Journal of Empirical Finance 58 (2020) 412–435

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/jempfin

Conditional extreme risk, black swan hedging, and asset prices ☆

S. Ghon Rhee [a,*], Feng (Harry) Wu [b,*]

[a] Shidler College of Business, University of Hawaii at Manoa, Room D-311b, 2404 Maile Way, Honolulu, HI 96822, USA
[b] Department of Accountancy, Lingnan University, SEK201/3, 8 Castle Peak Road, New Territories, Hong Kong SAR, China

ARTICLE INFO

Keywords:
Conditional extreme risk
Black swan hedging
Safety-first
Extreme value theory
Asset pricing

ABSTRACT

Motivated by the asset pricing theory with safety-first preference, we introduce and operationalize a conditional extreme risk (CER) measure to describe expected stock performance conditional on a small-probability market downturn (black swan). We document a significant CER premium in the cross-section of expected returns. We also demonstrate that CER explains the premia to downside beta, coskewness, and cokurtosis. CER provides distinct information regarding black swan hedging that cannot be captured by co-crash-based tail dependence measures. As we find that the pricing effect is stronger among black swan hedging stocks, this distinction helps explain the absence of premium to tail dependence.

1. Introduction

Financial economists have long recognized the importance of extraordinarily adverse market conditions in asset pricing (Roy, 1952; Menezes et al., 1980; Rietz, 1988). To risk-averse investors, an asset with declining value in a severely deteriorating market should be less valuable, while an asset performing well during rare market downturns (black swans) should be especially desirable. In other words, whether an asset contributes to small-probability (but potentially large) market calamities or hedges against black swans indicates its riskiness under conditions of extreme market drops. Conceptually, a straightforward way to express such risk is to gauge directly the expected payoffs of assets in the worst states of the market. We follow this intuition to introduce a novel conditional extreme risk (CER) measure that predicts individual stock performance conditional on a potential rare plummet of the market. We examine CER's asset pricing implication, its relations with comoment-based systematic risk measures (downside beta, coskewness, cokurtosis), and its role in black swan hedging which differentiates it from co-crash-based tail dependence measures.

Our study is motivated by the positive theory of equilibrium asset prices under the safety-first principle. Arzac and Bawa (1977) develop a generalized lexicographic form of the safety-first rule introduced by Roy (1952) and Telser (1955). Under this framework, risk-averse investors' objective is to maximize expected wealth while constraining the probability of failure (or substantial loss below a critical level). The Arzac–Bawa model shows that the expected return of an asset in the risk-averse safety-first (RASF) world is explained by its contribution to the Value-at-Risk (VaR) of the market portfolio associated with an admissible probability α (i.e., the α-quantile of the market return). This RASF factor can be expressed as the expected return of the asset conditional on extreme market return corresponding to its α-probability VaR (see the online appendix for details). As such, CER serves as a direct measure for the

☆ This paper was nominated for the "Best Paper Award" at 2015 Financial Management Association International (FMA) annual meeting. Feng (Harry) Wu gratefully acknowledges financial support from the General Research Fund (GRF) (No. 15507314) of the University Grants Committee of Hong Kong. This paper also benefits from the supports from General Research Fun (GRF), Hong Kong No. 15507015, Hong Kong Polytechnic University research funds (1-ZE46 and G-YN81), and Lingnan University research funds, Hong Kong. Ghon Rhee is grateful for the 2018 Shidler College of Business Summer Research Grant. Thanks go to Andrew Ang, Jeffrey Callen, Agnes Cheng, Louis Cheng, Timothy Chue, Campbell Harvey, David Hirshleifer, Byoung Kang, George Jiang, Huiwen Lai, Gang Li, Ji-Chai Lin, William Megginson, James Ohlson, Baochun Peng, Jean-Guy Simonato (FMA discussant), Nancy Su, Wilson Tong, Steven Wei, Kevin Zhu and participants of the FMA annual meeting (Orlando) and the seminar at Hong Kong Polytechnic University. The usual disclaimers apply.

* Corresponding authors.
E-mail addresses: rheesg@hawaii.edu (S.G. Rhee), harrywu@ln.edu.hk (F. Wu).

https://doi.org/10.1016/j.jempfin.2020.07.002
Received 15 November 2018; Received in revised form 23 April 2020; Accepted 1 July 2020
Available online 6 July 2020
0927-5398/

risk factor identified in the RASF model. A high CER indicates that the asset is expected to plummet together with the market, and a sufficiently low or negative CER implies that the asset's price would only drop slightly or even rise if the market experiences a rare crash. Therefore, CERs of individual assets reflect the expected hedge they provide against the market black swans.

A salient feature of the RASF risk factor is that it applies to arbitrary return structures, with beta in the capital asset pricing model (CAPM) as a special case for normal distribution. In the online appendix, we demonstrate that it also encompasses higher-order comoment-based coskewness and cokurtosis if the market portfolio is specified in terms of particular distribution characteristics. Consistent with its theoretical basis, the CER measurement adopts a conditional extreme value approach that has asymptotic properties suitable for any distribution. Such technique characterizes the behavior of one random variable (e.g., an individual asset return) given that another random variable (e.g., the market return) is approaching a rare and extraordinarily low level. CER thus holds the potential to explain the premia to the well-established comoment measures.

CER effectively predicts conditional stock performance when a rare market collapse occurs. In the period from 1931 to 2013, when the U.S. market experiences the 50 most severe monthly plummets (with average market drop of 11.72% per month), the lowest CER tertile stocks outperform the highest tertile stocks by 601 basis points (bps) per month on average (and by 865 bps for the 10 most severe market plummets). This evidence shows that black swan hedging stocks suffer much milder losses relative to high CER counterparts when market crashes, providing an effective hedge against the market black swans.

We find a significant and positive premium to CER. When large-scale market extreme losses are *predicted* (as opposed to crashes that are actually *realized*) and thus the disaster-avoidance motive is strong, top CER quintile stocks have expected returns substantially higher than those of the bottom CER quintile. For the top 10 percent months according to the magnitude of market VaR prediction, the top-minus-bottom quintile difference of expected return is 194.43 bps per month (26.00% per annum with monthly compounding) on an equal-weighted (EW) base. These high predicted extreme market risk months also include a large proportion of realized market crashes: 12 out of the 50 most severe market plummets happen in these months. Excluding realized market crashes boosts the EW CER premium to 335.01 bps per month (48.50% per annum). These results suggest that, when extreme market risk becomes a major concern and the black swan hedging attribute is especially desirable, investors trade off payoff to avoid the value of their assets plunging in a severely deteriorated market, and are content to accept a lower expected return in exchange for hedging market black swans. The high expected returns required to compensate for holding high CER stocks more than offset their price plunges during realized market crash times, leading to an overall positive CER premium. Such a CER effect also prevails throughout the full sample period with an EW average CER premium of 44.46 bps per month (5.47% per annum) and 84.30 bps per month (10.60% per annum) before and after the top 50 realized market crash months are excluded, respectively. These findings are robust under the value-weighted (VW) scheme, and are not driven by small or low-priced stocks.

CER explains the cross-sectional variation of stock returns in addition to market beta as well as size, book-to-market ratio, momentum, and liquidity. Alphas and pricing errors from the CAPM, the Fama–French 3-factor model, and the Fama–French-Carhart 4-factor model are greater when CER is higher, implying that these models are insufficient to price equities in the presence of extreme downside risk. Larger pricing errors are also observed in periods of larger CER premia, suggesting that CER explains part of what other factors do not.

We further find that CER explains the asset pricing effects of comoment-based systematic risk measures including downside beta, coskewness and cokurtosis. The positive premia to downside beta and cokurtosis become insignificantly negative and over 80 percent of the coskewness premium disappears as we control for CER. In contrast, the CER premium remains positive and significant after downside beta, coskewness, and cokurtosis are controlled. These findings suggest that investors process probabilistic information not confined to comoments. Emerging as a more comprehensive reflection of risk information, CER has a significant impact on asset prices over and beyond the comoment effects.

CER differs from the metrics used in the research on systematic tail risk (Kole and Verbeek, 2006; Spitzer, 2006; DiTraglia and Gerlach, 2013; Chabi-Yo et al., 2015; Van Oordt and Zhou, 2016). A salient commonality among these studies is that they all adopt the tail dependence method which constructs a co-crash measure to proxy for the case where *both* the market and the individual stock suffer from rare losses *simultaneously*, which is profoundly different from the CER measure that requires *only* the market to crash, i.e., CER assesses the riskiness not only when a stock plummets together with the market but also when the stock price remains stable in rare market crash scenarios (i.e., exhibiting the black swan hedging feature). Conceptually, under the situation where the market faces an extraordinarily negative state but the stock does not, only CER can deliver relevant information, whereas tail dependence measures are unable to do so. Such a distinction is critical since the ability to hedge market black swans can be highly desirable and can constitute the primary concern of investors.

We provide supporting evidence for the importance of CER's black swan hedging attribute in asset pricing. Using extreme value copula method, we estimate an asymptotic tail dependence measure based on sufficient co-exceedance return data below low thresholds (i.e., co-crash) for both the stock and the market, and find that it is positively correlated with CER but cannot account for the CER premium, suggesting that co-crash risk does not capture the complete riskiness reflected in CER. More importantly, we find that CER premium is almost twice as large among stocks without valid tail dependence estimates due to the lack of co-crash data, which is the case where the black swan hedging potential is greater. This implies that, when stock return is more likely to be asymptotically independent of the market and thus the black swan hedging property tends to be more prominent, investors are inclined to give up substantial expected compensation to avoid collapsing with the market. Our evidence also provides a hint that helps explain why existing tail dependence-based studies, which do not adequately capture the black swan hedging effect, generally fail to report a significantly positive premium to their measures.

Our work complements two strands of research on extreme risk and its pricing. One set of studies examine stock return's sensitivity to an aggregate tail index of the market (Chollete and Lu, 2011; Kelly and Jiang, 2014). This is equivalent in spirit to Ang et al. (2006a) and Chang et al. (2013), who measure stock return's sensitivity to innovations in market volatility and market skewness, respectively. Increased market volatility, negative skewness, and large tail index represent deteriorating investment opportunities with different emphases on normal or extreme cases. CER, similar to the tail dependence and comoment measures, connects the stock return with the market return in a more direct way, thus providing a more traditional aspect of tail risk pricing in contrast to Chollete and Lu (2011) and Kelly and Jiang (2014).

Another academic treatment of extreme risk includes introducing disaster states or jumps into the growth rate of consumption or returns of financial asset portfolios. Rietz (1988) and Barro (2006), among others, demonstrate that equity premium involves a compensation for potential crash risk of consumption. These works are corroborated by the risk premium uncovered from structural models that specify and estimate jump dynamics (e.g., Eraker et al., 2003; Wu, 2006; Bollerslev and Todorov, 2011). Complementing this body of work, our study employs the general safety-first preference framework with no need to specify consumption or return dynamics. Consistently, the empirical construction of CER based on extreme value algorithm is suitable for any return distributions, which greatly simplifies its estimation and expands its scope of application.

The remainder of this paper is organized as follows. Section 2 introduces the methodology for CER measurement. Section 3 reports statistics for the CER estimate, its validity as a measure of indicating price drops during realized rare market crashes, and the results for CER's ability to explain the cross-section of expected stock returns. Section 4 investigates the mutual influences between CER and downside beta, coskewness, and cokurtosis. Section 5 examines the difference between CER and tail dependence and their asset pricing effects. Section 6 provides robustness tests. Section 7 concludes.

2. Conditional extreme risk measurement

As a risk measure, the unique feature of CER is that it directly estimates the expected payoff of an individual stock if the market suffers from a small-probability but unusually large drop, which is consistent with the economic intuition of systematic extreme risk. Specifically, we construct CER as

$$CER_{\alpha,i} = \frac{E\left(R_i|R_M = VaR_{\alpha,M}\right)}{VaR_{\alpha,M}}, \tag{1}$$

where R_M refers to market return, and $VaR_{\alpha,M}$ is its α-probability VaR for a sufficiently small α, representing the rare but severe market calamity. $E(R_i|R_M = VaR_{\alpha,M})$ is the expected return of stock i conditional on the market return being equal to $VaR_{\alpha,M}$. We scale the conditional stock return by market VaR, which is common to all stocks in the cross-section, to be consistent with the risk factor identified in the RASF model as illustrated in the online appendix. Market VaR can be estimated via the extreme value theory (EVT) method which quantifies the stochastic behavior of a process at unusually large or small levels. The conditional extreme value approach developed by Heffernan and Tawn (2004) and Hilal et al. (2011) assesses inter-variable relations in a two-dimensional setting where only one of the random variables is extreme, providing a suitable statistical framework to estimate the stock's conditional expected return.

We start from estimating the marginal cumulative distribution functions (CDFs) of the log return series for stock i and the market M, that is, $X_i = \log\left(1+R_i\right)$ and $X_M = \log\left(1+R_M\right)$, by

$$\hat{F}(x) = \begin{cases} \tilde{F}(d)\left[1-\hat{\xi}\left(\dfrac{x-d}{\hat{\delta}}\right)\right]^{-1/\hat{\xi}} & \text{for } x < d \\ \tilde{F}(x) & \text{for } d \le x \le u, \\ 1-\left[1-\tilde{F}(u)\right]\left[1+\hat{\xi}\left(\dfrac{x-u}{\hat{\delta}}\right)\right]^{-1/\hat{\xi}} & \text{for } x > u \end{cases} \tag{2}$$

where the generalized Pareto distribution (GPD) with shape parameter $\hat{\xi}$ and scale parameter $\hat{\delta}$ is used to model the distribution of extreme tail observations beyond a high threshold u and a low threshold d. The empirical distribution $\tilde{F}$ is used to portray the non-tail part of the distribution. This distribution estimation extends Hilal et al. (2011) by describing both tails by GPD Hilal et al. (2011) only model one tail with GPD, see Eq. (1) of their paper on p. 2376). GPD is an appropriate specification in the extreme value paradigm to model the distribution of excesses over a suitably chosen threshold, and enjoys an asymptotic justification ensuring that the relevant estimates are invariant to the parent distributions.

The GPD model can be estimated by the maximum likelihood method, and parametric GPD facilitates the extrapolation for the magnitudes of rare events, which has been shown to give a superior estimate for VaR (Bali, 2003). A 1%-VaR represents the severity of a rare once-in-100-chance loss event which is used as a proxy for extraordinary market loss in our setting. To estimate VaR, we negate the log returns X_i and into $Y_i = -X_i$ and $Y_M = -X_M$ so that the upper tail region can be focused on. We adopt this treatment to estimate the CDFs in Eq. (2) for stock and market returns. The right tail of Eq. (2) gives the 1%-VaR of the negated log market return as

$$VaR_{1\%,M} = u_{Y_M} + \frac{\hat{\delta}_M}{\hat{\xi}_M}\left\{\left[100\left(1-\hat{F}_{Y_M}\left(u_{Y_M}\right)\right)\right]^{\hat{\xi}_M} - 1\right\}. \tag{3}$$

Since extreme risk by nature is of a long-term property, it is reasonable to treat tail risk estimated using the most recent historical observations as an approximation for the tail risk expected in the period immediately following. Therefore, the predicted market VaR of the original scale can be proxied as

$$MktVaR_{1\%} = \exp\left(-VaR_{1\%,M}\right) - 1. \tag{4}$$

To estimate a stock's expected return conditional on the market 1%-VaR, we first follow Hilal et al. (2011) to remove the effect of the margins by transforming (Y_i, Y_M) into another set of random variables (Z_i, Z_M) having common Laplace margins, as follows (the market margin can be analogously transformed):

$$Z_i = \begin{cases} \log\left[2\hat{F}_{Y_i}\left(Y_i\right)\right] & \text{if } \hat{F}_{Y_i}\left(Y_i\right) \le 0.5 \\ -\log\left[2\left(1-\hat{F}_{Y_i}\left(Y_i\right)\right)\right] & \text{if } \hat{F}_{Y_i}\left(Y_i\right) > 0.5 \end{cases}. \tag{5}$$

The asymptotic dependence structure of the transformed random variables (Z_i, Z_M) is determined by the conditional distribution of $Z_i|Z_M = z$ as $z \to \infty$. Based on a wide range of theoretical examples, Heffernan and Tawn (2004) show that there exist real normalizing functions $a_{i|M}(z)$ and $b_{i|M}(z)$ such that the standardized variable $S_{i|M} = \frac{Z_i - a_{i|M}(z)}{b_{i|M}(z)}$ satisfies

$$Z_i = a_{i|M}(z) + b_{i|M}(z) S_{i|M}, \text{ with } z = Z_M > u_{Z_M} \text{ for a high threshold } u_{Z_M}. \tag{6}$$

As shown by Hilal et al. (2011, p. 2377), for distributions with standard Laplace margins, the normalizing functions are given by $a_{i|M}(z) = \alpha_{i|M} z$ and $b_{i|M}(z) = z^{\beta_{i|M}}$ with $-1 \le \alpha_{i|M} \le 1$ and $\beta_{i|M} \le 1$, which suggests that Eq. (6) translates into

$$Z_i = \alpha_{i|M} z + z^{\beta_{i|M}} S_{i|M}, \text{ with } z = Z_M > u_{Z_M} \text{ for a high threshold } u_{Z_M}. \tag{7}$$

It naturally follows that the expected value of $Z_i|Z_M = z > u_{Z_M}$ is given by

$$E\left(Z_i|Z_M = z > u_{Z_M}\right) = \hat{\alpha}_{i|M} z + \hat{\mu}_{i|M} z^{\hat{\beta}_{i|M}}, \tag{8}$$

where $\hat{\mu}_{i|M}$ is the maximum likelihood estimate of the mean of $S_{i|M} = \frac{Z_i - \alpha_{i|M} z}{z^{\beta_{i|M}}}$ under the assumption made by Heffernan and Tawn (2004) and Hilal et al. (2011) that $S_{i|M} \sim N(\mu_{i|M}, \sigma^2_{i|M})$ (see Section 3.2.3 of Hilal et al. (2011, p. 2377) for more details about the estimation rationale and procedure).

Therefore, conditional on the 1%-VaR of the market return, a stock's expected return is estimated by first transforming the 99% quantile of Y_M (which corresponds to the 1%-VaR because we are making use of the upper tail data) into the corresponding value z of the standard Laplace variable Z_M, and then applying the Heffernan–Tawn theory to estimate $E(Z_i|Z_M = z)$. The conditional $E(Z_i)$ is then back transformed into $E(Y_i)$ via the Laplace-margin distribution and the CDF with GPD tails, and then into the return of the original scale $FirmRet_i|MktVaR_{1\%}$ for stock *i*, applying stock *i*'s versions of Eqs. (5), (2), and (4), sequentially.

We scale the conditional expected return of stock *i* by the corresponding market 1%-VaR, and compute CER as

$$CER_{1\%,i} = \frac{E\left[R_i|R_M = VaR_{1\%,M}\right]}{VaR_{1\%,M}} = \frac{FirmRet_i|MktVaR_{1\%}}{MktVaR_{1\%}}. \tag{9}$$

Some crucial considerations are in order for the CER measurement. First, the basic idea of CER is the expected stock performance when the market is experiencing a large plummet. This market plummet can be indicated by the magnitude of loss associated with a small probability. We use a 1%-probability loss magnitude as the indicator, which is by definition equal to the 1%-VaR. Here, we are not talking about the case when the market return is exactly equal to the value of VaR — such case does not exist because the probability of a continuous variable being equal to a single value is always zero. To put it another way, the expression in Eq. (1) is for the convenience of notation, which should be interpreted from an economic angle rather than in a statistical way. Another important reason to choose VaR as the indicator for market plummet is that it is required by the safety-first asset pricing model of Arzac and Bawa (1977) in which the market VaR appears in the theoretical expressions (as shown in Section I of the Online Appendix). Existing literature also has adopted the same way to represent the market crash condition, for example, Van Oordt and Zhou (2016).

Second, one may argue that using the expected shortfall (ES) (i.e., the event when the market return is lower than the VaR) to measure the condition of market collapse makes more statistical sense. Empirically, it is perfectly fine to use ES as the denominator in the CER construction, but doing so would be less consistent with the theoretical implication of Arzac and Bawa (1977). Another reason of using VaR is that it is always corresponding to the same probability level (i.e., 1%), but using ES would correspond to different probability levels. After all, given any value of loss magnitude, in order to estimate CER, we have to first find the quantile that corresponds to this loss value. Because different stock distributions have different tail thickness, even the same 1%-ES can be associated with different quantile values.

Third, our description of return distribution in Eq. (2) is based on the limiting arguments of GPD model which holds for stationary dynamics or series with limited long-range dependence at extreme levels (Coles, 2001). As shown by Longin and Solnik (2001), GPD tails can be obtained for return processes commonly used in finance, even though the process may not be strictly i.i.d. To make our estimation more robust, we also accommodate the conditional heteroscedasticity through a GARCH filtering of the time-series of market and individual returns and apply the GDP to the innovations that are closer to i.i.d., following McNeil and Frey (2000). Such an approach focuses on stocks' dependence on market tails isolated from the conditional volatility that also captures part of the heavy-tail characteristics (e.g., Poon et al., 2004; Jondeau et al., 2007), thus raising the possibility that the CER effect on expected returns is due to predicted volatility which prevails over the extremal dependence. The GARCH augmented estimation also involves more non-EVT parameters, exerting heavier computational burden and potentially more noises. Despite these concerns, we find results (available upon request) with similar asset pricing effects of CER in the GARCH scheme.

Fourth, we admit that despite their strong statistical advantage, the approaches to estimating the GPD marginal distribution tails and the conditional Heffernan–Tawn dependence depend on asymptotic characterizations of the probabilistic structure, and the entailed results based on VaR as a proxy for extreme event should not be treated as exact for finite sample. It is therefore important to check the sensitivity of CER estimate and its asset pricing implication to the rarity of market downturns (i.e., the probability level of market VaR). Additionally, in CER estimation, there are two sets of benchmark values: The first is for the GPD tail estimation (i.e., d and u) in Eq. (2), and the second is for the Heffernan–Tawn dependence estimation (i.e., u_{Z_M}) in Eq. (7). The overall findings tend to be more reliable if they are invariant to different benchmarking schemes. We address these issues in the following section as well as in the robustness tests.

3. Conditional extreme risk and the cross-section of stock returns

3.1. Data and statistical summary of CER estimates

Brownlees and Engle (2017) emphasize that financial crisis is a prolonged market decline and should entail a sufficiently long horizon and extreme threshold loss. "Otherwise, when the horizon is short and the threshold is modest, the role of risk is dramatically reduced" (Brownlees and Engle, 2017, p.55). They set the horizon to a month in order to compare more naturally with other monthly frequency indicators of distress. Barberis et al. (2016), in a prospect theory framework, suggest that "a natural mental representation of a stock's past return distribution is the distribution of its monthly returns over the previous five years" (Barberis et al., 2016, p.3070) and across various sources, "the daily and weekly fluctuations are not discernible, but the monthly fluctuations are, and they make a clear impression on the viewer" (Barberis et al., 2016, p.3076). We share the same considerations, and therefore adopt the past five year's monthly return data (a total of 60 for each estimation window) to estimate CER. As in most asset pricing tests, we use monthly stock excess returns as proxies for equity premia. Our corresponding CER estimates on a monthly basis are also consistent with this horizon. Although the fat-tailedness of monthly stock or index returns may be less pronounced than that of higher-frequency return series, technically the GPD specification for marginal tails and the dependence structure in the Heffernan–Tawn model are applicable to both fat- and thin-tailed random processes, which is ensured by their asymptotic properties independent of the parent distributions.[1] Consequently, the magnitudes of rare market plummets (and the corresponding conditional expected stock returns) can be estimated based on data from both volatile and tranquil periods. Our approach therefore differs from disaster-based studies that rely on historical realizations of rare macro events, phenomena less likely to be observed in tranquil times.

Using 60 months' returns to estimate a 1%-probability market event is obviously challenging for traditional methods. Our EVT-based method, however, with its unique property of dealing with small samples, can fulfill the goal. The limiting laws in extreme value methods are well-known for their ability to "predict the unpredictable" (Matthews, 1996, p.37), namely, to provide inference of extreme levels greater than those observed in historical data. For example, we can predict a once-in-100 chance event using less than 100 observations. This feature makes EVT especially suitable for studying small-probability risk. Combined with the fact that CER estimation only involves financial markets data for stocks and equity indexes, the CER approach effectively extenuates the "peso problems" often encountered in empirical studies relying on historical disaster events. Moreover, the extreme value approach adopted in this paper utilize real return data, rather than simulated data that are used in other crash studies (e.g., Brownlees and Engle, 2017), thus avoiding the heavy computational burden usually associated with simulations, especially for studies involving cross-sectional asset pricing which normally consider a large group of stocks over a long period. To further ease the concerns about the potential estimation noise due to small sample size, we will show that our EVT-based estimation has good predicting power of crash risk and also stocks' black swan hedging property. We also check the robustness of our CER measure in various benchmarking and data screening schemes. We have even tried the CER estimation using daily data, and find consistent, albert weaker, results (available upon request).[2] All these evidence suggests the reliability of our measure as an indicator of conditional extreme risk.

A main difficulty in our extreme value approach is the threshold choice, i.e., the benchmarks d and u to define marginal distribution tails for GPD specification in Eq. (2), and the threshold u_{Z_M} to describe conditional asymptotic distributions of the transformed returns in Eq. (7). If the threshold is much stringent, the asymptotic theory applies but the estimation involves too few observations which give large standard errors; on the other hand, a threshold that gives more observations can reduce the variance of estimator at the cost of increased biases, because the tail is contaminated by observations from the central part and the asymptotic property deteriorates. Although there are technical methods (Embrechts et al., 1997; Coles, 2001) to recommend the "optimal" threshold levels by trying different candidate values, we find it impractical to determine the benchmark set for a large number of individual stocks in each rolling estimation window by trial-and-error.[3] Instead, we follow the common practice in the literature by choosing the same percentile threshold for all stocks and the market. Considering the estimation bias and variance trade-off in the threshold selection, we use the top and bottom 20th percentiles as the up and down thresholds, respectively, to disentangle tails from the non-tail part, and we adopt the top 20th percentile of the market returns to separate out the stock-market pairs for their

[1] The literature has documented departures of monthly security returns from normality. See Campbell et al. (1997, p. 208), and the references therein.

[2] The weaker results could be due to the microstructure noise (e.g., nonsynchronous trading and bid–ask bounces) of daily return data (Campbell et al., 1997), or due to investors' less concern about daily performance relative to monthly performance (Barberis et al., 2016; Brownlees and Engle, 2017), both of which make the resulting CER estimates less reliable to capture systematic extreme risk. In addition, conditional one-period ahead predicted mean value as a risk measure using shorter-horizon (e.g., daily) returns may not match the risk premium horizon (e.g., monthly) in empirical asset pricing tests.

[3] An often used method for threshold choice is the minimum mean-squared error (MSE) criterion. The MSE of an estimator $\hat{\theta}$ of θ is defined by $MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2 = [E(\hat{\theta} - \theta)]^2 + \text{var}(\hat{\theta})$, which shows that MSE is the sum of squared bias and variance.

dependence description.[4] This operation gives 12 observations for each marginal distribution tail and Heffernan–Tawn dependence estimation (since some stocks have missing return data in certain estimation periods, we require a minimum of 10 observations). To ensure that our results are not sensitive to the specific thresholds selected, we also estimate CERs under alternative 15th and 25th percentile benchmarking schemes, and report their values and asset pricing effects in robustness checks.

We retrieve monthly return data from CRSP for all common stocks listed on the three major U.S. markets NYSE, NYSE MKT (former Amex), and Nasdaq from January 1926 to December 2013. We consider this long period because it covers many major macroeconomic and securities market turbulence times ranging from the Wall Street crash of 1929 to the recent subprime mortgage crisis in 2007–2008, and is therefore suitable for rare risk studies. We note that our main results remain robust for a shorter period starting from July 1963, which are reported below in analyses involving book-to-market ratio and liquidity beta that have estimates only after June 1963. These analyses also serve as a further subsample check of the CER effect. We use the CRSP value-weighted index return to proxy for market performance. At the end of each month *t*, we use the preceding 60 months' returns up to and including month *t*, to estimate market 1%-VaR$MktVaR$ and individual stocks' conditional expected returns $FirmRet|MktVaR$ for the next month t+1 as well as *CER*s as market VaR-scaled conditional expected stock returns. We focus on market VaR of a 1% probability after balancing the economic requirement of the conditional extreme value model and the technical precision of the estimation. A more severe crash would be more consistent with the asymptotic basis of the extreme value model, but the estimation becomes noisier if we use a limited number of observations to predict a rarer event. We also assess the results from using VaRs of various alternative probability levels in the robustness analyses.

Table 1 reports basic statistical summary of CER and its components. Before computing the statistics, we trim off the top and bottom 2.5% most extreme values of *CER* and $FirmRet|MktVaR$ estimates to exclude the influence of outliers because these values are unreasonably too big or too small.[5] Panel A shows that the average monthly market 1%-VaR *MktVaR* is −11.98%, suggesting that for a 1% probability, the market is predicted to drop by 11.98% or more in any given month. *MktVaR* is negatively skewed, as corroborated by the asymmetric maximum (−4.34%) and minimum (−31.63%) values around the median (−9.96%). The largest drop, as indicated by the minimum *MktVaR*, is remarkable and obviously not observed in history, which showcases GPD's ability to extrapolate extreme values beyond the range of historical observations. Conditional on the market having a 1%-VaR drop in the coming month, individual stocks' corresponding expected return $FirmRet|MktVaR$ is −12.82% on average, resulting in a mean *CER* of 1.1274.[6] The median of *CER* is at a similar level of 1.0810. Therefore, the majority (more than half) of the individual stocks severely drop together with the market during rare market down times, which makes the potential market index plummet a sign for common or systematic extreme risk. On the other hand, 45.62 percent (not reported in the table) of the stocks do not drop as much as the market (i.e., with *CER*s less than one), constituting a significant body of equities with black swan hedging potential of various degrees.

Not surprisingly, as shown in the correlation analysis in Panel B, larger conditional individual stock price drops are associated with higher CERs: The Pearson and Spearman correlation coefficients between $FirmRet|MktVaR$ and *CER* in the pooled sample are −0.8082 and −0.8492, respectively, as in the lower-left and upper-right triangles. $FirmRet|MktVaR$ is positively correlated with *MktVaR*, consistent with Panel A's evidence that individual stocks and the market in general tend to plunge together during a market crash. The average hedging ability of stocks, however, does not seem to substantially weaken amid a market-wide plummet, because the Pearson and Spearman correlations between *CER* and *MktVaR* are much smaller in magnitude (−0.0434 for Pearson and −0.0868 for Spearman). This evidence holds the promise that some stocks can de facto hedge away at least part of the market crash risk when the crash is considerably severe and the hedging is needed most. We will show whether investors take this black swan hedging characteristic into consideration when pricing equities.

In Fig. 1, we depict the average levels of *CER* and its components ($FirmRet|MktVaR$ and *MktVaR*) as a time series, along with the average monthly stock returns represented by the CRSP VW index returns. To make the plots easier to read, we smooth the time series values by taking the 12-month moving averages. Since the first *CER* and its components estimates are available at the end of December 1930 due to the 60-month rolling window scheme, 12-month moving average suggests that the time series in Fig. 1 start from November 1931. The left vertical axis represents return levels (for VaR and market index), and the right vertical axis refers to the scale for *CER*.

Consistent with the information from Table 1, Fig. 1 shows that over the time, although $FirmRet|MktVaR$ and *MktVaR* have closely related trends, the level of one variable, relative to the other, is not always higher or lower, resulting in a much volatile ratio of $FirmRet|MktVaR$ over *MktVaR*, i.e., *CER*. On average, *CER* is saliently lower, i.e., the black swan hedging feature is more pronounced, during mid-1940s, the decade after 1957, late-1990s (especially 1996 and 1998), and roughly the five years before the 2007–2008 financial crisis. In general, *CER* does not exhibit a strong association with the historical realized market returns (their Pearson correlation coefficient (untabulated) is only −0.0125), which is understandable because *CER* is estimated using the previous 60 months' returns, and the most recent month is unlikely to dominate its value. However, it is important to examine, before a large drop in market return, whether our ex ante *CER* measure, which is estimated before the market plunge, can effectively predict the performance of an individual stock during the extreme market down time. We address this issue in the following subsection.

---

4 McNeil and Frey (2000) and McNeil et al. (2005) use Monte Carlo simulations to investigate the MSE-based threshold choice and suggest sufficiently loose benchmarks for GDP models. Consistently, it is not uncommon to use the seemingly not-so-extreme thresholds for dependence modeling, as in Heffernan and Tawn (2004).

5 The trimming exercise is not imposed to *MktVaR* since it is a value common to all stocks and only has one observation each month. As will be explained later, its maximum and minimum values are not so extreme to be safely considered as unreasonable. Besides, we find that even using trimmed *FirmRet|MktVaR* to compute *CER*, the resulting *CER* still exhibits some unrealistically extreme values. We therefore resort to trimming raw *CER* directly.

6 Note that the ratio of mean *FirmRet|MktVaR* to mean *MktVaR* is slightly different from the reported mean *CER*. It is due to the trimming of unrealistically extreme observations for the *CER* and *FirmRet|MktVaR* estimate series.

Table 1
Summary statistics of CER and its components.

Panel A: Summary statistics

	Mean	StdDev	Skewness	Kurtosis	Minimum	25%	Median	75%	Maximum
CER	1.1274	0.7193	0.3269	−0.3899	−0.3606	0.5928	1.0810	1.6018	3.0922
FirmRet|MktVaR	−0.1282	0.0898	−0.4849	−0.4743	−0.3695	−0.1883	−0.1176	−0.0566	0.0303
MktVaR	−0.1198	0.0592	−1.0202	0.2688	−0.3163	−0.1582	−0.0996	−0.0795	−0.0434

Panel B: Correlations

	CER	FirmRet|MktVaR	MktVaR
CER	**1.0000**	**−0.8492**	**−0.0868**
FirmRet|MktVaR	**−0.8082**	**1.0000**	**0.4949**
MktVaR	**−0.0434**	**0.4787**	**1.0000**

Conditional extreme risk (*CER*) is measured as the expected return of an individual stock given the market return is at its predicted 1%-VaR level (*FirmRet*|*MktVaR*), scaled by the market 1%-VaR (*MktVaR*). CER measurement involves GPD specification of stock and market return distribution tails and dependence structure estimation based on a conditional multivariate extreme value approach. The estimation of CER and its components is conducted on a 60-month rolling window basis, using monthly return data retrieved from CRSP for all common stocks listed on NYSE, NYSE MKT (former Amex), and Nasdaq. Market returns are approximated by CRSP value-weighted index returns. The top and bottom 20th percentiles are used as thresholds to respectively identify right and left tail observations to be characterized by the GPD model, and the top 20th percentile is used as the minimum benchmark to isolate negated and transformed market returns for dependence estimation (see the text for details). Panel A presents full sample summary statistics of *CER*, *FirmRet*|*MktVaR*, and *MktVaR*. The top and bottom 2.5% observations of *CER* and *FirmRet*|*MktVaR* are trimmed off before the statistics are computed. Panel B reports their Pearson and Spearman correlation coefficients in the lower-left and upper-right triangles, respectively, with bold numbers indicating statistical significance at the 1% level.

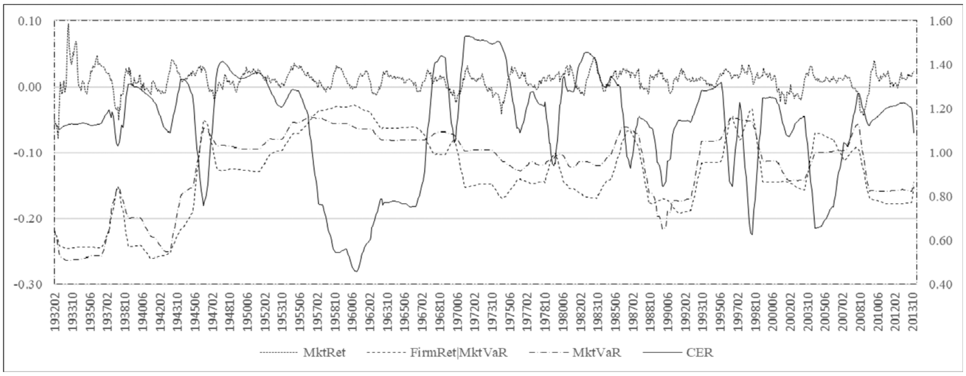

Fig. 1. The Time Series of Average Levels of CER (and Its Components) and Market Return. This figure presents the time series of the average levels of *CER* and its components (*FirmRet*|*MktVaR*, *MktVaR*), as well as the monthly CRSP value-weighted index returns (*MktRet*). A 12-month moving average scheme is applied before the time-series lines are plotted. The right vertical axis refers to the scale for *CER*, and the left vertical axis indicates the scale for other variables.

3.2. Validation of CER as a measure of indicating price drops during realized market crashes

Before testing the asset pricing implication of CER, we validate it as a measure of riskiness conditional on market crashes. By definition, high CER stocks are those expected to lose heavily if a rare market plunge actually occurs. They should perform worse than low CER stocks when small-probability but substantial negative market returns are realized. We examine whether this is the case. Specifically, we focus on the 50 months with the largest market drops during the 996-month sample period of 1931–2013 with valid month-beginning CER estimates. We identify five market crash scenarios with each consisting of 10 months: Scenario 1 contains the 10 lowest market return months, and scenario 2 involves the next 10 lowest returns, and so on. We classify stocks each month into tertile groups according to their CER estimates at the beginning of the month, with the highest (lowest) tertile containing stocks with high (low) CERs, relative to stocks in the middle tertile that have intermediate CERs. To assess the ability of CER in reflecting stock performance conditional on realized market crashes, we first check the probability that stocks belonging to a CER tertile at the beginning of the month also appear in the same tertile of realized price drops at the end of the month, where the price drop tertiles are created according to the magnitude of realized loss during the month, i.e., the highest (lowest) tertile consists of stocks with the largest (smallest) price drops. We compute the CER-price drop transition probabilities for CER tertiles under each market crash scenario. If CER has no power in predicting conditional stock performance when the market collapses, then these transition probabilities will all be around 33%. Instead, if high (low) CER can indicate large (small) price drops in a market crash scenario, the probabilities should exceed 33%. An alternative and more direct way to uncover CER's ability in reflecting potential risk conditional on extreme market plunges is to examine the average realized price drop of each CER tertile, and high CER stocks are expected to suffer from more substantial losses when the market crashes. We conduct both analyses and report average transition probabilities and price drops of different CER tertiles for each market crash scenario in Panels A and B of Table 2, respectively.

Consistent with the nature of CER as capturing risk under extreme market downturn conditions, Panel A shows that in scenarios with realized market crashes of various magnitudes, the transition probabilities of CER tertiles all exceed 33%, with the highest

Table 2
Performances of stocks with different CERs when the market experiences crashes.

Panel A: Average CER-price drop group transition probabilities					
	Market crash scenario 1	Market crash scenario 2	Market crash scenario 3	Market crash scenario 4	Market crash scenario 5
	Realized market return ∈ [−29.01%, −13.54%]	*Realized market return ∈ [−13.34%, −10.82%]*	*Realized market return ∈ [−10.53%, −9.61%]*	*Realized market return ∈ [−9.47%, −8.37%]*	*Realized market return ∈ [−8.32%, −7.42%]*
	average return = −19.88% n = 10	*average return = −11.95% n = 10*	*average return = −10.05% n = 10*	*average return = −8.83% n = 10*	*average return = −7.91% n = 10*
Low CER stocks	49.92%	50.98%	43.98%	43.10%	37.93%
Intermediate CER stocks	36.83%	36.70%	34.45%	33.64%	32.77%
High CER stocks	49.55%	49.69%	49.12%	43.35%	43.04%
Panel B: Average price drops in percentage					
Low CER stocks	−18.25%	−11.49%	−7.57%	−8.38%	−8.31%
Intermediate CER stocks	−22.59%	−16.07%	−10.19%	−10.79%	−8.61%
High CER stocks	−26.90%	−19.95%	−14.60%	−12.42%	−10.18%
High - Low	−8.65%	−8.46%	−7.04%	−4.05%	−1.86%
	[−13.10]	[−6.92]	[−7.59]	[−3.98]	[−1.69]

Five market crash scenarios, each containing 10 months, are identified from the 50 months with the largest market drops during the full sample period of 1931–2013. Scenario 1 includes the 10 lowest market return months, scenario 2 involves months with the next 10 lowest returns, and so on. Stocks are classified into tertile groups according to their CERs (estimated using the previous 60 months' monthly return data) at the beginning of each market crash month. The lowest tertile includes low CER stocks, the highest tertile includes high CER stocks, and the other tertile includes intermediate CER stocks. Panel A reports, for each CER group in each market crash scenario, average probability that stocks belonging to a CER group i (i = 1, 2, 3) at the beginning of the month also appear in tertile group i of realized price drops at the end of the month, where the price drop groups are created according to the magnitude of realized loss during the month, i.e., group 3 (1) consists of stocks with the largest (smallest) price drops. Panel B reports average price drops for stocks in each CER group under different market crash scenarios, along with the average differences in price drop "High - Low" and the associated t-statistics (in brackets).

values appearing in the top (i.e., high CER) and bottom (i.e., black swan hedging) tertiles. In crash scenario 1 that includes the 10 most severely collapsed months of the market, the probability of a stock in the highest CER tertile appearing in the largest price drop tertile and the probability of a black swan hedging tertile stock falling into the smallest price drop tertile are both close to 50%. A similar phenomenon is observed in the other four scenarios with relatively less severe market crashes, and is more persistent for high CER stocks. Therefore, there is clear evidence that when a disaster (and especially an extremely severe disaster) hits the market, high CER stocks are more likely to drop more and low CER ones are more likely to drop less (i.e., hedge the market black swan).

Panel B directly compares the price drops of CER tertiles. It is evident that stocks with high CERs experience price drops substantially larger than low CER stocks. This is especially true for scenarios with more extreme market plunges. For example, in crash scenario 1, which has an average market decline of 19.88% per month, high CER entails an extraordinary plummet of 26.90%. In contrast, low CER brings an average price drop of only 18.25%, which is 865 bps smaller (in magnitude) relative to high CER, thus effectively hedges the black swan in the market. In other words, black swan hedging (i.e., low CER) stocks indeed suffer much less relative to high CER counterparts during severe market crashes. This observation persists under other scenarios with less severe market plunges, although the price drop differences between high and low CERs become relatively smaller. In market crash scenario 2, black swan hedging stocks outperform high CER stocks by 846 bps per month. The corresponding price drop difference reduces to 704, 405, and 186 bps in scenarios 3 to 5. These values, although declining from more to less severe market crashes, are all substantial considering the fact that they are based on monthly returns. Indeed, these 50 crash months have a sizeable overall average market plummet of 11.72% per month, and high CER stocks underperform low CER stocks by 601 bps per month on average. It is thus reasonable to consider months with such large market drops as market crash situations. From another perspective, the declining trend of the price drop magnitude actually supports the validity of our CER measure since CER is constructed to reflect the relative riskiness under *extremely* negative market conditions. For a less extreme market down scenario, CER is not supposed to indicate conditional risk (and black swan hedging) as effectively as in a more extreme market crash situation. In fact, for months with no market crash, high CER no longer leads to large price drops. In these cases, high CER stocks are actually associated with high future returns. The reason for this is, as documented above, high CER stocks tend to plunge more in market crises and thus are much risky. If investors have serious concern about such risk, especially when such risk is more prominent, they would require a premium for bearing high CER and accept a discount for potential black swan hedging, as reflected in expected returns. We conduct detailed analyses on this issue in the following subsection.

3.3. CER and the cross-sectional variation of expected stock returns

We zero in on two aspects to examine the ability of CER (and equivalently, black swan hedging) to explain the cross-sectional variation of expected stock returns. First, when investors face high predicted market extreme risk or feel pessimistic about the potential harsh market environment, they have stronger motives to shun positions with which they would be exposed to catastrophic losses and therefore require large compensation for holding high CER assets. As the prospect of the market environment keeps changing, this means CER premium should vary with the *predicted* market extreme conditions. More specifically, when the market VaR is expected to be large, which indicates a more dreadful potential market crash, disaster-avoidance becomes investors' top-priority according to the safety-first principle, and the reward (premium) to high CER should be large. When a market disaster seems rather remote, the safety-first concern may be extenuated and the CER premium can diminish.

Second, as shown in the previous subsection, high CER stocks lose heavily when a severe market plunge actually takes place, that is, they perform much worse than low CER counterparts when substantially negative market returns are *realized*. Given this fact, the average premium to CER should be entirely due to the gains of high CER stocks during non-crash periods. This is consistent with the disaster risk argument by Rietz (1988, p. 118) who claims that "(r)isk-averse equity owners demand a high return to compensate for the extreme losses they may incur during an unlikely, but severe, market crash. To the extent that equity returns have been high with no crashes, equity owners have been compensated for the crashes that happened not to occur". In other words, the source of the CER premium is from non-crisis times, which outweighs the large losses a stock experiences during the worst market return months.

Table 3 provides empirical evidence regarding the above implications. In Panel A, we examine whether the CER effect is more prominent when the market crash is expected to be more severe. To do so, we examine mean excess returns of CER quintiles within sample months of different predicted market extreme risks as proxied by the market VaR estimates. Specifically, we consider months ranked within the top 10 percent, 20 percent, and 50 percent of all months in the sample period based on the magnitude of market 1%-VaR.[7] The top 10 percent represents the time when the market crash is expected to be the most devastating, and the top 20 percent refers to months with expectations of crash risk large enough but not as severe as in the top 10 percent subperiod, and so on. We also examine all months in the full sample period. In each case, we report average excess returns of CER portfolios along with the mean CER estimates and the high-minus-low (i.e., portfolio 5 minus portfolio 1) differences in return and CER. The results show a monotonic increase of excess return from low to high CER portfolios in each of the subperiods and the full period. The return spreads between the top and bottom CER quintiles are all significantly positive. Such a CER premium is especially pronounced in months with large predicted market extreme risk. For example, as reported in the "Excess return EW" rows, the EW average return spread between high and low CER portfolios within the top 10 percent predicted market VaR months is 194.43 bps per month (Newey–West t-value = 2.73), which translates into a 26.00% premium per annum based on monthly compounding. The CER premium declines as we extend the sample to include more months with relatively smaller predicted market VaR values. In the top 20 percent market VaR months, the monthly premium is 135.39 bps, and in the top 50 percent market VaR months, it reduces to 97.15 bps. The full-period average premium is 44.46 bps per month. This trend confirms the significance of the safety-first concern and the disaster-avoidance motive in investor preference, since such concern and motive become stronger with a larger predicted market crisis. Meanwhile, average CER levels have a narrower range when the market extreme risk is expected to be larger. Therefore, the CER pricing effect is more prominent when the predicted market crash is more severe, and investors require a higher compensation for bearing CER when they feel gloomy about the market. The black swan hedging attribute is especially valuable during these times and even a relatively small change in CER entails a large difference in expected stock return.

The last column of Panel A, also in the "Excess return EW" rows, shows the coefficients of CER in the Fama and MacBeth (1973) regressions of expected monthly excess returns (in percentage) on CER within each market VaR subperiod and the full period. We find a consistent pattern: The CER coefficient is 1.6194 in months with the top 10 percent market VaRs, and monotonically declines to 0.3407 in the full sample period.

We observe that in all periods with different levels of expected "extremeness" for market condition, the CER premia detected from the portfolio and regression analyses are significant in both statistical and economic terms. All the values are associated with large t-statistics. The monthly return differences between high and low CER portfolios are above 97 bps for the periods that include up to 50 percent of all sample months. The Fama–MacBeth regression loadings on CER in these periods are at least 0.6965, suggesting that one "unit" (which is close to one standard deviation of 0.7193 as in Table 1) increase in CER raises the equity premium by 69.65 bps per month (8.69% per annum) or more. Even in the full sample period, the CER effect is remarkable considering that CER estimation only involves a small portion (20 percent in the current measurement scheme) of observations in the joint distribution of stock and market returns, and reflects the risk conditional on a fairly small-probability event in the market — for the 1%-VaR case, there is a 99%-chance that such a rare event will not happen.

Panel B of Table 3 reports corresponding results after excluding the periods when market crashes actually happen and stocks with high CERs suffer from substantial losses. The purpose is to show that the compensation for bearing high CER completely comes from the non-crash times. We use the 50 months with the largest market drops to approximate realized market crashes, and reexamine the portfolio and univariate regression results for the CER effect after removing these market crash months from each market VaR subperiod as well as the full period. We first note that among these 50 crash months, 24 percent of them (i.e., 12 months) fall into the subperiod with the top 10 percent expected market VaRs, implying that our market VaR measure can effectively predict crises that are eventually realized in the market. Excluding these realized crash months from this subperiod dramatically boosts

[7] The 10th percentile of the total 996 predicted market VaRs during the full sample period of 1931–2013 should be around the 100th lowest value (note we do not negate the VaR). However, since market VaRs are estimated using the 20 percent largest market drops in the previous 60 months on a rolling window basis, for a series of adjacent estimation windows, it is possible that they all utilize the same market drop observations and thus have the same estimated VaR value. It turns out that the 100th, 101st, 102nd, and 103rd lowest market VaR estimates are the same. They are estimated from the adjacent estimation windows ending in April to July of 1938. This explains why there are 103 observations in the subperiod of top 10 percent market crash months, as shown in Panel A. Similar situation applies to the subperiod of top 20 percent market crash months.

Table 3
Excess returns of CER portfolios and Fama–MacBeth coefficients of CER.

Panel A: Excess returns of CER portfolios								
	No. of months	CER portfolios						Fama–MacBeth
		1 Low	2	3	4	5 High	High - Low	Coefficient of CER
	Top 10 percent predicted market VaR months							
CER		0.5218	0.9137	1.1472	1.3634	1.6729	1.1511	
Excess return EW	103	2.9465	3.0208	3.4220	4.2091	4.8907	1.9443	1.6194
							[2.73]	[2.91]
Excess return VW		1.1663	2.0364	2.3833	3.0364	3.2966	2.1303	1.9949
							[2.54]	[2.78]
	Top 20 percent predicted market VaR months							
CER		0.4920	0.9068	1.1683	1.4188	1.7950	1.3030	
Excess return EW	202	1.5714	1.6554	1.8140	2.4251	2.9253	1.3539	1.0908
							[3.03]	[3.05]
Excess return VW		0.6220	1.0736	1.2249	1.5787	1.8254	1.2034	1.0167
							[2.46]	[2.37]
	Top 50 percent predicted market VaR months							
CER		0.3939	0.8398	1.1445	1.4731	2.0121	1.6182	
Excess return EW	498	1.2331	1.3920	1.5306	1.8626	2.2046	0.9715	0.6965
							[3.19]	[3.50]
Excess return VW		0.5337	0.7372	0.8284	1.0231	1.2618	0.7281	0.5518
							[2.27]	[2.41]
	All months							
CER		0.3215	0.7512	1.0712	1.4295	2.0417	1.7202	
Excess return EW	996	0.9567	1.0448	1.0995	1.2719	1.4013	0.4446	0.3407
							[2.47]	[2.92]
Excess return VW		0.5847	0.7218	0.7495	0.7967	0.9059	0.3212	0.2740
							[1.76]	[2.16]
Panel B: Excess returns of CER portfolios after excluding months with realized market crashes								
	No. of months	CER portfolios						Fama–MacBeth
		1 Low	2	3	4	5 High	High - Low	Coefficient of CER
	Top 10 percent predicted market VaR months							
CER		0.5284	0.9157	1.1470	1.3611	1.6689	1.1405	
Excess return EW	91	4.7228	5.0279	5.8452	6.9871	8.0728	3.3501	2.7586
							[2.73]	[3.16]
Excess return VW		2.5997	3.6767	4.4911	5.4990	6.3582	3.7585	3.4918
							[2.72]	[3.04]
	Top 20 percent predicted market VaR months							
CER		0.4970	0.9096	1.1691	1.4182	1.7942	1.2972	
Excess return EW	182	2.9893	3.2971	3.8137	4.7041	5.4729	2.4836	1.9518
							[3.23]	[3.47]

(*continued on next page*)

the high-minus-low return spread of CER portfolios to 335.01 bps and the Fama–MacBeth CER loading to 2.7586, both of which are more than 70 percent larger than the corresponding values in Panel A, and exhibit similar or enhanced statistical significance. Similar patterns are observed in other subperiods and the full period. Panel B thus provides further information regarding the source of the CER premium documented in Panel A, and corroborates Rietz (1988) disaster-based equity risk premium proposition.

The above analyses are conducted on an EW base. We also report corresponding results on the VW base (in the "Excess return VW" rows), and find that all the patterns observed in the EW case persist in the VW case. Briefly, the CER premia from both the portfolio and regression analyses are significantly positive in each subperiod and also in the full period. Notably, we find no evidence that the asset pricing effect of CER is always weaker in the VW scheme than in the EW scheme. For example, in Panel A, the VW return spread between high and low CER portfolios and VW Fama–MacBeth regression coefficient of CER are both larger than those in the EW case in the top 10 percent market VaR months (the first subperiod), although in other subperiods and the full period the VW CER premia are smaller than the EW ones, but not by big margins. In Panel B, where realized market crash months are excluded, the difference between VW and EW results becomes even smaller. VW portfolio results are stronger in the first subperiod and slightly weaker in other subperiods and the full period, and VW regression results are stronger in the first two subperiods and almost identical to the EW results in the other subperiod and the full period. Therefore, the asset pricing effect of CER is less likely to be completely driven by small stocks. We will revisit the influence of firm size on CER pricing by explicitly controlling for market capitalization in both portfolio and regression analyses in the next subsection.

A relevant concern is that the detected CER effect on asset prices may be confined to micro-cap or low-priced penny stocks for which the microstructure issues may bring noise to the CER measurement. Although using monthly data to estimate CER largely

Table 3 (*continued*).

Excess return VW		1.8178	2.4273	2.9645	3.5901	4.2777	2.4598 [2.98]	2.0687 [2.98]
	Top 50 percent predicted market VaR months							
CER	462	0.3961	0.8413	1.1451	1.4729	2.0142	1.6181	
Excess return EW		2.1167	2.4175	2.7603	3.2537	3.7834	1.6667 [4.21]	1.1544 [4.20]
Excess return VW		1.2789	1.5913	1.8893	2.2966	2.8193	1.5404 [3.71]	1.1219 [3.45]
	All months							
CER	946	0.3213	0.7499	1.0696	1.4280	2.0429	1.7216	
Excess return EW		1.5644	1.7260	1.9020	2.1688	2.4074	0.8430 [3.61]	0.5960 [3.80]
Excess return VW		1.1128	1.2980	1.4493	1.6150	1.9035	0.7907 [3.32]	0.5917 [3.33]

Stocks are grouped into quintile portfolios each month based on their CER estimates. Panel A reports average CER levels and EW and VW monthly percentage returns in excess of the risk-free rate (proxied by one-month U.S. T-bill rate) of the following month for each portfolio in subperiods of months ranked within the top 10 percent, 20 percent, and 50 percent of all months in the 1931–2013 period according to the magnitude of predicted market 1%-VaR, as well as in the full sample period. The column "High - Low" represents mean differences in monthly excess return between the highest and lowest CER quintile stocks, with the associated Newey and West (1987) robust *t*-statistics reported in brackets. Coefficients of CER from EW and VW cross-sectional regressions of percentage excess returns on CER following the procedure in Fama and MacBeth (1973) are also reported, along with the Newey–West *t*-statistics in brackets. Sample size (no. of months) is indicated in the second column. Panel B presents corresponding portfolio and regression results after excluding the periods when market crashes are actually realized, i.e., the 50 months with the largest market downs.

mitigates potential microstructural biases, and the VW analyses further allay the concern to some degree, we still conduct a set of three additional tests to confirm the robustness of our results. First, we estimate CER by requiring that, for each stock, all its month-end market capitalizations in each estimation window are above the lowest NYSE size decile. Second, we estimate CER by requiring a stock's month-end price to be above $5 per share (a commonly used price threshold for penny stocks) in each estimation window. Third, we exclude all non-NYSE stocks from the CER estimation. We repeat the portfolio and regression analyses using the new set of CER estimates, and report the top and bottom CER quintile excess returns, their differences, and the Fama–MacBeth coefficients of CER in Table 4, for both EW and VW cases.

The results show that the CER premia are still significantly positive in all subperiods as well as the full period, and in both EW and VW schemes. For NYSE-only stocks, the premia are fairly close to those in Table 3. For the cases excluding small or penny stocks, their magnitudes become smaller in general, but this phenomenon is not universal as the CER effects are mostly stronger than in Table 3 in the subperiod of the largest market VaR months under the scenario excluding small stocks, especially when the realized market crash months are also excluded. Note that our data screening is rather aggressive since we require a stock's size or price to be above the threshold (lowest NYSE size decile or $5) in every month of each estimation window. This approach ensures that all the data inputs for CER estimation are subject to minimal microstructure noise.[8] Even in these strictly limited samples of truly large or high-priced stocks, the significantly positive relation between CER and expected stock returns still exists. Therefore, the asset pricing effect of CER is not confined to small-cap or low-priced stocks.

The overall implication of Tables 3 and 4 results is that, investors assign substantial weight to systematic extreme risk and require a sizeable compensation to hold equities suffering from large losses during potential rare market downturns. Equivalently, they are willing to sacrifice average returns of black swan hedging equities for the benefit of being protected in case of market plummets. This effect is more significant when the potential market calamities are expected to be severe, and the reward for bearing CER more than offsets the price drop that a stock may experience if the rare market disaster happens to be realized. Together, these findings, for the first time in the literature, provide supporting evidence for the asset pricing model based on the safety-first theory.

3.4. CER effect and commonly used risk variables

The CER measurement identifies stock risk conditional on the extreme downside part of market returns. It holds the potential to be associated with market beta, which measures a stock's sensitivity to the market for both extreme and non-extreme returns. CER could also be affected by firm characteristics related to the economic forces underlying stocks' different responses to market crashes. To disentangle the CER premium from the cross-sectional return effects of commonly used risk variables, we examine CER's associations with market beta, as well as other firm characteristics known to influence asset returns including size, book-to-market ratio, momentum, and liquidity. We estimate beta as the stock and market return covariance scaled by the variance of market returns, using the previous 60 monthly observations to make it comparable to the CER measurement (using the two-stage portfolio-based beta measures in Fama and French (1992) produces similar findings). Size is computed as market capitalization (in million U.S. dollars)

[8] Another important reason to exclude stocks whose market capitalizations or prices are higher than the corresponding thresholds in part of but not all of the estimation months is that, if we keep them in the sample and only exclude those small-cap or low price months, the number of observations in each 60-month estimation window is much reduced, especially in the subset with the 20 percent largest market drops, which makes the CER measurement subject to much estimation errors.

Table 4
Excess returns of CER portfolios and Fama–MacBeth coefficients of CER within samples excluding small, low-priced, and non-NYSE stocks.

Panel A: Excess returns of CER portfolios

	Exclude small stocks				Exclude low-priced stocks				NYSE stocks only			
	CER portfolios			Fama–MacBeth	CER portfolios			Fama–MacBeth	CER portfolios			Fama–MacBeth
	1 Low	5 High	High - Low	Coefficient of CER	1 Low	5 High	High - Low	Coefficient of CER	1 Low	5 High	High - Low	Coefficient of CER
	Top 10 percent predicted market VaR months											
Excess return EW	2.2521	4.2548	2.0027	1.5483	1.5331	3.0217	1.4886	1.2858	2.9462	4.9232	1.9770	1.6139
			[2.45]	[2.24]			[2.03]	[1.92]			[2.70]	[2.83]
Excess return VW	1.1466	3.3746	2.2280	1.9891	1.0397	2.8996	1.8599	1.8124	1.1714	3.3326	2.1612	2.0110
			[2.60]	[2.76]			[3.02]	[2.73]			[2.57]	[2.81]
	Top 20 percent predicted market VaR months											
Excess return EW	1.2703	2.3767	1.1065	0.8671	0.8557	1.7344	0.8787	0.7588	1.5963	2.7467	1.1504	0.9224
			[2.26]	[2.11]			[2.22]	[2.12]			[2.31]	[2.25]
Excess return VW	0.6301	1.8302	1.2001	1.0177	0.5307	1.5216	0.9909	0.9052	0.6281	1.8153	1.1872	1.0109
			[2.41]	[2.37]			[2.93]	[2.39]			[2.38]	[2.35]
	Top 50 percent predicted market VaR months											
Excess return EW	1.0122	1.7321	0.7199	0.5590	0.8712	1.4117	0.5406	0.4562	1.1645	1.9931	0.8286	0.6213
			[2.57]	[2.67]			[2.32]	[2.48]			[2.97]	[2.99]
Excess return VW	0.5385	1.1731	0.6347	0.5393	0.5218	1.0489	0.5271	0.4951	0.5371	1.1778	0.6408	0.5414
			[2.07]	[2.34]			[2.04]	[2.32]			[2.22]	[2.42]
	All months											
Excess return EW	0.8108	1.1786	0.3678	0.2877	0.7561	1.0211	0.2649	0.2298	0.8858	1.2982	0.4124	0.3181
			[2.31]	[2.45]			[1.93]	[2.19]			[2.53]	[2.69]
Excess return VW	0.5860	0.8734	0.2874	0.2745	0.5602	0.8113	0.2511	0.2601	0.5816	0.8609	0.2793	0.2778
			[1.66]	[2.16]			[1.67]	[2.20]			[1.68]	[2.24]

Panel B: Excess returns of CER portfolios after excluding months with realized market crashes

	Exclude small stocks				Exclude low-priced stocks				NYSE stocks only			
	CER portfolios			Fama–MacBeth	CER portfolios			Fama–MacBeth	CER portfolios			Fama–MacBeth
	1 Low	5 High	High - Low	Coefficient of CER	1 Low	5 High	High - Low	Coefficient of CER	1 Low	5 High	High - Low	Coefficient of CER
	Top 10 percent predicted market VaR months											
Excess return EW	3.8449	7.4352	3.5903	2.9402	2.8532	5.8403	2.9871	2.7343	4.7309	8.1099	3.3790	2.7719
			[2.54]	[2.56]			[2.53]	[2.48]			[2.66]	[2.97]
Excess return VW	2.5546	6.3488	3.7942	3.4919	2.5476	5.5430	2.9954	3.2078	2.6274	6.3989	3.7715	3.5112
			[2.76]	[3.03]			[3.57]	[3.18]			[2.73]	[3.06]
	Top 20 percent predicted market VaR months											
Excess return EW	2.5752	4.9282	2.3530	1.8950	2.0054	4.0741	2.0687	1.8059	3.0068	5.2891	2.2823	1.8119
			[2.81]	[2.80]			[3.02]	[2.87]			[2.75]	[2.86]
Excess return VW	1.8028	4.2104	2.4076	2.0739	1.7409	3.6942	1.9533	1.8935	1.8310	4.2557	2.4247	2.0643
			[3.04]	[2.99]			[3.95]	[3.11]			[2.92]	[2.97]
	Top 50 percent predicted market VaR months											
Excess return EW	1.8411	3.2954	1.4543	1.1036	1.6176	2.8668	1.2492	1.0113	2.0375	3.5567	1.5191	1.1113
			[3.66]	[3.56]			[3.79]	[3.51]			[3.84]	[3.77]
Excess return VW	1.2751	2.6687	1.3936	1.1134	1.2742	2.4210	1.1468	1.0407	1.2805	2.6943	1.4138	1.1153
			[3.56]	[3.41]			[3.98]	[3.54]			[3.59]	[3.44]
	All months											
Excess return EW	1.3832	2.1717	0.7885	0.5914	1.2908	1.9569	0.6662	0.5356	1.4819	2.2960	0.8141	0.5956
			[3.53]	[3.47]			[3.49]	[3.37]			[3.60]	[3.62]
Excess return VW	1.1094	1.8287	0.7194	0.5931	1.0912	1.7036	0.6124	0.5635	1.1052	1.8267	0.7215	0.5957
			[3.25]	[3.35]			[3.51]	[3.48]			[3.22]	[3.37]

Stocks are grouped into CER quintiles each month. Panel A reports the top and bottom quintile EW and VW percentage excess returns of the following month, their differences, and the EW and VW Fama–MacBeth coefficients of CER within samples excluding small, low-priced, and non-NYSE stocks, for the subperiods of different predicted market 1%-VaR months and the full sample period. Panel B reports corresponding results after excluding the 50 largest market down months. Newey–West *t*-statistics are in brackets.

of the previous month-end. Book-to-market ratio (B/M), measured after June 1963, is the book value (obtained from COMPUSTAT annual files) divided by the market value of equity of the last fiscal year ending in the preceding calendar year. Momentum is proxied by the past 11-month stock return after skipping the most recent month (Jegadeesh and Titman, 1993). We use the liquidity beta (i.e., the time-series regression loading on an aggregate liquidity innovation factor after controlling for Fama and French (1993) three factors using the previous 60 months' monthly data) from Pastor and Stambaugh (2003) to measure firm liquidity. Liquidity beta estimates are available from July 1963.[9]

[9] We adopt liquidity beta to represent liquidity risk mainly because it is more of a systematic measure, in the sense that it is constructed from the connection between a firm-level factor and an aggregate factor. This construction is more consistent with the measurement of CER which is based on the joint distribution of the firm and the market returns. Nevertheless, using the more firm-level liquidity measure as in Amihud (2002) does not change the main results and conclusions. For example, using Amihud illiquidity measure estimated using monthly data on a 60-month rolling window basis, the average return spread between the top

For the analyses from here on, to save space, we report EW results for the full sample period only, while noting that VW results are consistent and qualitatively identical to EW results. Another reason to focus on the full sample results is that, although the stronger pricing effect in subperiods of higher anticipated market crash risk holds for CER, there is no sufficient a priori basis for believing that it is also the case for the size, book-to-market effects, and so on, in this section, as well as the comoments and co-crash-based measures in the following sections. It is therefore fair to compare the asset pricing impacts of CER and other existing risk variables based on the full sample period without differentiating high and low expected market crash times.[10]

Panel A of Table 5 reports average values of the common risk variables in CER quintile portfolios, together with the Pearson correlation between CER and each variable. CER exhibits a strong and positive relation with beta, with a correlation coefficient of 0.6431. This is not surprising due to the CAPM's relation with the Arzac–Bawa model and the fact that both measures reflect the joint behavior of stock and market returns, with more or less focus on extremes. Small stocks tend to have higher CERs and are thus less likely to hedge black swan risk in the market, but the decrease of firm size from low to high CER portfolios is not monotonic, and the correlation coefficient between size and CER is only −0.0467. There is no obvious trend of B/M across CER portfolios. Past winners with higher momentum values appear in the lowest CER portfolio, but the difference in momentum between high and low CER quintiles is trivial and not statistically significant. This is understandable since the CER measure indicates losses conditional on market plunges over the past 60 months, while momentum only reflects unconditional returns in the most recent year. Although high CER stocks appear to be less liquid, the change of liquidity beta across CER portfolios is not monotonic but rather inverse U-shaped. Overall, CER seems to capture quite unique information as compared with book-to-market, momentum, and liquidity. Meanwhile, it also shows nontrivial associations with beta and firm size that can predict expected stock returns in the same direction as CER. It is therefore necessary to check to what extent the CER premium can be influenced by beta and firm characteristic variables (especially size) individually or collectively. We conduct the investigation in portfolio analyses controlling for other risk variables, and multivariate regressions including different explanatory variables simultaneously.

In Table 5, Panel B, we first sort stocks into five portfolios according to each of the risk (control) variables. Within each portfolio, we further form quintiles based on CER. The procedure is repeated every month, and the average excess return of each CER quintile across control variable-sorted portfolios is computed. We report time-series average excess returns for CER quintiles and the difference between the top and bottom quintiles, corresponding to each control variable. The results indicate that, after controlling for beta and each firm characteristic variable, the return spreads between high and low CER quintiles remain positive, although with more or less reduced magnitudes. Beta and size exercise relatively strong influences, but cannot explain a major part of the CER premium. For example, the remaining CER premium after controlling for beta (size) is 30.10 (31.70) bps per month, only reduced by less than one third from the 44.46 bps full-period average in Panel A of Table 3. Consistent with their weak associations with CER, B/M, momentum, and liquidity beta show less influences on the CER pricing effect. Noticeably, under each controlling scheme, the expected return still exhibits a monotonic increase from low to high CER portfolios, and the resulting CER premium becomes even more significant as the t-values are larger than the corresponding one in Panel A. Therefore, there is no evidence showing that beta and the selected firm characteristic variables can fully account for the CER premium. In particular, the weak influence of beta on the CER effect hints that CER reflects more information about systematic riskiness beyond beta, consistent with the implication of the Arzac–Bawa model that comoment measures can be incorporated into the more general RASF risk factor under additional assumptions, an issue that we will examine in greater detail in Section 4.

The inferences from Panel B are reinforced in Panel C where we report characteristic adjusted returns for CER portfolios, following the procedure in Daneil et al. (1997). Specifically, to adjust individual returns for size and B/M, we first sort stocks each month independently into size and B/M quintiles (we obtain similar results when stocks are first sorted by size, and then sorted by B/M within each size group). Then, we compute the average benchmark excess return for each of these 25 portfolios that is subtracted from the excess returns of individual stocks belonging to the corresponding size-B/M portfolio. Excess returns adjusted this way should have an expected value of zero if size and B/M completely describe the cross-sectional variation of expected returns. Evidence from the first row of Panel C shows, however, that there is still a positive relation between CER and the adjusted returns: The size-B/M adjusted return increases monotonically with CER, and the high-minus-low spread and univariate Fama–MacBeth coefficient of CER (using adjusted returns as the dependent variable) are both positive and significant. Similar results are found after returns are adjusted further by momentum or liquidity beta. To adjust the momentum effect in addition to size and B/M, we form 125 portfolios based on independent quintile sorting according to size, BM, and momentum, and use the average returns of these portfolios as the benchmarks to adjust individual stock returns. We report portfolio and regression analyses results using such three-variable adjusted returns in the second row and the similarly constructed size-B/M-liquidity beta adjusted return results in the third row. In both scenarios, we have findings consistent with the size-B/M adjusted case, namely, the positive premium to CER cannot be subsumed by the pricing effects of firm size, book-to-market ratio, momentum, and liquidity.

To further confirm the explanatory power of CER for expected stock returns beyond that of beta and firm characteristic variables, we run firm-level multivariate Fama and MacBeth (1973) regressions of expected excess returns on CER and other explanatory variables. This allows us to simultaneously control for more factors than in the above return adjustment scheme. Regression models

and bottom CER quintiles after controlling for Amihud illiquidity in a conditional two-way sorting analysis is 37.45 bps per month with a Newey–West t-value of 4.35. The average size-B/M-illiquidity adjusted return spread between the top and bottom CER quintiles is 29.17 bps with a Newey–West t-value of 1.86. The Fama–MacBeth regression coefficient of CER after controlling for Amihud illiquidity only is 0.3400 (Newey–West t-value = 2.90), after controlling for Amihud illiquidity together with beta, size, B/M, and momentum is 0.1395 (Newey–West t-value = 2.28). The Fama–MacBeth regression of size-B/M-illiquidity adjusted returns on CER has a CER coefficient of 0.1816 (Newey–West t-value = 2.10). Refer to the text for details of the analysis procedures.

[10] Although not reported, results about CER pricing in subperiods of higher predicted market crash months are consistently and substantially stronger for all analyses to be conducted.

Table 5
CER effect and commonly used risk variables.

Panel A: Common risk variables in CER portfolios

	CER portfolios						Correlation with
	1 Low	2	3	4	5 High	High - Low	CER
Beta	0.6370	0.8934	1.1160	1.3480	1.6948	1.0579	0.6431
						[34.38]	p<0.0001
Size	1188.9100	1394.0000	1195.7200	881.8300	517.8700	−671.04	−0.0467
						[−6.25]	p<0.0001
B/M	0.8734	0.8545	0.8751	0.8889	0.8788	0.0054	0.0300
						[0.44]	p<0.0001
Momentum	0.1575	0.1520	0.1498	0.1525	0.1556	−0.0019	−0.0226
						[−0.09]	p<0.0001
Liquidity beta	−0.0060	0.0010	0.0022	−0.0037	−0.0354	−0.0294	−0.0239
						[−6.74]	p<0.0001

Panel B: Excess returns of CER portfolios after controlling for common risk variables

	CER portfolios					
	1	2	3	4	5	High - Low
Controlling for beta	1.1007	1.0483	1.0615	1.1630	1.4017	0.3010
						[5.41]
Controlling for size	0.9744	1.1137	1.1769	1.2202	1.2913	0.3170
						[3.78]
Controlling for B/M	0.7181	0.8065	0.8647	0.9460	1.1320	0.4140
						[3.96]
Controlling for momentum	1.0042	1.0766	1.1221	1.2273	1.3353	0.3311
						[4.67]
Controlling for liquidity beta	0.7452	0.8489	0.9244	1.0124	1.0815	0.3363
						[3.30]

Panel C: Adjusted excess returns of CER portfolios

	CER portfolios						Fama–MacBeth
	1	2	3	4	5	High - Low	Coefficient of CER
Size-B/M adjusted	−0.1755	−0.0587	0.0052	0.0552	0.1732	0.3486	0.2143
						[1.95]	[2.15]
Size-B/M-momentum adjusted	−0.1319	−0.0482	0.0127	0.0418	0.1253	0.2572	0.1619
						[2.12]	[2.38]
Size-B/M-liquidity beta adjusted	−0.1597	−0.0565	0.0070	0.0505	0.1579	0.3175	0.1962
						[1.94]	[2.17]

Panel D: Coefficients from Fama–MacBeth regressions with CER as one explanatory variable

Model	CER	Beta	ln(Size)	ln(B/M)	Momentum	Liquidity beta
1	0.3779	−0.0621				
	[3.32]	[−0.40]				
2	0.1701	0.0255	−0.1326	0.2019		
	[2.66]	[0.21]	[−3.31]	[3.05]		
3	0.1614	0.0032	−0.1499	0.1792	0.6470	
	[2.64]	[0.03]	[−3.94]	[2.77]	[4.69]	

(*continued on next page*)

in Panel D show that, after additionally controlling for some or all of the risk variables (including beta, the logarithm of size *log(size)*, the logarithm of B/M *log(B/M)*, momentum, and liquidity beta), CER retains its significantly positive relation with expected stock returns cross-sectionally, consistent with the univariate regression result. Another observation from Panel D is that coefficients of other risk variables retain their signs and significance levels as predicted and documented in relevant studies. This is especially evident for the negative loading on *log(size)* and positive loadings on *log(B/M)* and momentum. The inference is that CER conveys information about risk that is distinct from that delivered by existing risk variables.

3.5. Alpha, pricing error, and CER

The asset pricing implication of CER can be further manifested by analyzing alphas and pricing errors from other pricing models. We report relevant results in Table 6. In Panel A, we run full-period time-series regressions with CER-sorted portfolio excess returns as the dependent variable and commonly used asset pricing factors as regressors. We separately examine the CAPM, the 3-factor model of Fama and French (1993), and the Carhart (1997) 4-factor model. Alphas from these models filter out influences of the risk factors therein, and a positive association between alpha and CER would suggest that the pricing of CER is not attributable

Table 5 (*continued*).

4	0.1638 [2.59]	0.0114 [0.10]	−0.1498 [−3.98]	0.1775 [2.74]	0.6553 [4.79]	0.0263 [0.32]

In Panel A, full-period average values of commonly used risk variables including beta, size, book-to-market ratio (B/M), momentum, and liquidity beta are reported for each CER quintile, along with the differences between the top and bottom quintiles. Beta is computed as the covariance between stock and market returns divided by the variance of market returns, using the previous 60 months' monthly data. Size is the market capitalization (in million U.S. dollars) of the previous month-end. B/M is the book value divided by the market value of equity of the last fiscal year ending in the preceding calendar year. Momentum is proxied by the past 11-month stock return after skipping the most recent month. Liquidity beta is the regression loading on an aggregate liquidity innovation factor after controlling for Fama–French three factors based on the previous 60 months' monthly data, following Pastor and Stambaugh (2003). For each risk variable case, CER portfolios are formed each month with non-missing observations for that variable. The variable's Pearson correlation coefficient with CER is reported in the last column with the associated *p*-value. In Panel B, stocks are first sorted each month by each of the risk variables into five groups, and within each group, further sorted into CER quintiles. Average monthly percentage excess returns across all risk variable groups are computed and the full-period time-series averages of them are reported for the CER quintiles, together with the high-minus-low spreads. In the first row of Panel C, stocks are independently sorted into size and B/M quintiles which form 25 portfolios each month. Average monthly excess return is computed for each of these 25 portfolios and is used as the benchmark return that is subtracted from each individual stock's excess return belonging to that particular portfolio. Full-period time-series average size-B/M adjusted percentage excess returns are reported for CER quintiles, as well as the high-minus-low spread and the univariate Fama–MacBeth coefficient of CER using adjusted percentage excess returns as the dependent variable. Individual stock returns are analogously adjusted by the benchmark returns of 125 portfolios formed by independent size-B/M-momentum or size-B/M-liquidity beta quintile sorting, and corresponding portfolio and regression analyses results are reported in the second and third rows of Panel C. In Panel D, multivariate Fama and MacBeth (1973) cross-sectional regressions are run each month with individual stock percentage excess returns as the dependent variable. Among the explanatory variables are CER and commonly used risk variables (beta, the logarithm of size *ln(Size)*, the logarithm of B/M *ln(B/M)*, momentum, and liquidity beta). Full-period time-series averages of the coefficients are reported. Newey–West *t*-statistics are reported in brackets.

to other factors.[11] We find that CAPM alpha shows a generally increasing trend from low CER to high CER portfolios, and the high-minus-low spread in alpha is significantly positive. Similar observations are obtained for the 3-factor and 4-factor alphas. The asset pricing effects of other commonly used factors are generally independent of CER, as their factor loadings (*MKTbeta*, *SMBbeta*, *HMLbeta*, *UMDbeta*) do not show monotonic changes across CER portfolios, and there is no clear direction of the trend in most cases. This suggests that CER delivers risk information quite different from the existing factors, which is further confirmed in the pricing error analysis below.

As demonstrated by Harvey and Siddique (2000), pricing errors of existing asset pricing models can be used to explore the pricing of another factor, namely, whether this factor can explain what existing models do not. Following their approach, we detect pricing errors of the CAPM by first running time-series regressions of stock excess returns on market excess returns based on a 60-month rolling window, which yields estimate of the coefficient of market excess return for each stock. This estimated factor loading is then used as the regressor in a cross-sectional regression with the following month's excess stock returns as the dependent variable. Pricing error is identified by the cross-sectional regression intercept. We compute average CAPM pricing error (as well as analogously estimated 3-factor model and 4-factor model pricing errors) of stocks in each CER quintile each month, and report their full-period means in Panel B of Table 6. Similar to alphas, pricing errors of all three models also exhibit an increasing pattern from low to high CER portfolios, and the differences between the two extreme quintiles are statistically significant. Therefore, CER shows its ability to describe the cross-sectional return variation not driven by pricing factors in existing models. This conclusion is reinforced by observing larger pricing errors in periods of higher CER premia, as demonstrated in Panel C, where we partition the full sample time into five subsamples according to return spread between the top and bottom CER quintiles. We find that subsamples with higher (lower) CER premia have larger (smaller) pricing errors, and the correlations between CER premium and pricing error (not reported in the table in the interest of brevity) are positive and significant in all models. Therefore, the periods when CER explains more cross-sectional return variation appear to be the periods when other models have larger pricing errors, suggesting that CER captures a special type of risk not reflected by existing asset pricing factors.

4. Conditional extreme risk and comoments

In this section, we investigate to what extent CER explains the premia to comoments (up to the fourth moment) and vice versa. As shown in the online appendix, comoments are nested in a more general RASF risk factor given additional assumptions about plausible distributional forms of the market tail. Hence, CER as a proxy for the RASF risk factor that is not restricted by any particular distribution, should capture the effects of comoment risks and deliver additional systematic risk information, but comoments would not completely reflect CER if the moments cannot fully describe the return distribution.

We first compare CER with comoment-based downside beta, coskewness, and cokurtosis, all of which have explanatory power in the cross-section of stock returns. Although downside beta is not explicitly shown as a special case of the RASF pricing model, it is a modified market beta and shares a similar covariance specification in its construction. More importantly, downside beta exhibits a more significant pricing effect (Ang et al., 2006b) than the traditional beta (Fama and French, 1992). It is therefore more

[11] Before carrying out the alpha tests, we filter out the effect of beta by computing monthly excess returns for quintile portfolios based on CER values orthogonal to beta. The reason is that alpha represents the part of return not captured by the market-driven portion, as in the CAPM. The market-driven return is determined by beta, which implies that high beta tends to drive down alpha. The negative association between beta and alpha is well-documented in the literature (Karceski, 2002; Baker et al., 2011; Frazzini and Pedersen, 2014). Since CER is highly correlated with beta, its relation with alpha tends to be biased toward a negative one, which adds noise to our alpha analyses. Because our main interest is in the pricing of risk reflected in CER beyond that in beta, it is necessary to remove the "beta noise" to obtain a meaningful inference about the CER-alpha relation.

Table 6
Alphas and pricing errors in CER portfolios.

Panel A: Alphas in CER portfolios

		CER portfolios					
		1 Low	2	3	4	5 High	High - low
CAPM alpha		0.2103	0.2814	0.3211	0.3191	0.4935	0.2832
							[3.34]
	MKTbeta	1.2917	1.1585	1.1741	1.2501	1.3336	0.0419
3-factor model alpha		−0.0722	0.0681	0.1071	0.0903	0.1973	0.2695
							[3.57]
	MKTbeta	1.0494	0.9780	0.9910	1.0441	1.0492	−0.0003
	SMBbeta	0.7904	0.5786	0.5951	0.7115	1.0518	0.2614
	HMLbeta	0.4988	0.3851	0.3798	0.3707	0.4189	−0.0799
4-factor model alpha		0.0257	0.1417	0.2088	0.2588	0.3983	0.3726
							[4.24]
	MKTbeta	1.0259	0.9603	0.9666	1.0036	1.0009	−0.0250
	SMBbeta	0.7883	0.5770	0.5929	0.7079	1.0474	0.2592
	HMLbeta	0.4532	0.3508	0.3324	0.2922	0.3252	−0.1280
	UMDbeta	−0.1011	−0.0760	−0.1051	−0.1740	−0.2077	−0.1066

Panel B: Pricing errors in CER portfolios

	CER portfolios					
	1 Low	2	3	4	5 High	High - Low
CAPM pricing error	0.7511	1.0291	1.0221	1.5482	1.7708	1.0197
						[3.80]
3-factor model pricing error	0.7704	1.0704	1.0959	1.8271	1.8624	1.0920
						[3.73]
4-factor model pricing error	0.7183	1.0652	1.0782	1.8096	1.8513	1.1331
						[3.80]

Panel C: Pricing errors and CER premia

	CER premium subsamples					
	1 Low	2	3	4	5 High	High - Low
CER premium	−6.0527	−1.8230	0.0209	1.8529	8.2575	14.3102
CAPM pricing error	−0.9341	0.4391	1.0065	1.4600	2.5434	3.4774
						[7.12]
3-factor model pricing error	−1.1327	0.4422	0.9252	1.5096	2.8714	4.0042
						[8.57]
4-factor model pricing error	−1.1042	0.4595	0.9178	1.4902	2.7264	3.8306
						[8.32]

In Panel A, monthly excess returns for quintile portfolios based on CER values orthogonal to beta are computed, and then are regressed on excess market returns (MKT) according to the CAPM, or on the three factors of Fama and French (1993) (MKT, SMB, HML), or on the four factors of Carhart (1997) (MKT, SMB, HML, UMD) over the full sample period. For each CER quintile, the time-series regression intercept (in percentage) is reported as *CAPM alpha*, *3-factor model alpha*, or *4-factor model alpha*, along with the factor coefficients (in percentage) denoted by *MKTbeta*, *SMBbeta*, *HMLbeta*, and *UMDbeta*. The corresponding alphas and factor loadings for the high-minus-low portfolios are also reported. In Panel B, pricing error of the CAPM, the 3-factor model, or the 4-factor model is estimated each month as the intercept of a cross-sectional regression of excess stock returns on factor loading(s). The factor loading(s) is(are) the coefficient(s) on the model factor(s) (MKT, SMB, HML, UMD) from a time-series regression of stock excess returns on pricing factor(s) using the previous 60 months' monthly data. The full-period time-series averages (in percentage) of *CAPM pricing error*, *3-factor model pricing error*, and *4-factor model pricing error* are reported for each CER quintile as well as the high-minus-low portfolio. In Panels A and B, Newey–West t-statistics are reported in brackets. In Panel C, the full sample period is divided into five subsamples according to CER premium, i.e., the return spread between the top and bottom CER quintiles. Average *CAPM pricing error*, *3-factor model pricing error*, and *4-factor model pricing error*, as well as CER premium (all in percentage) are reported for each CER premium subsample. Corresponding differences between high and low CER premium subsamples (and their t-statistics in brackets) are also reported.

meaningful to examine CER's influence on downside beta, which complements the relevant CER-market beta analysis in Section 3. For coskewness and cokurtosis measures, we follow the existing literature for their constructions (Kraus and Litzenberger, 1976; Friend and Westerfield, 1980; Barone-Adesi, 1985; Fang and Lai, 1997; Chung et al., 2006; Jurczenko and Maillet, 2006), which is in line with the Arzac–Bawa model implication.

Consistent with the CER measurement, we estimate these comoment-based measures using 60 months' monthly return data on a rolling window basis. Downside beta is the ratio of covariance between stock and market returns to the variance of market returns, conditional on the market returns being less than the sample mean. Coskewness is the covariance between stock returns and squared market returns scaled by the third power of the standard deviation of market returns. Cokurtosis is similarly calculated by dividing the covariance between stock returns and cubed market returns by squared market return variance.[12]

Panel A of Table 7 reports average values of downside beta, coskewness, and cokurtosis for the top and bottom CER quintile portfolios and the differences between them. High CER is associated with high downside beta, low coskewness, and high cokurtosis. Their differences between high and low CER quintiles are salient and highly significant, and their Pearson correlations with CER all have magnitudes higher than 0.37. These relations point to the same risk premium direction as CER because downside beta and cokurtosis are positively priced and coskewness is negatively priced. This is generally confirmed by the portfolio and regression analyses results in Panel B. The full-period average high-minus-low quintile return spreads for downside beta, coskewness, and cokurtosis are 31.78, −34.72, and 34.26 bps per month, respectively, and their univariate Fama–MacBeth regression coefficients are 0.1782, −0.1222, and 0.0272, respectively. Given the overlapping economic implications of these comoment-based measures with CER, it is necessary to delineate their potential interactions in asset pricing.

In Panels C and D, we explore the mutual impacts between downside beta and CER in their effects on the cross-section of stock returns. To examine how downside beta influences CER pricing, we categorize stocks each month into five groups according to their downside beta estimates and further assign stocks into CER quintile portfolios. We compute the cross-downside beta mean excess return for each CER portfolio, and report the values for the top and bottom quintiles, as well as the difference between them, in the left part of Panel C. We also report Fama–MacBeth regression coefficient of CER after downside beta is controlled. Clearly, downside beta does not exert a dominating influence over the CER premium since CER is still positively associated with expected stock returns after controlling for downside beta. The high-minus-low return spread for CER across all downside beta groups is 31.11 bps per month, only reduced by about 13 bps from the baseline result (44.46 bps) in Panel A of Table 3. The Fama–MacBeth coefficient of CER becomes even larger after controlling for downside beta: It increases from 0.3407 before downside beta is controlled to 0.5346 after downside beta is controlled in the regression. Both the return spread and the regression coefficient of CER after controlling for downside beta are statistically significant, with Newey–West t-statistics of 4.39 and 2.65, respectively.

In dramatic contrast, the pricing effect of downside beta disappears after controlling for CER, as shown in the corresponding left part of Panel D, in which we first create CER quintiles and then form five downside beta portfolios within each CER quintile. The cross-CER high-minus-low return spread for downside beta gives a negative and insignificant difference of −6.42 bps per month, which is reduced from the 31.78 bps (Panel B) before CER is controlled. The message from the Fama–MacBeth regression is equally clear and consistent: The significantly positive coefficient of downside beta in the univariate regression in Panel B becomes insignificantly negative after CER is included in the regressor list. These findings suggest that the asset pricing effect of downside beta may be largely due to its reflection of the more general RASF risk factor proxied by CER. Risk information embedded in CER goes beyond that in the comoment-based risk measure.

We conduct two-way sorting and bivariate Fama–MacBeth regression analyses in a similar manner to study CER premium's interactions with coskewness and cokurtosis in the middle and right parts of Panels C and D, and obtain similar results. To summarize, coskewness and cokurtosis cannot subsume the pricing of CER, but CER can largely explain the premia to coskewness and cokurtosis. This is especially true for cokurtosis. As shown in the right part of Panel C, the average return spread between the top and bottom CER quintiles across cokurtosis portfolios is positive and significant. In contrast, the average high-minus-low return spread for cokurtosis after controlling for CER is −4.86 bps in the right part of Panel D, a large drop from the 34.26 bps (Panel B) before CER is controlled. Regression analyses consistently show that controlling for CER drives the cokurtosis coefficient from positive to negative, while controlling for cokurtosis does not reduce the CER coefficient in either magnitude or significance level.

Comparable pictures, though slightly different, are observed from the middle parts of Panels C and D for the CER-coskewness interaction. The CER premium proxied by the high-minus-low quintile return difference remains significantly positive after being averaged across the five coskewness portfolios, although the magnitude is reduced by 54 percent to 20.49 bps from 44.46 bps before coskewness is controlled. On the other hand, CER exhibits relatively stronger explanatory power on the coskewness premium because the average cross-CER high-minus-low return spread for coskewness is reduced to −6.18 bps from the baseline number −34.72 bps in Panel B, that is, an 82-percent drop in magnitude. CER's influence on coskewness is equally substantial in the cross-sectional regressions. Bivariate Fama–MacBeth regressions reveal that the loading on coskewness is reduced by 84 percent in magnitude (from −0.1222 to −0.0190) after CER is controlled, and controlling for coskewness decreases the CER coefficient by only 19 percent (from 0.3407 to 0.2770).[13]

A final observation, although not reported in the table, is that CER loadings in the multivariate regressions remain significant even after further controls, including beta, size, B/M, momentum, and liquidity beta, are added in addition to downside beta,

[12] Some studies like Harvey and Siddique (2000) and Ang et al. (2006a) include the standard deviation of stock returns in a modified denominator to scale the numerator when estimating coskewness or cokurtosis, which makes the denominator uncommon to different stocks. We do not adopt this modification in our analyses because it is inconsistent with the CER measurement which has a common denominator. Nevertheless, using the modified coskewness and cokurtosis measures does not qualitatively change our results.

[13] Considering that the strong correlations between CER and comoment measures may not adequately disentangle the asset pricing inference of CER from comoments (and vice versa) in the simple two-way sorting scheme, when we check the mutual influences between CER and each of the comoment measures, we also use CER values orthogonal to the comoment (downside beta, coskewness, or cokurtosis) to form quintiles within each comoment portfolio, and use comoment values orthogonal to CER to form quintiles within each CER portfolio. We obtain similar results under this orthogonalization scheme. Specifically, the high-minus-low return spreads of CER quintiles after controlling for downside beta, coskewness, and cokurtosis are 31.12 bps (Newey–West t-value = 4.35), 14.66 bps (Newey–West t-value = 2.01), and 30.47 bps (Newey–West t-value = 5.16), respectively, while the high-minus-low return spreads of downside beta, coskewness, and cokurtosis quintiles after controlling for CER are −10.56 bps (Newey–West t-value = −1.93), −3.19 bps (Newey–West t-value = −0.50), and −12.07 bps (Newey–West t-value = −1.97), respectively.

Table 7
Mutual influences between CER and downside beta, coskewness, or cokurtosis.

Panel A: Downside beta, coskewness, cokurtosis in CER-sorted portfolios											
Downside Beta in CER portfolios			Correlation with	Coskewness in CER portfolios			Correlation with	Cokurtosis in CER portfolios			Correlation with
1 Low	5 High	High - Low	CER	1 Low	5 High	High - Low	CER	1 Low	5 High	High - Low	CER
0.4922	1.9870	1.4947	0.6777	−0.2346	−1.2805	−1.0459	−0.3707	2.6598	8.1678	5.5081	0.4114
		[35.67]	p<0.0001			[−12.80]	p<0.0001			[15.60]	p<0.0001
Panel B: Excess returns in downside beta-, coskewness-, cokurtosis-sorted portfolios											
Downside beta portfolios			Fama–MacBeth	Coskewness portfolios			Fama–MacBeth	Cokurtosis portfolios			Fama–MacBeth
1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient
1.0081	1.3259	0.3178	0.1782	1.3581	1.0109	−0.3472	−0.1222	0.9491	1.2918	0.3426	0.0272
		[2.02]	[2.32]			[−1.94]	[−1.14]			[1.89]	[0.86]
Panel C: Excess returns in CER-sorted portfolios after controlling for downside beta, coskewness, cokurtosis											
Controlling for downside beta			Fama–MacBeth	Controlling for coskewness			Fama–MacBeth	Controlling for cokurtosis			Fama–MacBeth
1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient
1.0864	1.3975	0.3111	0.5346	1.1315	1.3363	0.2049	0.2770	1.0839	1.3857	0.3018	0.4134
		[4.39]	[2.65]			[2.83]	[2.06]			[4.96]	[3.32]
Panel D: Excess returns in downside beta-, coskewness-, cokurtosis-sorted portfolios after controlling for CER											
Downside beta portfolios			Fama–MacBeth	Coskewness portfolios			Fama–MacBeth	Cokurtosis portfolios			Fama–MacBeth
1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient
1.2664	1.2021	−0.0642	−0.1799	1.2464	1.1846	−0.0618	−0.0190	1.2217	1.1730	−0.0486	−0.0221
		[−1.16]	[−1.54]			[−0.96]	[−0.19]			[−0.79]	[−0.70]

In Panel A, full-period average values of downside beta, coskewness, and cokurtosis are reported for the top and bottom CER quintiles, along with the differences between them. Downside beta is the ratio of covariance between stock and market returns to the variance of market returns, conditional on the market returns being less than the sample mean. Coskewness is the covariance between stock returns and squared market returns scaled by the third power of the standard deviation of market returns, and cokurtosis is the covariance between stock returns and cubed market returns scaled by squared market return variance. All these estimates are based on the previous 60 months' monthly data. For the case of each measure, CER portfolios are formed each month with non-missing observations for that measure. The measure's Pearson correlation coefficient with CER is reported in the last column with the associated p-value. Panel B reports full-period time-series average monthly percentage excess returns for the top and bottom quintiles (and their difference) formed each month on downside beta, coskewness, or cokurtosis, as well as the univariate Fama–MacBeth coefficient of each of them. In Panel C, stocks are first sorted each month into five groups based on downside beta, coskewness, or cokurtosis, and within each group, further sorted into CER quintiles. Average monthly percentage excess returns across all downside beta, coskewness, or cokurtosis groups are computed and the full-period time-series averages of them are reported for the top and bottom CER quintiles, together with the high-minus-low spreads. Also reported is bivariate Fama–MacBeth regression coefficient of CER after downside beta, coskewness, or cokurtosis is controlled. Panel D reports corresponding two-way sorting analysis results based on first sorting on CER and then on downside beta, coskewness, or cokurtosis, and the Fama–MacBeth regression coefficient of downside beta, coskewness, or cokurtosis after CER is controlled. Newey–West t-statistics are reported in brackets.

coskewness, or cokurtosis.[14] This highlights the distinct risk information CER captures and the robustness in the pricing of such risk. Overall, Table 7 shows that comoments represented by downside beta, coskewness, and cokurtosis, although highly correlated with CER, cannot subsume the CER premium, while CER can explain a major part of the premia to the comoment measures. These findings help achieve a better understanding of the asset pricing effects of downside beta, coskewness, and cokurtosis that have drawn extensive attention in the literature.

5. Conditional extreme risk and tail dependence

An important attribute of CER is its ability to signify hedging potential against extreme market risk, which differentiates it from tail dependence measures that mainly capture co-crash risk. In this section, we focus on how the black swan hedging aspect of CER manifests when tail dependence is less effective in reflecting such property, and how important black swan hedging is for the pricing of CER.

Tail dependence is the building block for existing systematic tail risk studies and is defined as the probability of all components of a multi-dimensional process being extreme simultaneously. Within our two-dimensional setting of individual stock i's return R_i and the market return R_M, the idea of tail dependence can be summarized in the following expression:

$$TDC_{iM} = \Pr\left(R_i < d_{\alpha,i} | R_M < d_{\alpha,M}\right), \tag{10}$$

[14] CER coefficients are 0.1613 (Newey–West t-value = 2.32), 0.1297 (Newey–West t-value = 1.81), and 0.1756 (Newey–West t-value = 2.38) in the multivariate regressions controlling for downside beta, coskewness, and cokurtosis, respectively, after further controlling for the beta, size, B/M, momentum, and liquidity effects.

where TDC_{iM} refers to tail dependence coefficient between stock i and the market M, $d_{\alpha i}$ and $d_{\alpha M}$ represent sufficiently low threshold returns represented by a low α-quantile of the stock and market return distributions (e.g., α-probability VaRs), respectively.[15] Obviously, this measure indicates the probability of the stock return dropping below a low threshold given that the market also drops below a low threshold. In the prospect that only the market crashes but the individual stock does not, tail dependence is close to zero or cannot be reliably estimated due to the lack of co-crash observations, and it is precisely the time when CER more prominently reflects the expected hedge the stock can provide against the market black swan.

The literature largely adopts copula methods to estimate tail dependence.[16] To be comparable with the CER measurement which is based on the limiting laws of extreme values, we estimate tail dependence by the Gumbel copula incorporated in the bivariate EVT model, a specification belonging to the parametric extreme value copula family. Tail dependence constructed this way highlights extreme risks and ensures the asymptotic property consistent with that of CER.[17]

Specifically, we use the bivariate logistic extreme value distribution to estimate tail dependence, following Longin and Solnik (2001) and Poon et al. (2004). These studies show that the asymptotic version of tail dependence in Eq. (10) can be expressed as

$$TDC_{iM} = 2 - 2^{\hat{\gamma}_i}, \quad 0 \le \hat{\gamma}_i \le 1, \tag{11}$$

and $\hat{\gamma}_i$ is the maximum likelihood estimate of γ_i in the joint co-exceedance distribution

$$F_{NR}\left[F_{NR_i}\left(NR_i\right), F_{NR_M}\left(NR_M\right)\right] = \exp\left\{ -\left[\left(-\ln\left(F_{NR_i}\left(NR_i\right)\right)\right)^{\frac{1}{\gamma_i}} + \left(-\ln\left(F_{NR_M}\left(NR_M\right)\right)\right)^{\frac{1}{\gamma_i}}\right]^{\gamma_i}\right\}, \tag{12}$$

where $F_{NR_i}(NR_i)$ and $F_{NR_M}(NR_M)$ are marginal distributions with GPD tails of negated log returns for stock i and the market M, respectively. We adopt the Longin–Solnik approach and use negated monthly return data left-censored at a high threshold for both the stock and the market in the construction of the likelihood function. Consistent with the CER measurement, we use a 60-month rolling window, set the top 20th percentile as the threshold, and require at least five stock-market pair observations in the co-exceedance subset for valid tail dependence estimation.[18]

We first check the relation between tail dependence and CER in Panel A of Table 8. The upper part of Panel A suggests a positive association between *TDC* and *CER* in the subsample of stocks with valid tail dependence estimates. Yet this association deserves further scrutiny. The values of CER have a large range from 0.4407 in the bottom CER quintile to 2.1111 in the top CER quintile, i.e., top quintile CER value is almost five times of that in the bottom quintile. In addition, in the bottom quintile, average *CER* is much lower than one, meaning that these stocks exhibit strong black swan hedging potential: They drop much less than the market during market plummets. As a comparison, average *CER* in the top *TDC* quintile (1.3405) is only 14 percent higher than that in the bottom *TDC* quintile (1.1733), and more importantly, both top and bottom *TDC* quintiles maintain a relatively high level of *CER* (higher than one). The evidence implies that, among the restricted subsample with valid tail dependence estimates, tail dependence may not effectively indicate stocks' black swan hedging attribute. This is consistent with the estimation procedure of tail dependence where estimates can only be obtained for stocks with sufficient stock-market co-exceedance data. In this case, co-crash risk is more pertinent than black swan hedging potential. Thus, tail dependence's inefficacy of reflecting the black swan hedging property is determined by its construct, and stocks with higher degrees of black swan hedging ability are more likely to appear in the cases where tail dependence estimates cannot be obtained, as we demonstrate below.

Unlike tail dependence, CER estimation does not require co-crash data (the stock does not have to crash), thus we can always have CER estimates even when tail dependence estimates are not available. It turns out that there is a nontrivial portion (37 percent) of sample stocks without valid tail dependence estimates due to the lack of co-exceedance observations. As shown in the lower part of Panel A, where CER levels are reported for the subsample without tail dependence estimates, stocks have much lower CERs compared with those in the subsample with *TDC* estimates (the upper part of Panel A). This is especially true for the bottom *CER* quintile where the average CER level for stocks without valid *TDC* estimates is only 0.1219, more than 70 percent lower than the corresponding level of 0.4407 among stocks with *TDC* estimates. Therefore, stocks that are associated with more salient black swan hedging feature are among those without tail dependence estimates. Hence, by design, tail dependence cannot reflect a stock's potential ability to hedge the market crash as adequately as CER. This is a key difference between our CER measure and the tail dependence measure in existing studies.

The fact that CER reflects black swan hedging better than tail dependence is critical because we find that black swan hedging plays an important role in asset pricing, which not only has material contribution to the CER premium, but also helps explain why tail dependence measures generally fail to exhibit a significant association with expected stock returns, as shown in most of the relevant studies in the literature.[19] To clarify this, we report the portfolio and regression analyses results for the asset pricing effects

[15] We are grateful to the referee for suggesting the notation of TDC.

[16] A copula is a multivariate probability function with uniform marginal distributions. The joint distribution of a random vector of variables contains both the marginal behavior and the dependence structure, and the copula approach isolates the description of the dependence structure.

[17] The extreme value copula family involves generalized extreme value distributions for margins that arise as the limit distributions for componentwise maxima, and the copula function proposed by Gumbel (1961). DiTraglia and Gerlach (2013) adopt a similar approach. For more references about extreme value dependence and extreme value copula, please refer to Poon et al. (2004) and McNeil et al. (2005).

[18] We follow Frahm et al.'s (2005) suggestion and use extremal data exceeding a threshold to estimate tail dependence without referring to other observations not belonging to the extremal set, since our tail dependence estimation method is of an extreme value copula type.

[19] Among the five tail dependence asset pricing studies (and the only five in the literature so far, to our best knowledge) by Kole and Verbeek (2006), Spitzer (2006), DiTraglia and Gerlach (2013), Chabi-Yo et al. (2015), and Van Oordt and Zhou (2016), only Chabi-Yo et al. (2015) document a significant premium to tail dependence measured by a weighted average copula carefully chosen from a total of 64 combinations of various copulas. Among other studies, Kole and Verbeek (2006) find no consistent evidence of a significantly positive tail dependence-expected return relation, while Spitzer's (2006), and DiTraglia and Gerlach's (2013), and Van Oordt and Zhou's (2016) results even show underperformances of portfolios with higher tail dependence-based measures.

Table 8
CER and tail dependence.

Panel A: CER levels of CER- and TDC-sorted portfolios					
CER portfolios			TDC portfolios		
1 Low	5 High	High - Low	1 Low	5 High	High - Low
Subsample with valid TDC estimates					
0.4407	2.1111	1.6703	1.1733	1.3405	0.1672
		[45.20]			[11.68]
Subsample without valid TDC estimates					
0.1219	1.7611	1.6392	N/A	N/A	N/A
		[43.51]			

Panel B: Excess returns in TDC-sorted portfolios and in CER-sorted portfolios before and after controlling for TDC											
CER portfolios			Fama–MacBeth	TDC portfolios			Fama–MacBeth	CER portfolios after controlling for TDC			Fama–MacBeth
1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient	1 Low	5 High	High - Low	Coefficient
Subsample with valid TDC estimates											
0.9377	1.3679	0.4302	0.3462	1.1499	1.0844	−0.0656	−0.0411	0.9145	1.3924	0.4780	0.3621
		2.27	[2.71]			[−1.05]	[−0.42]			[4.95]	[2.74]
Subsample without valid TDC estimates											
1.0629	1.9411	0.8783	0.6882	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
		3.37	[3.23]								

On the left side of Panel A, average values of CER are reported for the top and bottom CER quintiles, along with their differences, for the two subsamples with and without valid TDC estimates. TDC is proxied by the probability that both stock and market returns are approaching extreme, following an extreme value copula method in Longin and Solnik (2001). TDC is estimated based on the previous 60 months' monthly data and requires at least five stock-market pair observations in a co-exceedance sample beyond the top 20th percentiles of both the negated market and stock returns. The right side of Panel A reports average CER values for the top and bottom TDC quintiles (and their difference) within the subsample with valid TDC estimates. The left part of Panel B reports average monthly percentage excess returns for the top and bottom CER quintiles (and the differences between them), as well as the univariate Fama–MacBeth coefficients of CER, for the two subsamples with and without valid TDC estimates. The middle part of Panel B reports average monthly percentage excess returns for the top and bottom TDC quintiles (and their difference), along with the univariate Fama–MacBeth coefficient of TDC, within the subsample with valid TDC estimates. In the right part of Panel B, stocks with valid TDC estimates are first sorted each month by TDC into five groups, and within each group, further sorted into CER quintiles. Average monthly percentage excess returns across all TDC groups are computed and the time-series averages of them are reported for the top and bottom CER quintiles, together with the high-minus-low spread. Also reported is bivariate Fama–MacBeth regression coefficient of CER after TDC is controlled. Newey–West t-statistics are reported in brackets.

of tail dependence (if available) and CER (for both subsamples with and without *TDC* estimates) in Panel B. We first confirm that the variation of excess returns across tail dependence portfolios is rather modest, as the return difference between the top and bottom *TDC* quintiles is only −6.56 bps (Newey–West t-statistic = −1.05). This result is echoed by the univariate Fama–MacBeth regression coefficient of −0.0411 (t-statistic = −0.42) for *TDC*. The insignificant and seemingly negative association between *TDC* and stock returns is, again, consistent with the majority of the empirical findings about the tail dependence-return relation. In contrast, the return difference between the top and bottom CER quintiles and the univariate Fama–MacBeth regression loading on CER are both positive and statistically significant in the subsample with tail dependence estimates, consistent with the full sample results. This asset pricing effect of CER persists after controlling for tail dependence in the two-way sorting portfolio analysis and in the bivariate Fama–MacBeth regression.[20]

More importantly, by comparing across the upper and lower parts of Panel B, we find that the CER premium is much larger and more significant in the without-*TDC* subsample (high-minus-low return spread = 87.84 bps, Newey–West t-statistic = 3.37) than in the with-*TDC* subsample (high-minus-low return spread = 43.02 bps, Newey–West t-statistic = 2.27). This implies that the CER pricing effect manifests more strongly among stocks with lower CERs and greater black swan hedging potential. Consistently, in Fama–MacBeth regressions, CER exhibits an average coefficient of 0.6882 (Newey–West t-statistic = 3.23) in the without-*TDC* subsample, as compared with the coefficient of 0.3462 (Newey–West t-statistic = 2.71) in the with-*TDC* subsample. Therefore, the impact of CER on expected stock returns in the subsample without tail dependence estimates is twice as large as in the subsample with tail dependence estimates. As stocks with low CERs and thus clear black swan hedging feature mostly appear in the without-*TDC* subsample, these results imply that much of the CER premium is likely to be attributed to a more prominent presence of black swan hedging characteristics. Since tail dependence does not adequately capture the black swan hedging potential, this observation also partially explains why the risk premium to tail dependence-based measures appears absent.[21]

[20] Fama–MacBeth coefficient of CER keeps significantly positive even after beta, size, B/M, momentum, and liquidity beta are further controlled together with tail dependence: The coefficient is 0.1146, and the associated Newey–West t-statistic is 1.81.

[21] A recent study by Van Oordt and Zhou (2016) augment tail dependence by marginal distribution tails to generate a systematic tail risk measure. Although the marginal distributions deliver extra information beyond tail dependence, they only focus on extreme tails that are less informative for the black swan hedging

6. Robustness

6.1. Rarity of extreme market plummets

In the CER measurement, both the marginal tail and the Heffernan–Tawn dependence structure are based on asymptotic arguments. The GPD model enables extrapolation beyond data in the sample, which facilitates the estimation of the proxy for a very rare event but, at the same time, implies that the estimate is only an approximation. It is therefore important to test the sensitivities of our results to the extreme event approximation. We mainly use the expected return corresponding to the 1%-VaR as a measure for the severity of potential rare market drops. As a robustness check, we increase or decrease the rarity of extreme market drops by changing the probability level of VaR within the range of 0.5% to 5%. A lower-probability VaR indicates a rarer event and more extremeness, and a higher-probability VaR implies the opposite. Panel A of Table 9 reports selected summary statistics of CER measure and its components ($FirmRet|MktVaR$ and *MktVaR*) based on market VaRs of 0.5%, 1%, 2%, and 5%. Both market VaR and the corresponding conditional stock loss become more severe when the extreme market crash becomes rarer, as their average values are more negative from the 5%-VaR case to the 0.5%-VaR case. In contrast, CER level appears lower in more extreme market calamities, which may be due to stocks with stronger black swan hedging ability against larger crashes. This conjecture is supported by the fact that, while the maximum value of CER slightly (and non-monotonically) decreases from the 5%-VaR to the 0.5%-VaR cases, the minimum CER, which is negative, exhibits a large increase in magnitude, i.e., becomes more negative.[22] CERs based on different market VaRs are highly correlated with each other, with Pearson correlations no less than 0.7433 and Spearman correlations no less than 0.7668, as shown in the lower-left and upper-right triangles in Panel B, respectively. CERs of the adjacent VaR levels all have correlation coefficients higher than 0.91. Thus different VaR schemes yield consistent inferences regarding the relative CER levels among individual stocks. In Panel C, we find a consistently significant and positive CER pricing effect in all schemes of different VaR probabilities. Moreover, there is an increasing trend in the high-minus-low return spread of CER from the 5%-VaR to the 0.5%-VaR scenarios. The univariate Fama–MacBeth regressions show a similar pattern except that the 0.5%-VaR case does not have the largest coefficient of CER. This evidence suggests that the asset pricing effect of CER holds under different extreme market down conditions with higher or lower rarities, and that investors care more about the impacts of extreme market events that are rarer and more devastating.

6.2. Threshold levels in GPD tail and Heffernan–Tawn dependence measurements

As in empirical estimations of any asymptotic model, the characterization of an observation as being extreme is subject to the trade-off between bias and variance. In the cases of the GPD tail and Heffernan–Tawn dependence measurements, for the selection of extreme right tail observations, low thresholds generate more sample observations, but are more likely to violate the asymptotic basis; High thresholds give more extreme but smaller sample sizes, which increases the estimation variance. To facilitate the CER estimation and reduce computational workload, we employ the same 20th percentile benchmark levels for both tail and dependence estimations in the main part of this paper. To ensure that the results are robust to different benchmarking schemes, we alternatively adopt the 15th and 25th percentiles as thresholds for estimating GPD tails or Heffernan–Tawn dependence, and require a minimum of eight and 12 observations for estimations under the two schemes, respectively. We use symmetric thresholds for the left and right tails, but do not force the benchmarks for GPD and dependence estimations to be the same. In this way, along with the 20th percentile threshold, we have a total of nine combinations of benchmarks. We estimate CER under each combination, and report the full-period mean CER values, as well as the average high-minus-low return spreads and univariate Fama–MacBeth regression coefficients of CER in Panels A, B, and C of Table 10, respectively.

The overall findings indicate that CER estimate and its effect on cross-sectional stock returns are generally stable. Average CER levels are between 1.1037 and 1.1439 in Panel A, and become slightly smaller as the percentile threshold for dependence estimation decreases, implying that using more extreme market loss observations entails lower CER estimates. This echoes the evidence in Panel A of Table 9 that CERs based on more extreme market VaRs tend to be lower. There is no obvious trend for the pricing effect of CER across different threshold schemes. The majority of average CER premia from the portfolio analysis in Panel B of Table 10 are between 40 and 49 bps per month. The average cross-sectional regression coefficients of CER in Panel C are mostly around 0.3400, with the highest at 0.3625 and the lowest at 0.3176. Thus, our main conclusion of a positive relation between CER and expected stock returns holds under different benchmarking schemes for the CER measurement.

7. Concluding remarks

Asset pricing theories based on the safety-first principle highlight the importance of riskiness in the worst states of nature. Losses amid extreme market crashes are especially painful, and the potential to hedge market black swans is highly desirable. We gauge such features by introducing a CER proxy that predicts a stock's performance upon rare market plummets. We document a

---

case where the non-extreme distribution information is more relevant. Therefore, Van Oordt and Zhou's measure cannot effectively capture black swan hedging feature either. We report detailed analyses results about the difference between CER and Van Oordt and Zhou's measure (as well as its components) in the online appendix.

[22] Note in the case with $MktVaR_{0.5\%}$ in Panel A, the minimum value of *FirmRet|MktVaR* is larger than that of *MktVaR*. This is due to the fact that we trim off the top and bottom 2.5% observations of *FirmRet|MktVaR* in the full sample, but do not trim off the extreme values of *MktVaR*.

Table 9
Effects of CER measures corresponding to different probability levels of market VaR.

Panel A: Mean and range values of CER measures conditioning on different market VaRs

		Based on $MktVaR_{5\%}$	Based on $MktVaR_{2\%}$	Based on $MktVaR_{1\%}$	Based on $MktVaR_{0.5\%}$
Mean					
CER		1.2743	1.1753	1.1274	1.0939
FirmRet|MktVaR		−0.0888	−0.1120	−0.1282	−0.1433
MktVaR		−0.0775	−0.1019	−0.1198	−0.1384
Range (max & min)					
CER	Max	3.5796	3.1880	3.0922	3.0955
	Min	−0.2288	−0.2906	−0.3606	−0.4383
FirmRet|MktVaR	Max	0.0136	0.0213	0.0303	0.0422
	Min	−0.2698	−0.3270	−0.3695	−0.4156
MktVaR	Max	−0.0320	−0.0403	−0.0434	−0.0449
	Min	−0.1939	−0.2373	−0.3163	−0.6274

Panel B: Correlations of CER measures based on different market VaRs

	$CER_{5\%,i}$	$CER_{2\%,i}$	$CER_{1\%,i}$	$CER_{0.5\%,i}$
$CER_{5\%,i}$	**1.0000**	**0.9197**	**0.8420**	**0.7668**
$CER_{2\%,i}$	**0.9125**	**1.0000**	**0.9777**	**0.9321**
$CER_{1\%,i}$	**0.8260**	**0.9740**	**1.0000**	**0.9848**
$CER_{0.5\%,i}$	**0.7433**	**0.9214**	**0.9820**	**1.0000**

Panel C: Excess returns in portfolios of CER conditioning on different market VaRs

Excess return based on	CER portfolios			Fama–MacBeth
	1 Low	5 High	High - Low	Coefficient of CER
$CER_{5\%,i}$	0.9373	1.3728	0.4355	0.2835
			[2.18]	[2.46]
$CER_{2\%,i}$	0.9654	1.4022	0.4367	0.3318
			[2.33]	[2.81]
$CER_{1\%,i}$	0.9567	1.4013	0.4446	0.3407
			[2.47]	[2.92]
$CER_{0.5\%,i}$	0.9667	1.4191	0.4525	0.3291
			[2.61]	[2.96]

Market VaRs (*MktVaR*) associated with different probability levels of 0.5%, 1%, 2%, and 5% are predicted, and individual stocks' conditional expected returns (*FirmRet*\|*MktVaR*) are estimated. Mean, maximum, and minimum values of *MktVaR*, *FirmRet*\|*MktVaR*, as well as the corresponding *CER* are reported in Panel A under each probability scheme of market VaR, after the top and bottom 2.5% observations of *CER* and *FirmRet*\|*MktVaR* in their full samples are trimmed off. Panel B presents pooled sample Pearson and Spearman correlation coefficients in the lower-left and upper-right triangles, respectively, among CER measures corresponding to different probability levels of market VaR, with bold numbers indicating statistical significance at the 1% level. Panel C reports full-period time-series average percentage excess returns of the top and bottom quintile portfolios formed each month on CER under each market VaR scheme, the return differences between them, as well as the CER coefficients in univariate Fama–MacBeth regressions with individual stock percentage excess returns as the dependent variable. Newey–West t-statistics are reported in brackets.

significantly positive relation between CER and expected stock returns in the cross-section, which is more pronounced when the market calamities are expected to be more severe and thus hedging the extreme market risk is more valuable. We also find that CER is a driving factor for the cross-sectional effects of downside beta and cokurtosis, and it largely explains the premium to coskewness. CER exhibits a stronger asset pricing effect among stocks with greater black swan hedging potential, an attribute that cannot be effectively captured by co-crash-based tail dependence measures.

Our study sheds light on a few important considerations in respect to extreme risk. First, we provide empirical evidence supporting investors' safety-first concern in asset pricing, which has not been done before. Given the significant role of the safety-first theory in finance, our research, especially the novel methodology of CER measurement, contributes a relevant tool to future investigations in this field.[23] Second, with the juxtaposition of the already-documented pricing effect of systematic downside risk (downside beta) and the general failure to record a premium to systematic tail risk represented by tail dependence, there are arguments that investors may be concerned with the general downside risk of their investment rather than only the potential extreme

[23] Roy's (1952) article about safety-first rule appeared in the same year (with only a three-month lag) as Markowitz's (1952) groundbreaking paper on portfolio selection. Bernstein (1992, p. 212) notes that "(Roy's article) followed an almost identical line of argument (as Markowitz's)" and "the resembling between the two articles was all the more remarkable because no one had ever before tried to develop a theory of portfolio selection". Sullivan (2011, title) calls Roy "the forgotten father of portfolio theory". Markowitz (1999, p.5) himself also acknowledges Roy's contribution by commenting: "On the basis of Markowitz (1952), I am often called the father of modern portfolio theory (MPT), but Roy (1952) can claim equal share of this honor".

Table 10
Different benchmarking schemes in CER measurement and its pricing effect.

Panel A: Mean CERs constructed by using different percentile thresholds			
GPD estimation percentile threshold	Market return percentile threshold for Heffernan–Tawn dependence estimation		
	15th	20th	25th
15th	1.1037	1.1268	1.1439
20th	1.1109	1.1274	1.1404
25th	1.1073	1.1237	1.1349
Panel B: Return spreads of cers constructed by using different percentile thresholds			
GPD estimation percentile threshold	Market return percentile threshold for Heffernan–Tawn dependence estimation		
	15th	20th	25th
15th	0.4499	0.4065	0.4047
	[2.63]	[2.40]	[2.32]
20th	0.4899	0.4446	0.4485
	[2.79]	[2.47]	[2.45]
25th	0.4622	0.4596	0.4472
	[2.60]	[2.50]	[2.38]
Panel C: Fama–MacBeth coefficients of cers constructed by using different percentile thresholds			
GPD estimation percentile threshold	Market return percentile threshold for Heffernan–Tawn dependence estimation		
	15th	20th	25th
15th	0.3191	0.3176	0.3215
	[3.02]	[2.91]	[2.83]
20th	0.3439	0.3407	0.3401
	[3.09]	[2.92]	[2.85]
25th	0.3440	0.3625	0.3479
	[3.01]	[3.00]	[2.83]

In the CER measurement, different threshold levels of the top 15th, 20th, and 25th percentiles are used for selecting extreme observations for the estimations of GPD right tail and Heffernan–Tawn dependence (the bottom 15th, 20th, and 25th percentiles are used as benchmarks in GPD left tail estimation). Mean values of CER estimates based on different combinations of benchmarks (after the top and bottom 2.5% observations of the full sample are trimmed off) are reported in Panel A. Panels B and C respectively present full-period time-series mean return spreads (in percentage) between the top and bottom quintile portfolios formed each month on CER and CER's coefficients from univariate Fama–MacBeth regressions with percentage excess returns as the dependent variable and CER as the independent variable, for CER estimates under different benchmarking schemes. Newey–West t-statistics are reported in brackets.

risk (Van Oordt and Zhou, 2016). Our study, however, depicts a different picture. Our findings suggest that investors are not only concerned with extreme risk, but such a concern is so strong that it drives a significant part of the premium to downside risk. The weak effect of tail dependence-based measure is not due to investors' indifference to extreme risk but rather to its inadequacy of capturing the black swan hedging potential that turns out to play a significant role in asset pricing. Third, based on the covariance between asset payoff and a non-linear pricing kernel as a function of market return, higher-order comoment measures of coskewness and cokurtosis act as part of a remedy for the inefficacy of the CAPM in capturing tail risk. These measures incorporate the effects of non-Gaussian distribution tails, but to what extent they reflect extreme risk has yet to be investigated. Our work also helps obtain further insights on this issue. Finally, there is recent evidence that the comoment (especially coskewness) effect is not confined to the single asset class of stocks. Yang et al. (2010) find that the desirable positive skewness of bonds provides a good hedge against the undesirable negative skewness of stocks, and the coskewnesses between stock return and bond volatility and between bond return and stock volatility both command a negative premium. Yang et al.'s (2010) work suggests that, in an extreme risk framework, bond market holds great potential to hedge the black swan in stock market, and we expect that our CER-based measure works better to capture black swan hedging in a cross asset class (i.e., cross stock and bond) setting, within which the relation between CER and Yang et al.'s (2010) coskewness measure is worth further investigation in future research.

CRediT authorship contribution statement

S. Ghon Rhee: Writing - review & editing, Investigation. **Feng (Harry) Wu:** Conceptualization, Data curation, Formal analysis, Methodology, Writing - original draft.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jempfin.2020.07.002.

References

Amihud, Y., 2002. Illiquidity and stock returns: Cross-section and time-series effects. J. Financial Mark. 5, 31–56.
Ang, A., Chen, J., Xing, Y., 2006a. Downside risk. Rev. Financ. Stud. 19, 1191–1239.

Ang, A., Hodrick, R., Zhang, X., 2006b. The cross-section of volatility and expected returns. J. Finance 61, 259–299.
Arzac, E., Bawa, V., 1977. Portfolio choice and equilibrium in capital markets with safety-first investors. J. Financ. Econ. 4, 277–288.
Baker, M., Bradley, B., Wurgler, J., 2011. Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. Financ. Anal. J. 67, 40–54.
Bali, T., 2003. An extreme value approach to estimating volatility and value at risk. J. Bus. 76, 83–108.
Barberis, N., Mukherjee, A., Wang, B., 2016. Prospect theory and stock returns: An empirical test. Rev. Financ. Stud. 29, 3068–3107.
Barone-Adesi, G., 1985. Arbitrage equilibrium with skewed asset returns. J. Financ. Quant. Anal. 20, 299–313.
Barro, R., 2006. Rare disasters and asset markets in the twentieth century. Q. J. Econ. 121, 823–866.
Bernstein, 1992. Capital Ideas: The Improbable Origins of Modern Wall Street. The Free Press, New York, NY.
Bollerslev, T., Todorov, V., 2011. Estimation of jump tails. Econometrica 79, 1727–1783.
Brownlees, C., Engle, R., 2017. SRISK: A conditional capital shortfall measure of systemic risk. Rev. Financ. Stud. 30, 48–79.
Campbell, J., Lo, A., MacKinlay, C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton, NJ.
Carhart, M., 1997. On persistence in mutual fund performance. J. Finance 52, 57–82.
Chabi-Yo, F., Ruenzi, S., Weigert, F., 2015. Crash sensitivity and the cross-section of expected stock returns. Working paper. University of St. Gallen.
Chang, B.Y., Christoffersen, P., Jacobs, K., 2013. Market skewness risk and the cross section of stock returns. J. Financ. Econ. 107, 46–68.
Chollete, L., Lu, C.C., 2011. The market premium for dynamic tail risk. Working paper. University of Stavanger.
Chung, P., Johnson, H., Schill, M., 2006. Asset pricing when returns are nonnormal: Fama–French factors versus higher-order systematic comoments. J. Bus. 79, 923–940.
Coles, S., 2001. An Introduction To Statistical Modeling of Extreme Values. Springer-Verlag, London.
Daneil, K., Grinblatt, M., Wermers, R., 1997. Measuring mutual fund performance with characteristic-based benchmarks. J. Finance 52, 1035–1058.
DiTraglia, F., Gerlach, J., 2013. Portfolio selection: An extreme value approach. J. Bank. Financ. 37, 305–323.
Embrechts, P., Kluppelberg, C., Mikosch, T., 1997. Modelling Extremal Events for Insurance and Finance. Springer-Verlag, Berlin, Heidelberg, New York.
Eraker, B., Jahannes, M., Polson, N., 2003. The impact of jumps in volatility and returns. J. Finance 58, 1269–1300.
Fama, E., French, K., 1992. The cross-section of expected stock returns. J. Finance 47, 427–465.
Fama, E., French, K., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33, 3–56.
Fama, E., MacBeth, J., 1973. Risk, return, and equilibrium: Empirical tests. J. Polit. Econ. 81, 607–636.
Fang, H., Lai, T.Y., 1997. Co-kurtosis and capital asset pricing. Financ. Rev. 32, 293–307.
Frahm, G., Junker, M., Schmidt, R., 2005. Estimating the tail-dependence coefficient: Properties and pitfalls. Insurance Math. Econom. 37, 80–100.
Frazzini, A., Pedersen, L.H., 2014. Betting against beta. J. Financ. Econ. 111, 1–25.
Friend, I., Westerfield, R., 1980. Co-skewness and capital asset pricing. J. Finance 35, 897–913.
Gumbel, E., 1961. Bivariate logistic distributions. J. Amer. Statist. Assoc. 56, 335–349.
Harvey, C., Siddique, A., 2000. Conditional skewness in asset pricing tests. J. Finance 55, 1263–1295.
Heffernan, J., Tawn, J., 2004. A conditional approach for multivariate extreme values. J. R. Stat. Soc. Ser. B Stat. Methodol. 66, 479–546.
Hilal, S., Poon, S.H., Tawn, J., 2011. Hedging the black swan: Conditional heteroskedasticity and tail dependence in S & P500 and VIX. J. Bank. Financ. 35, 2374–2387.
Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. J. Finance 48, 65–91.
Jondeau, E., Poon, S.H., Rockinger, M., 2007. Financial Modeling under Non-Gaussian Distributions. Springer-Verlag, London.
Jurczenko, E., Maillet, B., 2006. The four-moment capital asset pricing model: Between asset pricing and asset allocation. In: Jurczenko, E., Maillet, B. (Eds.), Multi-Moment Asset Allocation and Pricing Models. John Wiley & Sons, Chichester.
Karceski, J., 2002. Returns-chasing behavior, mutual funds, and beta's death. J. Financ. Quant. Anal. 37, 559–594.
Kelly, B., Jiang, H., 2014. Tail risk and asset prices. Rev. Financ. Stud. 27, 2841–2871.
Kole, E., Verbeek, M., 2006. Crash risk in the cross section of stock returns. Working paper. Erasmus University Rotterdam.
Kraus, A., Litzenberger, R., 1976. Skewness preference and the valuation of risk assets. J. Finance 31, 1085–1100.
Longin, F., Solnik, B., 2001. Extreme correlation of international equity markets. J. Finance 56, 649–676.
Markowitz, H., 1952. Portfolio selection. J. Finance 7, 77–91.
Markowitz, H., 1999. The early history of portfolio theory: 1600–1960. Financ. Anal. J. 55, 6–16.
Matthews, R., 1996. Far out forecasting. New Sci. 2051 (October 12), 36–40.
McNeil, A., Frey, R., 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. J. Empir. Financ. 7, 271–300.
McNeil, A., Frey, R., Embrechts, P., 2005. Quantitative Risk Management: Concepts, Techniques and Tools. Princeton University Press, Princeton, Oxford.
Menezes, C., Geiss, C., Tressler, J., 1980. Increasing downside risk. Amer. Econ. Rev. 70, 921–932.
Newey, W., West, K., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55, 703–708.
Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected stock returns. J. Polit. Econ. 111, 642–685.
Poon, S.H., Rockinger, M., Tawn, J., 2004. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. Rev. Financ. Stud. 17, 581–610.
Rietz, T., 1988. The equity risk premium: A solution. J. Monetary Econ. 22, 117–131.
Roy, A., 1952. Safety first and the holding of assets. Econometrica 20, 431–449.
Spitzer, J., 2006. Market crashes, market booms and the cross-section of expected returns. Working paper. New York University.
Sullivan, E., 2011. A.D. Roy: The forgotten father of Portfolio Theory. In: Biddle, J., Emmett, R. (Eds.), Research in the History of Economic Thought and Methodology. In: Research in the History of Economic Thought and Methodology, vol. 29, Emerald Group Publishing Limited, Bingley.
Telser, L., 1955. Safety first and hedging. Rev. Econom. Stud. 23, 1–16.
Van Oordt, M., Zhou, C., 2016. Systematic tail risk. J. Financ. Quant. Anal. 51, 685–705.
Wu, L., 2006. Dampened power law: Reconciling the tail behavior of financial security returns. J. Bus. 79, 1445–1473.
Yang, J., Zhou, Y., Wang, Z., 2010. Conditional coskewness in stock and bond markets: Time-series evidence. Manage. Sci. 56, 2031–2049.