}_s} - \mathcal{U}_{t+T}^{EH} = \alpha^{\mathcal{M}_s} + \varepsilon_{t+T},$$

where $\mathcal{U}_{t+T}^{X} = \frac{\left(r_{p,t+T}^{X}\right)^{1-\gamma}}{1-\gamma}$ and $r_{p,t+T}^{X}$ is the cumulative portfolio return with the horizon T. Testing for the difference in the CERs boils down to a test for the significance in $\alpha^{\mathcal{M}_s}$. We flag in **bold font** those CER values that are significant at the 10% confidence level. The portfolio construction starts in February 1969.

	EH	OLS1	OLS2	OLS3	OLS4	NN1	NN2	NN3	LSTM
Panel A: 6-month horizon and quarterly rebalancing									
1969-2018									
CER	4.600	1.763	0.791	1.025	3.707	**6.762**	**6.601**	**6.236**	**7.253**
p-value		1.000	1.000	0.984	0.811	0.018	0.053	0.061	0.016
SR	0.025	0.010	0.018	0.059	0.057	0.135	0.140	0.132	0.165
Expansions									
CER	5.038	3.158	2.479	4.551	5.846	**7.237**	**7.314**	5.673	**6.734**
p-value		0.999	0.997	0.694	0.158	0.008	0.022	0.335	0.039
SR	0.090	0.059	0.045	0.070	0.068	0.139	0.138	0.141	0.161
Recessions									
CER	1.846	-6.771	-9.496	-18.688	-8.560	3.819	2.347	4.423	**17.657**
p-value		1.000	0.999	0.985	0.980	0.345	0.465	0.287	0.002
SR	-0.251	-0.253	-0.123	0.034	0.023	0.123	0.156	0.061	0.226
Panel B: 2-year horizon and annual rebalancing									
1969-2018									
CER	4.530	0.508	-2.573	-2.558	2.080	**5.788**	**5.136**	**7.038**	**6.477**
p-value		(1.000)	(1.000)	(1.000)	(1.000)	(0.026)	(0.068)	(0.000)	(0.000)
SR	0.023	0.008	-0.002	0.008	0.025	0.126	0.084	0.117	0.135
Expansions									
CER	4.448	0.246	-2.160	-2.174	3.324	**5.465**	4.675	**6.739**	**6.185**
p-value		(1.000)	(1.000)	(1.000)	(0.992)	(0.076)	(0.297)	(0.000)	(0.001)
SR	0.089	0.059	0.035	0.008	0.021	0.108	0.054	0.100	0.127
Recessions									
CER	5.089	2.275	-5.061	-4.914	-4.333	**8.133**	**8.538**	**9.073**	**8.409**
p-value		(0.999)	(1.000)	(1.000)	(1.000)	(0.023)	(0.004)	(0.003)	(0.009)
SR	-0.248	-0.259	-0.194	0.010	0.045	0.216	0.211	0.210	0.181

despite a weaker performance of the stock market during the period considered, monthly Sharpe ratios implied by NNs decrease marginally, with the drop approximately equal to 0.01 to 0.03 relative to the main results. Second, comparing NN1 through NN3 in terms of certainty equivalent returns, NNs yield statistically the same results. Although deeper networks generate slightly lower CERs than those predicted by shallower networks, the p-values indicate that these model-based values remain in the same equivalence class. Third, the LSTM still produces the most significant economic gains. Specifically, the annualized certainty equivalent yield is above 7% and monthly Sharpe ratios remain as high as 0.165. Finally, unlike a weak statistical evidence of the main results with recent data, the formal test of the results in this subsection demonstrates a strong statistical evidence in favor of NNs. The reason is that NNs use a 20-year rolling window for hyperparameter tuning, which helps to better learn non-linear relationships, short- and long-term dependencies (in case of LSTM) from the data.

5 Conclusion

In this paper, we evaluate the economic gains of using deep learning methods for the construction of optimal portfolios. To do so, we study the portfolio allocation of a long-horizon investor who uses neural networks to predict future returns when choosing an optimal allocation between a market portfolio and a risk-free asset. We propose and compare various architectures of neural networks including shallow and deep NNs as well as the LSTM specification, which is capable of learning the long-term relationships. Three key findings emerge from our investigation.

First, we demonstrate that a sound statistical performance of non-linear machine learning methods, such as neural networks, transmits to large and significant out-of-sample portfolio gains. These gains are robust to a variety of portfolio performance measures, the inclusion of transaction costs, borrowing and short-selling constraints. Second, we find that employing the forecasts of deeper networks does not necessarily translate into larger economic gains. In order to identify and benefit from the complex non-linear predictive relationship, the investor needs to harvest more data, while shallower NNs might be a better option in the setting with small samples. In terms of NNs, we further show that the novel LSTM is the best performing specification. This emphasizes the critical role of short- and long-term order dependencies in predicting stock returns, in addition to approximating the non-linear relationship. Finally, we document that NNs perform well even in the absence of additional ingredients, such as time-varying return volatility, which are commonly proposed by the literature studying the linear predictive regressions. Our results show that NNs are capable of identifying these complex features from the data in a non-parametric way and without any specific modelling assumptions.

Our analysis can be extended in a number of ways. It would be interesting to examine the interaction between NNs and alternative preference specifications. In particular, it is not clear whether the investor with a tail sensitive utility function or a preference for early resolution of uncertainty would be able to generate comparable economic gains. Van Binsbergen and Koijen (2010) present the evidence that additional economic restrictions can actually improve the model's performance. Our results point out a negative impact of restricting portfolio weights on the gains of the NNs. It would be interesting to examine if our evidence holds in the setting with other restrictions, in particular those proposed by Van Binsbergen and Koijen (2010). Finally, extending our analysis to multiple assets is a straightforward exercise, which would shed light on the economic significance of forecasting returns of different asset classes via NNs.

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