Economic Modelling 82 (2019) 168–184

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.journals.elsevier.com/economic-modelling

The relationship between trading activity and stock market volatility: Does the volume threshold matter?

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A R T I C L E I N F O

JEL classification:
G12
G17
C22
C58

Keywords:
Volatility
Trading volume
Threshold effect
Forecast evaluation
Trading profit

A B S T R A C T

This paper examines whether trading activity conveys valuable information about changes in market volatility dynamics. We use a modelling framework, in which the market smoothly switches from one state to another, according to the volume level. Results show that large volume drives the high volatility regime for most of the markets, quite consistently with the disagreement-in-beliefs hypothesis. The volume decomposition into normal trading activity and surprising information arrival reveals a reverse threshold linkage for emerging markets. Results support the sequential information arrival hypothesis and highlight the key role of asymmetric information and thin trading in modelling the volume-volatility relationship. The proposed volume-based models provide significant forecast improvements over competing models and offer scope for investors to earn substantial profits.

1. Introduction

The linkage between trading volume and return volatility has been at the centre of a plethora of academic publications. Various microstructure models have attempted to provide theoretical arguments for the relationship between price changes and trading volume. Competitive models hypothesize a positive volume-volatility nexus arising from the price impact of informed traders who operate large sized-orders (Easley and O'Hara, 1987; Holthausen and Verrecchia, 1990). On the other hand, strategic models predict that informed traders may prefer to execute small trades in an attempt to hide their private information, thereby generating a weaker or even a negative relationship between trade size and volatility (Kyle, 1985; Foster and Viswanathan, 1996; Chordia and Subrahmanyam, 2004). Competing theories underscore the basic role of either the information flow as suggested by the mixture of distributions hypothesis (MDH) pioneered by Clark (1973) and the sequential information arrival hypothesis (SIAH) introduced by Copeland (1976) or the dispersion-of-beliefs hypothesis (Harris and Raviv, 1993; Shalen, 1993) in modulating the volume-volatility relationship.

Under the MDH, price change and trading volume are jointly subordinate to the intensity of information flow to the market. A long-lasting explanation of the volatility clustering stylized fact is that the rate of information arrival is itself time-varying. Commonly considered as a proxy of latent information flow, the MDH predicts that contemporaneous trading volume is positively correlated with volatility (Epps and Epps, 1976; Tauchen and Pitts, 1983; Andersen, 1996). Thus, it may explain GARCH effects. Nevertheless, Li and Wu (2006) provide a theoretical support for a possible negative relationship between volume and volatility. Similarly to the MDH, the SIAH predicts a positive relationship, albeit driven by a sequential information arrival. Accordingly, lagged trading volume is expected to have significant forecasting power. In contrast, dispersion-of-beliefs theory supports an asymmetric relationship between volume and volatility arising from differential interpretation of the same public information flow among heterogeneous market participants. Additional evidence for such an asymmetric relationship is reported by Giot et al. (2010). They argue that "good" volatility (i.e., predictable volatility) is generally associated to high volume. "Bad" volatility (i.e., volatility spike) is accompanied by low volume. Balduzzi et al. (1996), Wagner and Marsh (2005) and Chen (2012) also reciprocate that the volume-volatility linkage may take various shapes as the market evolves across different phases of the business cycle.

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https://doi.org/10.1016/j.econmod.2019.01.003
Received 11 April 2018; Received in revised form 6 December 2018; Accepted 12 January 2019
Available online 21 January 2019
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Building on these insights, the present study brings novelties to the volume-volatility nexus by examining a possible threshold effect of trading volume on conditional volatility within a Smooth Transition FIGARCH (STFIGARCH) model. Specifically, the present study addresses the question of whether trading volume conveys information about the state of volatility and whether it has forecasting power. This is a relevant question because it provides new insights into regime-specific volume effects and bears several economic implications.

The primary economic motivations for the STFIGARCH model are as follows. First, there is mounting evidence that financial markets may be characterized by non-linear behaviours resulting from the presence of market frictions and decisions made by a large number of heterogeneous investors (e.g., informed *vs* noise traders and chartists *vs* fundamentalists). Depending on the speed of reaction of market participants and the degree of synchronicity of their decisions, the market moves gradually from transitory intermediate states until the final steady state is reached. The transition is activated by an observable and meaningful economic variable in a quite different way from Markov switching models, in which the transition is generated by a hidden Markov chain. In addition, the parameters of the model change smoothly as the market moves from one state to another according to the level of trading volume (i.e., above or below the threshold value). Therefore, the model embodies time-varying effects of trading volume. Second, the FIGARCH model is often used to accommodate the long-range dependence feature of volatility and has been proven highly accurate in modelling both economic and financial time-series (see Henry and Zaffaroni, 2003; Giraitis et al., 2005, for an excellent survey). Moreover, Bollerslev and Jubinski (1999) and Fleming and Kirby (2011) propose a long-run view of the MDH, seeing that the information flow, that drives price changes, exhibits itself long memory. Even though the aggregate information arrival process accommodates a mixture of numerous heterogeneous short-run processes, Andersen and Bollerslev (1997) show that the volatility process is likely to display long memory.

This paper contributes to the literature in a number of ways. First, our research question has important practical implications since investors and policymakers are strongly concerned with changing market dynamics. Though myriad economic and extra-economic forces may drive different phases of stock market cycles, our study shows how market regimes can be identified by trading activity. This can help regulatory authorities in limiting price manipulation and improving market resiliency. Second, in a significant departure from most prior research that has been limited to report on-average correlation and/or causal relationships, what we aim to examine in this study is how much volume is needed to generate high and low conditional volatility. On the methodological side, the present study owes a lot to McMillan (2007), albeit the latter investigates the return-volume linkage. Third, the smooth transition entails endogenous thresholds in contrast to the TARCH specification (Glosten et al., 1993; Zakoian, 1994) employed in previous related studies (Hadsell, 2006; Ureche-Rangau et al., 2011; Jawadi and Ureche-Rangau, 2013), where the threshold is *a priori* set to zero. Consequently, conditional volatility switches abruptly from one regime to another according to the sign of news. A sharp switching pattern may be fairly restrictive in the case where heterogeneous market participants with different trading motives and risk assessments are involved, implying different bands of inactivity around fundamental equilibrium. Moreover, even if new information is simultaneously released to all agents, their reaction may be more accurately described as taking place smoothly over time. Therefore, our modelling approach affords more flexible and thus realistic assumptions. Fourth, most conventional investigations of the MDH have focused on examining whether inserting the volume into the conditional variance function subsumes ARCH effects (e.g., Lamoureux and Lastrapes, 1990), which is weakly consistent with the MDH and most of market microstructure trading models that assume endogenous trading volume. This casts some doubt on the validity of prior conclusions, as noted by Fleming et al. (2006). We circumvent this specification bias by including volume as a transition variable. Fifth, the empirical analysis is conducted for developed and emerging countries to evaluate the extent to which our findings apply to different institutional settings and market microstructure.[1] Following Girard and Biswas (2007), we also consider expected and surprising volume components and extend their analysis to a non-linear framework. Finally, accurate estimates and forecasts of volatility are primary inputs for investment decision-making, risk management, derivatives pricing and hedging. We, therefore, examine the forecasting performance of our volume-based non-linear model and compare its statistical accuracy to both nested linear specifications and a highly competitive model that includes return shocks as the switching variable. Going one step further, we put forward the economic relevance of our study by devising simple trading strategies based on model estimates and volatility forecasts. The results provide strong support for the proposed volume-based framework not only on the grounds of statistical accuracy but also in terms of trading profits.

The rest of this article is structured as follows. Section 2 presents related literature while focusing on the methodological contributions of the current study. Section 3 introduces the econometric models. Section 4 describes the data and discusses the results of the empirical investigation. Section 5 concludes.

2. Literature review

Previous empirical research has provided strong support for the positive contemporaneous volume-volatility relationship as predicted by the MDH (Karpoff, 1987; Harris, 1987; Jones et al., 1994; Lamoureux and Lastrapes, 1990; Brailsford, 1996; Chan and Fong, 2000, 2006; Slim and Dahmene, 2015). Arago and Neito (2005) split total volume into expected and unexpected components and report a greater effect of unexpected volume on volatility. In a similar line, Girard and Biswas (2007) examine the role of differing trading systems on the relation between volume components and conditional volatility. The authors find a positive relationship between unexpected volume and volatility but a negative expected volume-volatility linkage in emerging markets. Among the different investigations of the SIAH, Brooks (1998) and Lee and Rui (2002) provide evidence of bidirectional feedback between volume and volatility. Nevertheless, Brooks (1998) argues that lagged volume leads to very poor improvements in forecasting performance. Fuertes et al. (2009) reciprocate similar limited information content of lagged volume when it is incorporated into the GARCH modelling framework for assigning market conditions. Empirical evidence in compliance with the SIAH is provided by Blume et al. (1994), Darrat et al. (2003), Le and Zurbruegg (2010), Chiang et al. (2010) and Mougoue and Aggarwal (2011), among others. Chuang et al. (2012) use a bivariate GJR-GARCH model to investigate simultaneously the contemporaneous and the causal relationship between volume and volatility in international financial markets. They conclude that the positive bidirectional causality is more likely to be found in developed markets rather than in developing markets.

All in all, past empirical research finds inconclusive evidence on the nature and even the shape of the relationship between volume and volatility which might be better characterized by non-linear models. For the benefit of readers, Table 1 summarizes key contributions to the price-volume nexus by using the non-linear approach to better highlight the contribution of the present study.

The early work of Gallant et al. (1993) has paved the way for further investigations along this line of research. The authors examine the joint dynamics of the S&P composite returns and the NYSE trading volume by using non-linear impulse response functions. Their results point to a very small feedback running from volume shocks to conditional volatility. Refinement of non-linear impulse response analysis

[1] The authors are grateful to an anonymous reviewer for suggesting to include emerging markets into the sample.

Table 1
Key studies addressing non-linearity.

Study	Market	Volatility measure	Source of non-linearity	Addresses regime-switching issue?	Evaluates forecasting performance?	Method	Main findings
Brooks (1998)	NYSE	Squared returns	Unspecified	No	Yes	Non-linear Granger causality, volume-augmented GARCH	Evidence of bidirectional causality, lagged volume leads to very limited improvements in forecasting performance
Chiang et al. (2010)	US stock market	Realized and conditional volatility	Unspecified	No	Yes	Linear and non-linear Granger causality tests, Volume-augmented EGARCH and GJR-GARCH	Strong bidirectional non-linear causality, evidence for the usefulness of volume to predict volatility
Damette (2016)	Forex	Realized volatility	Trading volume and volatility	Yes	No	STR	Strong relationship in turbulent periods
Gallant et al. (1993)	NYSE	Conditional volatility	Unspecified	No	No	Non-linear impulse response analysis	Very small feedback from volume shocks to volatility
Hiemstra and Jones (1994)	US stock market	EGARCH-filtered returns	Unspecified	Through dummies	No	Non-linear Granger causality	Bidirectional non-linear causality between returns and volume
Jawadi and Ureche-Rangau (2013)	Developed and emerging stock markets	absolute returns	Volume-volatility deviation	Yes	No	STR and TARCH	Non-linearity of smooth transition type, asymmetric relationship
Jawadi et al. (2016)	Major stock markets	Realized volatility, continuous volatility and jumps	Trading volume	Yes	No	TAR and threshold Tobit	Evidence for non-linearity, positive relationship between trading volume and realized volatility and its components
McMillan (2007)	Major stock markets	Stock returns	Trading volume	Yes	Yes	STAR and TAR	Non-linear return-volume relationship, suitability of non-linear models with volume as the switching variable in terms of trading profits
Mougoue and Aggarwal (2011)	Forex	Realized and conditional volatility	Unspecified	No	No	Linear and non-linear Granger causality tests	Significant non-linear lead-lag relationships
Tauchen et al. (1996)	NYSE	Conditional volatility	Unspecified	No	No	Non-linear impulse response analysis	Evidence of non-linear causality between volume and stock returns
This study	Developed and emerging stock markets	Conditional volatility	Trading volume	Yes	Yes	STFIGARCH	

leads Tauchen et al. (1996) to draw similar conclusions on disaggregated data on the NYSE. In the same vein, Brooks (1998) and Chiang et al. (2010) provide strong support for a non-linear Granger causality in the inter-temporal relationship between volume and volatility in the stock market, consistently with the SIAH. Their findings are corroborated by Mougoue and Aggarwal (2011) in the currency futures market. Using the same methodology, Hiemstra and Jones (1994) find evidence of strong bidirectional non-linear causality between volume and stock returns. While these studies emphasize the importance of using the non-linear approach, to capture the dynamic relationship between stock prices and trading volume, such an approach provides no guidance regarding either the appropriate non-linear functional form for the forecasting model or the source of non-linearity. Moreover, the forecasting power of trading volume has been based on statistical evaluation of volume-augmented GARCH models in Brooks (1998) and Chiang et al. (2010). This is weakly consistent with the evidence of non-linear dependence since trading volume enters linearly into the conditional variance equation of returns, which may explain the disappointing results in Brooks (1998).

In an attempt to reconcile the relationship between trading volume and regime-switching market behaviour, a promising line of research more closely related to the current paper, focuses on smooth transition models. The relevance of considering volume as a source of non-linearity within equity return dynamics is examined by McMillan (2007). The author considers a Smooth Transition Autoregressive Regression (STAR) model, in which lagged volume is used as the switching variable. Findings from this study indicate that such a non-linear model yields beneficial improvements over alternative linear and non-linear specifications, in terms of forecasting future returns, for major international stock markets. The present study is an attempt to extend the McMillan (2007)'s findings to the volume-volatility framework. Jawadi and Ureche-Rangau (2013) employ a Smooth Transition Regression (STR) model to capture threshold linkages in developed and emerging stock markets. Volume is included into the regression model as an additional explanatory variable and the optimal threshold variable corresponds to the lagged difference between volume and volatility. Based on goodness-of-fit tests, their findings point to the superiority of STR over TARCH model, in which the transition is abrupt. Even though results from this study appear to be country-specific, they point to a weak association between volume and low volatility while the relationship is stronger in the high volatility regime. However, the selected threshold variable provides no guidance as to the reaction of volatility to different volume levels. The framework we use enables us to apprehend the intensity of the linkage between extreme volatility (i.e., high and low) states and the level of trading activity (i.e., below and above average volume) in a parsimonious way. Based again on STR models, Damette (2016) points to the state dependency of the MDH in the currency market. While the positive association between volume and volatility cannot be rejected in normal market conditions, it tends to strengthen during market stress when both trading volume and realized volatility are at high levels. Jawadi et al. (2016) extend the study of Giot et al. (2010) to a non-linear framework and examine the effect of volume on realized volatility and its component (i.e., continuous volatility and jumps). They use discrete switching models, namely a Threshold Autoregressive (TAR) model and a two-regime threshold Tobit model in order to handle the issue of censored data regarding the jump component. This study finds a weak relationship between volume and both realized and continuous volatility in the low regime, and a stronger volume effect in the high regime. On the other hand, while the jump component is weakly related to trading volume in the low regime, the volume effect tends to disappear in the high regime. The authors conclude that sharp price changes are not necessarily associated to abnormal trading volume.

To sum up, the above-mentioned studies share the common conclusion that the information content of volume varies across the volatility regimes, with little emphasis on examining the extent to which it may improve the performance of non-linear volatility forecasting models. The present study attempts to fill this gap by evaluating the role of trading volume in generating future states of market volatility on the grounds of both statistical and economic metrics.

3. Econometric models

This section describes the models employed to examine the issue. The present study focuses on the non-linear impact of trading volume on market volatility. Accordingly, we propose an STFIGARCH model, which generalizes the standard FIGARCH, to allow for non-linear dynamics by introducing a smooth transition specification for the conditional variance process.

3.1. FIGARCH model

The FIGARCH(p, d, q) model is developed by Baillie et al. (1996) as an extension of the GARCH model to account for the long memory property observed in financial time-series as well as to distinguish between long and short memory in the conditional variance. For a discretely sampled return process $\varepsilon_t = \eta_t \sigma_t$, for $t = 1, \dots, T$, the conditional variance is given by the following infinite ARCH representation:

$$\sigma_t^2 = \frac{\omega}{\beta(1)} + \left[1 - \frac{\Phi(L)}{\beta(L)}(1-L)^d\right]\varepsilon_t^2 \tag{1}$$

where ε_t is the error at time t. The standardized residual η_t is independently and identically distributed following a Student-t distribution, with zero mean and unit standard deviation. $\Phi(L) = 1 - \sum_{i=1}^{q} \phi_i L^i$ and $\beta(L) = 1 - \sum_{i=1}^{p} \beta_i L^i$ are polynomials in the lag operator L with all their roots lying outside the unit circle. The long memory parameter, $0 < d < 1$, indicates that shocks to the conditional variance dissipate slowly following a hyperbolic rate of decay. The conditions for non-negativity of conditional variance are given in Bollerslev and Mikkelsen (1996) and Conrad and Haag (2006).

3.2. Smooth transition FIGARCH model

From the discussion in Section 2, the nature of the relationship between index returns and volume appears to support a threshold-type model whereby the dynamics of volatility alters, according to the volume size. As such, we consider the STFIGARCH model (Kiliç, 2011) which is particularly designed to jointly characterize long-range dependence and regime-switching volatility dynamics in a parsimonious way as follows:

$$\sigma_t^2 = \frac{\omega}{1 - \beta\,(1 - G\,(z_{t-s})) - \beta^* G\,(z_{t-s})} + \left[1 - \frac{(1-\phi L)\,(1-L)^d}{1 - \beta\,(1 - G\,(z_{t-s}))\,L - \beta^* G\,(z_{t-s})\,L}\right]\varepsilon_t^2 \tag{2}$$

where σ_t^2 is the conditional variance of ε_t and $G\,(z_{t-s})$ is the transition function with z_{t-s} being the transition variable, which is typically lagged returns, though in the present paper this is extended to also include lagged volume. The suggested logistic form of the transition function is given by:

$$G(z_{t-s}) = \frac{1}{1 + \exp(-\gamma(z_{t-s} - c))} \tag{3}$$

where s is the delay parameter and c is the threshold or location parameter. The transition parameter γ determines the smoothness of change in the value of the transition function between regimes and is assumed to be positive for identification purposes. The smooth transition approach also nests an abrupt transition as a special case if $\gamma \to \infty$. The transition function is bounded between 0 and 1 and allows the parameters to change monotonically with z_{t-s}. Eq. (2) shows that the infinite ARCH terms depend on the volatility regime in a given date t and so does the constant term, which changes smoothly and takes values between $\omega/(1-\beta)$ and $\omega/(1-\beta^*)$ across extreme regimes $G(z_{t-s}) = 0$ and $G(z_{t-s}) = 1$, respectively. As $z_{t-s} \rightarrow +\infty$, $G(z_{t-s}) \rightarrow 1$, and hence the STFIGARCH(1,d,1) behaves locally like a FIGARCH(1,d,1) process with volatility dynamics parameter β^*. When $z_{t-s} \rightarrow -\infty$, $G(z_{t-s}) \rightarrow 0$, the STFIGARCH(1,d,1) reduces to a FIGARCH(1,d,1) model with parameter β. When $z_{t-s} \rightarrow c$ or when $\gamma = 0$, then $G(z_{t-s}) \longrightarrow \frac{1}{2}$, and hence the dynamic behaviour of STFIGARCH(1,d,1) becomes similar to that of a FIGARCH(1,d,1) model with parameter $\frac{\beta+\beta^*}{2}$. The parameter β corresponds to the lower regime where the function $G(z_{t-s})$ takes a value of zero, while β^* characterizes the upper regime where $G(z_{t-s})$ equals unity. Thus, the terms β and β^* can be used to depict the volatility responsiveness to the considered threshold variable.

The properties of the non-linear STFIGARCH model can be better described by considering its implied news impact curve (Pagan and Schwert, 1990) which shows the relationship between the current shock ε_t and conditional variance in the next period, σ^2_{t+1}, keeping all other information constant. The news impact curve (NIC) for the STFIGARCH(1,d,1) model is given by the following equation:

$$NIC\left(\varepsilon_t \mid \sigma_t^2 = \sigma^2\right) = \omega + \left[\beta + \left(\beta^* - \beta\right) G\left(z_{t-s}\right)\right]\sigma^2 \\ + \left[1 - \beta - \left(\beta^* - \beta\right) G\left(z_{t-s}\right)\right]\varepsilon_t^2 - \left[(1-\phi)\sum_{i=0}^{\infty}\pi_i(d)\right]\varepsilon_t^2 \tag{4}$$

where $\pi_i(d) = \frac{\Gamma(i-d)}{\Gamma(i+1)\Gamma(-d)}$ and $\Gamma(.)$ denotes the Gamma function. The impact of a shock on the next period's conditional volatility depends on the volatility regime and evolves smoothly between the extreme regimes. Here, a change in the value of σ^2 also moves the NIC vertically, similar to the FIGARCH model, but the size of these moves depends on the volatility regime. If $\beta < \beta^*$, the impact of a shock would be higher when $z_{t-s} \rightarrow -\infty$ $(G(z_{t-s}) = 0)$ than $z_{t-s} \rightarrow +\infty$ $(G(z_{t-s}) = 1)$ and inversely if $\beta > \beta^*$.

Estimation of the parameters of the STFIGARCH model is carried out by the method of quasi-maximum likelihood estimation, where the log-likelihood is numerically maximized with respect to the vector of parameters $(\omega, \phi, \beta, \beta^*, d, c, \gamma, \nu)'$, where ν is the degree of freedom parameter under the Student-t distribution.[2] The main distinctive feature of our analysis is to consider three transition variables, namely the lagged index return, the lagged volume and the lagged volume variation.

The established volatility-volume relationship motivates the use of volume as a threshold variable in our non-linear volatility model. The market alternates between calm and volatile periods as a response to the intensity of information arrival. Provided that trading volume is an appropriate surrogate for the unobservable information flow, tracking its behaviour over time can afford a broad portrayal of the market state. More picturesque, one might expect the high volatility regime to be driven by heightened volume and the low volatility regime to be accompanied by moderate volume. Regime-switching can also be caused by threshold response of investors to news arrivals as follows. Each agent compares the signal he receives with his individual threshold and undertakes a decision. Heterogeneity ensures that individual thresholds are different and specific to each agent depending on the information nature and trading behaviour (i.e., chartists *vs* fundamentalists and informed *vs* uninformed), which leads a fraction of agents to being relatively more active in any time period. The heterogeneous reactions of investors induce a continuum of regimes and accommodate time-varying volume-volatility sensitivities depending on changes in the proportion of active trader's type in the market. The volume threshold determines the extreme regimes, associated to the extreme values of transition function, and hence reflects the average response to information arrival. Finally, behavioural switching can be regarded as an additional trigger for regime shifts. In Lux and Marchesi (2000), an outbreak of volatility occurs if the fraction of chartists exceeds a certain threshold value, but the market returns to its tranquil mode of operation by stabilizing tendencies, and DeLong et al. (1990) suggest that the preponderance of noise traders exacerbates excess volatility.

In the subsequent empirical investigation, we compare in-sample and out-of-sample performance of the proposed non-linear specifications along with the baseline FIGARCH model.

4. Empirical results and discussion

4.1. Data and preliminary analysis

For the empirical analysis, our sample consists of five developed markets including France (CAC 40), Germany (DAX), Japan (NIKKEI 225), UK (FTSE 100) and USA (DJIA), and four emerging markets, namely China (SSEC), Mexico (MXSE IPC), India (S&P SENSEX) and South Korea (KOSPI COMPOSITE).[3] The data are obtained from Thomson Reuters Eikon over the sample period from January 3, 2000 to August 31, 2015. For each market, the data are daily and consist of trading volume (the number of shares traded of market index constituents) and closing price of the stock market index. The inclusion of data from different venues is expected to enrich the analysis given their diverse trading and regulation systems, firms ownership structures and market depth. We use long time-series to ensure both reliable in-sample estimates and sufficient data to evaluate out-of-sample forecasts. More importantly, the coverage of a long time period enables us to reflect actual trading conditions wherein the market alternates between episodes of prosperity and market meltdowns.

Summary statistics for the volume measures and the index percentage returns, computed on a continuously compounded basis, are presented in Table 2. These summary statistics reveal the usual characteristics of stock returns, namely a mean value which is dominated by the standard deviation value and evidence of non-normality. Moreover, the Ljung-Box $Q^2(20)$ statistics for the squared returns are statistically significant, providing strong evidence for volatility persistence. These salient features of daily returns justify the use of FIGARCH-type models based on fat-tailed distributions. In order to ensure stationarity of the volume variable, we follow the convention in Gallant et al. (1992) and detrend the raw volume. The detrended volume series are the residuals inferred from the OLS regression of the raw log-volume series on both linear and non-linear time trends. The volume variation variable is calculated as the first-difference of the raw log-volume. The unit root test confirms that both volume series are stationary.

4.2. Linearity tests

The prerequisite for smooth transition models is to test the null hypothesis of linearity against its alternative hypothesis of non-linearity. However, under the null hypotheses $\beta = \beta^*$ and $\gamma = 0$, the STFIGARCH model is not identified and the asymptotic distribution of t-statistics and *Wald* tests are not valid (Davies, 1987). To circumvent the identification problem, the test strategy consists in expanding the transition function by using Taylor series expansions around $\gamma = 0$ (see Luukkonen et al., 1988; González-Rivera, 1998; Lundbergh and Teräsvirta, 2002, and references therein). We follow Kiliç (2011) and conduct robust *Wald* tests for testing the null hypothesis of FIGARCH against the alternative STFIGARCH specification. We

[2] Conditional volatility is non-negative provided that $\omega > 0$, $\phi \leq (1-d)/2$ and $d + \phi - \beta \geq 0$ for some $0 < \beta < 1$, and $d + \phi - \beta^* \geq 0$ for the case $0 < \beta^* < 1$, in the limiting regimes. The non-negativity conditions also hold in the intermediate regimes provided that they are satisfied in the extreme regimes (Kiliç, 2011).

[3] The selection of emerging markets is based on the MSCI market classification criteria as of June 2018 (https://www.msci.com/market-classification).

Table 2
Summary statistics.

Index Return	Mean	S.D.	Skew	Kurt	Min	Max	$Q^2(20)$	J-B
Developed markets								
France	−0.007	1.510	−0.017	7.755#	−9.472	10.595	3254#	3755#
Germany	0.010	1.563	−0.093‡	7.600#	−10.163	10.797	3425#	3499#
Japan	0.017	1.534	−0.563#	10.363#	−12.111	13.235	3820#	7397#
UK	−0.002	1.237	−0.238#	9.452#	−9.266	9.384	4200#	6856#
US	0.010	1.194	−0.068*	11.010#	−8.201	10.508	4896#	10460#
Emerging markets								
China	0.021	1.645	−0.271#	7.388#	−9.256	9.401	1129#	3075#
Mexico	0.047	1.363	0.052	8.007#	−8.267	10.441	2589#	4090#
India	0.040	1.564	−0.194#	9.981#	−11.809	16.115	1514#	7912#
South Korea	0.015	1.618	−0.556#	8.902#	−12.805	11.284	1775#	5793#
Volume	**Mean**	**S.D.**	**Skew**	**Kurt**	**Min**	**Max**	**PP**	**J-B**
Developed markets								
France	7.33E-16	0.366	−0.674#	6.435#	−2.497	1.318	−30.90#	3053#
Germany	3.31E-15	0.347	0.123	6.245#	−2.183	2.509	−33.75#	1888#
Japan	1.92E-14	0.281	0.582#	4.038#	−1.114	1.356	−21.05#	259#
UK	−7.14E-17	0.327	−1.558#	11.417#	−2.763	1.230	−29.58#	17783#
US	4.41E-15	0.292	0.475#	5.464#	−1.629	1.392	−32.15#	1321#
Emerging markets								
China	−1.98E-15	0.059	0.085‡	2.410	−0.225	0.204	−12.34#	59.42#
Mexico	2.32E-16	0.026	−1.529#	9.610#	−0.192#	0.112	−39.98#	8652#
India	−4.90E-16	0.028	−0.063	6.382#	−0.203	0.140	−27.42#	1854#
South Korea	−8.30E-17	0.019	0.328#	3.657#	−0.055	0.083	−15.78#	138.7#
Volume variation	**Mean**	**S.D.**	**Skew**	**Kurt**	**Min**	**Max**	**PP**	**J-B**
Developed markets								
France	4.94E-04	0.321	−0.290#	7.987#	−2.574	1.657	−91.96#	4187#
Germany	2.57E-04	0.328	−0.175#	9.587#	−2.636	2.425	−92.30#	7185#
Japan	4.37E-04	0.196	−0.062	4.679#	−0.922	1.032	−79.31#	376#
UK	1.24E-04	0.279	−0.047	10.409#	−2.371	2.341	−88.34#	8992#
US	−4.66E-05	0.267	−0.281#	8.525#	−2.159	1.411	−93.93#	5027#
Emerging markets								
China	1.22E-03	0.230	0.687#	7.477#	−1.319	1.865	−77.60#	3451#
Mexico	5.95E-04	0.512	0.092‡	8.655#	−2.954	2.663	−92.66#	5220#
India	−1.08E-04	0.372	−0.337#	24.476#	−4.412	3.914	−98.84#	74732#
South Korea	1.31E-04	0.184	0.443#	5.675#	−0.925	1.438	−87.19#	1276#

This table reports main statistics for daily returns, detrended volume and volume variation over the full sample period from January 3, 2000 to August 31, 2015. $Q^2(20)$ denotes the Ljung-Box test statistics for up to 20th-order serial correlation in squared returns and J-B denotes the Jarque-Bera test statistics for normality. The PP statistic is the Phillips-Perron test including a constant, with the bandwidth chosen by the Newey and West (1994)'s automatic selection method. #, ‡ and * indicates statistical significance at 1%, 5% and 10% level, respectively.

estimate STFIGARCH models with our three presumed transition variables, i.e., detrended volume (V_{t-s}), volume variation (ΔV_{t-s}) and index return (ε_{t-s}) with a delay parameter s up to five. In all series, the choice of $s = 1$ is found to provide the best fit. The test statistics W_1 and W_3, reported in Table 2, have an asymptotic χ^2 distribution with one and three degrees of freedom and correspond to first and third Taylor expansion of STFIGARCH around $\gamma = 0$, respectively.

The results from the W_3 test indicate that the null hypothesis of linearity is significantly rejected for all the markets, regardless of the selected transition variable. However, the W_1 test fails to reject the null in favour of the non-linear specification with ΔV_{t-1} as the transition variable for the case of France, Germany and US. Not surprisingly, this result is consistent with the literature (Lundbergh and Teräsvirta, 2002; Kiliç, 2011) that W_3 performs considerably better than W_1 in discriminating the FIGARCH from the STFIGARCH model.[4] Further, the p-values associated to the Lagrange Multiplier (LM) tests are higher than standard significance levels for all the markets, except for Japan. Thus, the null hypothesis of no remaining non-linearity of logistic form in residuals cannot be rejected, which confirms that switching regime in the volatility dynamics is guided by trading volume. Finally, the Ljung-Box test shows no evidence of remaining serial correlation for the fitted non-linear models.

4.3. In-sample results

4.3.1. Total volume

In light of linearity test results, we discuss the estimation results of the corresponding STFIGARCH models. We purposely focus our analysis on the coefficient estimates for β and β^* because they characterize the intensity of the volume-volatility relationship in the limiting regimes.[5]

Table 3 shows that the values of the long memory parameter d are statistically significant. However, it is well documented in the financial econometrics literature that the presence of non-linearity such as level shifts, regime shifts or occasional structural breaks might be easily confused with the observed long memory in volatility (Baillie et al., 1996; Diebold and Inoue, 2001). As a result, even though inference on the standard FIGARCH model shows that d is statistically significant, it could be caused by neglected non-linearity. Consistently with

[4] The power of W_3 increases, especially with respect to the smoothness parameter γ and the difference between the autoregressive parameters β and β^* (Kiliç, 2011).

[5] Note that in smooth transition models, one can only observe the extreme regimes, the intermediate states being a mixture of them.

Table 3
Estimation results of linear and non-linear FIGARCH models for daily index returns.

Country	Model	ω	ϕ	β	β^{*}	d	c	γ	ν	LL	W_1	W_3	LM	$Q^2(5)$
Developed markets														
France	FIGARCH	$0.037^{\#}$	$0.082^{\ddagger}$	$0.659^{\#}$		$0.609^{\#}$			$9.602^{\#}$	−6575				3.364
		(0.012)	(0.041)	(0.070)		(0.080)			(1.407)					[0.644]
	STFIGARCH(ε)	$0.065^{\#}$	$0.042^{\#}$	$0.518^{\#}$	$0.638^{\#}$	$0.542^{\#}$	$1.358^{\#}$	$4.672^{\ddagger}$	$10.543^{\#}$	−6572	15.35	22.45	3.288	3.066
		(0.018)	(0.005)	(0.023)	(0.002)	(0.003)	(0.272)	(1.877)	(3.706)		[0.000]	[0.000]	[0.193]	[0.690]
	STFIGARCH(V)	$0.025^{\ddagger}$	$0.081^{\ddagger}$	$0.683^{\#}$	$0.598^{\#}$	$0.585^{\#}$	$0.163^{\#}$	$31.41^{\#}$	$9.823^{\#}$	−6572	7.094	11.33	3.592	2.732
		(0.012)	(0.039)	(0.080)	(0.089)	(0.085)	(0.021)	(10.50)	(1.485)		[0.008]	[0.010]	[0.166]	[0.741]
	STFIGARCH(ΔV)	$0.037^{\#}$	0.076^{*}	$0.538^{\#}$	$0.670^{\#}$	$0.622^{\#}$	$-0.676^{\#}$	$24.46^{\ddagger}$	$9.699^{\#}$	−6570	0.926	13.68	3.508	3.605
		(0.012)	(0.041)	(0.131)	(0.071)	(0.085)	(0.040)	(11.43)	(1.432)		[0.336]	[0.003]	[0.173]	[0.608]
Germany	FIGARCH	$0.038^{\#}$	0.048	$0.670^{\#}$		$0.648^{\#}$			$9.765^{\#}$	−6617				5.540
		(0.013)	(0.041)	(0.091)		(0.106)			(1.528)					[0.354]
	STFIGARCH(ε)	$0.049^{\#}$	0.026	$0.482^{\#}$	$0.624^{\#}$	$0.509^{\#}$	$1.569^{\#}$	7.612	$11.04^{\#}$	−6607	12.703	28.43	2.349	2.587
		(0.019)	(0.035)	(0.086)	(0.071)	(0.059)	(0.254)	(4.723)	(1.881)		[0.000]	[0.000]	[0.309]	[0.763]
	STFIGARCH(V)	$0.022^{\ddagger}$	0.049	$0.721^{\#}$	$0.561^{\#}$	$0.639^{\#}$	$0.258^{\ddagger}$	$13.12^{\ddagger}$	$9.935^{\#}$	−6609	4.064	115.92	2.074	3.870
		(0.011)	(0.039)	(0.088)	(0.105)	(0.106)	(0.118)	(6.334)	(1.586)		[0.044]	[0.000]	[0.354]	[0.568]
	STFIGARCH(ΔV)	$0.032^{\#}$	0.034	$0.733^{\#}$	$0.637^{\#}$	$0.698^{\#}$	$0.228^{\#}$	$21.36^{\#}$	$9.785^{\#}$	−6609	1.044	18.32	1.546	4.856
		(0.011)	(0.061)	(0.152)	(0.153)	(0.199)	(0.038)	(4.343)	(1.529)		[0.307]	[0.000]	[0.462]	[0.434]
Japan	FIGARCH	$0.048^{\#}$	$0.111^{\ddagger}$	$0.641^{\#}$		$0.573^{\#}$			$9.356^{\#}$	−5398				31.01
		(0.018)	(0.054)	(0.112)		(0.122)			(1.539)					[0.000]
	STFIGARCH(ε)	$0.036^{\ddagger}$	0.095^{*}	$0.471^{\#}$	$0.721^{\#}$	$0.552^{\#}$	−1.703	$0.447^{\#}$	$10.08^{\#}$	−5390	8.444	8.392	179.6	5.316
		(0.017)	(0.049)	(0.126)	(0.063)	(0.086)	(1.893)	(0.166)	(1.785)		[0.004]	[0.039]	[0.000]	[0.379]
	STFIGARCH(V)	0.019	0.078	$0.680^{\#}$	$0.431^{\#}$	$0.492^{\#}$	$0.185^{\ddagger}$	$6.963^{\ddagger}$	$9.307^{\#}$	−5388	7.788	8.349	69.20	8.253
		(0.022)	(0.053)	(0.105)	(0.132)	(0.090)	(0.093)	(2.919)	(1.500)		[0.005]	[0.039]	[0.000]	[0.143]
	STFIGARCH(ΔV)	0.043^{*}	0.063	$0.655^{\#}$	0.428	$0.486^{\#}$	0.063	5.395^{*}	$9.645^{\#}$	−5393	18.83	20.57	14.07	6.884
		(0.023)	(0.073)	(0.136)	(0.267)	(0.113)	(0.319)	(3.199)	(1.600)		[0.000]	[0.000]	[0.001]	[0.229]
UK	FIGARCH	$0.024^{\#}$	$0.102^{\ddagger}$	$0.577^{\#}$		$0.554^{\#}$			$10.93^{\#}$	−5540				3.697
		(0.008)	(0.045)	(0.064)		(0.059)			(1.763)					[0.594]
	STFIGARCH(ε)	0.013	0.003	$0.226^{\#}$	$0.533^{\#}$	$0.477^{\#}$	-1.028^{*}	$1.665^{\ddagger}$	$11.53^{\#}$	−5521	25.44	35.73	4.171	1.207
		(0.010)	(0.002)	(0.086)	(0.051)	(0.049)	(0.569)	(0.789)	(1.964)		[0.000]	[0.000]	[0.124]	[0.944]
	STFIGARCH(V)	$0.018^{\ddagger}$	0.080^{*}	$0.626^{\#}$	$0.515^{\#}$	$0.545^{\#}$	0.047^{*}	30.49^{*}	$10.71^{\#}$	−5536	3.883	17.66	4.035	3.905
		(0.009)	(0.048)	(0.061)	(0.073)	(0.059)	(0.027)	(15.71)	(1.677)		[0.049]	[0.001]	[0.133]	[0.563]
	STFIGARCH(ΔV)	$0.026^{\#}$	$0.103^{\ddagger}$	$0.545^{\#}$	$0.639^{\#}$	$0.549^{\#}$	0.291	10.33^{*}	$11.14^{\#}$	−5535	15.23	18.18	0.513	2.583
		(0.009)	(0.051)	(0.081)	(0.079)	(0.059)	(0.193)	(6.080)	(1.829)		[0.000]	[0.000]	[0.774]	[0.764]
US	FIGARCH	$0.026^{\#}$	0.008	$0.696^{\#}$		$0.701^{\#}$			$7.158^{\#}$	−5304				9.182
		(0.006)	(0.027)	(0.068)		(0.084)			(0.832)					[0.102]
	STFIGARCH(ε)	$0.025^{\#}$	0.027	$0.683^{\#}$	$0.712^{\#}$	$0.686^{\#}$	$1.393^{\#}$	28.59^{*}	$7.284^{\#}$	−5303	34.35	102.4	3.752	7.224
		(0.007)	(0.042)	(0.090)	(0.063)	(0.105)	(0.340)	(16.38)	(0.881)		[0.000]	[0.000]	[0.153]	[0.205]
	STFIGARCH(V)	$0.025^{\#}$	0.024	$0.713^{\#}$	$0.504^{\#}$	$0.688^{\#}$	$0.736^{\#}$	10.75^{*}	$7.179^{\#}$	−5300	4.864	30.21	3.166	6.939
		(0.006)	(0.032)	(0.063)	(0.140)	(0.087)	(0.192)	(5.796)	0.847		[0.027]	[0.000]	[0.205]	[0.225]
	STFIGARCH(ΔV)	$0.025^{\#}$	0.004	$0.707^{\#}$	$0.612^{\#}$	$0.707^{\#}$	0.428^{*}	20.08^{*}	$7.189^{\#}$	−5299	0.368	24.72	4.469	6.995
		(0.006)	(0.004)	(0.060)	(0.118)	(0.069)	(0.239)	(11.80)	(0.844)		[0.544]	[0.000]	[0.107]	[0.221]
Emerging markets														
China	FIGARCH	$0.066^{\ddagger}$	$0.156^{\ddagger}$	$0.700^{\#}$		$0.580^{\#}$			$5.060^{\#}$	−6629				2.957
		(0.032)	(0.061)	(0.170)		(0.211)			(0.423)					[0.707]
	STFIGARCH(ε)	0.066^{*}	0.120^{*}	$0.551^{\ddagger}$	$0.694^{\#}$	$0.574^{\ddagger}$	$-3.020^{\#}$	13.18	$4.952^{\#}$	−6625	1.166	18.88	0.551	2.989
		(0.039)	(0.070)	(0.232)	(0.229)	(0.283)	(0.705)	(39.05)	(0.448)		[0.280]	[0.000]	[0.458]	[0.702]
	STFIGARCH(V)	$0.055^{\ddagger}$	$0.178^{\#}$	$0.736^{\#}$	$0.638^{\#}$	$0.557^{\#}$	0.290	3.394	$5.065^{\#}$	−6626	1.901	53.74	0.010	2.767
		(0.022)	(0.057)	(0.098)	(0.119)	(0.120)	(0.760)	(2.292)	(0.391)		[0.168]	[0.000]	[0.920]	[0.736]
	STFIGARCH(ΔV)	$0.070^{\ddagger}$	$0.164^{\#}$	$0.677^{\#}$	$0.606^{\#}$	$0.544^{\#}$	$-0.868^{\ddagger}$	18.44	$5.140^{\#}$	−6627	11.88	31.30	0.425	2.266
		(0.030)	(0.050)	(0.175)	(0.138)	(0.151)	(0.369)	(103.2)	(0.423)		[0.001]	[0.000]	[0.515]	[0.811]
Mexico	FIGARCH	$0.027^{\ddagger}$	$0.194^{\#}$	$0.719^{\#}$		$0.612^{\#}$			$6.894^{\#}$	−6071				5.094
		(0.010)	(0.052)	(0.117)		(0.156)			(0.686)					[0.404]
	STFIGARCH(ε)	$0.027^{\ddagger}$	$0.176^{\#}$	$0.575^{\#}$	$0.702^{\#}$	$0.589^{\#}$	$-3.245^{\#}$	5.115	$6.850^{\#}$	−6069	12.60	12.83	0.741	5.305
		(0.010)	(0.048)	(0.163)	(0.123)	(0.154)	(0.788)	(4.917)	(0.687)		[0.000]	[0.005]	[0.690]	[0.380]
	STFIGARCH(V)	$0.030^{\#}$	$0.184^{\#}$	$0.720^{\#}$	$0.811^{\#}$	$0.632^{\#}$	$1.635^{\#}$	$150.0^{\#}$	$6.920^{\#}$	−6069	12.33	70.49	0.694	4.952
		(0.011)	(0.067)	(0.147)	(0.149)	(0.209)	(0.025)	(51.63)	(0.729)		[0.000]	[0.000]	[0.707]	[0.422]
	STFIGARCH(ΔV)	$0.025^{\#}$	0.108	$0.800^{\#}$	$0.880^{\#}$	$0.784^{\#}$	$0.941^{\#}$	$124.4^{\#}$	$6.765^{\#}$	−6062	8.895	62.63	0.670	9.081
		(0.007)	(0.068)	(0.051)	(0.040)	(0.099)	(0.014)	(32.50)	(0.647)		[0.003]	[0.000]	[0.716]	[0.106]
India	FIGARCH	$0.062^{\#}$	0.088^{*}	$0.586^{\#}$		$0.571^{\#}$			$7.693^{\#}$	−6530				3.389
		(0.019)	(0.051)	(0.105)		0.097			(0.882)					[0.640]
	STFIGARCH(ε)	$0.049^{\#}$	0.034	$0.352^{\ddagger}$	$0.576^{\#}$	$0.543^{\#}$	$-2.420^{\#}$	8.823	$7.553^{\#}$	−6521	9.588	10.38	1.380	3.119
		(0.018)	(0.064)	(0.170)	(0.153)	(0.127)	(0.473)	(18.76)	(0.873)		[0.002]	[0.016]	[0.240]	[0.682]
	STFIGARCH(V)	$0.060^{\ddagger}$	$0.096^{\ddagger}$	$0.678^{\#}$	$0.597^{\#}$	$0.581^{\#}$	$-0.928^{\ddagger}$	16.50	$7.686^{\#}$	−6529	24.50	57.14	0.555	3.031
		(0.019)	(0.047)	(0.271)	(0.112)	(0.110)	(0.424)	(10.45)	(0.882)		[0.000]	[0.000]	[0.456]	[0.695]
	STFIGARCH(ΔV)	$0.065^{\#}$	0.090^{*}	$0.615^{\#}$	$0.553^{\#}$	$0.550^{\#}$	$0.898^{\#}$	102.9	$7.774^{\#}$	−6527	2.575	8.499	1.606	3.729
		(0.021)	(0.049)	(0.107)	(0.092)	(0.090)	(0.020)	(69.20)	(0.889)		[0.109]	[0.037]	[0.205]	[0.589]

(*continued on next page*)

Table 3 *(continued)*

Country	Model	ω	ϕ	β	β^*	d	c	γ	ν	LL	W_1	W_3	LM	$Q^2(5)$
S. Korea	FIGARCH	0.029‡	0.119#	0.585#		0.484#			7.721#	−6479				6.305
		(0.013)	(0.037)	(0.079)		(0.069)			(0.905)					[0.178]
	STFIGARCH(ε)	0.020*	0.124#	0.502#	0.615#	0.491#	−1.990#	124.4#	7.636#	−6472	9.333	16.35	0.179	6.487
		(0.012)	(0.044)	(0.096)	(0.091)	(0.087)	(0.021)	(28.29)	(0.894)		[0.002]	[0.001]	[0.671]	[0.166]
	STFIGARCH(V)	0.031‡	0.105‡	0.530#	0.574#	0.475#	−0.441#	20.49#	7.726#	−6479	3.001	15.91	0.007	6.095
		(0.014)	(0.042)	(0.100)	(0.082)	(0.068)	(0.078)	(6.688)	(0.899)		[0.083]	[0.001]	[0.933]	[0.192]
	STFIGARCH(ΔV)	0.028‡	0.113#	0.602#	0.530#	0.492#	0.176#	63.09‡	7.732#	−6470	5.204	18.32	0.015	7.737
		(0.012)	(0.035)	(0.081)	(0.084)	(0.074)	(0.032)	(25.16)	(0.913)		[0.023]	[0.000]	[0.903]	[0.102]

This table reports estimation results of FIGARCH and STFIGARCH models. We consider three sources of non-linearity: index return (STFIGARCH(ε)), trading volume (STFIGARCH(V)) and volume variation (STFIGARCH(ΔV)). LL is the maximized log-likelihood value for each model and numbers in parentheses are White robust standard errors. W_1 and W_3 are the robust *Wald* tests based on the first and third order Taylor series expansion of the STFIGARCH around $\gamma = 0$. LM denotes the test statistics for no remaining non-linearity of logistic form in residuals with delay parameter 1 (see Lundbergh and Teräsvirta, 2002) and $Q^2(5)$ is the Ljung-Box test for autocorrelation of order 5 in squared standardized residuals. The p-values associated to all the four tests are reported in brackets. #, ‡ and * denotes statistical significance at 1%, 5% and 10% level, respectively.

Kiliç (2011), estimates of d from the STFIGARCH models are lower than those from the FIGARCH model, thereby indicating that an upward bias in the long memory property is due to neglecting the smooth transition non-linearity generated by both lagged returns and trading volume.

Our outcomes confirm that the use of a regime-dependent approach is appropriate, since we get quite different behaviours in both extreme regimes across the considered threshold variables and markets. The speed of the transition between the two extreme regimes, measured by γ, is slow and points in favour of a gradual transition under the STFIGARCH(ε) model in most cases. However, it turns out to be fairly fast in some cases, most markedly when it comes to volume as the source of non-linearity, suggesting that the switching between the two states is almost instantaneous and points to a nearly discontinuous transition. These cases are for Mexico, India and South Korea. It is also worth noting that the speed of transition reflects how fast the market adjusts to changes in the transition variable. The results suggest that as the news conveyed by trading volume hit the markets, an increased uncertainty about the volatility regime leads to abrupt price adjustments. This corroborates the conventional wisdom that emerging stock markets are more vulnerable and may collapse much more suddenly than developed markets. The estimated threshold values also differ owing to the fact that different market indexes exhibit different degrees of volatility.

When we compare the values of the parameter β in the linear FIGARCH to those of the regime-specific parameters β and β^* in the non-linear FIGARCH models as well as parameter estimates across the volatility regimes, two results deserve particular mention. First, the persistence of conditional volatility is significantly reduced in the STFIGARCH(V) for the group of developed countries and China when volume exceeds its threshold value. The decline in the persistence ranges from 6.14% (France) to 21.06% (Japan). On the other hand, the low regime (i.e., when volume is below the threshold value) reports the opposite inference. For the remaining emerging markets, the volume slightly reduces the persistence of conditional volatility in both regimes in the case of South Korea, and solely in the low volatility regime in the case of Mexico. Second, the significant values of both persistence coefficients β and β^* for the volume-based non-linear model points to an asymmetric realization in the markets. In other words, there is regime-switching of volatility depending on the degree to which the transition variable is smaller or greater than the threshold value. Moreover, conditional volatility responds differently to the volume level as evidenced by the difference between the coefficient estimates for β and β^*. It is worthwhile noting that the lower the value of the persistence coefficient the higher is the volume impact. Specifically, we find that $\beta > \beta^*$ for developed countries and two emerging countries (China and India). This indicates that the inclusion of volume as a trigger variable into the non-linear FIGARCH model is not only more informative, but also conditional volatility appears to be more responsive to trading volume in the limiting regime of high trading activity.

In order to better illustrate the non-linear size effect of volume, Fig. 1 reports scatter plots of the estimated conditional volatility from both the FIGARCH and STFIGARCH(V) models along with the estimated

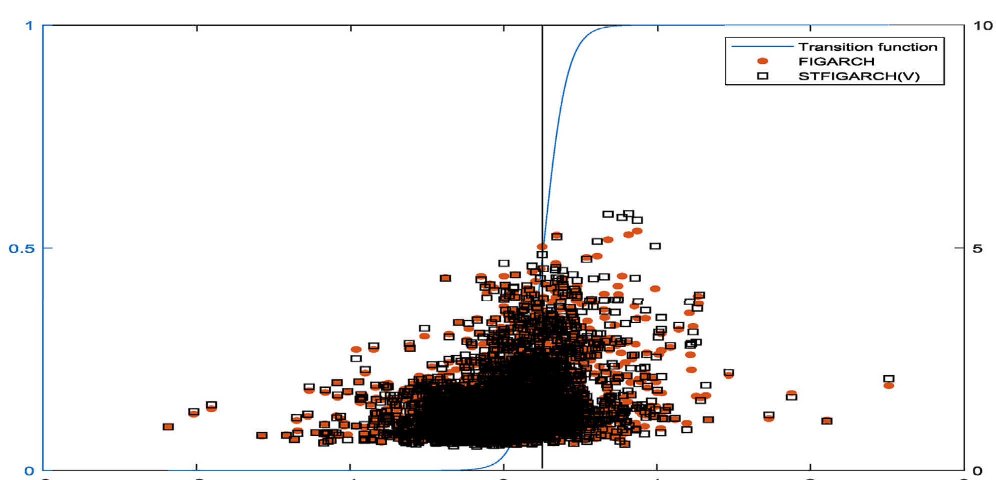

Fig. 1. Conditional volatility from FIGARCH and STFIGARCH(V) models and transition function over the transition variable for Germany. Vertical line indicates the threshold value for volume.

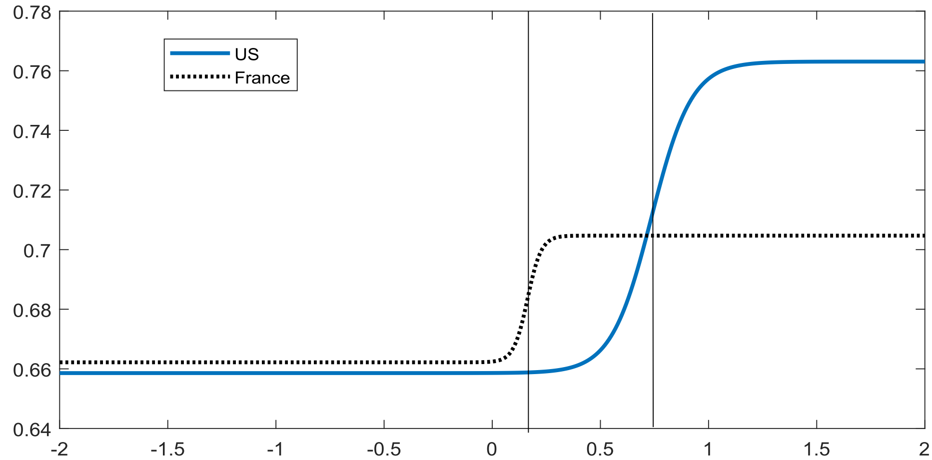

Fig. 2. News impact curves for STFIGARCH(V) models for US and France as a function of volume, given $\varepsilon_t = 1$. Vertical lines depict the corresponding threshold values for the transition variable.

transition function for Germany over the volume.[6] For large volume, the STFIGARCH model produces higher conditional volatility than the FIGARCH model whereas the two models generate fairly close volatility when volume falls below the threshold. Besides, Fig. 2 depicts the NIC of the STFIGARCH(V) model as a function of trading volume in the range $[-2, 2]$, at $\varepsilon_t = 1$, using parameter estimates for US and France. Displayed plots clearly show an asymmetric response of conditional volatility to the level of trading volume being below or above the threshold. Plots also reveal that as the degree of non-linearity (i.e., $\beta - \beta^*$) increases so does the asymmetric response.

Overall, our findings shed light on the extent to which trading volume could proxy the information flow in bull and bear market states (i.e., low and high volatility, respectively). Total volume appears to be more informative in bear market. The fact that volatility responds more strongly to large trading volume, for the bulk of the markets, may arise from disagreement-in-beliefs among investors, consistently with Shalen (1993) and Banerjee and Kremer (2010) who suggest that periods of major disagreements are periods of high volume and volatility. Moreover, owing to the conjecture that large trading volume reflects high uninformed trading activity (Easley et al., 1996; Weston, 2001), the asymmetric volatility response is likely to be due to the involvement of noise traders with a greater dispersion-of-beliefs as compared to informed traders. The stealth trading hypothesis suggests that informed traders use median trading volume to maximize their private information's value (Barclay and Warner, 2001). The rationale behind this assumption is that heavy trading volume will expose their favoured information too quickly while light trading volume will induce high transaction costs. Accordingly, profit-maximizing informed investors attempt to camouflage their information by spreading trades over time.

On the opposite side, conditional volatility turns out to be more responsive to small volume in the case of Mexico and South Korea. The high volatility regime is driven by small volume and the low regime is generated by large volume. The latter result is more related to the Kandel and Pearson (1995)'s view of the differential interpretation of news. The authors argue that differential interpretation of public information disclosure is likely to elucidate why abnormal volume may be associated to relatively small absolute equilibrium price changes. On the other hand, the evidence reported for the high volatility regime supports the findings of Ning and Wirjanto (2009). They point out that periods of market stress are not necessarily accompanied by a large trading volume in emerging East-Asian stock markets. The observed asymmetric response of volatility to the volume size may be also explained by both insider trading and market tightness. Given the weak insider trading laws and the limited price reporting mechanisms, especially due to market fragmentation, we expect to observe a high level of asymmetric information in emerging markets. Market makers can incur considerable losses when the transactions are made with a cluster of informed traders. Asymmetric information will reduce the incentive of the market makers to trade. Consequently, they proceed to revise their reference prices driving up bid-ask spreads. Given the inverse relation between trading volume and spreads (see for instance Cai et al., 2004), we expect to observe large price adjustments that occur with small volume arising from the speculation of informed traders. The involvement of less informed traders increases both trading volume and liquidity, improves price discovery and reduces volatility.

Similar findings broadly hold when it comes to employ volume variation as the transition variable in the STFIGARCH(ΔV) model. Large volume variation reduces the persistence of conditional volatility for six out of nine markets. The persistence decline, as compared to the linear FIGARCH, ranges from 3.24% (India) to 21.31% (Japan). We can also see a stronger volatility response to trading volume above the threshold.

The coefficient estimates with lagged returns as the transition variable are statistically significant, yet they follow another trend. In sharp contrast to trading volume, conditional volatility is found to be more responsive to lagged returns below the estimated threshold value for all the markets. Not surprisingly, the STFIGARCH(ε) model produces a higher conditional volatility associated to large negative shocks as compared to positive shocks with the same magnitude, thus reflecting the widely-documented leverage effect. Such an effect is mainly due to an increase of financial leverage at the firm level, as measured by the debt-to-equity ratio (Black, 1976; Christie, 1982). Volatility tends to increase as equities become riskier. We also notice that the degree of the asymmetric return-volatility relationship is higher in developed markets than in emerging markets. The difference between β and β^* ranges from −30.73% (UK) to −25,01% (Japan), whereas it ranges from −22,48% (India) to −11,29% (South Korea), in emerging markets. This result is possibly due to more active price adjustments, when bad news hit the market, triggered by short-selling in developed markets.[7]

4.3.2. Volume components

Prior empirical evidence suggests that linking volatility to total volume does not extract all information (Bessembinder and Seguin, 1993; Arago and Neito, 2005; Girard and Biswas, 2007). In order to improve

[6] To save space, similar plots for the remaining countries are not reported. They are available from the authors upon request.

[7] While several emerging countries have established policies allowing short-sales, developed countries have relatively more liberal short-selling rules.

the analysis, we investigate whether the effect of volume on volatility is homogeneous by breaking down total volume into its expected and unexpected components and allowing each component to have a separate non-linear effect on conditional volatility. Specifically, we apply the following ARMA(p, q) process:

$$V_t = \sum_{i=1}^{p} a_i V_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \alpha D_t + u_t \tag{5}$$

where V_t is the observed volume on day t and D_t is a dummy variable that controls for the day of the week effect. Unexpected volume (UV) on day t is the residual term u_t and expected volume (EV) is $V_t - UV_t$. UV is linked to private information while EV is not. For each market index, we separately introduce both volume components as a transition variable into the STFIGARCH model. The introduction of surprises in the level of market activity enables us to analyze the response of volatility to the unexpected arrival of new information, thus eliminating the distortions produced by market activity resulting from purely liquidity needs.

We begin by comparing the outcomes of such a decomposition in Table 4, to those for total volume reported in Table 3. Interestingly, the long memory parameter d is higher in both volume component-based models. Therefore, it appears that total volume can better describe the long memory effect, suggesting that the interaction among market participants, particularly long-term investors such as fundamentalists, is needed to reach the steady state fairly quickly. The results also show that the volume-volatility relationship keeps the same pattern for developed markets. Again the values of β are greater than β^* in both EV and UV-based models. Consistently with our earlier findings, conditional volatility is more responsive to large volume, regardless of the volume component.

These findings may be attributed to economic mechanisms at work in highly regulated and efficient markets wherein information shocks are quickly and fully integrated into the price. Indeed, a number of features such as abundant information disclosure, regular fundamental analysis, arbitrage trading and feedback from the derivative market make that it needs much volume to generate large price deviations. Even in presence of asymmetric information, conditional volatility is not so much affected by informed trading activity given the relatively higher liquidity in developed markets compared to emerging markets. The results also echo those of Tauchen and Pitts (1983). They suggest a positive volume-volatility relationship in mature and liquid markets where the number of active traders is relatively large. We notice, nevertheless, that the regime-specific sensitivities of conditional volatility are restricted to be positive in our modelling framework. This is quite different from Jawadi and Ureche-Rangau (2013), Jawadi et al. (2016) and Damette (2016) where trading volume enters as an additional exogenous variable in the non-linear regression model. Thus, we can hardly contend positive or negative volume-volatility nexus. We firmly

Table 4
Estimation results of STFIGARCH models with volume components as transition variables.

Country	Model	ω	ϕ	β	β^*	d	c	γ	ν	LL	W_1	W_3	LM	$Q^2(5)$
Developed markets														
France	STFIGARCH(*EV*)	0.032$^{\#}$	0.093‡	0.707$^{\#}$	0.565$^{\#}$	0.614$^{\#}$	0.576	2.269	9.703$^{\#}$	−6575	10.17	11.06	1.846	2.980
		(0.012)	(0.044)	(0.095)	(0.190)	(0.088)	(1.111)	(3.432)	(1.485)		[0.001]	[0.011]	[0.397]	[0.703]
	STFIGARCH(*UV*)	0.032$^{\#}$	0.067‡	0.673$^{\#}$	0.604$^{\#}$	0.607$^{\#}$	0.279$^{\#}$	50.000*	9.559$^{\#}$	−6573	12.18	13.76	5.426	3.313
		(0.011)	(0.040)	(0.076)	(0.085)	(0.088)	(0.033)	(29.825)	(1.406)		[0.000]	[0.003]	[0.066]	[0.652]
Germany	STFIGARCH(*EV*)	0.037$^{\#}$	0.059	0.713$^{\#}$	0.676$^{\#}$	0.654$^{\#}$	−0.324$^{\#}$	50.000	9.886$^{\#}$	−6616	5.908	11.68	1.834	4.349
		(0.014)	(0.052)	(0.222)	(0.093)	(0.107)	(0.049)	(232.289)	(1.599)		[0.015]	[0.015]	[0.400]	[0.500]
	STFIGARCH(*UV*)	0.025*	0.036	0.764$^{\#}$	0.594‡	0.728‡	0.315$^{\#}$	21.359$^{\#}$	10.071$^{\#}$	−6606	7.425	13.76	3.275	4.564
		(0.013)	(0.084)	(0.241)	(0.265)	(0.328)	(0.092)	(5.005)	(1.705)		[0.006]	[0.003]	[0.194]	[0.471]
Japan	STFIGARCH(*EV*)	0.036*	0.130‡	0.688$^{\#}$	0.593$^{\#}$	0.558$^{\#}$	0.029	15.632‡	9.498$^{\#}$	−5394	9.788	29.31	1.591	2.391
		(0.019)	(0.060)	(0.107)	(0.158)	(0.142)	(0.025)	(7.631)	(1.592)		[0.002]	[0.000]	[0.451]	[0.793]
	STFIGARCH(*UV*)	0.036	0.104*	0.636$^{\#}$	0.227	0.532$^{\#}$	0.490	8.547*	10.019$^{\#}$	−5390	3.085	10.21	6.427	4.637
		(0.022)	(0.061)	(0.151)	(1.384)	(0.122)	(0.539)	(5.172)	(1.777)		[0.079]	[0.017]	[0.040]	[0.462]
UK	STFIGARCH(*EV*)	0.021‡	0.105*	0.653$^{\#}$	0.530$^{\#}$	0.548$^{\#}$	−0.062	33.431	11.072$^{\#}$	−5532	16.96	43.51	5.174	1.546
		(0.009)	(0.059)	(0.087)	(0.082)	(0.066)	(0.425)	(149.111)	(1.847)		[0.000]	[0.000]	[0.075]	[0.908]
	STFIGARCH(*UV*)	0.023$^{\#}$	0.111‡	0.682$^{\#}$	0.594$^{\#}$	0.570$^{\#}$	−0.365$^{\#}$	50.000	10.993$^{\#}$	−5539	19.13	48.92	2.975	2.914
		(0.008)	(0.045)	(0.239)	(0.070)	(0.071)	(0.041)	(32.761)	(1.807)		[0.000]	[0.000]	[0.226]	[0.713]
US	STFIGARCH(*EV*)	0.024$^{\#}$	0.011	0.706$^{\#}$	0.671$^{\#}$	0.699$^{\#}$	0.175$^{\#}$	50.000	7.194$^{\#}$	−5305	7.858	14.67	4.643	8.171
		(0.006)	(0.012)	(0.063)	(0.067)	(0.068)	(0.031)	(46.357)	(0.835)		[0.005]	[0.002]	[0.098]	[0.147]
	STFIGARCH(*UV*)	0.023$^{\#}$	0.012	0.716$^{\#}$	0.298	0.704$^{\#}$	0.680$^{\#}$	9.825*	7.171$^{\#}$	−5300	9.722	25.14	4.490	5.417
		(0.006)	(0.024)	(0.058)	(0.286)	(0.071)	(0.194)	(5.304)	(0.850)		[0.002]	[0.000]	[0.106]	[0.367]
Emerging markets														
China	STFIGARCH(*EV*)	0.056$^{\#}$	0.181$^{\#}$	0.699$^{\#}$	0.591$^{\#}$	0.524$^{\#}$	0.602$^{\#}$	124.476‡	5.185$^{\#}$	−6623	3.561	68.26	5.667	2.862
		(0.020)	(0.048)	(0.069)	(0.083)	(0.081)	(0.013)	(51.558)	(0.393)		[0.059]	[0.000]	[0.059]	[0.721]
	STFIGARCH(*UV*)	0.067‡	0.169$^{\#}$	0.693$^{\#}$	0.739$^{\#}$	0.570$^{\#}$	0.992$^{\#}$	71.095‡	5.111$^{\#}$	−6628	13.44	90.53	6.429	2.679
		(0.030)	(0.057)	(0.144)	(0.137)	(0.175)	(0.017)	(33.856)	(0.423)		[0.000]	[0.000]	[0.040]	[0.749]
Mexico	STFIGARCH(*EV*)	0.028$^{\#}$	0.195$^{\#}$	0.794$^{\#}$	0.703$^{\#}$	0.600$^{\#}$	−1.832$^{\#}$	145.589$^{\#}$	6.926$^{\#}$	−6071	13.17	24.89	0.743	4.698
		(0.009)	(0.046)	(0.077)	(0.093)	(0.117)	(0.014)	(53.397)	(0.676)		[0.000]	[0.000]	[0.690]	[0.454]
	STFIGARCH(*UV*)	0.030‡	0.189$^{\#}$	0.715$^{\#}$	0.811$^{\#}$	0.623‡	1.820$^{\#}$	123.529‡	6.907$^{\#}$	−6068	28.98	23.11	0.798	5.149
		(0.013)	(0.066)	(0.203)	(0.232)	(0.270)	(0.010)	(58.611)	(0.802)		[0.000]	[0.000]	[0.671]	[0.398]
India	STFIGARCH(*EV*)	0.057$^{\#}$	0.084*	0.586$^{\#}$	0.541$^{\#}$	0.551$^{\#}$	0.156$^{\#}$	192.978‡	7.771$^{\#}$	−6530	11.23	10.25	5.726	3.611
		(0.019)	(0.047)	(0.097)	(0.114)	(0.094)	(0.006)	(78.865)	(0.890)		[0.001]	[0.017]	[0.057]	[0.607]
	STFIGARCH(*UV*)	0.066$^{\#}$	0.092*	0.560$^{\#}$	0.636$^{\#}$	0.558$^{\#}$	1.455$^{\#}$	191.501‡	7.830$^{\#}$	−6528	20.26	30.01	5.272	3.539
		(0.020)	(0.051)	(0.113)	(0.109)	(0.098)	(0.007)	(74.588)	(0.904)		[0.000]	[0.000]	[0.072]	[0.618]
S. Korea	STFIGARCH(*EV*)	0.029‡	0.121$^{\#}$	0.599$^{\#}$	0.451$^{\#}$	0.495$^{\#}$	1.961$^{\#}$	47.565‡	7.745$^{\#}$	−6479	3.003	23.85	1.002	6.383
		(0.012)	(0.036)	(0.081)	(0.137)	(0.073)	(0.052)	(24.052)	(0.904)		[0.083]	[0.000]	[0.606]	[0.172]
	STFIGARCH(*UV*)	0.029‡	0.112$^{\#}$	0.512$^{\#}$	0.586$^{\#}$	0.485$^{\#}$	−0.357‡	36.000$^{\#}$	7.693$^{\#}$	−6479	5.883	16.19	0.883	5.755
		(0.013)	(0.039)	(0.120)	(0.083)	(0.071)	(0.139)	(13.307)	(0.898)		[0.015]	[0.001]	[0.643]	[0.218]

This table reports estimation results of STFIGARCH model with either expected volume (EV) or unexpected volume (UV) as separate source of non-linearity. LL is the maximized log-likelihood value for each model and numbers in parentheses are White robust standard errors. W_1 and W_3 are the robust *Wald* tests based on the first and third order Taylor series expansion of the STFIGARCH around $\gamma = 0$. LM denotes the test statistics for no remaining non-linearity of logistic form in residuals with delay parameter 1 (see Lundbergh and Teräsvirta, 2002) and $Q^2(5)$ is the Ljung-Box test for autocorrelation of order 5 in squared standardized residuals. The p-values associated to all the four tests are reported in brackets. $^{\#}$, ‡ and * denotes statistical significance at 1%, 5% and 10% level, respectively.

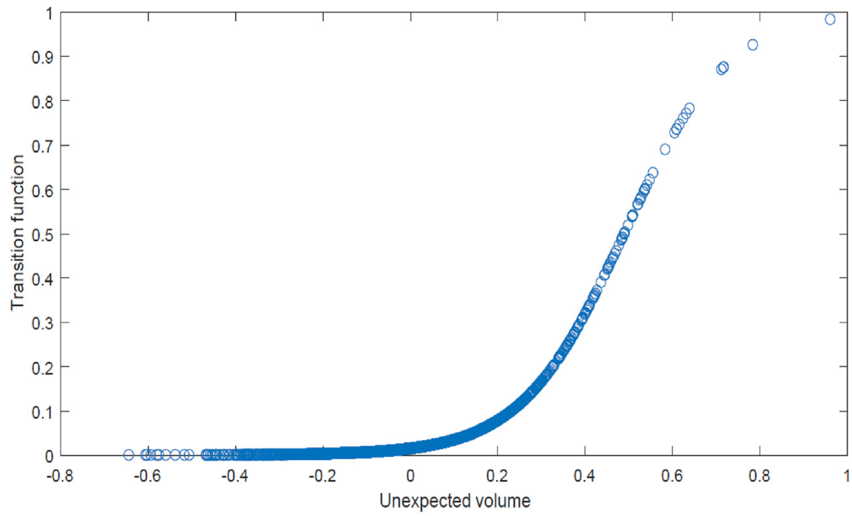

Fig. 3. Estimated transition function for Japan.

report that conditional volatility is relatively more responsive to large volume as opposed to small volume. Consequently, we argue that trading volume is more informative in the upper volatility regime. We also notice that the difference between the regime-specific coefficients is higher in the UV-based model for most of developed markets. This seems to indicate that the asymmetry between extreme regimes is more pronounced with the unexpected part, implying that large surprises in trading activity convey more information than large expected volume.

Here, two markets (i.e., US and Japan) deserve a thorough analysis. In these markets, when UV exceeds its threshold value, β^* is not significant. This result indicates that large UV completely vanishes the GARCH effect which is strongly supportive of the MDH. The intensity of the UV-volatility linkage results in sharp price fluctuations such that conditional volatility cannot be properly shaped by a FIGARCH model. Fig. 3 shows that the transition function does not reach unity, in the upper regime, suggesting that large UV hurdles against the convergence towards a stable dynamics. One may classify Japan and US as the most efficient markets, because large surprising information are more quickly and fully integrated into the price. Intensive price adjustments may explain the strong relationship between UV and volatility with unspecified dynamics. We deduce that the remaining persistence observed with total volume, in the upper regime, is mainly driven by the expected trading activity. In this case, pressure from fundamentalists to correct the market can generate large and highly persistent absolute stock returns, at least in the short-run (see Jawadi and Koubaa, 2007, and references therein). Moreover, during the adjustment process, the involvement of heterogeneous noise traders, who engage in momentum trading, leads to a market overreaction following the arrival of surprising information, such that price deviations will exceed that required by the news. The aggregate actions of the two groups of market participants destabilize the market and worsen the deviation. Therefore, it appears that it is not so much the size of information shocks (UV) but the level of expected trading activity that causes the persistence of volatility and delays of the mean-reversion in the upper regime.

Turning to the group of emerging markets, pairwise comparisons of the estimated coefficients β and β^* for EV and UV-based specifications as well as across the volatility regimes, in each model, yield the following verdict. The STFIGARCH(*EV*) coefficients are smaller than the corresponding STFIGARCH(*UV*) coefficients in the upper regime, with differences ranging from −9.49% (India) to −14.75% (China). In the lower regime, the difference turns out to be positive ranging from 0.59% (China) to 8.76% (South Korea). On the other hand, similar to developed markets, we can see that β is significantly higher than β^* for the EV-based model, and conversely for the UV-based model. Therefore, conditional volatility is more responsive to EV in the upper regime. The outstanding feature is the difference between the coefficients across regimes in the UV-based model. These differences vary from −4.60% (China) to −9.62% (Mexico), which indicate a reverse UV-volatility relationship compared to that when EV is included as the transition variable. More precisely, when UV is below the threshold, volatility tends to be higher than when UV is above the threshold. This uncovered asymmetric relationship can be explained by the rate of information arrival and the number of active traders in the market as follows. First, a sequential information dissemination process is likely to prevail in emerging markets, in which information disclosure and fundamental analysis are relatively limited. Initially, only well informed traders take positions to realize speculative profits. Their trading has a sizeable price impact which drives the high volatility regime. After a series of intermediate transient equilibrium, where information disseminate gradually to more and more traders, the market expands until a final stable equilibrium is reached, resulting in a lower volatility.

Our results appear at odds with those reported by Girard and Biswas (2007). They find a stronger positive relation between UV and volatility in emerging markets compared to developed ones. They explain their findings by the presence of noise trading and speculative bubbles. However, the single-regime GARCH model they use is limited to report on-average relationship between UV and volatility. Our regime-switching framework allows us to find out a greater price impact driven by a relatively small group of informed traders and we attribute this effect to a number of aspects that characterize emerging markets including unstructured regulatory systems, market tightness and liquidity shortages.

In sum, the volume decomposition reveals a high degree of homogeneity in the results which was obscured with total volume. While the results from total volume show an asymmetric volatility response akin to developed markets for China and India and a reverse asymmetric response for Mexico and South Korea, the outcome from UV contributes soundness to our conclusions, regarding emerging markets.[8]

[8] In order to check the robustness of our analysis to the presence of crises, we employ outlier-robust techniques to test linearity and re-estimate the non-linear models. In addition, we remove the major sub-prime crisis period (2007–2009) and re-estimate the models over both the pre-crisis and the post-crisis periods. In light of the results summarized in an unpublished supplementary appendix, we draw the same conclusions regarding the non-linear volume-volatility relationship, even after dampening the impact of financial crises. We thank an anonymous referee for raising this issue.

4.4. Forecasting performance

In this section, we evaluate the out-of-sample forecasting performance of linear and non-linear volatility models with statistical tools. We also consider an augmented FIGARCH model (FIGARCH + V) by adding a one-period lag of trading volume, V_{t-1}, to the right-hand side of Eq. (1) as an incremental variable. Only one lag is considered here, since it is likely to have a more profound effect on the current value of volatility. The effects from earlier lags are negligible. The volatility forecasting procedure is implemented as follows. First, we split the total sample of 3191 observations into two parts. We use the first 2500 daily observations, from January 3, 2000 to November 10, 2009 to initialize the volatility forecasting models. Then, we use the rolling forecasting methodology to generate the one-day-ahead volatility forecasts from the competing models over the out-of-sample period from November 11, 2009 to August 31, 2015, yielding 1391 forecasted data points. All the models are updated on a weekly basis. To evaluate the accuracy of the volatility forecasts, we employ the mean squared-error (MSE), the mean absolute error (MAE) and the quasi-likelihood (QLIKE) loss functions. These loss functions are invariant to noise in the proxy for the true unobserved volatility as proved in Patton (2011). The loss functions are

Table 5
Model confidence set test.

		FIGARCH	FIGARCH + V	STFIGARCH(ε)	STFIGARCH(V)	STFIGARCH(ΔV)
Developed markets						
France	MSE	1.4039	1.4096	1.4092	1.3394	1.4299
		(0.543)	(0.543)	(0.543)	**(1.000)**	(0.316)
	MAE	0.9887	0.9962	0.9905	0.9824	0.9916
		(0.561)	(0.107)	(0.561)	**(1.000)**	(0.520)
	QLIKE	0.2730	0.2750	0.2722	0.2707	0.2744
		(0.646)	(0.371)	(0.646)	**(1.000)**	(0.371)
Germany	MSE	1.1369	1.1531	1.0693	1.0693	1.1146
		(0.560)	(0.174)	**(1.000)**	**(1.000)**	(0.512)
	MAE	0.8810	0.8883	0.8721	0.8668	0.8910
		(0.489)	(0.183)	(0.489)	**(1.000)**	(0.092)
	QLIKE	0.2700	0.2721	0.2636	0.2628	0.2712
		(0.082)	(0.058)	(0.748)	**(1.000)**	(0.027)
Japan	MSE	2.2296	2.1537	2.2645	2.0990	2.1630
		(0.107)	(0.691)	(0.107)	**(1.000)**	(0.691)
	MAE	1.3815	1.3575	1.3718	1.3424	1.3643
		(0.003)	(0.292)	(0.165)	**(1.000)**	(0.169)
	QLIKE	0.5907	0.5823	0.5786	0.5702	0.5827
		(0.031)	(0.315)	(0.315)	**(1.000)**	(0.093)
UK	MSE	0.3799	0.3793	0.3826	0.3788	0.3756
		(0.343)	(0.462)	(0.343)	(0.859)	**(1.000)**
	MAE	0.5506	0.5523	0.5540	0.5574	0.5474
		(0.060)	(0.060)	(0.060)	(0.060)	**(1.000)**
	QLIKE	0.3218	0.3254	0.3197	0.3220	0.3192
		(0.008)	(0.008)	(0.791)	(0.406)	**(1.000)**
US	MSE	1.8822	1.8885	1.8986	1.8883	1.8754
		(0.507)	(0.111)	(0.111)	(0.507)	**(1.000)**
	MAE	0.6167	0.6262	0.6193	0.6161	0.6167
		(0.859)	(0.089)	(0.637)	**(1.000)**	(0.859)
	QLIKE	0.3719	0.3990	0.3822	0.3749	0.3673
		(0.463)	(0.198)	(0.432)	(0.544)	**(1.000)**
Emerging markets						
China	MSE	3.5682	3.4905	3.8027	3.0699	3.5606
		(0.263)	(0.263)	(0.263)	**(1.000)**	(0.263)
	MAE	1.2563	1.3178	1.2769	1.2253	1.2625
		(0.157)	(0.039)	(0.039)	**(1.000)**	(0.157)
	QLIKE	0.2980	0.3019	0.3067	0.2840	0.2993
		(0.068)	(0.068)	(0.035)	**(1.000)**	(0.068)
Mexico	MSE	1.3740	1.3691	1.3872	1.3784	1.3738
		(0.720)	**(1.000)**	(0.426)	(0.426)	(0.720)
	MAE	0.6339	0.6344	0.6343	0.6372	0.6334
		(0.984)	(0.984)	(0.984)	(0.262)	**(1.000)**
	QLIKE	0.3626	0.3635	0.3686	0.3649	0.3602
		(0.536)	(0.536)	(0.424)	(0.132)	**(1.000)**
India	MSE	0.3756	0.3841	0.3741	0.3803	0.3724
		(0.524)	(0.102)	(0.782)	(0.166)	**(1.000)**
	MAE	0.6467	0.6489	0.6392	0.6485	0.6439
		(0.165)	(0.186)	**(1.000)**	(0.052)	(0.287)
	QLIKE	0.2689	0.2679	0.2645	0.2677	0.2675
		(0.047)	(0.365)	**(1.000)**	(0.365)	(0.365)
South Korea	MSE	0.8175	0.8084	0.8606	0.8064	0.8172
		(0.373)	(0.893)	(0.373)	**(1.000)**	(0.373)
	MAE	0.7359	0.7388	0.7368	0.7346	0.7347
		(0.908)	(0.908)	(0.908)	**(1.000)**	(0.960)
	QLIKE	0.3818	0.3902	0.3751	0.3803	0.3813
		(0.092)	(0.077)	**(1.000)**	(0.143)	(0.094)

This table reports the mean losses for the different volatility models over the out-of-sample period (November 11, 2009–August 31, 2015) with respect to three evaluation criteria (MSE, MAE and QLIKE). Consistent p-values of the model confidence set (MCS) test are reported in parentheses under the selected loss functions. The model with the highest p-value (in bold face) is given the best rank.

given by:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(RV_t - \sigma_t^2\right)^2, \tag{6}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|RV_t - \sigma_t^2\right|, \tag{7}$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}\log\left(\sigma_t^2\right) + \frac{RV_t}{\sigma_t^2}, \tag{8}$$

where T is the number of forecasting data points. RV_t and σ_t^2 refer to the realized variance and the variance forecast from a particular model, respectively. Daily realized variance constructed using five-minute intraday returns is obtained from the Oxford-Man Institute realized library (https://realized.oxford-man.ox.ac.uk).

To statistically compare the results of the one-day-ahead forecasting accuracy of the volatility models, we utilize the model confidence set (MCS) test of Hansen et al. (2011) under the selected loss functions. The MCS test has the null hypothesis of no difference in the expectation of the difference in the loss function, among the various forecasting models, and draws p-value for the null hypothesis. If the model is rejected at the significance level α, then it is deemed worse than the other models in the set. Thus, the model with p-value equal to one survives to the last in the set and is the best among the competing forecasting models.

Table 5 reports the MCS results from the comparison of volatility forecasts by presenting the p-values of the MCS tests as well as the average loss for each model. The test results indicate that the non-linear FIGARCH models with either lagged volume or lagged volume variation as transition variables have superior forecasting abilities than their competitors. This result holds, regardless of the considered loss function, for nearly all the markets with a few exceptions. The first exception is for Germany, for which the STFIGARCH(ε) cannot be rejected under the MSE. The second exception is for Mexico, for which the volume-augmented FIGARCH is selected under the MSE. The third exception is for India, for which the STFIGARCH(ε) is selected under both the MAE and the QLIKE loss functions. The last exception is for South Korea, for which the STFIGARCH(ε) is selected as the best performing model under the QLIKE loss function. Even in these cases, we notice that the volume-based non-linear models cannot be rejected at standard significance levels. More precisely, the MCS test provides compelling evidence in favour of the STFIGARCH(V) for five markets (France, Germany, Japan, China and South Korea) whereas the STFIGARCH(ΔV) is selected for three markets (UK, US and India). Both the FIGARCH and the volume-augmented FIGARCH models are never selected, thereby highlighting the switching role of trading volume in forecasting conditional volatility surges.

4.5. Trading profits

In light of the promising statistical results, regarding the predictive power of trading volume, we examine whether the volatility forecasts of a given stock market index are useful as decision-making tools to devise profitable trading strategies. This question is of most relevance for investors and market participants because it provides a direct practical implication of our findings. Following McMillan (2007), Narayan and Sharma (2014), Phan et al. (2015) and Kumar (2015), we employ economic gains generated by simple trading strategies as a further metric to gauge the forecasting accuracy of competing models. In our experimental set-up, we devise two trading rules based upon the properties of our non-linear models, discussed in Section 3.2, and the empirical evidence gathered so far. Both trading strategies employ the recursive window estimates and a decision is made every week whether to buy or to sell the index portfolio using the available information set.

The first trading strategy is that if the observed trading volume is below the estimated threshold value and if the next period volatility forecast from the non-linear volume-based models is lower than the volatility forecast generated by the linear FIGARCH model, then purchase the index portfolio or else sell the fund. The opposite rule holds for the non-linear FIGARCH model with lagged returns as the transition variable. Therefore, if the observed index return is below the estimated threshold value and if the volatility forecast from the STFIGARCH(ε) model is greater than the linear FIGARCH volatility forecast then the index portfolio is sold or else the fund is purchased. In the second trading strategy, we remove the conditions on conditional volatility and we

Table 6
Average cumulative returns (%) from trading strategies.

		B&H	STFIGARCH(ε)	STFIGARCH(V)	STFIGARCH(ΔV)
Developed markets					
France	Strategy 1	18.14	18.84	19.49	**21.09**
	Strategy 2		20.04	17.49	**21.02**
Germany	Strategy 1	33.08	39.64	35.52	**43.35**
	Strategy 2		38.80	31.20	**38.96**
Japan	Strategy 1	52.54	69.83	65.31	**73.96**
	Strategy 2		33.07	56.34	**61.95**
UK	Strategy 1	2.20	−0.41	−3.10	**8.80**
	Strategy 2		−2.15	−1.57	**10.30**
US	Strategy 1	18.38	28.92	30.72	**33.31**
	Strategy 2		21.56	**32.18**	31.71
Emerging markets					
China	Strategy 1	22.31	**44.87**	42.85	44.08
	Strategy 2		42.63	**55.02**	20.03
Mexico	Strategy 1	10.35	7.27	**11.34**	7.68
	Strategy 2		7.60	**12.74**	10.45
India	Strategy 1	31.07	37.14	39.14	**39.48**
	Strategy 2		38.18	37.51	**38.61**
South Korea	Strategy 1	−2.04	−2.79	**0.22**	−0.47
	Strategy 2		**1.14**	−0.93	−1.15

This table reports average cumulative returns (%) over various investment horizons based on a buy-and-hold (B&H) strategy and two sets of switching rules as follows. The first set (Strategy 1) is to buy if the volume is below the threshold and if the next period forecast of volatility from the non-linear model is lower than the corresponding linear forecast or else sell. The opposite rule holds for the non-linear model with returns as the transition variable. In the second set (Strategy 2), we remove the conditions on volatility forecasts and only rely on the value of the transition variable as compared to the estimated threshold. The results are based on a switching cost of 0.1%. Bold numbers indicate the most profitable strategy.

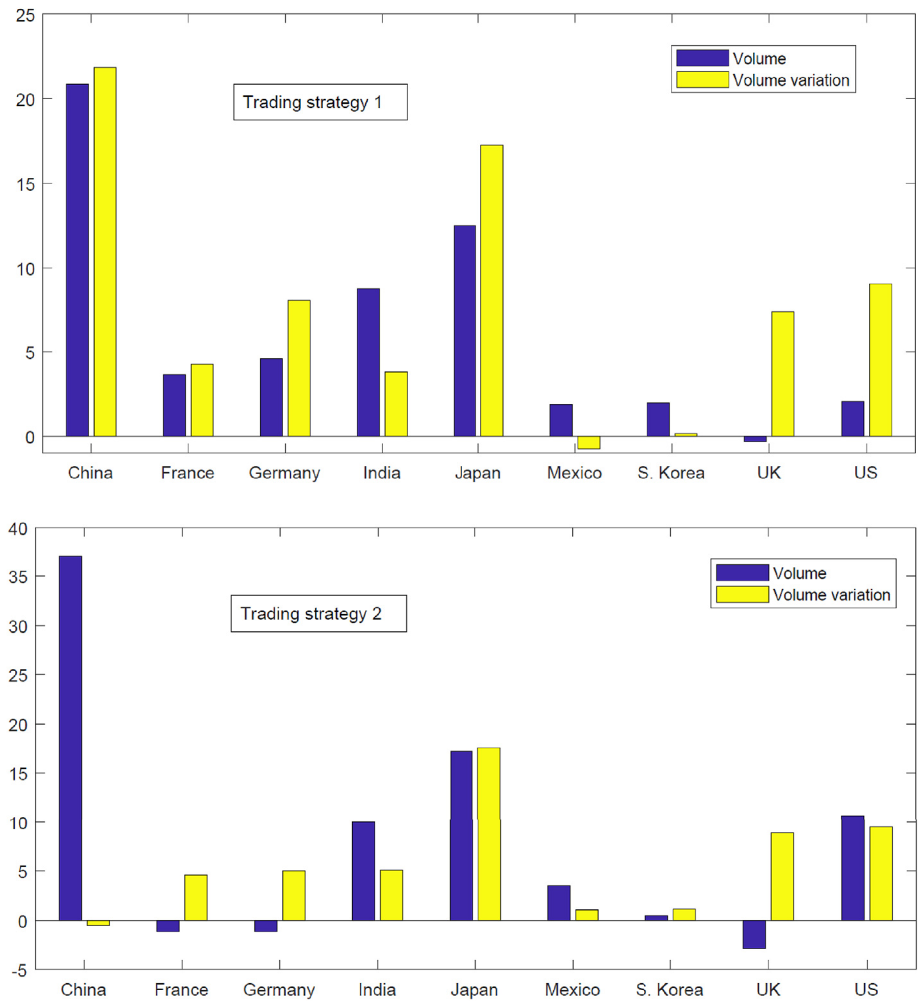

Fig. 4. Average improvements in cumulative returns (%) from using volume-switching rules over the B&H strategy. The results are based on 1000 randomly drawn holding periods ranging from one year to nearly six years.

only rely on the observed value of the transition variable as compared to its threshold value. This less restrictive trading strategy allows us to investigate whether the sole observation of the presumed source of non-linearity is sufficiently informative to earn as much profit as the first full-fledged trading strategy. These switching rules arise from our earlier findings that conditional volatility is more responsive to trading volume and return shocks in the upper and lower regime, respectively.[9]

In our assessment, we compare cumulative returns of threshold-based strategies to those generated by a simple buy-and-hold (B&H) strategy. Since the trading performance is likely to depend on the holding period determined by the investor, we consider various investment horizons. To this end, we begin by a short-term investment of one year which is recursively expanded by a quarter. This process continues until the end of the out-of-sample period is reached. We also assume transaction costs of 0.1% to mirror actual market conditions, consistently with McMillan (2007), Kumar (2015), Marquering and Verbeek (2004) and Phan et al. (2015). Table 6 reports the average cumulative return, over the different investment periods, from the given trading strategy for all the non-linear models as well as for the B&H strategy. We find compelling evidence of substantial economic gains from the threshold-based investment strategies. On the whole, the average returns across all the markets for the STFIGARCH(ε), the STFIGARCH(V) and the STFIGARCH(ΔV) are 27.04%, 26.83% and 30.14%, respectively, from strategy 1, and 22.32%, 26.66% and 25.76%, respectively, from strategy 2. The B&H strategy, however, incurs the least average profit of 20.67%. We notice that average cumulative returns are significantly positive for most of the markets. This is due to the upward trend over the out-of-sample period. The country-specific analysis also reveals that the volume-based models provide the largest returns, for nearly all the cases. The biggest winners are investors who use switching rules based on the volume variation measure in France, Germany, Japan, UK, US and India. The largest profits are earned by investors who utilize the volume measure in China, Mexico and South Korea. However, we notice that the return-based strategy 2 provides a slight improvement over the two volume measures for South Korea.

All in all, the results indicate that investors gain by using the volume-based threshold models, whatever the considered volume measure, compared to both the B&H and the non-linear model that uses returns as the transition variable. Interestingly, for both the STFIGARCH(V) and the STFIGARCH(ΔV) models, the portfolio performance does not appear to be much affected by using the second (less restrictive) trading strategy. This result confirms our earlier finding that the volume threshold provides genuine information about the future state of the market.

[9] Note that the condition on the volatility forecasts in the more restrictive trading strategy could be replaced by conditioning on both β and β^* as predicted by the NICs in Eq. (4). This translates in the following rules. Purchase (sell) the fund if $\beta < \beta^*$, otherwise sell (purchase) the fund for volume-based (return-based) model.

To further alleviate the effect of the timing of market entry and exit as well as the effect of the upward trend, we randomly draw 1000 holding periods ranging from one year to nearly six years. This allows the models to compete on a more equal footing. For the sake of brevity, the results from this exercise are reported in Fig. 4 as improvements in cumulative returns from using the STFIGARCH(V) and the STFIGARCH(ΔV) models over the B&H strategy. Again investors generally gain by using non-linear models to devise trading strategies. We also notice that the performance improvement depends on the volume measure, albeit we can hardly distinguish any pattern regarding the most suitable volume measure across all the markets. The heterogeneity of the results points towards the unique amount and quality of information conveyed by each measure of trading activity. This may also be attributed to some loss of information induced by trend removal from the raw volume.

5. Conclusion

The relationship between stock return volatility and trading volume has been at the centre of the debate among researchers, market participants and policymakers over the past decades. The conflicting empirical evidence surrounding their precise relationship has added fuel to this debate paving the way for further research in the non-linear context. While most of prior studies find out a non-linear inter-temporal relationship, they provide no clue regarding the suitable functional form for the non-linear model. In addition, the mounting evidence that financial markets may be characterized by both non-linear and persistent behaviour, resulting from the interaction among heterogeneous investors, reinforces the need to apprehend how the market switches from one state to another according to trading activity.

It is within this context that the present study provides a number of methodological and empirical contributions. First, this study examines the volume-volatility relationship, while accounting for both long memory and non-linear dynamics in conditional volatility. Such a non-linear dynamics is generated by trading volume in a way that conditional volatility smoothly switches from intermediate regimes to the final regimes according to an endogenous volume threshold. The STFIGARCH model can capture the volatility response to different volume levels via the degree of volatility persistence in the extreme regimes. The lower the value of the persistence coefficient the higher is the volume impact. This is an important departure from traditional assessments of the volume-volatility linkage through the on-average correlation sign. Second, total volume is disentangled into expected and unexpected components to examine the volatility response to normal trading activity and surprising information arrival. Third, we investigate the relative forecasting accuracy of volume-based non-linear models against linear and non-linear alternative specifications. This is a relevant question because researchers remain sceptical regarding the out-of-sample performance of non-linear models, albeit their suitability in terms of in-sample diagnostics (see McMillan, 2007, and references therein for a detailed discussion). Fourth, as the main practical implication from volatility forecasting models, we devise threshold-based trading strategies and compare their trading profit to that generated by a B&H strategy.

The main empirical findings from developed and emerging markets are summarized as follows. First, the results reveal an asymmetric relationship between volume and volatility. While prior research typically assigns non-linear volatility dynamics to return shocks, this study shows that such a dynamics can be driven by trading volume. The volume effect varies according to the volatility regime. For most of the markets, large (small) volume drives the high (low) volatility regime, thereby indicating that it needs much volume to reveal information. Small volume is likely to be induced by informed traders who operate strategically to reduce the price impact of their transactions and profit from their information advantage, consistently with the stealth trading hypothesis. The greater volatility response to large volume may be explained by the major disagreement-in-beliefs among heterogeneous and less informed market participants. Second, the volume decomposition confirms the stronger volume-volatility linkage in the upper regime, for the case of developed markets. More interestingly, results from UV provide an overwhelming evidence of a reverse asymmetric volatility response in emerging markets. That is to say, conditional volatility is relatively more responsive to small UV than large UV. Such an asymmetric relationship supports the SIAH, which can be basically explained by the high degree of asymmetric information and thin trading. Third, the results of multiple forecast evaluation based on noise-robust loss functions indicate that the non-linear models based on either total volume or volume variation outperform the non-linear model that uses return shocks as the transition variable and both linear specifications with and without volume. Finally, the economic significance analysis based on various investment horizons indicates that investors can improve average trading profits by around 5%–6% and 6%–10% if they were to use the first and the second volume-based strategy, respectively, over the B&H strategy.

While this provides a direct practical implication for investors and portfolio managers, our study also bears broader economic implications. Tracking the behaviour of surprising volume over time can enrich the information set available to regulators and policymakers. Our findings suggest that timely and promptly information disclosure, by listed companies, has to be strictly imposed to dampen large price impact, resulting from the speculative behaviour of informed traders, in emerging markets. The results also show that trading volume can be utilized to identify the future volatility state. This affords an additional tool for financial institutions to devise more sophisticated internal risk models to efficiently fulfil regulatory requirements, in compliance with the Basel IV Accord (Basel Committee on Banking Supervision, 2018). Indeed, the improvement of the predictive ability of market risk forecasting models may give potential to reduce the cost of risk management, since it can help financial institutions compute tailored regulatory capital provision. On the other hand, the regulatory framework emphasizes the need to properly account for liquidity risk. A common practice is to measure the difference between the liquidation time forecast and the actual time to close the position. The more challenging part of measuring liquidity risk is how to measure the impact of trading volume on price change. Professional traders can also learn from the volume-volatility relationship and adjust algorithmic trading tools according to the volume threshold such that the submitted order size would have the intended impact on the price of the assets they are monitoring.[10]

Acknowledgements

We are grateful to the editor and three anonymous reviewers for their helpful comments and suggestions. Any remaining errors or omissions are ours.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2019.01.003.

[10] Algorithmic trading, also referred to as automated trading, is a smart trading system that utilizes advanced statistical and mathematical models to make high-speed optimal decisions. As such, algorithms are able to recognize profitable patterns and execute trading instructions under particular conditions in volume, price and timing. Algorithmic trading is widely used by institutional traders to execute large orders while minimizing transaction costs and market impact.

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