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The Forced Safety Effect: How Higher Capital Requirements Can Increase Bank Lending

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ABSTRACT

Government guarantees generate an implicit subsidy for banks. A capital requirement reduces this subsidy, through a simple liability composition effect. However, the guarantees also make a bank undervalue loans that generates surplus in states of the world in which it defaults. Raising the capital requirement makes the bank safer, which alleviates this problem. We refer to this mechanism, which we argue is empirically relevant, as the *forced safety effect*.

SINCE THE GLOBAL FINANCIAL CRISIS, bank capital requirements have been substantially tightened.[1] The merits of these reforms have been fiercely debated. Critics argue that higher capital requirements raise banks' cost of funds, thereby reducing credit provision and dampening economic activity.[2] Others, however, argue that increases in banks' private cost of funds are not necessarily relevant from a normative perspective (see, for example, Hanson, Kashyap, and Stein (2011) and Admati et al. (2013)). Nonetheless, the idea

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[1] Specifically, minimum tier-one capital requirements were raised from 4% to 6% of risk-weighted assets, but additional "buffers" were created to account, inter alia, for the systemic importance of the institution, for the economic cycle, and to prevent accidental breaches of the minimum. Effective requirements for large global banks are now in the double digits as a percentage of risk-weighted assets.

[2] See, for instance, p. 10 of Institute of International Finance (2011).

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that such increases result in a reduction in lending has seeped into conventional wisdom.

In this paper, we challenge such conventional wisdom. We develop a model in which capital is costly from a bank's perspective due to an implicit subsidy from a government guarantee. At a given level of lending, a higher capital requirement reduces the value of the subsidy and hence it increases the bank's weighted average cost of funds. But it also makes the bank safer, which can actually make the marginal loan more appealing and therefore induce an increase in lending.

How can the marginal loan become more appealing under a higher capital requirement? To build intuition, it is useful to consider why the marginal loan may not have been financed in the first place. Despite implying a subsidy, a government guarantee can generate a mechanism analogous to the debt overhang problem in Myers (1977): the bank undervalues a loan (and potentially passes on it) if a portion of its surplus, in effect, accrues to the taxpayer, who is backing the guarantee. We refer to this problem as the *guarantee overhang* problem.

Making the bank safer means that loan surplus accrues to the bank's shareholders in more states of the world. If, for the marginal loan, this surplus is positive in these specific states, forcing the bank to be safer makes the marginal loan more appealing as doing so alleviates the guarantee overhang.

Our model has a single period in which a representative bank faces a capital requirement and finances loans with a mix of liabilities that can be interpreted as deposits and capital. The bank starts with existing loans and can make new ones.

The bank maximizes the expected payoff of initial shareholders. Deposits are insured by the government with no fee, and hence are implicitly subsidized. This has two implications. First, the capital requirement is binding in equilibrium: the bank chooses lending and adjusts capital to meet the requirement. Second, the objective function can be written as the sum of the economic surplus from lending and a term that captures the value of the implicit subsidy (as in Merton (1977)).

We first assume that the payoffs to *new* loans are perfectly correlated and that new lending yields an increasing and strictly concave aggregate payoff. This design allows us to study how the equilibrium level of lending responds to marginal changes in the capital requirement using a first-order approach (we refer to the response to an increase in the capital requirement as the *lending response*). The derivative of the subsidy with respect to lending, the marginal subsidy, is a wedge in the bank's first-order condition. This wedge captures the underlying moral hazard problem arising from the guarantee. Economic surplus is independent of the capital requirement. Hence, if an increase in the requirement increases the marginal subsidy, the bank increases lending.

Increasing the capital requirement has two effects on the marginal subsidy. First, a smaller fraction of the marginal loan is financed by deposits. This generates a well-understood composition effect: the bank substitutes subsidized deposits with capital, decreasing the marginal subsidy. This effect also exactly captures how the capital requirement increases the bank's weighted average cost of funds.[3]

However, the change in the capital requirement also affects whether the bank defaults in any given state. To go further, we note that the appropriate measure of surplus from the marginal loan is its residual cash flow. This variable, which we denote by Z, is the marginal loan's realized payoff minus the repayment on the deposits raised to finance it. In the states in which the bank survives, Z comes as an addition to the shareholder's payoff. But if the bank defaults, Z accrues to the taxpayer.

We can now elaborate on the second effect, which we argue is overlooked by conventional wisdom. Consider the default boundary, that is, the set of states in which the bank can just repay depositors. Increasing the requirement increases the buffer against losses and shifts this boundary. As a result, there are more states in which Z accrues to shareholders. In particular, increasing the capital requirement makes shareholders internalize the expected value of Z along the default boundary.

This second effect reflects the fact that the requirement forces the bank toward safety. Because the bank could have chosen to be safer and to internalize these cash flows (by operating at a higher capital ratio than the requirement) but preferred not to, we refer to the second effect the *forced safety effect* (FSE).

If, in expectation, the residual cash flows along the default boundary are positive, the bank is internalizing cash flows that increase shareholders' payoff. In this case, the FSE is positive and increases the value of the marginal subsidy. If, in contrast, the residual cash flows are negative, the FSE makes the bank internalize more losses, decreasing the value of the marginal subsidy and reinforcing the composition effect.

Our main theoretical contribution is to show that (i) the FSE can be positive and (ii) the FSE can dominate the composition effect, which is why lending can increase with the capital requirement.

In equilibrium, the bank may optimally choose to finance negative net present value (NPV) loans and/or not to finance positive NPV loans. Showing that the guarantee overhang can lead to the latter is a third contribution of this paper. Reasoning in terms of residual cash flows also helps clarify the link between the guarantee and debt overhang. At the heart of both is that a portion of the residual cash flows from investment accrue to another stakeholder.[4] In Myers (1977), residual cash flows can only be positive.[5] In our context, residual cash flows can be positive or negative. This is why government guarantees can

[3] The change in average funding costs is relevant in determining the impact on the bank's profit. See Kisin and Manela (2016) for a quantification.

[4] Papers that link bank underlending to the debt overhang problem include Hanson, Kashyap, and Stein (2011), Admati et al. (2018), and Jakucionyte and van Wijnbergen (2018). Bank behavior exhibiting symptoms of an overhang problem has been noted in different contexts in recent literature. See, for instance, Gropp et al. (2019), who provide evidence from stress tests, or Duffie, Andersen, and Song (2019), who show that funding-value adjustments correspond to the transfer to existing debtholders associated with debt overhang.

[5] Investment is fully financed with equity, and hence the residual cash flow is the cash flow itself.

lead a bank to undervalue positive NPV loans and/or overvalue negative NPV loans. The latter is typically interpreted in terms of risk-shifting (Kareken and Wallace (1978)). We argue that the former has a similar interpretation: the bank undervalues positive NPV loans precisely because some of the surplus they generate only reduces the risk shifted onto the taxpayer.

We calibrate the model and find an economically significant, positive FSE under plausible conditions: targeting the situation facing a global bank in 2017, we find that the FSE fully offsets the negative forces (i.e., the lending response is slightly positive). However, at levels of capital requirements prevailing before the global financial crisis, lending responses are more likely to have been negative.

Overall, our sensitivity analysis reveals that lending responses are likely to exhibit substantial variation. As a result, one should not expect a homogeneous relationship between capital requirements and bank lending. In Section IV, we discuss the empirical predictions that arise from our analysis and we argue that our findings help reconcile results in the empirical literature.

The key ingredient for a positive FSE is a form of residual cash flow heterogeneity. In particular, a positive FSE requires that, in equilibrium and in expectation along the default boundary, the residual cash flow of the marginal loan (i.e., Z) must be greater than the residual cash flow of the average asset on the bank's balance sheet (which is exactly zero on the default boundary). For the marginal subsidy to be positive (i.e., the bank passes on positive NPV loans) the same is required but the expectation is conditional on the entire default region, and not just the default boundary. This subtle difference implies that the FSE and the marginal subsidy can have different signs. Yet they both rely on residual cash flow heterogeneity which, as we argue, is missing from many models in which a negative lending response always arises.

The relevant residual cash flow heterogeneity can come from an imperfect correlation between legacy and new loans. It can also come from other sources such as heterogeneity among new loans, differences in capital requirements, or background risk. We consider examples that depart from the first-order approach to show that a positive FSE (and/or a positive marginal subsidy) also can arise (i) when *all* loans are perfectly correlated or (ii) when the bank starts from scratch (i.e., has no legacy loans or debt).

The idea that tighter capital requirements increase banks' average costs of funds and leads to a credit contraction has been formalized by Thakor (1996), among others. As Suarez (2010) discusses, a usual way to capture such an effect is to assume an exogenous cost of issuing outside equity. This effectively makes the capital requirement a tax on lending. In the extreme case in which aggregate bank capital is in fixed supply, a higher capital requirement can only shrink banks' balance sheets. We do not restrict equity issuance in our model.

Many models linking capital requirements and bank lending assume that loans are infinitesimal, face the same capital requirement, and, conditional on an aggregate state, have independent and identically distributed (iid) payoffs. This payoff structure is convenient in that it enables aggregation to a representative bank. However, it also implies that idiosyncratic risk is automatically diversified: the default region and the default boundary are pinned down by the aggregate state. As a result, residual cash flows on all loans are identically distributed over the default boundary. Since, by definition, the default boundary is the locus where the average residual cash flow is nil, it must be the case that it is nil in expectation for any given loan, including the marginal one. Therefore, the FSE can only be nil. Such a structure assumes away the relevant heterogeneity.[6]

In Repullo and Suarez (2004) and Martinez-Miera and Suarez (2014), there are two types of conditionally *iid* loans, but banks fully specialize in equilibrium. So, endogenously, all of the loans on a given bank's balance sheet are identically distributed, meaning that the FSE is again nil. Full specialization is also the key feature that implies a nil FSE in Rochet (1992) and Harris, Opp, and Opp (2017). In practice, however, it is impossible for banks to fully specialize, especially if they wish to be large.

To preserve tractability in our first-order approach, we also impose some structure in the payoff of new loans (i.e., perfect correlation and diminishing returns). In isolation, a portfolio of such new loans can only exhibit a negative FSE. But imperfectly correlated legacy loans introduce sufficient heterogeneity to make a positive lending response possible.[7]

The main takeaway from our paper for the policy debate is that, whether the FSE dominates the composition effect or not, in many cases it will make the lending response substantially less negative than otherwise. Overlooking this effect is tantamount to confusing how average, rather than marginal, costs of funds are affected by changes in capital requirements.

The remainder of the paper is organized as follows. In Section I, we outline the environment used in most of the analysis. In Section II, we use a first-order approach to study the lending response and introduce the FSE. In Section III, we consider specific cases to explain the mechanisms behind the FSE and in Section IV we consider the FSE's empirical relevance.

I. The Environment

In this section, we discuss the environment that we will use in most of our analysis. In Section III, we consider slight deviations from this environment, which we will specify in due course.

[6] Models following such an approach include Repullo and Suarez (2013), Corbae and D'Erasmo (2017), Elenev, Landvoigt, and van Nieuwerburgh (2017), Malherbe (2020), and Malherbe and McMahon (2019).

[7] Begenau (2020) potentially has the required heterogeneity for a positive FSE. However, in the spirit of Thakor (1996), she proxies for the implicit subsidy using a reduced-form function of which the capital requirement is not an argument. Hence, the cross-partial is nil. Nonetheless, she finds that a positive aggregate lending response can arise due to a general equilibrium effect: higher capital requirements can *decrease* banks' funding costs because they reduce the aggregate supply of deposits (which carry an endogenous convenience yield).

Assets	Liabilities
(new loans) x	$\kappa + c = \gamma_b x + \gamma_a \lambda$ (capital)
(legacy loans) λ	$d = (1-\gamma_b)x + (1-\gamma_a)\lambda$ (deposits)

Figure 1. The bank's balance sheet. The parameters γ_a and γ_b are the capital requirements on legacy loans (λ) and new loans (x), respectively, κ is existing capital, d is deposits raised, and c is net capital issuance (which can be negative). The equalities on the liabilities side of the balance sheet arise because capital requirement constraint is always binding in equilibrium (see Section I.B).

A. The Baseline Model

There are two dates, 1 and 2. There is a bank, a continuum of households that own the bank's liabilities, and a government. Households are risk-neutral and do not discount the future, and they supply funds perfectly elastically with an opportunity cost of funding equal to one. We focus on the date 1 decision of the bank. The random variables A and B capture the realized state of the economy at date 2, and are distributed according to a joint density function $f(A, B)$ with support $[a_L, a_H] \times [b_L, b_H]$, where $a_L, b_L \geq 0$ and $\mathbb{E}[A] = \mathbb{E}[B] = 1$. Figure 1 summarizes the bank's balance sheet.

Predetermined Variables: At date 1, legacy loans are on the bank's balance sheet. Their total book value is $\lambda \geq 0$, and they generate a risky date 2 payoff of $A\lambda$. Without loss of generality, the bank holds no cash. The bank has existing deposits that can be withdrawn at par at date 1. Given deposits are supplied perfectly elastically, it is only necessary to consider their level at the end of date 1. Thus, we do not define notation for existing deposits. The book value of capital at the beginning of date 1 is denoted by $\kappa \geq 0$.

Decision Variables: The bank decides how much to lend. We denote the total amount of new lending by $x \geq 0$. New loans also mature at date 2 and yield a stochastic payoff. Our baseline assumption is that new loans have a continuous payoff function $BX(x)$, which is increasing, strictly concave in x, and twice differentiable in the strictly positive domain, with $X(0) = 0$, and $\lim_{x\to 0} X_x(x) = \infty$. In Section III, we also consider linear and discrete lending opportunities.

At the same time, the bank adjusts its liabilities, namely, capital and deposits. We denote the change in capital by c. The change in capital can be negative (as long as $\kappa + c > 0$). In this case, the change can be interpreted as a dividend payment or the value of a share repurchase. If c is positive, it should be interpreted as the bank raising more capital. In this case, an amount c is raised in exchange for date 2 cash flow rights. The corresponding total repayment is denoted C. This repayment is determined in equilibrium and can be contingent on any realized variable. The bank's chosen level of deposits is denoted by d.

Even though they all correspond to households, we use different terms for holders of different bank-issued liabilities. Initial shareholders own the initial (i.e., inside) equity, investors hold new capital, and depositors hold deposits.

Deposit Insurance, Taxes, and the Capital Requirement Constraint: The government insures bank deposits with no premium: in the event the bank has insufficient cash flows to repay depositors at date 2, the government makes depositors whole and breaks even via an ex post lump-sum tax on households. This is the source of moral hazard in the model. Given this guarantee, deposits pay no interest. If the bank defaults on deposits, no payment to any other liability is allowed.

The bank faces a capital requirement constraint that takes the form

$$\kappa + c \geq (\gamma_a \lambda + \gamma_b x), \tag{1}$$

where $\gamma_a, \gamma_b \in (0, 1)$ are the capital requirements on legacy and new loans, respectively. Both γ_a and γ_b are set by the government. To operate at date 1, the bank must have a book value of capital $(\kappa + c)$ sufficient to satisfy the capital requirement constraint. Otherwise, the government forces the bank to close, initial shareholders walk away with zero, and we impose $x = c = 0$. The assumption $\mathbb{E}[A] = 1$ ensures that, in equilibrium, it is never profitable for the bank to close.[8]

In real-world regulation, typically there is no distinction between legacy and new loans, as new loans become legacy loans immediately after being made. Hence, in most of what follows, we assume that $\gamma_a = \gamma_b = \gamma$, and we refer to γ as *the requirement*. Still, the case of multiple requirements, which we study separately below, allows us to clarify some concepts. Our main result is that equilibrium lending may increase with γ. This result continues to hold for γ_a and γ_b when considered separately.

B. Setting up the Analysis

Date 2 Default on Deposits: If date 2 cash flows are too low to repay the depositors, the bank defaults on them. This happens when

$$\underset{\text{promised repayment}}{d} > \underset{\text{total cash flow}}{BX + A\lambda.}$$

We can define two functions, either of which can be used to define the set of states in which the bank is on the brink of default:

$$a_0(B) \equiv \frac{\big((1-\gamma_b)x + (1-\gamma_a)\lambda\big) - BX}{\lambda}; \; b_0(A) \equiv \frac{\big((1-\gamma_b)x + (1-\gamma_a)\lambda\big) - A\lambda}{X}. \tag{2}$$

[8] We study date 1 bank closure in Appendix A.

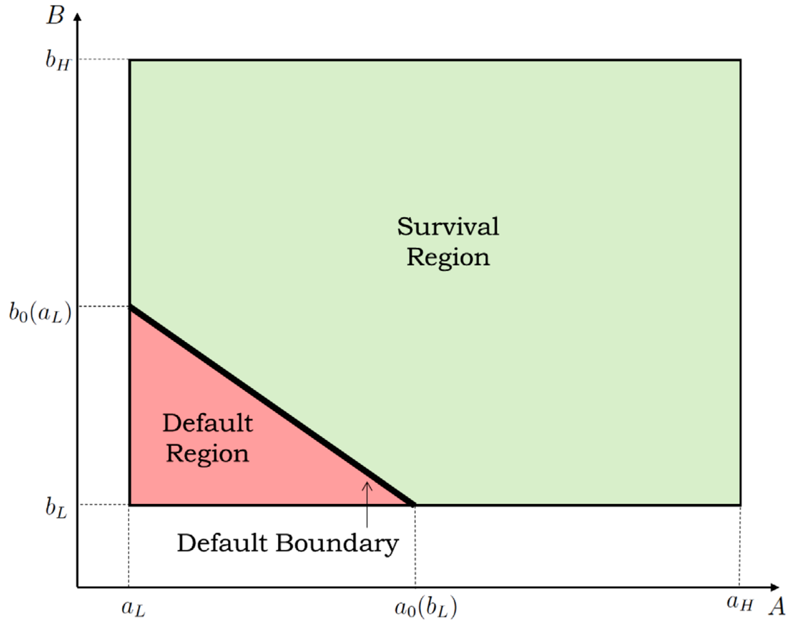


Figure 2. Default region and boundary. This figure illustrates the default boundary and the default region in the state space $\{[a_L, a_H] \times [b_L, b_H]\}$. The default region is the set of points such that $BX + A\lambda - (1 - \gamma_b)x - (1 - \gamma_a)\lambda \leq 0$. The default boundary is the locus at which this condition just binds. The parameters γ_a and γ_b are the capital requirements on legacy loans (λ) and new loans (x), respectively. The term BX denotes the payoff to new loans and the term $A\lambda$ denotes the payoff to legacy loans. The functions $a_0(B)$ and $b_0(A)$ are defined in equation (2). (Color figure can be viewed at wileyonlinelibrary.com)

We refer to these states as the *default boundary*. Figure 2 depicts the default boundary (the thick black line) and the entire default region (the red triangle) as subsets of the state space $\{[a_L, a_H] \times [b_L, b_H]\}$. Define the set Δ as all pairs $\{A, B\}$ in the (triangular) default region, the set $\diagdown\!\!\diagdown$ as all pairs along the default boundary, and the operators $\mathbb{E}[. \mid \diagdown\!\!\diagdown]$ and $\mathbb{E}[. \mid \Delta]$ as conditional expectations taken over these sets. The probability that the bank does not default is given by $p = \int\int_{\{A,B\}\notin\Delta} f(A, B)dAdB$.

Pricing of New Capital: Investors act competitively, so that, in equilibrium, they just break even in expectation. For the moment, assume that the bank issues new capital (i.e., $c \geq 0$). Denoting by $C(A, B)$ the contingent date 2 repayment to new capital, we have

$$\int_{b_L}^{b_H}\int_{a_L}^{a_H} C(A, B)f(A, B)dAdB = c. \tag{3}$$

To be able to interpret c as capital issuance, the underlying securities should be junior to deposits. We therefore impose

$$C(A, B) \leq 0,\ \forall\{A, B\} \in \Delta. \tag{4}$$

We also impose limited liability for investors, which implies $C(A, B) \geq 0$, and for initial shareholders, which implies $C(A, B) \leq BX + A\lambda - d$. However, we do not restrict new capital to be a particular form of security. What matters is that capital will absorb losses before the guarantee is called. In practice, one can think of it as, for instance, seasoned equity or subordinated debt.

Initial Shareholders' Payoff: If c is positive, the expected final wealth of initial shareholders is

$$w \equiv \int\int_{\{A,B\}\notin\Delta} \left[BX(x) + A\lambda - d - C(A, B)\right] f(A, B)dAdB.$$

Substituting in break-even condition (3) gives

$$w = \int\int_{\{A,B\}\notin\Delta} \left[BX(x) + A\lambda - d\right] f(A, B)dAdB - c.$$

If c is negative, w is identical to the above, as initial shareholders will receive $-c$ with certainty at date 1. In the absence of frictions that affect the contracting between initial shareholders and investors in new capital, the shadow value of initial capital is equal to the price of new capital. Hence, it is unnecessary to treat positive and negative c as separate cases in what follows. Accordingly, there is no need to distinguish between the owners of different classes of bank capital. For simplicity, we refer to both the initial shareholders and the investors in new capital collectively as *the shareholders*.

The Bank's Problem: If the bank is safe (i.e., $p = 1$), shareholders are locally indifferent between any mix of capital and deposits that satisfies the capital requirement. If the bank defaults with strictly positive probability in equilibrium, the capital requirement always binds. From the bank's point of view, deposits are cheaper (depositors always break even, but sometimes at the expense of the taxpayer). Hence, the bank's problem boils down to finding a level of lending x^* that solves

$$\max_{x\geq 0} \int\int_{\{A,B\}\notin\Delta} \left[BX(x) + A\lambda - \left((1-\gamma_b)x + (1-\gamma_a)\lambda\right)\right] \times f(A, B)dAdB - \left((\gamma_b x + \gamma_a\lambda) - \kappa\right). \tag{5}$$

We refer to x^* as the equilibrium level of lending.

II. The First-Order Approach

We now impose $\gamma_a = \gamma_b = \gamma$. For simplicity, our propositions focus on the cases in which the first-order condition uniquely pins down an implicit function

$x^*(\gamma)$, and $p^*(x^*(\gamma), \gamma) < 1$, so that the capital requirement is relevant.[9] To ease readability, we often omit function dependencies on x and γ. In addition, we use subscripts for partial derivatives in these two variables and stars to indicate where functions are evaluated in equilibrium. For instance $p_x^* \equiv p_x(x^*, \gamma)$.

A. The Sign of the Lending Response

The bank's objective function can be rewritten as

$$w(x) = \underbrace{X - x}_{\text{economic surplus}} + \underbrace{\int \int_{\{A,B\} \in \Delta} \big((1-\gamma)(x+\lambda) - BX - A\lambda\big) f(A,B) dA dB}_{\equiv s(x,\gamma)\text{, that is, the implicit subsidy}} + \kappa. \tag{6}$$

Intuitively, the first term in equation (6) captures the economic surplus generated by new loans.[10] The second term integrates, over all of the default states, the difference between the promised repayment to the depositors, $(1-\gamma)(x+\lambda)$, and the total cash flow available to the bank, $BX + A\lambda$. Under unlimited liability, this term would be the expectation of how much, ex post, shareholders would have to pay into the bank to make depositors whole. Here, it is the taxpayer that footings the bill. This is why s should be interpreted as the implicit subsidy to the bank's shareholders arising from the government guarantee.

REMARK 1: *The implicit subsidy corresponds to the expected net worth of the bank when it is negative. As Merton (1977) shows, deposit insurance can be interpreted as a (free, or at least mispriced) put option on the bank's equity, with a strike price of zero. The implicit subsidy is therefore equal to the value of such an option.*

The first-order condition can be written as

$$\underbrace{(X_x^* - 1)}_{\text{NPV}} + \underbrace{s_x^*}_{\text{marginal subsidy}} = 0. \tag{7}$$

The first term represents economic surplus maximization. The second captures how the marginal loan affects the implicit subsidy (we derive s_x^* below).

At this stage, three points are in order. First, absent the implicit subsidy, only the opportunity cost of funds in the economy matters for investment decisions. The bank would then choose a level of lending consistent with

[9] Because of the truncation, the objective function in (5) may have multiple peaks, or exhibit jumps or kinks. In these knife-edge cases, no or several x's may solve the first-order condition. The FSE cannot be isolated in these circumstances. Ignoring these cases enables a first-order approach: focusing on small increases in γ helps build intuition and establish the possibility results in our propositions. However, as will become clear in Section III, our main results do not hinge on this assumption.

[10] Since we assume $\mathbb{E}[A] = 1$, legacy loans are valued on the balance sheet at their expected value. Hence, $(\mathbb{E}[A] - 1)\lambda = 0$ and the surplus from these loans does not appear in equation (6).

proposition 3 in Modigliani and Miller (1958). Denoting this level x_{MM}, we have $X_x(x_{\mathrm{MM}}) = 1$.

Second, an increase in the capital requirement unambiguously decreases shareholders' expected payoff: $\forall x,\ w_\gamma \le 0$. Intuitively, for any x, the expected transfer from the taxpayer shrinks as the share of deposits in the bank's liabilities goes down. Formally

$$w_\gamma = s_\gamma = -(1-p)(x+\lambda). \tag{8}$$

Third, the gap between x_{MM} and x^* is due solely to s_x^*. Intuitively, if an increase in γ increases the extent to which the marginal loan is subsidized, the *lending response* is positive (that is $\frac{dx^*}{d\gamma} > 0$). Formally,

LEMMA 1 (The sign of the lending response):

$$\frac{dx^*}{d\gamma} \lesseqqgtr 0 \Leftrightarrow s_{x\gamma}^* \lesseqqgtr 0.$$

PROOF: Proofs are provided in Appendix B. □

B. The FSE

We have established that the sign of the lending response is that of $s_{x\gamma}^*$. We now turn to the underlying economic mechanism that can lead to $s_{x\gamma}^* > 0$.

Property Rights and the Marginal Residual Cash Flow: Issuing the marginal loan affects the bank's cash flows: the bank's date 2 revenue increases by BX_x, and the repayment due to depositors increases by $1-\gamma$. Let Z denote the *residual* (i.e., net of deposit repayments) cash flow associated with the marginal loan,

$$Z(x, B) \equiv BX_x - (1-\gamma).$$

Now, which stakeholder is entitled to Z depends on the realization of the state variables. If the bank survives, the shareholders are the residual claimants. But if the bank defaults, shareholders walk away with zero, and the taxpayer becomes, in effect, the residual claimant.[11] What determines the bank's survival is the sign of the *total residual cash flow* $(A\lambda + BX - (1-\gamma)(x+\lambda))$, which is not the same object as Z. As we will see, Z plays a key role in our analysis.

The Cross-Partial Derivative of the Subsidy: We state our key result in the following proposition.

[11] Technically, the taxpayer is a claimant, in the sense that there is a reduction in the transfer needed to make depositors whole.

PROPOSITION 1:

$$s^*_{x\gamma} = \underbrace{-(1-p^*)}_{<0} + \underbrace{p^*_\gamma z^*_0}_{FSE \gtrless 0}, \tag{9}$$

where

$$z^*_0 \equiv \mathbb{E}\left[Z^* \mid \mathbb{\backslash}^*\right] = \frac{\int_{b_L}^{b_0^*(a_L)} Z^* f(a_0^*(B), B)dB}{\int_{b_L}^{b_0^*(a_L)} f(a_0^*(B), B)dB}$$

is the expected marginal residual cash flow conditional on being on the equilibrium default boundary. It is the case that (i) z^*_0 can be positive, which implies (ii) a positive FSE and can lead to (iii) a positive lending response: $s^*_{x\gamma} > 0$.

The cross-partial derivative is the sum of two components. The first term is negative. Increasing γ reduces the portion of the marginal loan that is financed with (subsidized) deposits. Since the bank must substitute deposits (which it repays with probability p^*) with capital (which, in expectation, it repays in full), this change in the composition of liabilities reduces the marginal subsidy. We refer to this effect as *the composition effect*.

The second component reflects the fact that, keeping x^* constant, an increase in γ makes the bank safer: $p^*_\gamma > 0$. This corresponds to a shift in the default boundary. That is, there are states of the world where the bank would have defaulted if not for the extra capital. In these states, the rights to the residual cash flow from the marginal loan (Z^*) switch from the taxpayer to the shareholders. In expectation, this raises the marginal subsidy by $p^*_\gamma z^*_0$. Since this term follows from the fact that the bank is forced to be safer (something it could have always chosen to do), we refer to this effect as the *forced safety effect*. To the best of our knowledge, this paper is the first to highlight such a mechanism.

Proposition 1 indicates that z^*_0 can be positive, which implies a positive FSE. The states in which $Z^* > 0$ are defined by a threshold value $\hat{b}^* \equiv \frac{1-\gamma}{X^*_x}$ for B. Figure 3 depicts this threshold value in relation to the default boundary. It follows that z^*_0 will be positive if there is sufficient probability mass concentrated on the corresponding upper segment of the boundary. But how should one interpret $z^*_0 > 0$? This condition means that, in expectation along the default boundary, the residual cash flows on the marginal loan are greater than those on the average loan.

By construction, on the default boundary, the residual cash flow on the bank's average loan, over the whole balance sheet, is nil ($BX^* + A\lambda - (1-\gamma)(x^* + \lambda) = 0$). If the marginal loan is exactly equivalent to the average loan on the balance sheet, then we would have $Z^* = 0$ at all points on the boundary and the FSE would be nil. But if the marginal loan differs along some dimension (e.g., it may have a different return, or simply a different capital requirement), then Z^* can be positive along the boundary. In other words, for a

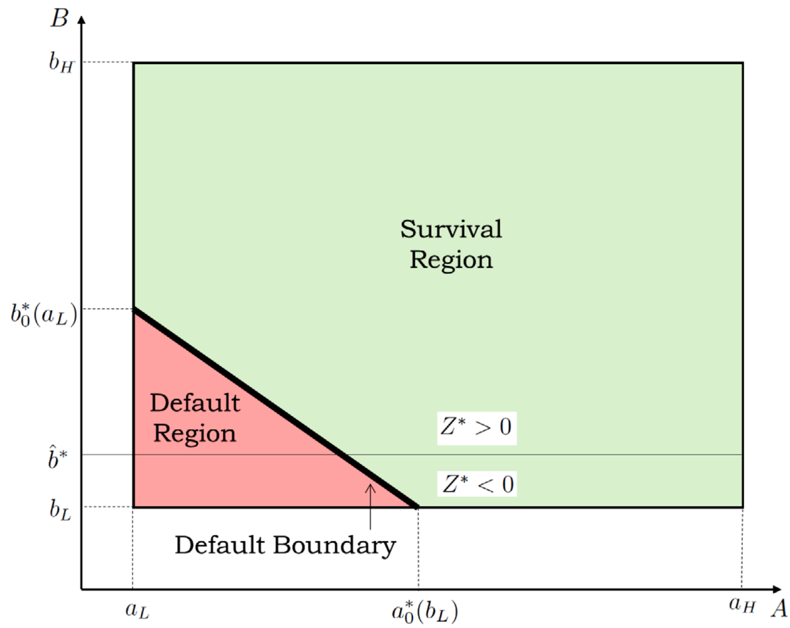


Figure 3. Residual cash flows and the default boundary. This figure illustrates the default boundary and the default region in equilibrium. The former is the set of points such that $BX^* + A\lambda - (1-\gamma)(x^* + \lambda) \leq 0$. The latter is the locus at which this condition binds. The parameter γ is the capital requirement. The term $Z^* = BX_x^* - (1-\gamma)$ is the equilibrium residual cash flow on the marginal loan, where BX_x^* denotes the payoff to the marginal loan. The threshold $\hat{b}^* \equiv \frac{1-\gamma}{X_x^*}$ is such that $B > \hat{b}^* \Rightarrow Z^* > 0$. See also the notes to Figure 2. (Color figure can be viewed at wileyonlinelibrary.com)

nonzero FSE, heterogeneity in the residual cash flows generated by the bank's assets is required.

A common assumption in the macrobanking literature is that loans on a bank's portfolio are *iid* conditional on the aggregate state. Thus, the marginal loan is ex post different from the average loan but is identically distributed (and faces an identical capital requirement). Hence, by assumption, $z_0^* = 0$ and the FSE is nil.

The Marginal Subsidy: Heterogeneity in residual cash flows, or a lack thereof, is also key to the sign of the wedge in the first-order condition, that is, the marginal subsidy.

The marginal subsidy is given by

$$s_x^* = -(1-p)z_\Delta^*, \tag{10}$$

where

$$z_\Delta^* = \mathbb{E}\left[Z^* \mid \Delta^*\right] = \frac{\int\int_{\{A,B\}\in\Delta^*} Z^* f(A,B)dAdB}{\int\int_{\{A,B\}\in\Delta^*} f(A,B)dAdB}.$$

The sign of the marginal subsidy is determined by the expected residual cash flows from the marginal loan in the default states, z^*_Δ. If $z^*_\Delta < 0$, the taxpayer subsidizes the marginal loan by making good the losses the loan would otherwise impose on depositors. Correspondingly, $x^* > x_{\mathrm{MM}}$, that is the bank finances negative NPV loans (as per Kareken and Wallace (1978); this is a well-understood manifestation of risk-shifting). If $z^*_\Delta > 0$, then the positive residual cash flows instead reduce the amount the taxpayer needs to chip in. So, *at the margin*, the subsidy is negative and acts like a tax, reducing equilibrium lending below x_{MM}. This reflects the guarantee overhang that we refer to in the introduction, and will discuss in more detail in Section III.

The default region is defined by the states in which the average loan has negative residual cash flows. Assuming that loans are conditionally *iid* (and face identical capital requirements) is akin to assuming that $z^*_\Delta < 0$. Hence, under such an assumption, the marginal subsidy is always positive. As we now show, however, the contrary can be true as well.

A Representative Example: The left panel in Figure 4 depicts a case in which $x^*(\gamma)$ is U-shaped and the lending response is positive for intermediate values of γ. As we discuss in Section IV, this is a representative example of how the moral hazard generated by government guarantees can play out in the model. The right panel shows how the positive lending response translates into movements in the bank's objective. The fact that $w_\gamma < 0$ implies the payoff function associated with a higher capital requirement (denoted by γ') is below the initial one. But $w_\gamma < 0$ does not tell us whether the payoff function peaks to the left or the right of the initial optimum. If $s^*_{x\gamma} > 0$, it peaks to its right, which means that the lending response is positive. As we discuss above and in the introduction the tractability assumptions commonly made in the literature, would restrict our model's equilibrium outcomes to $s^*_x < 0$ and $s^*_{x\gamma} < 0$. Relaxing such assumptions can lead to any sign combinations for these two objects. We explain why other sign combinations can arise and illustrate this via specific examples in Section III.

C. Multiple Capital Requirements

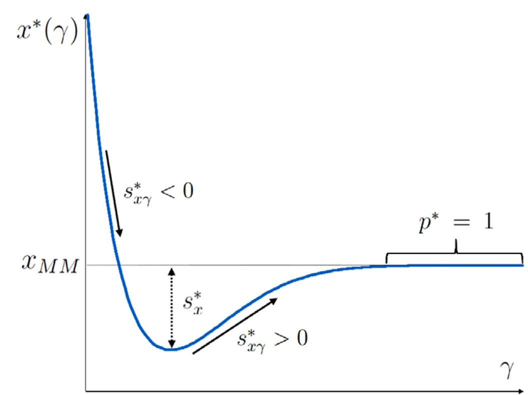
If the bank faces different capital requirements on legacy loans (γ_a) and new loans (γ_b), we obtain the following result.

PROPOSITION 2:

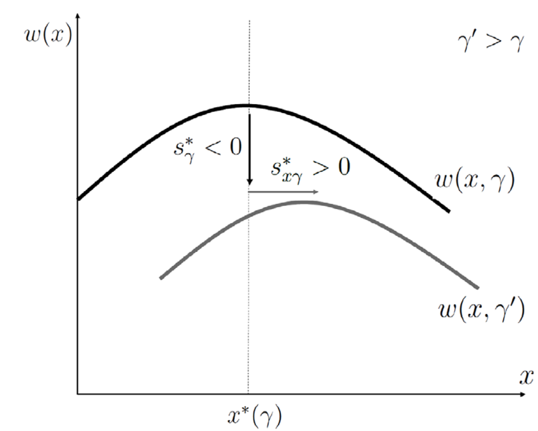
$$s^*_{x\gamma_a} = \underbrace{p^*_{\gamma_a} z^*_0}_{\gamma_a FSE} \tag{11}$$

$$s^*_{x\gamma_b} = -(1-p^*) + \underbrace{p^*_{\gamma_b} z^*_0}_{\gamma_b FSE}. \tag{12}$$

Both $s^*_{x\gamma_a}$ and $s^*_{x\gamma_b}$ can be positive.



Panel A. $x^*(\gamma)$



Panel B. $w(x)$

Figure 4. An example of $x^*(\gamma)$ and the bank's objective. Panel A provides a representative numerical example of $x^*(\gamma)$, the bank's equilibrium level of lending as a function of the capital requirement. The calibration is as given in Table I, except that the correlation between A and B and the tax rate are both set to zero. As explained in Section II, the sign of the slope of $x^*(\gamma)$ is given by the sign of $s^*_{x\gamma}$, the cross-partial derivative of the implicit subsidy arising from the government guarantee with respect to lending and the capital requirement. The difference between the Modigliani-Miller level of lending, x_{MM}, and $x^*(\gamma)$ is a negative function of s^*_x, the derivative of the subsidy with respect to lending only. Panel B shows how the bank's objective function, $w(x, \gamma)$, shifts downward in response to an increase in the capital requirement when the lending response is positive at the initial equilibrium, $x^*(\gamma)$. If, instead, the lending response is negative, then $w(x, \gamma^{'})$ peaks to the left of $x^*(\gamma)$. (Color figure can be viewed at wileyonlinelibrary.com)

The first term in $s^*_{x\gamma_b}$ corresponds to the composition effect. That this effect is present in $s^*_{x\gamma_b}$ and not in $s^*_{x\gamma_a}$ makes perfect sense: only γ_b affects how the marginal loan is financed. However, both $s^*_{x\gamma_b}$ and $s^*_{x\gamma_a}$ contain a term that corresponds to an FSE. The intuition for the γ_a FSE is straightforward: a higher capital requirement on legacy loans makes the bank safer, in which case it internalizes the residual cash flows on the boundary, z^*_0. The intuition for the γ_b FSE is similar. Thus, whether an extra capital buffer is associated with legacy or inframarginal loans is irrelevant: in both cases it makes the bank safer. It follows that if $z^*_0 > 0$, raising either requirement could boost lending. As a result, as we show in Section III.C, a positive lending response can arise without legacy loans.

The relative strength of these two FSEs depends on how much the boundary shifts when either requirement is raised (given by $p^*_{\gamma_a}$ and $p^*_{\gamma_b}$) and therefore on the shares of new loans and legacy loans on the balance sheet ($p^*_{\gamma_a}/p^*_{\gamma_b} = \lambda/x^*$). This relation allows us to decompose the effect of γ on the marginal subsidy. For instance, starting from $\gamma_a = \gamma_b = \gamma$, we have $s^*_{x\gamma} = -(1-p^*) + p^*_{\gamma_a} z^*_0 + p^*_{\gamma_b} z^*_0$.

III. Residual Cash Flow Heterogeneity

To illustrate how the FSE operates, why the implicit subsidy may act like a tax and lead banks to pass on positive NPV loans, and more generally to highlight the key role played by residual cash flow heterogeneity, we organize our discussion around four specific cases.

A. Perfectly Homogeneous Residual Cash Flows

A useful starting point is the special case in which the FSE is always nil. This may occur if new loans are identical to the legacy loans. Here, the deviation from our baseline assumption is that the bank chooses $x \in (0, \bar{x})$, which yields the payoff

$$BX(x) = Ax.$$

So the bank's choice is simply whether to scale up or not (with $\bar{x}$ the maximum possible increase in scale). Considering a single capital requirement, we have

$$s = \int_{a_L}^{a_0} (1 - \gamma - A)(x + \lambda) f(A) dA.$$

The default boundary is given by $a_0 = (1-\gamma)$, and we assume $a_L < (1-\gamma)$, so that the bank defaults with positive probability (note that p is independent of x). We then have

$$s_x = -(1-p)\underbrace{\left(\mathbb{E}[A \mid A < a_0] - (1-\gamma)\right)}_{<0} > 0.$$

Here, the marginal subsidy is positive and makes lending more appealing. Abusing notation somewhat, define x^* as the bank's optimum:

$$x^* = \begin{cases} \bar{x} \ ; \mathbb{E}[A] + s_x > 1 \\ 0 \ ; \mathbb{E}[A] + s_x < 1. \end{cases}$$

If $\mathbb{E}[A] < 1 < \mathbb{E}[A] + s_x$, we have a typical case of bank risk-shifting (Kareken and Wallace (1978)): a new loan has a negative NPV, but the bank finances it because an expected loss (equal to s_x) can be shifted to the taxpayer.

Here,

$$s_{x\gamma} = -(1-p) < 0. \tag{13}$$

The composition effect operates as usual, but the FSE is nil. Changes in γ still shift the default boundary (a_0 decreases and therefore p increases with γ), but $z_0^* = z_0 = a_0 - (1-\gamma) = 0$. In this scaling-up example, marginal and average residual cash flows are the same and the average residual cash flow is, by definition, nil along the boundary.

Let us define the *average subsidy* as $\frac{s}{x+\lambda}$. We have that

$$\frac{\partial}{\partial \gamma}\left(\frac{s}{x+\lambda}\right) = -(1-p) < 0. \tag{14}$$

The expression is the same as in equation (13). This is intuitive: when residual cash flows on all loans are identical, a change in the capital requirement has exactly the same effect on the marginal subsidy and the average subsidy. Equation (14) is not specific to the present example. It holds in our model in general and always corresponds to the composition effect.

Our interpretation is that the composition effect is behind the claim (made repeatedly by bank lobbies and sometimes by policymakers (see, for instance, Brooke et al. 2015, p.5)) that increasing capital requirements would (i) increase bank funding costs and (ii) naturally lead to less lending. Equation (14) shows that the first part of the claim applies in our model: the average subsidy always falls in response to a higher capital requirement.

However, our analysis shows that the second part of the claim is a *non sequitur*. We believe that it confounds the effect of the capital requirement on the *average* subsidy and the *marginal* subsidy. In the current example, there is no difference between these two objects. But in general they are different. As a comparison between equations (14) and (9) makes clear, this difference is precisely the FSE.

Indeed, as we now show, $z_0^* = 0$ is a special case. Marginal and average residual cash flows can differ for several reasons, so that $z_0^* \neq 0$.

B. Capital Requirements as a Source of Heterogeneity

Even when new loans are identical to legacy loans, the FSE can be positive if the loans are subject to different requirements. Otherwise keeping the same structure as in the previous example, we have

$$s = \int_{a_L}^{a_0} \big[(1-\gamma_a - A)\lambda + (1-\gamma_b - A)x\big] f(A)dA,$$

and

$$s_x = -(1-p)\big(E[A \mid A < a_0] - (1-\gamma_b)\big).$$

The default boundary is now given by $a_0(x) = (1-\gamma_a)\frac{\lambda}{x+\lambda} + (1-\gamma_b)\frac{x}{x+\lambda}$. The marginal residual cash flow on the boundary is $z_0(x) \equiv a_0(x) - (1-\gamma_b) = \gamma_b - \frac{\lambda\gamma_a + x\gamma_b}{\lambda+x}$. So, if $\gamma_b > \gamma_a$, then $z_0 > 0$ and the FSE will be positive:[12]

$$p_{\gamma_a} z_0 > 0,\ p_{\gamma_b} z_0 > 0.$$

This simple example is useful to illustrate how residual cash flows can be heterogeneous in spite of the loan cash flows being perfectly correlated. (We discuss the role of correlation in more detail in Appendix D.)

C. Heterogeneity among New Loans

Residual cash flow heterogeneity can, of course, arise from heterogeneity in loan cash flows rather than capital requirements. As will soon become apparent, the U-shape in Figure 4 is due to heterogeneity between legacy loans and new loans. Here, we show that if the bank starts from scratch (i.e., $\lambda = \kappa = 0$), heterogeneity among new loans can also lead to a positive FSE.

A Portfolio Composition Problem: In this example, we deviate from our baseline model and assume that there are n different unit lending opportunities indexed by i, and m equiprobable states indexed by j. Let B denote the $n \times m$ matrix of realized payoffs and $\boldsymbol{x} = \{x_1, x_2, \ldots, x_n\}$, where $x_i \in \{0, 1\}$, denote the bank's portfolio decision ($x_i = 1$ means that the loan is financed). In our general notation, the total realized payoff associated with a portfolio is given by

$$BX(\boldsymbol{x}) \equiv \boldsymbol{x}B.$$

The Effect of γ_b on Portfolio Decisions: Unless n and m are small, the bank's maximization problem is intractable analytically. But it can be solved numerically to show that three key features typically emerge (since there are only new loans, only γ_b is relevant):

(i) Unless γ_b is high enough, the bank generally optimally chooses to both *finance negative NPV loans* and *not finance positive NPV loans*.
(ii) A loan that is not financed under a capital requirement γ_b may end up being financed under a capital requirement $\gamma_b' > \gamma_b$, and vice versa.
(iii) Total lending for the bank, as a function of γ_b, follows a sort of *U shape*.

[12] Note that at $x = 0$, there are no inframarginal loans, so $p_{\gamma_b} = 0$.

Figure 5 provides an example of optimal portfolio choices across values of γ_b (the bank chooses among $n = 20$ loans whose payoffs in the $m = 100$ states are independently drawn; Appendix C provides more details). We draw payoffs so that loans 11 to 20 have a positive NPV and would form the optimal portfolio in the absence of the guarantee. As we can see, γ_b dramatically affects how the bank deviates from this benchmark. Understanding why requires looking at residual cash flows in the default region.

Residual Cash Flows and Risk-Shifting: Any loan is potentially marginal. Consider, for instance, a loan k that is not included in the equilibrium portfolio. In equilibrium, the marginal subsidy associated with this loan can be expressed as

$$s_k^* \equiv -\underbrace{\frac{1}{m}\sum_{j\in\Delta^*} Z_{kj}}_{\text{expected Z over }\Delta^*} + \underbrace{\frac{1}{m}\sum_{i\in\{\Pi^*\cup k\}}\sum_j H_{jk}^* Z_{ij}}_{\text{“boundary shift” effect}}. \tag{15}$$

The first term is simply minus loan k's expected residual cash flow over the equilibrium default region and is equivalent to equation (10). The additional second term arises as we are now in a discrete case. Financing loan k shifts the default boundary, which also affects the marginal subsidy. The second term captures the resulting change in the expected residual cash flow of the equilibrium portfolio (where Π^* denotes the set of loans by number included in the optimal portfolio and H_{jk}^* is a function that takes the value -1, 1, or 0 depending on whether the inclusion of loan k in the bank portfolio adds state j to the default region, excludes it from the region, or does not affect its presence in it).[13]

Loans are included in the optimal portfolio if[14]

$$\text{NPV}_i + s_i^* > 0,$$

that is the bank finances negative NPV loans if their marginal subsidy is large enough. As above, there is a classic risk-shifting interpretation as the bank shifts losses on these loans onto the taxpayer.

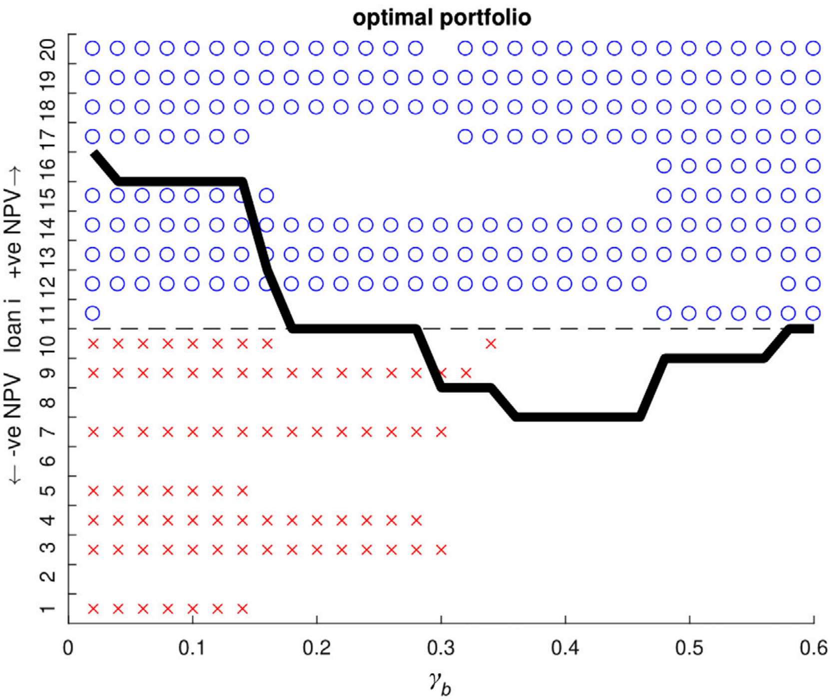
Loans are excluded from the optimal portfolio if

$$\text{NPV}_i + s_i^* < 0,$$

that is the bank passes on positive NPV loans if their marginal subsidy is sufficiently negative. If the bank were to add one of these loans to its portfolio, it would increase the portfolio NPV, but this would be more than offset by the

[13] To see that the first term corresponds to the marginal subsidy in the continuous case, denote by $z_\Delta^*(k)$ the conditional expectation of Z_{kj} over the default region and note that $\frac{1}{m}\sum_{j\in\Delta^*}(-Z_{kj}) = -(1-p^*)z_\Delta^*(k)$. The equivalent of the second term is nil under the first-order approach. Indeed, by definition, the *total* residual cash flow is nil on the default boundary.

[14] For such loans, the marginal subsidy can be written as $s_k^* \equiv -\frac{1}{m}\sum_{j\in\Delta^*} Z_{kj} + \frac{1}{m}\sum_{i\in\{\Pi^*\backslash k\}}\sum_j (-H_{jk}^*)Z_{ij}$.



($\gamma_b = 10\%$; selected loans $i = \ldots$)

	1	2	15	16
NPV_i	−4.5%	−3.9%	2.1%	2.7%
s_i^*	6.6%	−2.6%	2.1%	−6.1%
$\mathrm{NPV} + s_i^*$	2.1%	−6.5%	4.3%	−3.4%
Financed?	yes ×	no	yes ○	no

Figure 5. Discrete new loans with cash flow heterogeneity. The top panel provides an example of optimal portfolio choices across values of γ_b, the capital requirement, with $n = 20$ assets and $m = 100$ states (see Appendix C for details). On the x-axis is the capital requirement at two-percentage-point increments. On the y-axis is the corresponding portfolio choice. Loans are sorted accorded to their NPV in increasing order. Loans 1 to 10 have negative NPV and loans 11 to 20 have positive NPV. For a given capital requirement, a circle (for positive NPV loans) or a cross (for negative NPV loans) means that the corresponding loan is in the optimal portfolio of the bank. The solid line indicates how many loans are in the portfolio. For instance, at $\gamma_b = 10\%$, 15 loans are financed, including loan 1 and six others that all have negative NPV. In contrast, loans 11 and 16, which have positive NPV, are not financed. For this case, the bottom panel compares the NPV and marginal subsidy, s_i^*, of loans 1 and 16 to two loans of comparable NPV but for which the bank's decision is in line with an NPV criterion. (Color figure can be viewed at wileyonlinelibrary.com)

decline in the expected transfer from the taxpayer. In a sense, financing these loans would provide hedging, but this hedging would only benefit the taxpayer. Deciding not to hedge is comparable to deciding to increase risk. Hence, we argue that not financing positive NPV loans can also be interpreted as risk-shifting. (Below we provide an interpretation related to guarantees generating an overhang problem.)

The Composition and Forced Safety Effects: Holding the portfolio and default region fixed, an increase in γ_b reduces the marginal subsidy. The partial derivative of the first term in equation (15) is $-(1-p^*)$, that is, the composition effect.

However, an FSE is present. The intuition follows Section II. More capital shifts the default region, which affects s_i^* for all loans. Moreover, in this discrete case, this leads the bank to reshuffle its portfolio, which affects the default region again, and so on. The net result is that any loan (with positive or negative NPV) that is not financed at some γ_b may be financed at some $\gamma_b^{'} > \gamma_b$, and vice versa.

The U-shape: Total lending exhibits a U-shape in Figure 5 (comparable to the example in Figure 4). To see how this shape emerges, note first that at low levels of γ_b, many negative NPV loans are made, bringing the total number of loans substantially above x_{MM}. As γ_b increases, the number of negative NPV loans in the portfolio tends to decrease. In addition, at low or high levels of γ_b most positive NPV loans are financed, but at intermediate values many of them are passed on, leading to a U-shape in positive NPV lending.

This example relies on heterogeneity in the payoffs associated with new loans. Building on Section III.B, we could instead consider discrete identical (positive NPV) loans that have different capital requirements (or risk weights). As we show in Appendix D, this also delivers a U-shape.

D. Legacy Loans as a Source of Heterogeneity

Our final example provides analytic intuition for why U-shapes emerge in many cases.

We return to our baseline assumption where $BX(x)$ is strictly increasing and strictly concave, which allows us to use the first-order approach. However, we assume that new loans are safe, we normalize $B = 1$, and we assume that $\lambda > 0$ and $a_L < 1 - \gamma$, so that the bank may fail in equilibrium. The first-order condition, with a single capital requirement γ, reads

$$X_x^* - 1 + (1 - p^*)\underbrace{((1-\gamma) - X_x^*)}_{=-Z^*} = 0.$$

Rearranging, we obtain

$$X_x^* = (1-\gamma) + \frac{\gamma}{p^*}. \tag{16}$$

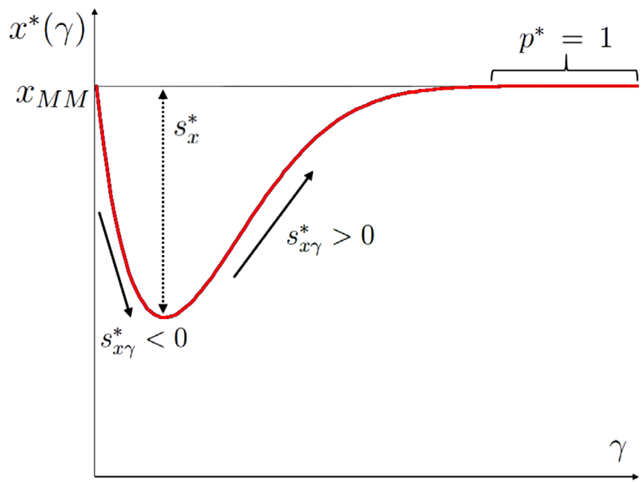


Figure 6. Example of $x^*(\gamma)$ in the case with safe new loans: a well-behaved U-shape. This figure illustrates a representative numerical example of $x^*(\gamma)$, the bank's equilibrium level of lending as a function of the capital requirement, when new loans are safe. The calibration is defined in Table I, except that the tax rate and σ_B^2, the standard deviation of new loan returns, are set to zero. As explained in Section II, the sign of the slope of $x^*(\gamma)$ is given by the sign of $s_{x\gamma}^*$, the cross-partial derivative of the implicit subsidy arising from the government guarantee with respect to lending and the capital requirement. The difference between the Modigliani-Miller level of lending, x_{MM}, and $x^*(\gamma)$ is a negative function of s_x^*, the derivative of the subsidy with respect to lending, which is itself weakly negative in this case. Hence, $x_{\text{MM}} \geq x^*(\gamma)$. The term p^* denotes the probability of the bank surviving in equilibrium. (Color figure can be viewed at wileyonlinelibrary.com)

This equilibrium condition makes clear that as soon as $p^* < 1$ (and $\gamma > 0$), we have $X_x^* > 1$: the bank passes on positive NPV new loans. The intuition is similar to the example above, but now with more structure on the cash flows. Legacy loans can make the bank default. New loans are safe, with $X_x^* > 1$. Hence, new loans produce positive residual cash flows in all states: $Z^* > 0$. Intuitively, the marginal subsidy is negative. It acts as a tax, and the bank's optimal lending cutoff is below x_{MM}.

From first-order condition (16), when $\gamma = 0$, or when γ is high enough (so that $p^* = 1$), $x^* = x_{\text{MM}}$. Between these two cases, $p^* < 1$ and x^* is strictly below x_{MM}. This provides analytic intuition for a U-shaped relationship and therefore the existence of a positive lending response. Figure 6 provides an example in which $x^*(\gamma)$ is a well-behaved U-shape. Adding risk to new loans typically makes the lending response intersect the y-axis above x_{MM} (as in Figure 4), which links back to our observation in the previous example that, at very low values of γ, many negative NPV loans are financed.

E. Overhang Problems

Positive residual cash flows in the default region reduce the expected transfer from the taxpayer. So, as we established in Section II.B, if $z_\Delta^* > 0$, the marginal subsidy is negative and acts like a tax, reducing equilibrium lending below x_{MM}. This is similar to what happens in the classic debt overhang problem of Myers (1977). Furthermore, a positive FSE implies that the bank now internalizes some of the positive residual cash flows in the default region. In this sense, the FSE can be interpreted as alleviating an overhang problem. However, this interpretation only works for a subset of the meanings that an "overhang problem" can have in the literature. In particular, an overhang problem may refer (explictly or not) to (i) the presence of risky (long term) debt; (ii) an implicit transfer of positive cash flows to creditors; (iii) the resulting underinvesment problem; or the combination of the three. In the context of our model, only (ii) is fully consistent with the proposed interpretation.

In what follows, we formalize this statement and explore further the links with Myers (1977).

DEFINITION 1: An overhang problem occurs in equilibrium if $\exists A \in \Delta^*, Z(A) > 0$.

This means that there are *some* states in the equilibrium default region where the marginal loan has positive residual cash flows. As described in Section II.B, these positive cash flows are, in effect, transferred to the taxpayer. Hence, our definition corresponds to notion (ii) above.

Ceteris paribus, an overhang problem leads the bank to undervalue the marginal loan. However, nothing prevents Z from being negative in *other* default states. These negative residual cash flows are *losses* that are *shifted* onto the taxpayer. Ceteris paribus, they lead the bank to overvalue the marginal loan. Whether the overhang problem or loss-shifting problem dominates overall depends on the sign of z^*_{Δ}, which is the expectation of Z over *all* default states. As discussed, this expectation pins down the sign of the marginal subsidy and whether the bank lends more or less than x_{MM}.

Here, importantly, the sign of the FSE does not hinge upon the sign of z^*_{Δ}. Instead, it depends on the sign of $z^*_{\partial\Delta}$, that is, it depends only on whether the overhang problem dominates along the default *boundary*. If it does, then the FSE is positive and, indeed, encourages more lending *because* it alleviates the overhang problem. But this can also happen when $z^*_{\Delta} > 0$. That is, a positive FSE can exacerbate an overlending problem. This is why the interpretation that a positive FSE alleviates an overhang problem does not work well with notion (iii) above.

Moreover, a higher capital requirement can also increase the extent of underlending. To see this, it is useful to draw further links with the classic debt overhang problem.

In the model of Myers (1977), a firm has existing risky assets and existing debt, on which it will default in some states of the world. The firm can choose to raise equity to finance a positive NPV investment but may pass on it because the cash flow it generates accrues to existing debtholders in the default states. This can be mapped into a special case of our model. To see this, just impose that $\gamma_b = 1$ (i.e., new investment must be fully financed with capital). Then the residual cash flow on the investment is the cash flow itself. Hence, $Z > 0$. As a result, (i) the marginal subsidy can only be negative, (ii) the FSE can only be positive, and (iii) increasing γ_a can only increase lending. Reinterpreting $1/\gamma_a$ as the initial leverage of the bank (where a fraction $1-\gamma_a$ of legacy loans are funded with uninsured legacy debt), we obtain the classic interpretation: less initial leverage generates more investment.

However, banks can finance new investment with insured deposits (i.e., $\gamma_b < 1$). Hence, for banks, Z^* can have either sign. Our main focus is on a positive FSE, which occurs when $Z^* > 0$. But $Z^* < 0$ yields an implication that is at odds with what happens in the classic model—an increase in γ_a can make the bank pass on a positive NPV loan.[15]

Another difference with Myers (1977) is that the guarantee overhang cannot be fixed by shortening debt maturity. In the classic debt overhang problem, if existing debt were to mature before investment took place (or if its interest rate was renegotiable), the problem would not occur, as the price of debt would reflect its market value (taking into account new investment). In our model, debt takes the form of demand deposits that can be raised at the time investment take place. As a result debt maturity is not part of the problem and notion (i) above is not relevant here. Indeed in Section III.C we establish that an overhang problem occurs (and may lead to underlending) even in the absence of initial debt. This is why we find it more appropriate to refer to the problem as *a guarantee overhang* rather than a debt overhang.

IV. Empirical Relevance

In real-world regulation, there is generally a single capital requirement that applies to risk-weighted assets. Accordingly, we now interpret the potential difference in requirements as a difference in risk weights. Formally, we just rewrite the constraint as

$$\kappa + c \geq \gamma(\alpha\lambda + \beta x),$$

where $\alpha \equiv \frac{\gamma_a}{\gamma}$ and $\beta \equiv \frac{\gamma_b}{\gamma}$ capture risk weight parameters.

A. Extending the model

To provide a meaningful calibration, we extend the baseline model with two additional features.

[15] Consider a simple numerical example in which there are three equiprobable states {I, II, III}, there are 10 legacy loans with vector total payoff {7, 9, 14} and there is a single new lending opportunity with payoff vector {1.4, 0.5, 1.25}. All loans are of size one, so the new loan has positive NPV. Initially, $\gamma_a = \gamma_b = 0.1$. Facing these parameter values, the bank fails in states I and II and optimally decides to finance the new loan. Raising γ_a to 0.2, for instance, makes the bank survive in state II. This means that the bank internalizes the negative marginal residual cash flow in this state ($Z_{\mathrm{II}} = 0.5 - 0.9 = -0.4$). This more than offsets the positive Z_{III} and the bank now prefers to pass on the new loan.

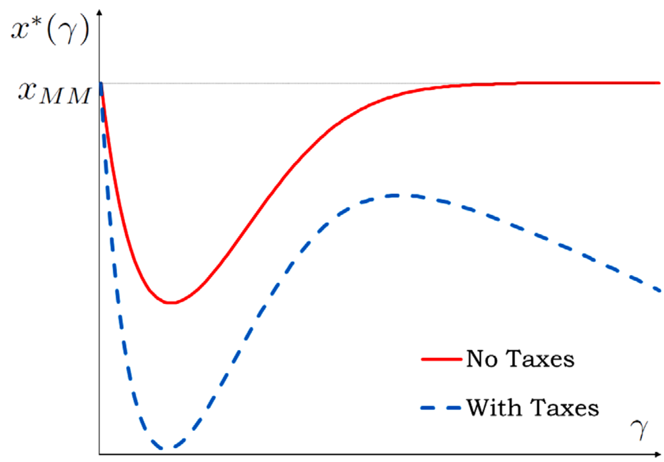


Figure 7. The shape of $x^*(\gamma)$ with corporate income tax. This figure illustrates the effect of taxes on $x^*(\gamma)$, the equilibrium level of lending as a function of the capital requirement, γ. The solid red line is equivalent to that in Figure 6. The blue dashed line corresponds to the same calibration with a corporate income tax of $\tau = 50\%$ and an interest rate of $\rho = 8\%$. The horizontal dotted line is the Modigliani-Miller level of lending. (Color figure can be viewed at wileyonlinelibrary.com)

Taxes and Tax Shields: We first introduce the tax advantage of debt—another reason banks may find capital relatively costly. To do so, we assume that the bank faces a tax rate τ on positive profits, net of interest expenses on deposits. To introduce a meaningful tax shield, we also assume that households have an opportunity cost of funds $1 + \rho > 1$, so that the interest rate on deposits is ρ. To maintain our normalization, we now set $\mathbb{E}[A] = \mathbb{E}[B] = 1 + \rho$.

We formally describe how the tax interacts with $x^*(\gamma)$ in Internet Appendix.[16] Here, we only illustrate the main qualitative effect of the tax shield on the lending response in Figure 7. The solid red line depicts an example of $x^*(\gamma)$ without the tax ($\tau = 0$). This is the U-shaped relationship of Section III.D (where new loans are safe). The blue dashed curve is the case with the tax. As we can see, the tax shield tilts the relationship clockwise.

This is intuitive, as with tax deductibility, an increase in γ leads to a composition effect that is similar to the one above and is stronger the higher the tax rate and the interest rate paid on deposits. (To make the effect visually obvious, we set both the tax and the interest rates at very high levels in Figure 7.)

Competition and Aggregate Demand for Loans: We next model competition between banks. Our baseline assumption has been that the bank faces a downward-sloping demand for loans, independent of the capital requirement. In practice, however, the loan demand for a bank is affected by the loan supply of other banks and therefore by the capital requirements they face.

We capture imperfect competition in a Cournot fashion: there is a given number ν of identical banks, all face the same capital requirement γ, and all pick

[16] The Internet Appendix may be found in the online version of this article.

their optimal level of lending taking other banks' decisions as given.[17] Up to a normalization that we introduce later, the payoff function of the representative bank takes the form

$$BX(x) = Bx\big((x + x')^{-\eta}\big),$$

where $x^{'}$ captures the *total* lending by other banks and η captures the elasticity of aggregate loan demand.

Our equilibrium concept is a symmetric Nash equilibrium. Assuming that it is unique, it corresponds to the fixed point (i.e., $x^* = x^{'}/(\nu - 1)$) that solves the representative bank's first-order condition.[18]

B. Calibration

Our benchmark calibration aims to capture a plausible situation facing a major international bank in 2017. Table I summarizes this calibration.

The capital requirement is not a straightforward object to calibrate. In the model, what matters is loss-absorbing liabilities as a percentage of the bank's assets. Even after accounting for risk weights, this is not necessarily the same object as the headline regulatory capital requirement that the policy debate focuses on. Relevant considerations include: (i) banks have hybrid liabilities that may or may not count toward the requirement and may or may not have implicit guarantees attached to them, (ii) different types of capital face different requirements, (iii) banks hold voluntary buffers above the requirements (for instance, to prevent small shocks from leading to violations), and (iv) requirements vary across jurisdictions, types of banks (for example, banks deemed to be globally systemic now face higher requirements), and macroeconomic conditions (this is the role of countercyclical capital buffers).

To circumvent this issue, we present our results for a wide range of values of γ, that is, we display the $x^*(\gamma)$ functions. Still, we need a reference value to center the calibration. For ease of interpretation, we use a headline number of 13% of risk-weighted assets for the requirement. Under Basel III, this corresponds roughly to the Tier 1 capital requirement (including systemic, conservation, and pillar 2 buffers) that globally systemically important banks face.[19]

[17] A Cournot approach is analytically convenient, but we also believe that it is particularly meaningful if one considers that banks first choose their level of capital (which, given the capital requirement, creates a capacity constraint) and then compete in price (i.e., in the interest rate) in the market for loans. Schliephake and Kirstein (2013) show that this results in an elegant application of Kreps and Scheinkman (1983): the equilibrium outcome corresponds to that under Cournot competition. Other papers using Cournot competition for banks include Corbae and D'Erasmo (2017) and Jakucionyte and van Wijnbergen (2018).

[18] In our numerical explorations, we have not encountered multiple fixed points.

[19] See Basel Committee on Banking Supervision (2017) and European Banking Authority (2017) for recent assessments; table A in Bank of England (2015) describes a breakdown of different requirements.

Table I

Benchmark Calibration

This table shows the benchmark calibration of the parameters used to investigate the empirical relevance of the forced safety effect in Section IV. The calibration aims to capture a plausible situation facing a major international bank in 2017.

Parameter	Value	Definition	Calculation	Source(s)
γ	0.13	Capital requirement	Tier 1 Risk-based minimum capital requirement of globally systemically important banks.	Basel Committee on Banking Supervision (2017) - table B.4
α	0.5	Risk weight on new loans	Average risk weights.	Mariathasan and Merrouche (2014)
β	0.5	Risk weight on legacy loans		
x_{MM}	1	MM level of lending	Normalization.	
ρ	0.012	Interest rate	Average one-year constant maturity U.S. Treasury yield (2017).	Federal Reserve Board - Release H.15
σ_A	0.041	Standard deviation of $\log(A)$	Target $p = 0.97$: annual frequency of banking crises in OECD countries 1970 to 2012.	Valencia and Laeven (2012)
σ_B	0.041	Standard deviation of $\log(B)$		
μ_A	$\log(1+\rho) - 0.5\sigma_A^2$	Expectation of $log(A)$	$E[A] = 1 + \rho$, existing loans fairly valued.	
μ_B	$\log(1+\rho) - 0.5\sigma_B^2$	Expectation of $\log(B)$	$E[B] = 1 + \rho$, implies $x_{MM} = 1$.	
λ	4	Book value of legacy loans	x_{MM} normalized, and $x_{MM}/\lambda \Rightarrow 20\%$ of loans maturing per year.	Van den Heuvel (2007)
ν	12	Number of banks	Loan spread over deposit rate = 2%.	Bernanke, Gertler, and Gilchrist (1999)
η	0.2	Interest elasticity of demand	Interest elasticity of demand on mortgage debt estimated from U.K. loan-to-value notches.	Best et al. (2020)
τ	0.24	Corporate tax rate	OECD average corporate tax rate, 2005 to 2017.	OECD tax database

Average risk weights are typically around 50% (see Mariathasan and Merrouche (2014)), so we use this number for α and β. Again, in practice, there is variation across banks and over time.

We calibrate τ to match the average statutory corporate tax rate among OECD countries; this corresponds to 24% in 2017. We interpret the period in our model as one year. Hence, we calibrate the interest rate ρ to match the average one-year constant maturity U.S. Treasury bond yield: 1.2% in 2017.

We select parameters so that $x_{\text{MM}} = 1$ in the benchmark calibration and any alternatives presented. To this end, we rescale the representative bank's gross return function:

$$BX(x) = Bkx\big((x + x')^{-\eta}\big), \tag{17}$$

where $k = \nu^{\eta}/(1 - \frac{\eta}{\nu})$, and $\mathbb{E}[B] = 1 + \rho$.

We calibrate η to match the interest elasticity of demand on residential mortgage debt estimated from U.K. loan-to-value notches (Best et al. (2020)). We choose ν to target the average spread on new loans in the model ($\mathbb{E}[B]\frac{X^*}{x^*} - 1 - \rho$), which we calibrate at 2%, consistent with Bernanke, Gertler, and Gilchrist (1999). This gives $\nu = 12$.

We calibrate the book value of legacy assets (λ) to 4 such that, if the bank chooses $x = x_{\text{MM}}$, 20% of loans on the balance sheet were made in the current period. This value is in line with the literature (see, for example, Van den Heuvel (2007)). To abstract from bank closure (see Appendix A), we assume that $\kappa > \gamma\lambda$.

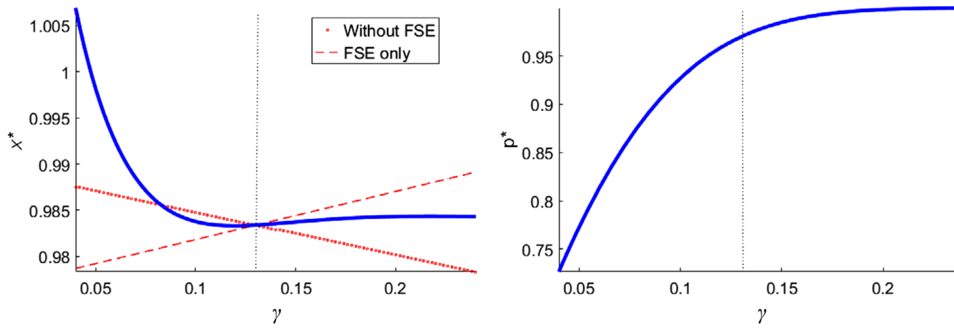
We model the joint distribution of $f(A, B)$ as a log-normal. We assume that legacy loans are held at fair value on the bank's balance sheet, that is, $\mathbb{E}[A] = 1 + \rho$. We further assume that A and B have identical standard deviations, which we calibrate by targeting the bank's equilibrium default probability ($1 - p^*$ in the model).

The appropriate calibration for $1 - p^*$ is the probability that the implicit subsidy is in the money and creditors benefit from a taxpayer transfer. Determining the value of p^* from the price of bank securities is challenging, due to the need to strip out the value of any expected transfer. Instead, we use realized frequencies. Valencia and Laeven (2012) find that there have been 40 banking crises among the 34 OECD members over the period 1970 to 2012, which suggests a target value of $p^* = 0.97$ or a 3% annual probability of default (Martinez-Miera and Suarez (2014) use a similar value in their calibration).

Last, we set the correlation parameter in the joint distribution of A and B equal to 0.5. This choice is arbitrary, and thus we run sensitivity analysis on it below.

C. Results

Benchmark Example: Figure 8 displays the $x^*(\gamma)$ curve (the left panel) and the associated probability of survival $p^*(\gamma)$ (right panel) for our benchmark calibration. As we can see, $x^*(\gamma)$ is slightly upward-sloping when γ is within



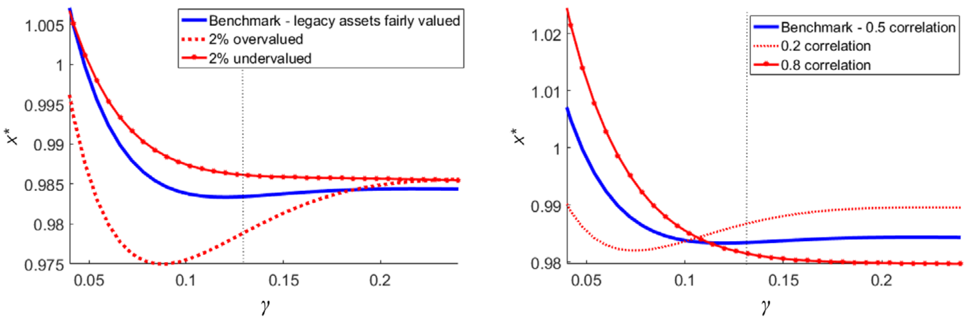
Panel A. $x^*(\gamma)$ Panel B. $p^*(\gamma)$

Figure 8. Equilibrium lending and survival probability under the benchmark calibration. This figure depicts outcomes for the calibration defined in Table I. Panel A shows equilibrium lending, $x^*(\gamma)$, for the representative bank under the alternative levels of γ, the capital requirement. The downward sloping dotted red line depicts the slope $x^*(\gamma)$ would have, at $\gamma = 13\%$, absent the FSE; the upward sloping dashed red line depicts the slope of $x^*(\gamma)$ at $\gamma = 13\%$ if only the FSE was operating. Panel B shows the equilibrium survival probability for the representative bank, $p^*(\gamma)$, under the alternative levels of γ. In both panels, the vertical line corresponds to the reference level of the capital requirement, $\gamma = 13\%$. (Color figure can be viewed at wileyonlinelibrary.com)

the range of 11% to 21%. At lower values of γ, the slope is negative (and steeper at very low values). From values 21% onward, the curve is downward sloping again: the bank is in fact very safe, and the effect of the tax shield dominates.

At the reference value ($\gamma = 13\%$, indicated by the vertical dotted line), the slope is positive. However, the lending response has little economic significance. For instance, a capital requirement increase from 13% to 14% would generate an increase in lending of 0.02%. But, this is still very different from conventional wisdom and the typical concern that such a policy change would induce a cut in lending. The reason for the absence of a cut is the FSE. To illustrate this effect, we add two counterfactual slopes to the left panel of Figure 8. The dotted line shows what the slope of $x^*(\gamma)$ would be (at $\gamma = 13\%$) absent the FSE (i.e., if the slope was driven exclusively by the composition effect plus the tax shield). The dashed line shows what the slope if only the FSE is active. The true tangent to $x^*(\gamma)$ at this point is essentially flat. Thus, the FSE can have the same magnitude as the forces that pull toward a negative lending response and can even overcome them. On this basis, we argue that the effect is quantitatively relevant.

Now consider instead an 8% capital requirement. The curve is downward sloping and steeper, which is more in line with conventional wisdom. Given that this percentage is the one mandated by Basel I, the regulation in place in most countries throughout the 1990s and most of the 2000s, it constitutes a plausible situation that banks faced before the global financial crisis. In this case, going from a capital requirement of 7% to 8% leads to a lending cut of 0.2% and an increase in lending spreads of 5bps. Note also the higher



Panel A. Legacy Loan Valuation Panel B. Correlation

Figure 9. Sensitivity analysis on equilibrium lending. The figure depicts equilibrium lending, $x^*(\gamma)$ for the representative bank under alternative levels of the capital requirement, γ, when the benchmark calibration given in Table I is modified in a single dimension. In both panels, the solid blue line corresponds to benchmark calibration, and the vertical dashed line corresponds to the reference level of the capital requirement, $\gamma = 13\%$. In Panel A, the red line with markers corresponds to legacy loans being undervalued by 2% ($E(A) = 1.02(1 + \rho)$) and the red dashed line is legacy loans overvalued by 2% ($E(A) = 0.98(1 + \rho)$). The term A denotes the stochastic gross return on legacy loans and ρ is the interest rate. In Panel B, the red line with markers corresponds to $\log(A)$ and $\log(B)$ having a correlation of 0.8 and the red dashed line corresponds to $\log(A)$ and $\log(B)$ having a correlation of 0.2. The term A denotes the stochastic portion of the gross return on new loans. (Color figure can be viewed at wileyonlinelibrary.com)

probability of a bailout in the right panel. A stronger composition effect is the root cause of the negative lending response.[20]

Discussion and Sensitivity Analysis: We believe that our benchmark numbers for the calibration above are plausible. However, they constitute only one example; and thus our results should not be taken as indicating that raising capital requirements today would necessarily induce most banks to increase lending slightly. Nor do we claim, on the basis of the discussion above, that all banks would have shown steep negative lending responses in the run-up to the global financial crisis.

Rather, we argue that lending responses are likely to display a lot of variation in both the time series and in the cross section, as well as in both sign and magnitude. To provide evidence of such variation, Figure 9 shows how lending by the representative bank in the benchmark example changes when we perturb parameter values one at a time. It is possible to generate much steeper positive lending responses. For instance, one can make the legacy loans overvalued (think of a large stock of nonperforming loans), as in Panel A, or

[20] Empirical evidence from precrisis sample periods generally points to a negative lending response (see, for instance, Hancock and Wilcox (1994), Francis and Osborne (2012), Aiyar et al. (2014), and Aiyar, Calomiris, and Wieladek (2014)). Our model predicts that a negative lending response is more likely at low levels of the capital requirement. It is therefore conceivable that future empirical research, with sample periods under the stricter requirements of the new Basel III regime, will obtain different findings.

decrease the correlation between A and B as in Panel B. Conversely, undervalued legacy loans and a higher correlation can make the lending response more negative, and relatively steep in some cases. Combining the two, we find that even with a high correlation, there is positive response when legacy assets are overvalued (see Figure IA2 in Internet Appendix II).

In Appendix D, we consider heterogeneity among banks and show that small differences in initial conditions, or bank-specific capital requirements, can lead to very different responses, as competition provides a feedback effect.[21]

D. Empirical Predictions and Links to the Literature

Our results show that one should not necessarily expect a stable relationship between capital requirements and lending. This may explain why empirical analyses that look at similar regulatory interventions but different samples arrive at different conclusions (compare, for instance, the evidence in Gropp et al. (2019) and Bassett and Berrospide (2018) on stress tests).[22] Extrapolating evidence from specific settings or time periods should be done with caution.

In spite of the unstable relationship, we can use our model and calibration exercise to formulate four empirical predictions:

1. *Banks that have a high risk of failure are likely to have a negative lending response because of the strength of the composition effect*. One implication of this prediction is that if bankers argue that higher capital requirements would lead to a substantial decrease in lending, they must believe that the composition effect, and hence default probabilities, are large. This would mean that banks are receiving substantial subsidies in the first place.
2. *Banks for which marginal loans are likely to deliver good cash flows when the rest of their assets perform badly are more likely to have a positive lending response due to the FSE*. This could occur because legacy loans and new loans are different in some respects, for instance, in the correlation of their returns or because legacy loans are overvalued in some way (in the spirit of Panels A and B in Figure 9).[23] One particular case in which such a situation may occur is after a major economic crisis, when new opportunities may open up even as banks are struggling with their legacy loans. Since new capital regulation typically arises after crises, this may be a fortuitous coincidence from a policy perspective.

[21] In addition, Internet Appendix I discusses the sensitivity of our results to the parameters governing the strength of the tax shield.

[22] The authors study stress-test-induced increases in capital requirements in Europe in 2011 and in the United States in 2013 to 2016, respectively. Both use difference-in-differences estimators where size-based stress test eligibility criteria determine treatment. Despite the similar settings, however, they obtain a very different conclusion: Gropp et al. (2019) find a reduction in lending, whereas Bassett and Berrospide (2018) find that, if anything, lending goes up.

[23] Our model also predicts that banks will overvalue new loans that are similar to legacy loans. Landier, Sraer, and Thesmar (2015) provide evidence of this behavior from a precrisis U.S. subprime lender.

However, as we have argued, heterogeneity in cash flows may not simply be an issue of legacy loans versus new loans. The bank may face sectoral heterogeneity in its lending opportunities that leads to differences in the cash flows of potential new loans.[24] Alternatively, a securities or investment banking division may generate cash flows orthogonal to the bank's loans.

3. *Within a portfolio, a difference-in-differences approach can identify the composition effect:* Looking at our model, imagine that the bank could make two types of new loans, and that the risk weight on just one of them was increased. The bank would issue less of that type of lending relative to the other type. If this risk weight change makes the bank safe enough, it could still expand its balance sheet, which would reflect a strongly positive FSE. Interestingly, if these loans were otherwise identical, the relative lending response would also be proportional to the composition effect.[25] This would be consistent with results in Behn, Haselmann, and Wachtel (2016), who show that banks cut back on loans in portfolios that faced an increase in capital requirements *relative* to other similar loans in untreated portfolios.
4. *Targeted regulatory interventions can generate FSEs that affect all bank lending (and investment) decisions.* Consider a bank that has two main lines of business. Imagine that the regulator substantially restricts risk-taking in one of them. This will make the bank safer and make it internalize the residual cash flows of the other line of business in more states. If those cash flows are positive (perhaps because this line of business generates relatively safe returns), the bank will expand in that dimension. This prediction is consistent with evidence in Acharya et al. (2018), who finds that when the Central Bank of Ireland imposed restrictions on the issuance of risky loans to urban borrowers, banks that were initially heavily exposed aggressively expanded their issuance of loans to safer borrowers in rural counties.

[24] One specific case is geographical variation in lending. For example, Puri, Rocholl, and Steffen (2011) show that state banks in Germany that were heavily invested in U.S. subprime loans cut back on loans to German retail borrowers during the 2007 to 2009 financial crisis. Given the relative safety of German retail borrowers we conjecture that the residual cash flows on these loans was very different from U.S. subprime lending and a higher capital requirement may have prevented this cut back.

[25] To see this, imagine that new loans x_1 and x_2 have identical payoff functions and residual cash flows in all states of the world, and x_1 must be financed with γ_1 of capital. We then have

$$\frac{dx_1^*}{d\gamma_1} = \left(-(1-p^*) + p_{\gamma_1} z_0^*\right)/\left(-w^*_{x_1x_1}\right)^{-1}, \; \frac{dx_2^*}{d\gamma_1} = \left(p_{\gamma_1} z_0^*\right)/\left(-w^*_{x_2x_2}\right)^{-1},$$

where z_0^* is the expected residual cash flow along the default boundary for both loans. The assumptions mean that $w^*_{x_2x_2} = w^*_{x_1x_1}$, so the differential effect of γ_1 on x_1^* and x_2^* is proportional to the composition effect.

V. Conclusion

In this paper, we contribute to the policy debate by showing that capital being costly for banks does not necessarily imply a negative lending response to higher capital requirements. The FSE can counteract the liability composition effect, which overturns conventional wisdom. Even if the FSE is not strong enough to dominate, a more conservative interpretation of our results is that, in some circumstances, the FSE can be an important countervailing force that mitigates the contractionary effect of tighter regulation.

The necessary ingredient to generate this effect in our model is heterogeneity in the bank's residual cash flows. Our calibration introduces this through differences between legacy loans and new loans. We view such differences as a form of heterogeneity that is relevant for banks that, after all, are going concerns with long-term loans. However, as we have argued, other sources of heterogeneity can make the FSE positive.

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Appendix A: Participation Constraint and Bank Closure

In our model, the shadow value of initial capital is equal to the price of new capital. Therefore, how much capital comes from new shareholders versus how much was already on the bank's books is irrelevant. This is why the value of κ does not affect the value of x^*.

Still, initial shareholders have the option to close the bank at date 1 and walk away with zero. This means that κ alters their participation constraint. To see this, note that the participation constraint is

$$w^* = (\mathbb{E}[B]X^* - x^*) + (\mathbb{E}\big[A\big]\lambda - \lambda) + s^* + \kappa \geq 0. \tag{A.1}$$

Under our assumption that $\mathbb{E}[A] = 1$, the second term disappears and the constraint is always satisfied ($X(0) = 0$, $s(0) \geq 0$, so at worst the constraint boils down to $\kappa \geq 0$). However, if $\mathbb{E}[A] < 1$, the participation constraint can be violated and κ becomes relevant. First, consider $\kappa \geq \gamma\lambda$. Here, closing the bank is never the best option for shareholders. This is because, under limited liability, even operating at $x = 0$ gives the bank's shareholders a positive payoff in expectation. However, if $\kappa < \gamma\lambda$, shareholders must first raise new capital if the bank is to operate. When the option value of operating the bank is low (i.e., when $\mathbb{E}[A]$ is low and new loans do not generate much surplus), operating may not be worth the cost of recapitalization. The participation constraint is then violated.[26]

[26] To see this, rewrite the participation constraint as

$$\underbrace{\int\int_{\{A,B\}\notin\Delta^*} \big(BX^* + A\lambda - (1-\gamma)(x^* + \lambda)\big) f(A,B)dAdB}_{\text{option value of operating}} \geq \underbrace{(x^* + \lambda)\gamma - \kappa}_{\text{new capital needed}}.$$

The key point we want to make is that when $\mathbb{E}[A] < 1$, increasing γ may make the bank close at date 1. Formally

PROPOSITION A1: *Assume $a_L, b_L > 0$. If $\kappa + (X(x_{MM}) - x_{MM}) < (1 - \mathbb{E}[A])\lambda$, then there exists a $\overline{\gamma} < 1$ such that for all $\gamma \geq \overline{\gamma}$, the bank closes at date 1.*

PROOF: If $a_L, b_L > 0$, then $\forall x$, there exists a $\overline{\gamma} < 1$ such that $s(x) = 0$. Then $x^* = x_{\mathrm{MM}}$ and the participation constraint (A.1) simplifies to $(X(x_{\mathrm{MM}}) - x_{\mathrm{MM}}) + (E[A] - 1)\lambda + \kappa \geq 0$. The condition in the proposition ensures that the participation constraint is violated. □

To understand Proposition A1, first note that a high γ makes it more likely that $\kappa < \gamma\lambda$. Second, γ reduces the subsidy (see equation (8)), so that $s^* = 0$ for some sufficiently large γ. Given $s^* = 0$, if initial equity plus the surplus on new loans is insufficient to cover the expected losses on legacy assets, then the bank will shut down. The implication is that, for distressed banks, it is possible that $x^*(\gamma)$ is only upward-sloping when $\gamma > \overline{\gamma}$ and the bank would always choose to close rather than increase lending in response to an increase in capital requirements.

Appendix B: Proofs

LEMMA 1: *The sign of the lending response*

$$\frac{dx^*}{d\gamma} \lesseqqgtr 0 \Leftrightarrow s^*_{x\gamma} \lesseqqgtr 0.$$

PROOF: This result follows directly from the implicit function theorem applied to the first-order condition. □

PROPOSITION 1:

$$s^*_{x\gamma} = \underbrace{-(1 - p^*)}_{<0} + \underbrace{p^*_\gamma z^*_0}_{FSE \gtrless 0},$$

where

$$z^*_0 \equiv \mathbb{E}\left[Z^* \mid \backslash\backslash^*\right] = \frac{\int_{b_L}^{b^*_0(a_L)} Z^* f(a^*_0(B), B)dB}{\int_{b_L}^{b^*_0(a_L)} f(a^*_0(B), B)dB}$$

is the expected marginal residual cash flow conditional on being on the equilibrium default boundary. It is the case that (i) z^*_0 can be positive. This implies (ii) a positive forced safety effect and can lead to (iii) a positive lending response: $s^*_{x\gamma} > 0$.

PROOF: The subsidy is

$$s = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} \big((1 - \gamma)(x + \lambda) - BX - A\lambda\big) f(A, B)dAdB.$$

Define $\Omega(x,B) \equiv \int_{a_L}^{a_0(B)}((1-\gamma)(x+\lambda)-BX-A\lambda)f(A|B)dA$. So

$$s = \int_{b_L}^{b_0(a_L)} \Omega(x,B)f(B)dB.$$

The marginal subsidy is given by

$$s_x = \frac{\partial b_0(a_L)}{\partial x}\underbrace{\Omega(x,b_0(a_L))}_{0} + \int_{b_L}^{b_0(A_L)} \Omega_x(x,B)f(B)dB,$$

with

$$\Omega_x(x,B) = \frac{\partial a_0(B)}{\partial x}\underbrace{\big((1-\gamma)(x+\lambda)-BX-a_0(B)\lambda\big)}_{0} - \int_{a_L}^{a_0(A_L)} \underbrace{BX_x-(1-\gamma)}_{Z(x,B)} f(A)dA.$$

So

$$s_x = -\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} Z(x,B)f(A,B)dAdB = -(1-p)z_\Delta,$$

where

$$z_\Delta = \mathbb{E}[Z \mid \Delta] = \frac{\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} Z(x,B)f(A,B)dAdB}{\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} f(A,B)dAdB}.$$

Alternatively, we can write $s_x = (1-p)(1-\gamma) + \int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB$. The cross partial derivative is therefore

$$s_{x\gamma} = -p_\gamma(1-\gamma) - (1-p) - \frac{\partial}{\partial\gamma}\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB.$$

Since $a_0(b_0(a_L)) = a_L$,

$$\frac{\partial}{\partial\gamma}\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB = -\int_{b_L}^{b_0(a_L)} \frac{(x+z)}{z} BX_x f(a_0(B),B)dB.$$

Also,

$$p_\gamma = \frac{(x+z)}{z}\int_{b_L}^{b_0(a_L)} f(a_0(B),B)dB,$$

so we have

$$s_{x\gamma} = -(1-p) + p_\gamma z_0,$$

where

$$z_0 \equiv \mathbb{E}[Z \mid \diagdown] = \frac{\int_{b_L}^{b_0(a_L)}(BX_x-(1-\gamma))f(a_0(B),B)dB}{\int_{b_L}^{b_0(a_L)} f(a_0(B),B)dB}.$$

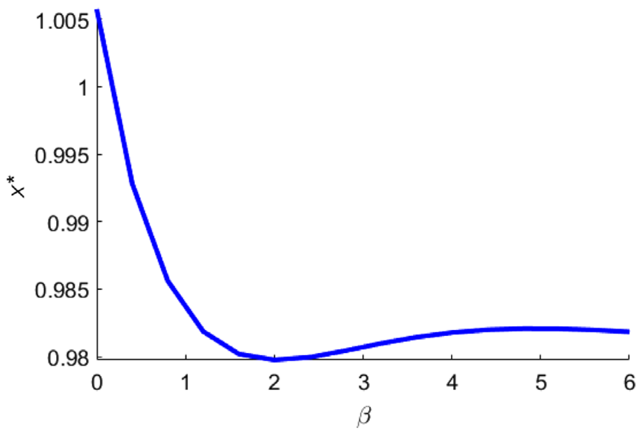

Figure B1. Lending and the capital requirement on new loans. This figure plots x^* as a function of β, the risk weight on new loans (which is set to 0.5 in the baseline calibration). All other parameters adhere to the calibration in Section IV, in particular, $\gamma = 13\%$. (Color figure can be viewed at wileyonlinelibrary.com)

The remainder of the proof is by example. We provide examples of positive lending responses in Section IV. Now, having proved (iii) by example, (ii) must be true, and therefore (i) as well, since the composition effect is always negative. □

PROPOSITION 2:

$$s^*_{x\gamma_a} = \underbrace{p^*_{\gamma_a} z^*_0}_{FSE} \tag{B.1}$$

$$s^*_{x\gamma_b} = -(1-p^*) + \underbrace{p^*_{\gamma_b} z^*_0}_{FSE}. \tag{B.2}$$

Both $s^*_{x\gamma_a}$ and $s^*_{x\gamma_b}$ can be positive.

PROOF: Starting from

$$s = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} \big((1-\gamma_b)x + (1-\gamma_a)\lambda - BX - A\lambda\big) f(A,B)dAdB,$$

we get

$$s_x = (1-p)(1-\gamma_b) + \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB$$

$$s_{x\gamma_b} = -(1-p) - p_{\gamma_b}(1-\gamma_b) - \frac{\partial}{\partial \gamma_b} \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB$$

$$s_{x\gamma_a} = -p_{\gamma_a}(1-\gamma_b) - \frac{\partial}{\partial \gamma_b}\int_{b_L}^{b_0(a_L)}\int_{a_L}^{a_0(B)} BX_x f(A,B)dAdB.$$

Noting that $a_0(B) \equiv \frac{((1-\gamma_b)x+(1-\gamma_a)\lambda)-BX}{\lambda}$ and rearranging in a similar way as in the proof of Proposition 1 gives equations (B.1) and (B.2). That $s^*_{x\gamma_a}$ can be positive follows from Proposition 1 as $s^*_{x\gamma} > 0 \Rightarrow s^*_{x\gamma_a} > 0$. We prove that $s^*_{x\gamma_b}$ can be positive by example. Recall that in our calibration in Section IV, we have $\gamma_b = \beta\gamma$. Figure B1 presents x^* as a function of β holding γ fixed at its calibration of 13% in Table I. All other parameters also adhere to the calibration in Table I. As can be seen, at the baseline level of $\beta = 0.5$, lending is decreasing in the capital requirement on new loans. But for β between 2 and 5 (or γ_a between 26% and 65%), lending is increasing in the capital required against new loans. □

Appendix C: Solving the Problem with Discrete Loans

The numerical example in Figure 5 is set up and solved as follows. We set $n = 20$ and $m = 100$, each state being equiprobable. Let B be the $n \times m$ matrix of returns. We first independently draw each element of B from a *Lognormal*$(0, 1)$. We then rescale payoffs to assign loans different NPV: we draw 10 numbers from a $U(0.95, 1)$ and 10 from a $U(1, 1.05)$. Using these numbers (in ascending order), we rescale the columns in B such that the averages of these columns match these numbers. We then solve for the optimal portfolio for different γ by grid search over all possible portfolio combinations.

Appendix D: Additional Numerical Results

Discrete Loans and Heterogeneity in Capital Requirements: Here we provide an example of a positive lending response when the bank can only choose to make identical new loans that face heterogeneous capital requirements. We reconsider the model in Section III.C but impose the requirement that the matrix B is composed of n identical $m \times 1$ columns such that the loans always deliver identical payoffs. We then rescale B such that the loans all have a small positive NPV of 0.2%. Last, loan i is given a specific risk weight such that its effective capital requirement, γ_i, is given by $\gamma_i \equiv \frac{2i}{20} \times \gamma$. This corresponds to risk weights from 10% to 200% in 10% increments. We again solve for the optimal portfolio through grid search.

Figure D1 presents the relationship between total lending and γ for a randomly drawn B. The black line plots the total level of lending and a black circle indicates whether a particular loan is financed.

First, note that since the payoffs across loans are identical, for any given state j, loan 1 has the smallest realized residual cash flow. Using our definition of the marginal subsidy in a discrete state space (equation (15)), this means that $s^*_1 \geq s^*_2 \geq \ldots \geq s^*_{20}$. Given that all loans are equal NPV, this generates a clear pecking order: loans with the lowest risk weight are always financed first.

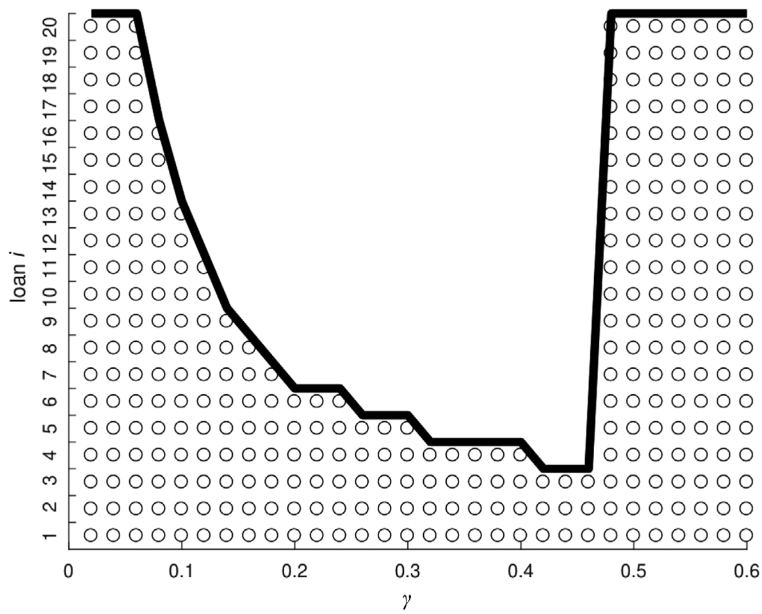

Figure D1. Numerical example: Discrete new loans with risk weight heterogeneity. The figure provides an example of the bank's optimal portfolio choices for different values of γ, the capital requirement, with $n = 20$ assets and $m = 100$ states. On the x-axis is the capital requirement at two percentage point increments. On the y-axis is the corresponding portfolio choice. Loans are sorted in increasing order according to their risk weight (which is given by $\frac{2i}{20}$). That is, loan 20 has the highest risk weight. For a given capital requirement, a circle means that the corresponding loan is in the optimal portfolio of the bank. The solid line indicates the number of loans in the portfolio.

Second, a clear U-shape emerges. To understand where the familiar U-shape comes from, first consider the two extremes. If the capital requirement, γ, is very high, the bank never defaults, which means that $s_i^* = 0, \forall i$ and since all loans are positive NPV they are all financed. Now consider the other extreme when the capital requirement is very low: now the bank defaults with positive probability and even the 20th loan, with the highest risk weight, generates negative residual cash flows in the default states. This means that $s_i^* > 0, \forall i$ and so all loans are financed. Next consider an intermediate level of γ. The bank will still default but the capital requirement will be sufficiently high that s_i^* is negative, and greater than the NPV in absolute terms, for loans with a high risk weight. These loans therefore are not financed.

The Role of Correlation: To illustrate the link between the correlation of asset payoffs and the distortion arising from government guarantees, Figure D2 revisits the numerical example in Figure 5. In particular it presents for each loan $i \in \{1, 2, \ldots, 20\}$ and two different levels of the capital requirement ($\gamma_b = 6\%$ and $\gamma_b = 40\%$) (i) the correlation between loan i's payoff and the aggregate

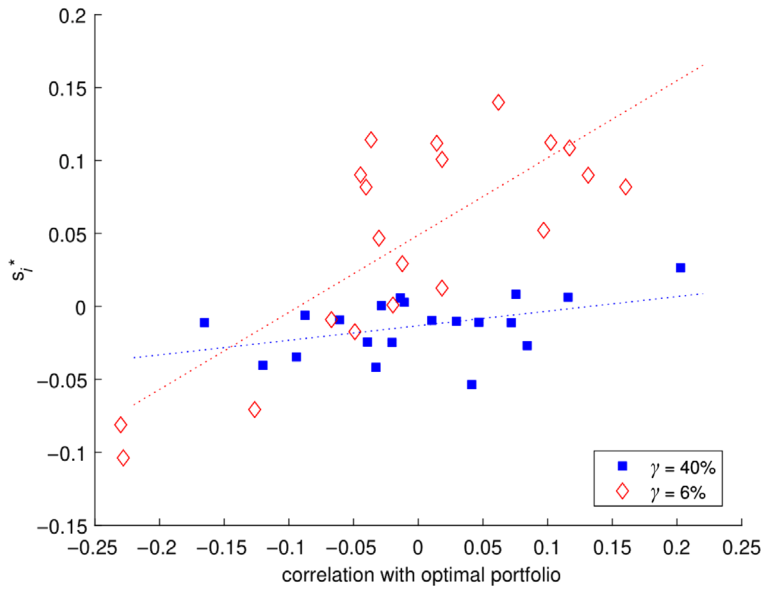

Figure D2. Numerical example: The link between correlation and the marginal subsidy. The graph presents for loans $i = \{1, 2, \ldots, 20\}$ in Figure 5 the relationship between (i) the correlation between loan i's payoff and the aggregate payoff on all other loans in the bank's portfolio (x-axis) and (ii) the marginal subsidy, s_i^* (y-axis). Red diamonds indicate observations when $\gamma_b = 6\%$ the blue squares indicate observations when $\gamma_b = 40\%$. (Color figure can be viewed at wileyonlinelibrary.com)

payoff on all other loans in the bank's portfolio; versus (ii) the marginal subsidy, s_i^*.

Three points stand out. First, the correlation is positively related to the marginal subsidy. If the loan is highly correlated with the rest of the bank's portfolio, it is more likely to have low residual cash flows when the bank is in default resulting in a higher s_i^*.

Second, the correlation is not a sufficient statistic for s_i^*. There are occasions when a loan has a high s_i^* even though the loan's correlation with the overall portfolio is relatively low. This is because s_i^* depends only on the residual cash flows in the default region. So this outcome is perfectly plausible if the loan is only weakly related to the overall portfolio most of the time but generates low residual cash flows in expectation when the bank fails.

Third, the relationship between the correlation and s_i^* is weakening in the level of the capital requirement. The intuition for this is straightforward: the higher the capital requirement, the safer the bank and thus default is more of a tail event. This means that the overall correlation, across all states of the world, between cash flows is less informative about whether loan i will perform badly in the default region.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication code.