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Forecasting daily conditional volatility and *h*-step-ahead short and long Value-at-Risk accuracy: Evidence from financial data

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Abstract

In this article we evaluate the daily conditional volatility and h-step-ahead Value at Risk (VaR) forecasting power of three long memory GARCH-type models (FIGARCH, HYGARCH & FIAPARCH). The forecasting exercise is done for financial assets including seven stock indices (Dow Jones, Nasdaq100, S&P 500, DAX30, CAC40, FTSE100 and Nikkei 225) and three exchange rates vis-a-vis the US dollar (the GBP- USD, YEN-USD and Euro-USD). Because all return series are skewed and fat tailed, each conditional volatility model is estimated under a skewed Student distribution. Consistent with the idea that the accuracy of VaR estimates are sensitive to the adequacy of the volatility model used, h-step-ahead VaR forecasts are based on the skewed Student-t AR(1)-FIAPARCH (1,d,1). This model can jointly accounts for the salient features of financial time series. Our findings reveal that the skewed Student AR (1) FIAPARCH (1.d.1) relatively outperforms the other models in out-of-sample forecasts for one, five and fifteen day forecast horizons. However, there is no difference for the AR (1) FIGARCH (1.d.1) and AR (1) HYGARCH (1.d.1) models since they have the same forecasting ability. Results indicate also that skewed Student-t FIAPARCH (1,d,1) model provides more accurate one-day-ahead VaR forecasts for both long and short trading positions than those generated using alternative horizons (5-day and 15-day-ahead). This result holds for each of the financial time series.

JEL classification: G11; L94; E39; G15

Keywords: Value-at-Risk; Forecasting volatility; Skewed student distribution; Long-range memory

1. Introduction

ARCH model was born in the literature with the publication paper of,[1] soon the model was generalized (GARCH) by Bollerslev (1986). While ARCH was developed to model the changing volatility of inflation series, the model and its later extensions were quickly adopted for modeling conditional volatility of financial returns,[2] The main advantage of GARCH models is that they captured jointly heavy tails and volatility clustering: "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes".[3] To account for some financial time series stylist facts, many variants of GARCH class models were proposed such as EGARCH, GJR-

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GARCH, APARCH, FIGARCH, HYGARCH … etc. The models and their later extensions were quickly found to be relevant for the conditional volatility of financial returns. More precisely, those models are usually used for both volatility modeling and forecasts of financial time series. See Bollerslev, Chou and Kroner,[4] Bollerslev, Engle and Nelson,[5] Bera and Higgins (1995) and Diebold and Lopez.[6] Ghysels, Harvey and Renault,[7] Allen et al (2005), Angelidis et al,[8] Assaf (2009), Chiu et al (2006), Cheong (2008), Shieh and Wu (2007), Lee and Saltoglu (2002), Awartani Corradi (2005), Gençay Selçukc (2004), Fan et al (2004), Tse[9] Giot and Laurent,[10] Bollerslev,[11] Wright, (2008), Bali Bali,[12] Engle,[13] Hamilton and Susmel (1999), Gallo and Pacini (1998). In general, models that allow for volatility asymmetry come out well in the forecasting contest because of the strong negative relationship between volatility and shock. Cao and Tsay,[14] Heynen and Kat,[15] Lee[16] and Pagan and Schwert[17] favor the EGARCH model for volatility of stock indices and exchange rates, whereas Brailsford and Faff (1996) and Taylor Taylor, J.[18] find GJR-GARCH outperforms GARCH in stock indices. During the last decades long-memory processes (the presence of statistically significant correlations between observations that are a large distance apart.) have evolved into a vital and important part of the time series analysis. A long memory series has autocorrelation coefficients that decline slowly at a hyperbolic rate. These features change dramatically the statistical behavior of estimates and predictions. An important property of fractionally integrated GARCH models is their ability to capture both volatility clustering and long memory in financial time series. During recent years, several researches have been concerned with the long-range memory on both price variations and price volatilities. More precisely, the empirical literature is focused on volatility modeling when studied time series are governed by a long memory process. See Tang and Shieh (2007), Yu So (2010), Assaf (2009), Chiu et al (2006), Cheong (2008), Shieh and Wu (2007), Lee and Saltoglu (2002), Kang and Yoon,[19] Mabrouk and Aloui,[20] Mabrouk and Saadi (2012) … etc. These studies showed that stock market and exchange market volatility are governed by a long memory process. They concluded that the long memory GARCH class models outperform the other models. The long memory characteristic of financial market volatility has important implications for volatility forecasting and option pricing. Comparing forecasting performance of studied models is crucial for any forecasting exercise. In contrast to the efforts made in the construction of volatility models and forecasts, little attention has been paid to forecast evaluation in the volatility forecasting literature. Figlewski[21] finds GARCH superiority confined to the stock market and for forecasting volatility over a short horizon only. Vilasuso (2002) tested FIGARCH against GARCH and IGARCH for volatility prediction for five major currencies. Vilasuso (2002) finds FIGARCH produces significantly better 1- and 10-day-ahead volatility forecasts for five major exchange rates than GARCH and IGARCH. Zumbach[22] produces only one-day-ahead forecasts and find no difference among model performance. In most applications, the excess kurtosis implied by the GARCH class model under a normal density is not enough to mimic what we observe on real data. Other distributions are possible. Bollerslev[2] proposed to use the Student-t distribution, since it implies conditional leptokurtosis and, therefore, stronger unconditional leptokurtosis. To account for excess kurtosis, the generalized error (GE) distribution was proposed by Nelson.[23] As reported by Pagan,[24] the use of symmetric heavy-tailed distributions (such as Student-t distribution and the generalized error distribution) is common in the finance literature. In particular, Bollerslev,[2] Hsieh (1989), Baillie and Bollerslev[25] and Palm and Vlaar (1997) among others show that these distributions perform better in order to capture the excess kurtosis. However, many financial times series returns are fat tailed and skewed. To account for both asymmetric and fat tail in the empirical density, Fernandez and Steel (1998) proposed skewed-Student density which has been extended by Lambert and Laurent (2000), Lambert and Laurent.[27] The last decade has seen a spectacular development in market risk management techniques. VaR has become a popular method of risk quantification. Indeed, the VaR is adopted by several financial institutions and risk managers as being an effective tool to measure the market risk. Thanks to its conceptual simplicity, VaR has also become a standard risk measure used in financial risk management. Therefore, VaR is widely used to assess exposure to investment risk. VaR can be defined as the maximum potential loss for a given period and at a fixed confidence level. At first the determination of VaR is based on the normal distribution of returns. However, the series returns are leptokurtic (see 3,4,28–32. Therefore, the normal distribution fails to provide good results. To improve the VaR results, empirical studies suggest other distributions such as the Student distribution,[33–36,8] Cheong, 2008), and distribution (GED).[8] The goal is to capture the fat tails of returns. However, the financial asset time series returns are usually both asymmetric and fat tailed. The skewed distribution Student is recommended for estimating VaR since it takes into accounts both asymmetry and fat tail of the return distribution. Therefore, have accurate VaR requires that the volatility model jointly accounts for the salient features of financial time series: fat tails, asymmetry, volatility clustering and long memory. The aim of this paper is to contribute to the finance literature on volatility forecasting and VaR accuracy of financial time series. Thus, our goal is to

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compare the volatility forecasting ability and out-of-sample VaR of same non-linear models for different horizons. Since all studied financial time series returns are skewed, fat tailed, exhibit ARCH effect and long memory, we adopt GARCH-type models with asymmetric innovation distributions (a skewed Student distribution) to forecast financial time series volatility for three horizons. More specifically, we are concerned with three GARCH-type models: the FIGARCH, FIAPARCH and HYGARCH. To the best of our knowledge, this paper represents one of the first studies focused on volatility forecasting of financial assets using the FIGARCH, FIAPARCH and HYGARCH processes under a skewed Student-t distribution. In addition, we compare the forecast accuracy under same alternative approaches. In our empirical application, we search for models that capture the features of the analyzed data and that provide accurate out-of-sample forecasts. Thus, our analysis has greater emphasis on in-sample fit, while our forecasting exercise will necessarily concentrate on out-of-sample outcomes. We select the best model that fit the data based on several model selection criteria. We then assess the performance of the selected model in estimating h-step-ahead VaR for both short and long trading positions using failure rate, the Kupiec's[37] likelihood ratio test and Engle and Manganelli's[38] dynamic quantile test. Our main result confirms that the quality of the forecast depends on the used assumptions. When the assumptions are realistic, the results are good and vice versa. In fact, we find that skewed Student-t FIAPARCH (1,d,1) model provides more accurate forecasts of one-day-ahead VaR returns of both long and short trading positions than those generated using others horizons. The rest of this paper is organized as follows: Section 2 provides a description of GARCH-class models, density model, forecasting criteria evaluation, VaR model backtesting tests employed in this paper are presented in the third section. Section 4 presents the data and the empirical results of out-of-sample volatility forecasting. Section 5 provides the results of the out-of-sample VaR model accuracy. Section 6 summarizes this paper.

2. Long-memory models

2.1. The fractional integrated GARCH model

Bailey, Bollerslev and Mikkelsen Bailey, Bollerslev and Mikkelsen[39] applied the concept of fractional integration to the conditional variance of a time series, proposing the fractionally integrated GARCH model (FIGARCH). Unlike the GARCH model is I(0) and IGARCH model that is I (1), the integrated fractional process I (d), distinguishes between short memory and long memory in time series.

According to Bailey et al,[39] the FIGARCH (pdq) model is defined by:

$$y_t = \mu + \varepsilon_t,\ \ \varepsilon_t = z_t\sigma_t,\ \ z_t \sim N(0,1)$$

$$\varnothing(L)(1-L)^d\varepsilon_t^2 = \omega + [(1-\beta(L)]v_t \tag{1}$$

with, $v_t = \varepsilon_t^2 - \sigma_t^2$; all the roots of $\varnothing(L)$ *and* $[(1-\beta(L)]$ is outside the unit circle.

(1) can be rewritten as follows:

$$\sigma_t^2 = \omega + \left(1 - [1-\beta(L)]^{-1}\varnothing(L)(1-L)^d\right)\varepsilon_t^2 \tag{2}$$

$$\sigma_t^2 = \omega + \lambda(L)\sigma_t^2 \tag{3}$$

with, $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + \ldots and\ 0 \le d \le 1$.

For the FIGARCH process (pdq) is well defined and that the conditional variance is positive for all, all the coefficients of ARCH representation shall be positive.

$\lambda_j \ge 0$ for $j = 1.2\ldots$ Consider the condition proposed by Bollerslev and Mikkelsen,[39] which is necessary and sufficient to ensure the non negativity λ_j:

$$\beta_1 - d \le \varnothing_1 \le \frac{2-d}{3}\ \ and\ d\left[\varnothing_1 - \frac{1-d}{2}\right] \le \beta_1(\varnothing_1 - \beta_1 + d) \tag{4}$$

For FIGARCH model (p.d.q), the persistence of the conditional variance or the long memory degree is measured by the parameter d. Thus, the model FIGARCH (p.d.q) will be attractive for $0 < d < 1$, which is an intermediate situation (finite long memory).

2.2. The hyperbolic GARCH model

Davidson has developed in 2004 a model called Hyperbolic GARCH which represents an extension of FIGARCH model. In fact, this model is based on the fact to test how non-stationary model of FIGARCH. Extending from the model HYGARCH, the FIGARCH resides in the addition of weight. The conditional variance of HYGARCH model can be formulated as follows:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \left\{1-[1-\beta(L)]^{-1}\rho(L)\left[1+\alpha\left\{(1-L)^d\right\}\right]\right\}\varepsilon_t^2 \tag{5}$$

The HYGARCH model becomes a simple GARCH when $\alpha = 0$ and a model FIGARCH in case $\alpha = 1$. Therefore, GARCH and FIGARCH models are only special cases of HYGARCH model.

2.3. The fractional integrated asymmetric power ARCH model

The FIAPARCH model can be considered an extension of the FIGARCH model with the APARCH model of Ding, Granger and Engle.[40] This model can capture both long memory and asymmetry in the conditional variance. The FIAPARCH model (p.d.q) can be specified as follows:

$$\sigma^\delta = \omega + [1-(1-\beta(L))^{-1}(1-\varnothing(L)(1-L)^d](|\varepsilon_t| - \gamma\varepsilon_t)^\delta \tag{6}$$

$\delta > 0$, $-1 < \gamma < 1$ and $0 < d < 1$. when, $\gamma > 0$, a negative shock increases volatility than a positive shock and vice versa.:

The FIAPARCH model becomes a FIGARCH model when $\delta = 2$ and $\gamma = 0$. That 's why we can say that the FIAPARCH model is a generalization of FIGARCH model.

2.4. Density model

In order to overcome the shortcomings of the symmetric Student-t distribution and to take into account both the skewness and excess kurtosis, we consider the skewed Student-t distribution proposed by Lambert and Laurent.[27] The latter distribution captures both asymmetry and thick tail (fat tail).

If $z \sim SKST(0,1,k,\nu)$, the log probability distribution function skewed Student-t is formulated as follows

$$\begin{aligned} L_{Skst} = {} & T\left\{ln\Gamma\left(\frac{\upsilon+1}{2}\right) - ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}ln[\pi(\nu-2)] + \ln\left(\frac{2}{k+\frac{1}{k}}\right) + \ln(s)\right\} \\ & -\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(\sigma_t^2\right) + (1+\nu)ln\left[1+\left(1+\frac{(sz_t+m)^2}{(\nu-2)}\right)k^{-2I_t}\right]\right] \end{aligned} \tag{7}$$

with, $I_t = 1 \; si \; z_t \geq m/s \;\; or \; I_t = -1 \; si \; z_t < m/s$, k is a parameter of asymmetry, the constant $m = m(k,\nu) \; and \; s = \sqrt{s^2(k,\nu)}$, are the mean and standard deviation of the distribution skewed Student-t, respectively:

$$m(k,\nu) = \frac{\Gamma\left(\frac{\nu-1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left(k-\frac{1}{k}\right) \tag{8}$$

$$s^2(k,\,\nu) = \left(k^2+\frac{1}{k^2}-1\right) - m^2 \tag{9}$$

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The value of $ln(k)$ denotes the degree of asymmetry in the distribution of the residual term. Where, $ln(k) > 0$, the density is asymmetric to right. If $ln(k) < 0$, the density is asymmetric left. When $ln(k) = 0$, that's means $k = 1$, the skewed Student-t distribution reduces to a general distribution of Student-t.

2.5. Forecast evaluation criteria

Since volatility itself is unobservable, the comparison of volatility forecasts relies on an observable proxy for the latent volatility process. Several criteria for measuring the predictive ability of the models were developed namely MSE (Mean Squared Error), MAE (Mean Absolute Error), TIC (Theil Inequality Coefficient) and the Mincer-Zarnowitz (MZ) regression Mincer-Zarnowitz (MZ) regression,[41] which involves regressing the realization of a variable on a constant and its forecast. In our study we will use these four evaluation criteria to measure the predictive capabilities of the following long memory models: FIGARCH, HYGARCH and FIAPARCH.

The MZ regression is based on regression of realized volatility on a constant and expected volatility. Formally, the regression of MZ regression may be presented as follows

$$\sigma_{\mathrm{realized}(t+1)} = \alpha + \beta \times \sigma_{forecast(t+1)} + \varepsilon_t \tag{10}$$

The MZ regression allows to evaluate two different aspects of the volatility forecast. First, the MZ regression allows to test the presence of systematic over or under predictions, that is, whether the forecast is biased, by testing the joint hypothesis $H_{(0)}\alpha = 0 \cup \beta = 1$. Second, being the R^2 of Equation (10), an indicator of the correlation between the realization and the forecast, it can be used as an evaluation criterion of the accuracy of the forecast. Indeed, the model with the R^2 closer to 1indicates a great predictive power compared to those with R^2 near 0. The R^2 of the MZ regression has frequently been used as a criterion for ordering over a set of volatility forecasts.[42,43]

To assess the forecasting power of the GARCH- type models under skewed Student-t distribution, we used different loss functions rather than make an individual choice. To compare the predicative power of studied models, many researchers used, Mean Square Error (MSE), Mean Absolute Error (MAE), Theil Inequality Coefficient (TIC) among other. see, Hansen and Lunde,[44] Patton,[45] Wang and Wu[46] and Byun and Cho (2013). These three criteria may be presented respectively as follows:

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|\varepsilon_t| = \frac{1}{N}\sum_{t=1}^{N}\left|\widehat{\sigma_t} - \sigma_t\right| \tag{11}$$

$$MSE = \frac{1}{N}\sum_{t=1}^{N}\varepsilon_t^2 = \frac{1}{N}\sum_{t=1}^{N}\left(\widehat{\sigma_t} - \sigma_t\right)^2 \tag{12}$$

$$TIC = \frac{\sum_{t=1}^{N}\left(\widehat{\sigma_t} - \sigma_t\right)^2}{\sum_{t=1}^{N}\left(\widehat{\varepsilon_t^{BM}} - \sigma_t\right)^2} \tag{13}$$

The forecasting exercise will focus on three horizons are; 1-day, 5-day and 15-day ahead.

3. The Value-at-Risk

We should mention that unlike previous studies focused on one-day-ahead VaR computations, we estimate h-step-ahead out-of-sample VaRs based on skewed Student AR(1)−FIAPARCH models. We present in this section the VaR model under a FIAPARCH model with skewed Student-t distribution innovation. Let's consider that

$$r_t = \mu_t + \varepsilon_t \tag{14}$$

$$\mu_t = \mu + \sum_{i=1}^{m} \xi_i r_{t-i} + \sum_{j=1}^{n} \theta_j \varepsilon_{t-j} \tag{15}$$

The $\varepsilon_t = z_t \sigma_t$ is governed by a FIAPARCH (p, d, q) process and the innovations are assumed to follow the skewed Student-t distribution if:

$$f(z_t|k, v) = \begin{cases} \dfrac{2}{k + \frac{1}{k}} sg(k(sz_t + m)|v) \\ \dfrac{2}{k + \frac{1}{k}} sg((sz_t + m)/k|v)_t \end{cases}, \quad \text{if } \begin{matrix} z_t < -m/s \\ z_t \geq -m/s \end{matrix} \tag{16}$$

In the above equation, $g(.|v)$ denotes the symmetrical Student-t density and k is the asymmetry parameter. The estimated VaR for the long and short trading positions can be expressed as follows:

$$\alpha = P(r_t < VaR_{t,L}) = P\left(\frac{r_t - \mu_t}{\sigma_t} < \frac{VaR_{t,L} - \mu_t}{\sigma_t}\right) \tag{17}$$

$$\alpha = P(r_t > VaR_{t,s}) = P\left(\frac{r_t - \mu_t}{\sigma_t} > \frac{VaR_{t,s} - \mu_t}{\sigma_t}\right) \tag{18}$$

VaR in Eqs. (18) and (19), $VaR_{t,L}$ and $VaR_{t,s}$ are forthe long and the short trading positions, respectively. More specifically $VaR_{t,L}$ and $VaR_{t,s}$ are as follows:

$$VaR_{t,L} = \mu_t + st_\alpha(v,k)\sigma_t \tag{19}$$

$$VaR_{t,s} = \mu_t + st_{1-\alpha}(v,k)\sigma_t \tag{20}$$

where $st_\alpha(v,k)$ is the left quantile at the $\alpha\%$ of the skewed Student-t distribution innovation. Correspondingly, $st_{1-\alpha}(v,k)$ is the right quantile of the skewed Student-t distribution.[1] According to Lambert and Laurent[27] and Wu and Shieh,[47] p. 252), we can compute the one-day-ahead VaR estimated at time (t−1) for the long and the short trading positions. Under the hypothesis of skewed Student-t distribution, the one-day-ahead VaR for the long and the short trading positions are as follows:

$$\widehat{VaR}_{t,L} = \widehat{\mu}_t + st_\alpha(v,k)\widehat{\sigma}_t \tag{21}$$

$$\widehat{VaR}_{t,s} = \widehat{\mu}_t + st_{1-\alpha}(v,k)\widehat{\sigma}_t \tag{22}$$

3.1. Statistical accuracy of model-based VaR forecasts

The VaR quality estimation is sensitive to the methodology chosen to model the volatility of asset returns. Therefore it is important to evaluate the performance of VaR model. In order to back-test the accuracy for the estimated VaRs we using two approaches. The first is to examine the accuracy of VaR estimate consists at backtesting VaR using Kupiec LR test. In order to test the accuracy and to evaluate the performance of the model-based VaR estimates, Kupiec[37] provided a likelihood ratio test (LR_{UC}) for testing whether the failure rate of the model is statistically equal to the expected one (unconditional coverage). Consider that $N = \sum_{t=1}^{T} I_t$ is the number of exceptions in the sample size T. Then,

[1] For more details, see Lambert and Laurent,[27] Giot and Laurent[55] and Wu and Shieh[47].

$$I_{t+1} = \begin{cases} 1 & \text{if} \quad r_{t+1} < VaR_{t+1|t(\alpha)} \\ 0 & \text{if} \quad r_{t+1} \geq VaR_{t+1|t(\alpha)} \end{cases} \tag{23}$$

follows a binomial distribution, $N \sim B(T,\alpha)$. If $p = E\left(\frac{N}{T}\right)$ is the expected exception frequency (i.e. the expected ratio of violations), then the hypothesis for testing whether the failure rate of the model is equal to the expected one is expressed as follows: $H_0{:}\alpha{=}\alpha_0$. α_0 is the prescribed VaR level. Thus, the appropriate likelihood ratio statistic in the presence of the null hypothesis is given by:

$$LR_{uc} = -2\log\left\{\alpha_0^N(1-\alpha_0)^{T-N}\right\} + 2\log\left\{\left(\frac{N}{T}\right)^N\left(1-\left(\frac{N}{T}\right)\right)^{T-N}\right\} \tag{24}$$

Under the null hypothesis, LR_{uc} has a $\chi^2(1)$ as an asymptotical distribution. Thus, a preferred model for VaR prediction should provide the property that the unconditional coverage measured by $p = E\left(\frac{N}{T}\right)$ equals the desired coverage level p_0.

Our second and last approach to evaluate the performance of the model-based VaR estimates is through the conditional coverage test proposed by Engle and Manganelli.[38] Engle and Manganelli develop the Dynamic Quantile (DQ) test building upon a linear regression model based on the process of centred hit function:

$$\delta_t^\alpha = Hit_t(\alpha) \equiv I(y_t < -VaR_t(\alpha)|\Omega_{t-1}) - \alpha \tag{25}$$

Conditional on pre-sample values the dynamic of the hit function is modeled as:

$$\delta_t^\alpha = \theta_0 + \sum_{i=1}^{p}\theta_i\delta_{t-i}^{(\alpha)} + \sum_{\tau=1}^{m}\vartheta_i\delta_{t-i}^{(\tau)} + \mu_t \tag{26}$$

where μ_t is an IID process. The DQ test is defined under the hypothesis that the regressors in Equation (26) have no explanatory power:

$$H_0 = \Psi = \left(\theta_0,\ \theta_1,\ \ldots,\ \theta_p,\ \vartheta_0,\ \vartheta_1,\ \ldots,\ \vartheta_m\right)^T = 0$$

For backtesting, the DQ test statistic, in association with Wald statics, is as follows:

$$DQ = \frac{\widehat{\Psi}^T X^T \widehat{\Psi}}{\alpha(1-\alpha)} \xrightarrow{\ell} \chi^2_{1+p+m} \tag{27}$$

where X denotes he regressors matrix in Equation (26).

4. Empirical analysis

4.1. Data description

To investigate the volatility forecasting power of GARCH- class models, the data employed in this study consists of daily closing prices of seven stock indexes (Dow Jones, Nasdaq, S&P500, DAX30, FTSE100, CAC40 and Nikkei 225) and three exchange rates vis-a-vis the US dollar (the GBP- USD, YEN-USD and Euro-USD). The raw data sets are downloaded from the web http://finance.yahoo.com. For each financial asset's time series, the sample period and the number of observations are displayed in Table 1. The continuously compounded daily returns are computed as follows:

$$r_t = 100\ \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{28}$$

where r_t and P_t are the return in percent and the closing price on day (t), respectively. We should mention that for each asset the data set is subdivided into two subsets. The last 1000 day returns are reserved for the out-of-sample analysis.

Table 1
Data sets.

asset	Noum	Sample period	Observations
Indices	DAX	01/02/1990 – 10/10/2008	4498
	DOW JONES	01/02/1990 – 10/10/2008	4734
	NASDAQ	01/02/1990 – 10/10/2008	4720
	NIKKEI	01/02/1990 – 10/10/2008	4607
	CAC40	01/02/1990 – 10/10/2008	4678
	FTSE100	01/02/1990 – 10/10/2008	4727
	S&P500	01/02/1990 – 10/10/2008	4720
Exchange rates	EURO/USD	12/31/1998 – 10/10/2008	2498
	GBP/USD	01/02/1990 – 10/10/2008	4777
	YEN/USD	01/02/1990 – 10/10/2008	4777

Table 2
Descriptive statistics, unit root and stationarity tests.

Asset	Mean	Median	SD	Skewness	Kurtosis	Jarque−Béra	ADF	PP	KPSS
DAX	0.0262	0.0741	1.3733	−0.394	7.6728	4209	−46.91***	−66.24***	0.25***
DOW JONES	0.0239	0.0476	1.009	−0.387	8.139	5327	−48.81***	−68.53***	0.39***
NASDAQ	0.0266	0.1014	1.5784	−0.291	10.1645	10159	−50.12***	−71.4***	0.34***
NIKKEI	−0.0317	−0.0232	1.3951	0.158	6.3244	2140	−38.96***	−64.96***	0.12***
CAC40	0.0112	0.0591	1.464	−0.362	9.9244	9448.45	−50.22***	−74.65***	0.21***
FTSE100	0.0120	0.0262	1.1053	−0.046	8.1293	5183.79	−50.55***	−73.64***	0.26***
S&P500	0.0198	0.0435	1.042	−0.432	8.630	6382.04	−49.29***	−69.68***	0.45***
EURO	0.0077	−0.00004	0.5905	0.013	3.8602	77.108	−35.64***	−49.67***	0.36***
GBP	0.0021	0.003	0.586	−0.405	8.811	6851.96	−49.03***	−69.21***	0.05***
YEN	0.0061	0.0077	0.672	−0.460	7.056	3444.92	−47.58***	−67.38***	0.08***

SD is the standard deviation. For all the time series, the descriptive statistics for cash daily returns are expressed in percentage.
MacKinnon's 1% critical value is −3.435 for the ADF and PP tests. The KPSS critical value is 0.739 at the 1% significance level.
ADF is the Augmented Dickey-Fuller[48] unit-root test statistic. PP is the Phillips-Perron[49] unit-root test statistic. KPSS is the Kwiatkowski, Phillips, Schmidt and Shin[50] stationarity test statistic. P-values are given into brackets. *,*** denotes significance at 1% level.

4.2. Preliminary analysis

Descriptive statistics and unit root and stationarity tests for all the daily returns are reported in Table 2.

As it’s shown on the table above, all the daily return series have a positive mean unless the Nikkei stock index return which have a negative one. Furthermore, those time series data are not normally distributed in fact that the 3rd and the 4th moment respectively are different from zero and three. More precisely, the return series are skewed and fat tailed. The same conclusion is confirmed by the Jarque − Bera statistic which indicates the non normality of our time series. As it is given by the table above, the results indicate that for all the time series the null hypothesis of presence of unit root is absolutely rejected by the Augmented Dickey-Fuller[48] (ADF), Phillips-Perron[49] (PP)[2] unit root tests. The Kwiatkowski, Phillips, Schmidt and Shin[50] (KPSS) stationarity test[3] indicates that all time series returns are stationary at a 1% significant level.

4.3. Graphical data analysis

In Fig. 1, we present graphs of the daily returns. The graph of the return series clearly shows that there are periods of low volatility followed by periods of high volatility (some tranquil periods as well as turbulent ones) which suggests volatility clustering and confirm the presence of ARCH effect.

[2] The lag length or the ADF test regressions is set using the Schwarz information criteria (SIC) and the bandwidth for the PP test regressions is set using a Bartlett Kernel.

[3] These unit root and stationary test results could be considered with caution because these tests have been later refined by several authors including Elliot et al (1996), Ng and Perron (2001).

Table 3
Long memory tests.

Panel. a	$\lvert r_t \rvert$				r_t^2			
	DAX	DOW JONES	NASDAQ	NIKKEI	DAX	DOW JONES	NASDAQ	NIKKEI
GPH Test								
$m = T^{0.5}$	0.62	0.48	0.49	0.25	0.59	0.35	0.45	0.26
$m = T^{0.6}$	0.52	0.44	0.47	0.35	0.47	0.35	0.38	0.27
$m = T^{0.7}$	0.45	0.43	0.38	0.29	0.41	0.37	0.36	0.24
Lo's RS Test								
	2.62746	1.89117	2.9925	0.90332	3.37581	2.56252	3.80299	1.61096
	{<0.005}	{<0.025}	{<0.005}	{<0.95}	{<0.005}	{<0.005}	{<0.005}	{<0.02}

Panel. b	$\lvert r_t \rvert$			r_t^2		
	CAC40	FTSE100	S&P500	CAC40	FTSE100	S&P500
GPH Test						
$M = T^{0.5}$	0.50	0.45	0.38	0.33	0.36	0.20
$M = T^{0.6}$	0.44	0.40	0.26	0.29	0.33	0.16
$M = T^{0.7}$	0.39	0.42	0.24	0.28	0.32	0.17
Lo's RS Test						
	1.70413	1.53375	1.52429	2.35328	2.00091	1.83822
	{<0.1}	{<0.2}	{<0.2}	{<0.005}	{<0.025}	{<0.05}

Panel. c	$\lvert r_t \rvert$			r_t^2		
	EURO	GBP	YEN	EURO	GBP	YEN
GPH						
$m = T^{0.5}$	0.34	0.45	0.41	0.28	0.30	0.31
$m = T^{0.6}$	0.26	0.37	0.41	0.21	0.33	0.40
$m = T^{0.7}$	0.16	0.26	0.32	0.10	0.25	0.29
Lo's RS Test						
	0.93629	1.67277	1.01412	1.56165	2.94725	2.02818
	{<0.9}	{<0.1}	{<0.9}	{<0.2}	{<0.01}	{<0.025}

Notes: (r_t), (r_t^2), and $\lvert r_t \rvert$ are respectively log return, squared log return and absolute log return. (m) denotes the bandwidth for the Geweke and Porter-Hudakk's[52] test.

4.4. Long range memory tests

Testing the presence of long range memory is important. Indeed, like many previous studies we use the absolute returns and the daily squared volatility returns as two proxies of daily volatility. To test long memory we employed two long-range memory tests: Lo's[51] test and the log-periodogram regression (GPH) of Geweke and Porter-Hudak Geweke and Porter-Hudak.[52]

Table 3 displays the results of long-memory tests including two tests: Lo's R/S and GPH test for three BANDWITH $m = T^{0.5}$, $m = T^{0.6}$ and $m = T^{0.7}$. As shown, Lo's R/S that tests in null hypothesis H_0 for the presence of short memory vs. Long memory (long dependence) indicates the presence of long memory in both absolute log return and squared log return. Furthermore, the GPH test rejects the null hypothesis of short memory. Indeed, the two time series are governed by long-memory process. Thus, the long memory ARCH type models are highly recommended for the volatility modeling of the studied return series. For each return series, we fit three specific models, namely FIGARCH, HYGARCH and FIAPARCH. The models are estimated on a rolling basis, using a window of 1000 observations, and under a skewed Student-t distribution. The three models are then used to produce one, five and fifteen -day-ahead variance forecasts. The models are compared using some of the methods described in the previous section.

5. Forecasts evaluation results

In our study we consider a multiple comparison without control test of Hansen et al (2011), where all the forecasts are compared against each other.

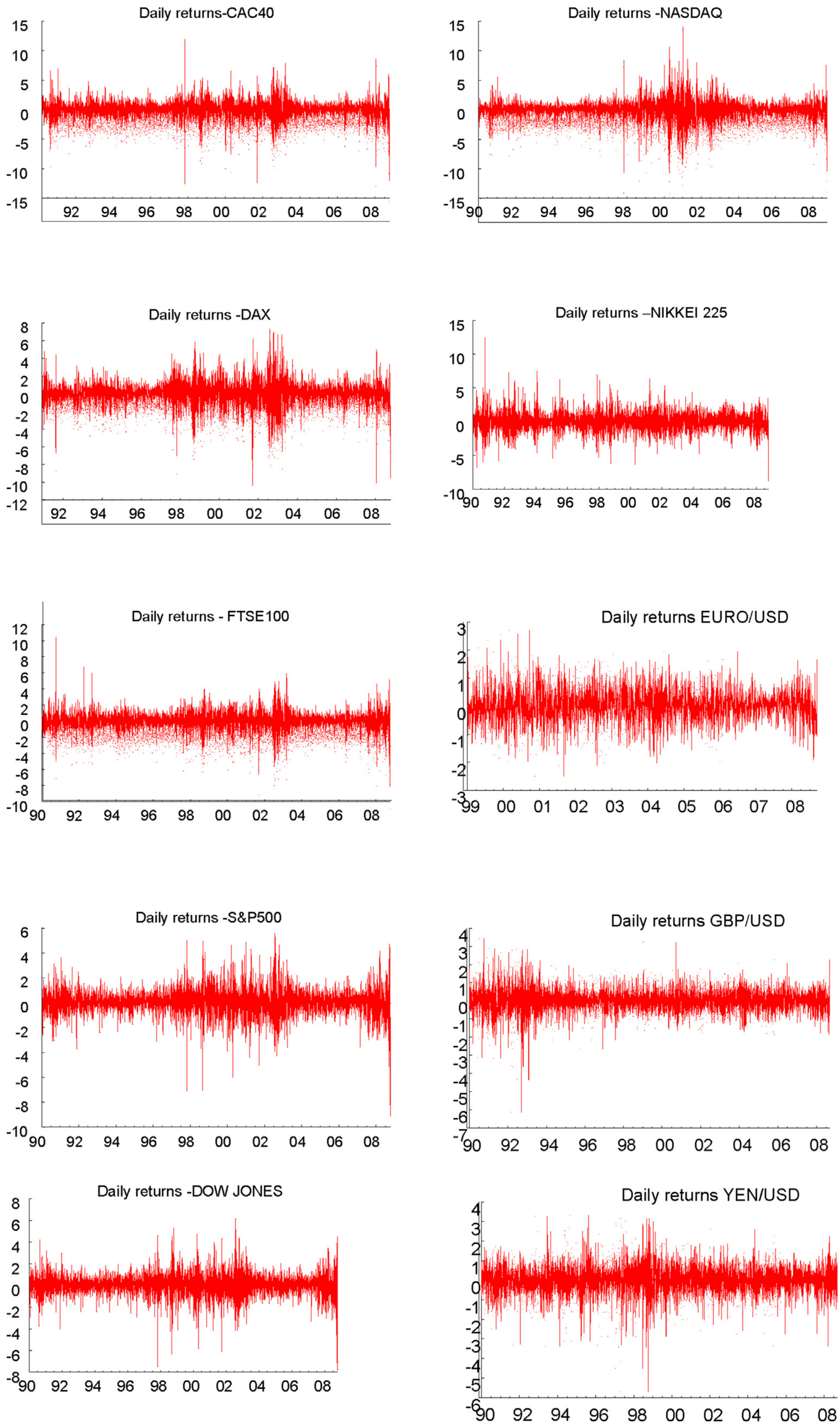

Fig. 1. Financial assets daily returns.

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Table 4
The predictive power of the FIGARCH, HYGARCH and FIAPARCH models under the skewed Student-t distribution (1-day, 5-day and 15-day horizons).

Horizons		Indices							Exchange rate		
		DAX	Dow Jones	NASDAQ	NIKKEI	CAC40	FTSE 100	S&P 500	Euro	GBP	Yen
AR(1) − FIGARCH											
1-day	MSE	0.210	0.22	19.14	1.079	2.003	10.43	2.789	0.410	0.029	0.076
	MAE	0.458	0.469	4.375	1.039	0.909	3.23	1.67	0.640	0.172	0.276
	TIC	0.459	0.796	0.715	0.964	0.690	0.798	0.621	0.460	0.833	0.854
5-day	MSE	1.365	0.644	4.391	3.391	1.429	2.174	0.701	0.156	0.163	0.069
	MAE	0.957	0.641	1.534	1.467	0.977	0.401	0.642	0.358	0.299	0.251
	TIC	0.538	0.690	0.606	0.581	0.500	0.714	0.523	0.402	0.593	0.415
15-day	MSE	0.998	0.799	2.091	1.944	1.049	0.844	0.348	0.178	0.113	0.095
	MAE	0.848	0.692	1.058	1.185	0.887	0.560	0.465	0.370	0.265	0.289
	TIC	0.470	0.552	0.458	0.482	0.454	0.620	0.452	0.475	0.541	0.496
AR(1) − HYGARCH											
1-day	MSE	0.207	0.218	18.95	1.117	1.905	10.37	2.726	0.411	0.029	0.076
	MAE	0.455	0.466	4.353	1.057	1.38	3.221	1.651	0.641	0.171	0.277
	TIC	0.458	0.795	0.709	0.964	0.479	0.794	0.610	0.461	0.832	0.854
5-day	MSE	1.364	0.646	4.367	3.389	1.4	2.168	0.691	0.156	0.163	0.069
	MAE	0.954	0.738	1.539	1.48	0.985	0.897	0.646	0.395	0.298	0.251
	TIC	0.539	0.484	0.600	0.577	0.488	0.709	0.512	0.403	0.594	0.415
15-day	MSE	0.996	0.798	2.08	1.948	1.042	0.848	0.355	0.178	0.113	0.095
	MAE	0.846	0.691	1.065	1.196	0.89	0.568	0.472	0.371	0.264	0.289
	TIC	0.471	0.553	0.478	0.479	0.443	0.616	0.445	0.475	0.542	0.496
AR(1) − FIAPARCH											
1-day	MSE	0.266	0.422	19.19	1.132	1.76	10.54	2.501	0.150	0.031	0.082
	MAE	0.516	0.650	4.380	1.064	1.327	3.246	1.582	0.671	0.176	0.286
	TIC	0.489	0.844	0.716	0.964	0.452	0.805	0.570	0.493	0.836	0.858
5-day	MSE	1.381	0.538	4.425	3.363	1.356	2.187	0.667	0.163	0.164	0.071
	MAE	1.007	0.691	1.555	1.463	1.005	0.880	0.665	0.366	0.300	0.253
	TIC	0.520	0.393	0.594	0.578	0.465	0.722	0.475	0.419	0.591	0.414
15-day	MSE	1.056	0.833	2.149	1.932	1.065	0.830	0.411	0.179	0.113	0.099
	MAE	0.889	0.780	1.109	1.181	0.902	0.540	0.529	0.370	0.265	0.296
	TIC	0.457	0.500	0.452	0.482	0.421	0.646	0.434	0.482	0.540	0.496

Moving to the out-of-sample comparison, we start from the outcomes of the MSE, MAE and TIC loss functions. In order to evaluate model performance across different market, we consider last 1000 observations for our out-of-sample forecasting exercise.

The evaluation results of the predictive ability of the FIGARCH, FIAPARCH and HYGARCH models adjusted by the skewed Student-t distribution for different horizons are included in the following tables.

Table 5
The predictive ability of the FIGARCH, HYGARCH and FIAPARCH models under the skewed Student-t distribution based on MZ regression (1969).

		AR(1) − FIGARCH(1.d.1)			AR(1) − HYGARCH(1.d.1)			AR(1) − FIAPARCH(1.d.1)		
		α	β	R^2	α	β	R^2	α	β	R^2
Indices	DAX	0.11	−0.04	0.000533006	0.11	−0.04	0.000537348	0.08	−0.01	9.61483e-005
	DOW JONES	−0.16	0.38	0.00643073	−0.16	0.38	0.00641623	−0.14	0.306	0.00722727
	NASDAQ	−0.033	0.07	0.000642019	−0.03	0.07	0.000691542	−0.02	0.06	0.000665357
	NIKKEI	0.068	−0.002	1.57335e-006	0.06	0.000	2.12012e-007	0.04	0.01	8.50767e-005
	CAC40	0.06	−0.01	9.88671e-005	0.07	−0.01	9.80916e-005	0.033	0.026	0.00028156
	FTSE 100	0.04	0.01	9.27359e-005	0.04	0.01	6.49319e-005	0.02	0.05	0.0010579
	S&P 500	−0.11	0.28	0.00369096	−0.11	0.29	0.00383687	−0.13	0.29	0.0070623
Exchange rate	Euro	0.03	−0.08	4.03567e-005	0.04	−0.10	5.87205e-005	−0.002	0.03	1.14566e-005
	GBP	0.17	−0.64	0.00614568	0.18	−0.65	0.00619664	0.21	−0.77	0.00943436
	Yen	0.03	−0.08	8.88553e-005	0.03	−0.08	8.78054e-005	0.03	−0.08	0.000116371

Table 6
Out-of-sample short and long VaR estimation results (1-day, 5-day and 15-day-ahead).

	Short trading position							Long trading position				
	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value	Quantile	Failure rate	Kupiec LRT	P-value	DQT	P-value
Panel a.												
CAC 40												
15	0.95	0.965	3.892	0.046	7.068	0.419	0.05	0.071	4.916	0.025	18.26	0.011
	0.99	0.998	6.83	0.01	5.294	0.622	0.01	0.019	6.471	0.009	18.99	0.007
5	0.95	0.952	0.541	0.455	7.529	0.371	0.05	0.056	1.614	0.201	12.77	0.075
	0.99	0.996	3.099	0.081	2.811	0.932	0.01	0.014	0.834	0.366	11.33	0.130
1	0.95	0.965	5.268	0.021	9.717	0.205	0.05	0.066	4.918	0.026	15.16	0.033
	0.99	0.993	1.015	0.313	2.372	0.936	0.01	0.023	12.485	0.000	33.65	0.000
Panel b.												
DAX												
15	0.95	0.969	7.777	0.007	10.54	0.159	0.05	0.056	0.986	0.318	8.069	0.324
	0.99	0.999	9.629	0.003	6.694	0.461	0.01	0.019	9.284	0.005	41.6	0.002
5	0.95	0.967	7.780	0.004	10.64	0.154	0.05	0.062	1.984	0.155	8.376	0.299
	0.99	1.000	13.478	0.001	28.25	0.001	0.01	0.018	6.470	0.009	38.91	0.000
1	0.95	0.965	5.268	0.021	10.21	0.176	0.05	0.056	0.730	0.392	7.345	0.393
	0.99	0.998	9.626	0.001	6.694	0.461	0.01	0.014	1.437	0.230	23.84	0.001
Panel c.												
Dow Jones												
15	0.95	0.963	2.75	0.1	7.211	0.402	0.05	0.063	2.391	0.123	17.45	0.012
	0.99	0.995	0.436	0.513	12.42	0.080	0.01	0.03	17.987	0.01	53.32	0.001
5	0.95	0.958	1.421	0.236	6.51	0.465	0.05	0.066	3.34	0.071	24.21	0.004
	0.99	0.997	3.092	0.081	2.594	0.921	0.01	0.019	4.096	0.046	24.52	0.009
1	0.95	0.954	0.345	0.556	6.103	0.527	0.050	0.061	2.387	0.122	22.22	0.002
	0.99	0.992	0.433	0.510	12.27	0.091	0.010	0.013	0.830	0.362	2.480	0.928
Panel d.												
FTSE 100												
15	0.95	0.969	7.778	0.004	10.49	0.161	0.05	0.058	0.99	0.319	8.076	0.323
	0.99	0.999	9.629	0.004	6.695	0.463	0.01	0.022	9.282	0.004	41.61	0.001
5	0.95	0.969	7.776	0.005	10.59	0.153	0.05	0.064	1.988	0.160	8.377	0.299
	0.99	0.999	13.476	0.001	28.31	0.001	0.01	0.021	6.474	0.011	38.93	0.001
1	0.95	0.965	5.268	0.021	10.21	0.176	0.05	0.056	0.730	0.392	7.345	0.393
	0.99	0.998	9.626	0.001	6.694	0.461	0.01	0.014	1.437	0.230	23.84	0.001
Panel e.												
NASDAQ												
15	0.95	0.973	10.865	0.001	11.35	0.121	0.05	0.06	0.989	0.322	2.351	0.939
	0.99	0.994	1.016	0.3111	1.014	0.992	0.01	0.017	3.074	0.080	28.35	0.001
5	0.95	0.968	7.776	0.004	9.611	0.214	0.05	0.070	7.529	0.007	10.55	0.154
	0.99	0.996	1.889	0.171	1.789	0.969	0.01	0.016	2.187	0.14	28.4	0.001
1	0.95	0.951	0.021	0.884	7.653	0.363	0.050	0.055	0.510	0.474	3.360	0.849
	0.99	0.990	0.000	1.000	0.420	0.999	0.010	0.014	1.437	0.230	28.54	0.000
Panel f.												
Nikkei 225												
15	0.95	0.957	0.540	0.462	4.386	0.735	0.05	0.042	1.079	0.23	11.55	0.119
	0.99	0.995	1.012	0.311	4.092	0.771	0.01	0.013	1.435	0.232	8.349	0.301
5	0.95	0.946	0.082	0.769	7.299	0.399	0.05	0.048	0.085	0.770	13.63	0.060
	0.99	0.994	3.091	0.080	4.235	0.751	0.01	0.008	0.102	0.749	11.24	0.129
1	0.95	0.949	0.020	0.884	6.611	0.470	0.050	0.048	0.085	0.770	13.63	0.058
	0.99	0.995	3.093	0.078	4.222	0.753	0.010	0.010	0.000	1.000	9.692	0.206
Panel g.												
S&P 500												
15	0.95	0.963	5.265	0.022	8.169	0.315	0.05	0.065	3.301	0.07	26.16	0.000
	0.99	0.996	1.884	0.170	1.881	0.964	0.01	0.019	9.279	0.003	27.71	0.001
5	0.95	0.962	2.743	0.095	9.294	0.230	0.05	0.068	4.915	0.025	32.57	0.000
	0.99	0.998	6.823	0.009	4.990	0.661	0.01	0.015	0.832	0.363	7.259	0.399

1	0.95	0.954	0.342	0.559	7.971	0.333	0.05	0.066	4.346	0.039	31.39	0.000
	0.975	0.985	4.777	0.028	6.584	0.473	0.025	0.031	1.373	0.241	21.97	0.002
	0.99	0.996	4.706	0.030	3.838	0.798	0.01	0.011	0.097	0.754	0.947	0.995
Panel h.												
EURO												
15	0.95	0.96	1.078	0.301	5.821	0.561	0.05	0.046	0.029	0.889	10.61	0.159
	0.99	0.995	1.884	0.171	1.731	0.970	0.01	0.013	0.102	0.741	0.653	0.996
5	0.95	0.940	1.613	0.199	8.226	0.311	0.05	0.057	0.331	0.562	10.10	0.187
	0.99	0.994	0.436	0.503	6.434	0.471	0.01	0.013	0.099	0.752	6.260	0.511
1	0.95	0.942	1.284	0.257	8.656	0.278	0.05	0.055	0.510	0.474	13.21	0.068
	0.99	0.993	1.015	0.313	4.696	0.696	0.01	0.012	0.379	0.537	9.802	0.200
Panel h.												
Yen												
15	0.95	0.959	1.085	0.300	6.190	0.514	0.05	0.055	0.510	0.472	7.206	0.409
	0.99	0.984	0.302	0.756	0.671	0.995	0.01	0.014	1.437	0.234	16.22	0.022
5	0.95	0.956	0.541	0.462	5.951	0.548	0.05	0.061	2.387	0.120	12.18	0.096
	0.99	0.996	3.095	0.080	2.954	0.891	0.01	0.011	0.004	0.554	18.82	0.007
1	0.95	0.955	0.543	0.460	5.995	0.540	0.050	0.061	2.387	0.122	12.13	0.096
	0.99	0.995	3.093	0.078	2.942	0.890	0.010	0.010	0.000	1.000	18.7	0.008
Panel k.												
GBP												
15	0.95	0.967	6.047	0.009	8.699	0.272	0.05	0.055	0.407	0.568	4.439	0.730
	0.99	0.994	1.019	0.315	1.155	0.988	0.01	0.017	5.224	0.019	11.55	0.117
5	0.95	0.964	6.035	0.014	8.531	0.289	0.05	0.059	1.282	0.256	5.819	0.564
	0.99	0.998	6.823	0.010	4.969	0.661	0.01	0.012	0.099	0.755	0.929	0.992
1	0.95	0.958	1.421	0.233	7.702	0.359	0.050	0.055	0.510	0.474	5.563	0.591
	0.99	0.994	1.886	0.169	1.969	0.961	0.010	0.010	0.000	1.000	0.603	0.998

Table 4 reports the full set of empirical results. Focusing on the MSE and MAE loss functions, all empirical models seem very similar for all indices and exchange rate, with the null hypothesis of zero loss function differential being rejected only in few cases. When we consider the TIC loss function, the null hypothesis is rejected more frequently, with the finding seemingly independent of the sample used for model evaluation. In this case, there are some differences across exchange rate and indices, but the outcomes suggest a preference of FIGARCH and FIAPARCH over HYGARCH. All models are equivalent as they are all included in the confidence set independently of the loss function used for their evaluation. In summary, there is not a clear preference for a specific model. Model preference depends on the loss function under consideration and on the sample period used for model evaluation.

Table 5 highlights that FIAPARCH (1,d,1) is always preferred to its FIGARCH(1,d,1) and HYGARCH (1,d,1) counterpart for all return series. The results for FIGARCH (1,d,1) and HYGARCH (1,d,1) are quite similar. Therefore, FIAPARCH is the preferred conditional volatility model. This is confirmed by a relatively higher R^2 of the MZ regression MZ regression[41] for FIAPARCH model than those of FIGARCH and HYGARCH models. This finding is not surprising as FIAPARCH is more flexible than FIGARCH and HYGARCH, can exhibit long memory, volatility clustering, asymmetry and leverage, and there are no restrictions on the parameters of the model.

The main conclusion is that when we give a great importance to volatility spikes, most models seem relevant, and simple specifications may perform as well as their more flexible counterparts. If we consider the evolution over time of the conditional volatility, then more flexible models are to be preferred. Our findings are consistent with those of Chortareas et al (2011), Balaban (2004) Bollerslev, Poon and Granger (2003) and Xekalaki and Degiannakis[53] who they show that the predictive ability of the AR (1) -APARCH (1.1) adjusted by the skewed Student-t distribution outperforms those of the AR (1) -GJR (1.1), and AR (1) -IGARCH (1,1) models. Bollerslev et al,[5] Diebold and Lopez (1996) and Lopez,[54] confirm that none of the GARCH models can out-perform all of the others under the criteria of different loss functions.

6. Out-of -sample VaR estimation

In this sub-section, we estimate the out of sample h-Step-ahead VaR using AR(1)-FIAPARCH (1.d.1) model under the skewed Student-t innovation's distribution and this for each of the seven stock index and the three exchange rate returns. We compute the failure rate for both long and short trading position for different three horizons. The failure rate for the short trading position denotes the percentage of positive returns larger than the VaR prediction. However for the long trading positions, the failure rate is the percentage of negative returns smaller than the VaR prediction. We also compute the Kupiec's[37] LR tests and the Dynamic Quantile (DQ) test. For every horizon, two VaR levels are considered for each trading position. More precisely, (α) equals to 0.05 or 0.01 for the short trading position. However, its equals to 0.95 or 0.99 for the long trading position.

Table 6 below provides the out of sample h-Step-ahead VaR results for financial time series returns using AR(1)-FIAPARCH (1.d.1) model under the skewed Student-t distribution. The out-of-sample VaR estimates are computed based on 1000 observations (i.e. last five years). We re-estimate the AR(1)-FIAPARCH model every 50 observations in the out-of-sample period. We report the failure rate along with the Kupiec's[37] LR test and the Dynamic Quantile test and their corresponding p-values. The results clearly indicate that for the horizon of 15-day- forecasting exercise, the VaR model based on the skewed Student-t AR (1)-FIAPARCH model fails to model positive and negative returns. In fact, the hypothesis of model adequacy is usually rejected as evidenced by the considerable difference between the prefixed level (α) and the failure rate. Results indicate that for five-day-ahead VaR forecasts, the asymmetric FIAPARCH provides better results. The accuracy of VaR forecast under the skewed Student-t FIAPARCH is even higher when we consider an horizon of one-day-ahead. In fact, that hypothesis of correct VaR model is strongly accepted for short trading position as well as long trading position. Therefore, the skewed Student-t FIAPARCH model provides high short and long VaR forecast results for a short horizon (one day).

In summary, the obtained results confirm that long range memory GARCH- type models and especially the FIAPARCH model have strong predictive power of the conditional volatility. The backtesting exercise confirmed the skewed Student-t FIAPARCH model provides more accurate VaR for a short horizon whether for a long or short position. This can be explained by the fact that this model takes into account both the main stylized facts of financial time series: fat tails, asymmetry, clustering of volatility and a long memory. Our results support works that confirm that the quality of the VaR forecasts depends on the ability of volatility model to take into account the stylized facts of the studied time series (eg Stiglitz, 2010; Roldán, 2009).

7. Conclusion

In this paper, at first we focused on the predictive capacity of three conditional variance models with long memory. More precisely, our study involved ten sets of financial assets. The skewed Student-t distribution was used to adjust the conditional volatility models to take account of the asymmetry criterion of the daily return series. We focused on forecast evaluation and comparison where the forecast accuracy is measured by a statistical criterion. We tried to study the predictive ability of FIGARCH, HYGARCH and FIAPARCH models adjusted by the skewed Student-t distribution for three horizons (one, five and fifteen days). The results of the forecasting exercise show that the three models have the same predictive power. However, result based on the MZ regression (1969) confirms that the FIAPARCH model has relatively better predictive ability compared to FIGARCH and HYGARCH models. Then, we estimated the h-day-ahead VaR for both short and long trading positions based on the skewed Student-t FIAPARCH model. The forecasting exercise is done for three different horizons (1-day; 5-day and 15-day). Backtesting VaR results indicate that this model provides more correct short and long VaR for one-day-ahead forecast than other horizons. We conclude that considering jointly for volatility clustering, asymmetry, long range memory in volatility and skewed distributed return innovations performs better the VaR forecasts for both short and long trading positions. These results may have potential implications for risk management and construction of hedging strategies.

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