Economic Modelling 80 (2019) 260–274

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.journals.elsevier.com/economic-modelling

Combining the minimum-variance and equally-weighted portfolios: Can portfolio performance be improved?

Chonghui Jiang [a], Jiangze Du [a,*], Yunbi An [b]

[a] School of Finance, Jiangxi University of Finance and Economics, Nanchang, 330013, China
[b] Odette School of Business, University of Windsor, Windsor, Ontario, N9B 3P4, Canada

ARTICLE INFO

JEL classification:
G11
G15

Keywords:
Portfolio performance
Estimation errors
Minimum-variance portfolio
Equally-weighted portfolio

ABSTRACT

It is documented in the literature that due to estimation errors, mean-variance efficient portfolios deliver no higher out-of-sample Sharpe ratios than does the naïve equally-weighted portfolio (EWP). This paper demonstrates how the out-of-sample performance of the minimum-variance portfolio (MVP) can be improved in the presence of estimation errors by combining the MVP and EWP. Our results indicate that an appropriate combination of the MVP and EWP can enhance Sharpe ratios under any scenarios considered, and can also reduce the portfolio risk if short-selling is allowed. However, the combination strategy is not able to generate a lower risk level than the MVP when a short-selling restriction is imposed. We find that the optimal combination coefficient depends on the factors that greatly impact estimation errors in the MVP, including sample size, estimation method, no-short-selling restriction, and length of the out-of-sample period under consideration.

1. Introduction

Since Markowitz's (1952) portfolio selection theory was proposed, mean-variance analysis has become an important portfolio management approach in both academics and practice. However, practical implementation of the mean-variance approach requires accurate estimation of means, variances, and covariances of individual asset returns. The estimated mean-variance efficient portfolio can deviate substantially from the true efficient portfolio due to estimation errors in input parameters, and the effect of estimation errors is particularly pronounced for highly frequently-rebalanced dynamic trading strategies (Zhang et al., 2017). It is well documented in the literature that estimated mean-variance efficient portfolios deliver poor out-of-sample performance (Broadie, 1993; Chopra and Ziemba, 1993; Jorion, 1985; Kan and Smith, 2008; Klein and Bawa, 1976; Michaud, 1989), and that these portfolios perform even worse than the naïve 1/N portfolio (DeMiguel et al., 2009a) or equally-weighted portfolio (EWP).

The standard approach to portfolio selection ignores estimation risk by simply plugging sample means and covariances into the mean-variance model and solving for portfolio weights. The Bayesian method, on the other hand, accounts for estimation uncertainty in the portfolio selection problem, and thereby can improve out-of-sample performance relative to the standard approach. Under the Bayesian approach, the unknown parameters are assumed to follow a prior distribution, and then the predictive distribution of asset returns is recovered. The Bayesian optimal portfolio weights are obtained by maximizing utility with respect to the predictive distribution. Early applications of this approach are based on uninformative diffuse-priors (Barry, 1974) or Bayes-Stein shrinkage priors (Jobson and Korkie, 1980; Jorion, 1986, 1991), while recent studies rely on asset pricing models to form the prior belief (Pastor, 2000; Pastor and Stambaugh, 2000).

While Bayesian portfolio analysis can effectively reduce estimation risk, it still requires estimates of both the means and covariance matrix of asset returns. It is known that means are more difficult to estimate than the covariance matrix, and errors in the mean estimates impact portfolio weights more significantly than do the errors in the covariance matrix (Merton, 1980; Best and Grauer, 1992; Black and Litterman, 1992). For this reason, many studies are devoted to the minimum-variance portfolio (MVP), as it relies solely on estimates of the covariance matrix. However, while the MVP has a relatively low risk, there is empirical evidence that it usually delivers poor out-of-sample returns (Fletcher, 2009).

In the EWP (also called the 1/N portfolio) each of the N assets is assigned an equal weight, 1/N. Construction of the EWP does not require estimation of any unknown variables, nor is the optimization technique needed to obtain portfolio weights. Thus, the EWP is widely used by pension funds to allocate assets in practice (Benartzi and Thaler, 2001),

* Corresponding author.
E-mail addresses: jiangchonghui@jxufe.edu.cn (C. Jiang), jiangzedu@jxufe.edu.cn (J. Du), yunbi@uwindsor.ca (Y. An).

https://doi.org/10.1016/j.econmod.2018.11.012
Received 23 October 2017; Received in revised form 27 August 2018; Accepted 15 November 2018
Available online 19 November 2018
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and usually serves as a benchmark portfolio with which other portfolio investment strategies are compared in the literature (Duchin and Levy, 2009; DeMiguel et al., 2009b; Fletcher, 2009; Jiang et al., 2013). Many previous empirical studies demonstrate that the EWP yields a higher out-of-sample Sharpe ratio than the optimal portfolio implied by portfolio selection models, but is more risky than the MVP in terms of out-of-sample variance (Disatnik and Benninga, 2007; Clarke et al., 2006; Fletcher, 2009; Duchin and Levy, 2009).

To reduce the impacts of estimation errors and improve the out-of-sample portfolio performance, we propose a strategy that combines the MVP with the EWP. There are mainly three reasons why such a combination strategy can help improve performance. First, the construction of the MVP relies only on the estimation of the covariance matrix of asset returns, while any other mean-variance efficient portfolios rely not only on the estimation of the means of asset returns but also on the estimation of the covariance matrix. Thus, combining the MVP instead of any other mean-variance efficient portfolios with the EWP can effectively avoid the estimation of the means of asset returns, thereby reducing the impact of estimation errors on the out-of-sample performance of the portfolio strategy. Second, in the combination, the construction of MVP requires only estimates of the covariance matrix, while the construction of the EWP does not require estimates of either returns or the covariance matrix. Thus, the combination strategy is less affected by estimation errors than the single MVP strategy as long as the combination coefficientϖ fall in the interval of (0,1).[1] Finally, in terms of out-of-sample returns, risk and Sharpe ratio, the combination strategy is likely to improve its performance relative to a single MVP strategy. For any given combination coefficient ϖ in (0,1), the out-of-sample returns of the combination strategy is the weighted average of the out-of-sample returns of the MVP and EWP, and thus can be higher than the returns of the MVP. Due to the imperfect correlation between the returns of these two portfolios, the risk of the combination strategy can be lower than the risk of the EWP. If the correlation coefficient between the two portfolios is small, the risk of the combination portfolio may be lower than the risk of any single portfolio. Consequently, combining both the MVP and EWP can lead to an increase in the Sharpe ratio compared with the single EWP or MVP strategy.

Given that the MVP depends only on the estimate of the covariance matrix, and that shrinkage estimators of the covariance matrix usually perform relatively well (Disatnik and Benninga, 2007; Fletcher, 2009), we consider the cases in which MVPs are estimated based on the sample covariance matrix, the shrinkage estimator of Ledoit and Wolf (2003), the shrinkage estimator of Ledoit and Wolf (2004), and the simple average of the above three estimators. In addition to the method used to estimate covariance matrix, the sample size used for such estimation also impacts estimation errors. Thus, we estimate the covariance matrix using the most recent 60 (5 years), 120 (10 years), and 180 (15 years) monthly returns. Moreover, given that incorporating portfolio weight constraints into the portfolio optimization process trades off the reduction of sampling errors and the loss of sample information (Behr et al., 2013; Jagannathan and Ma, 2003), we construct the MVP with and without no-short-selling constraints.

In this study, we first examine the out-of-sample performance of the combination strategy with the combination coefficient $\varpi \in [0, 1]$ under various scenarios, and see whether the combination strategy can outperform the single EWP or MVP strategy in terms of risk reduction and Sharpe ratio improvement. Second, to detect the greatest possible improvement in the performance of the combined portfolio, we solve for the optimal combination coefficient such that the out-of-sample Sharpe ratio/variance of the portfolio is maximized/minimized. Fig. 1 provides a flowchart to demonstrate our research. From the figure, we can see that the combination of the MVP and EWP can take advantage of both the EWP's high return and MVP's low estimation errors and low risk, while minimizing the disadvantages of the EWP's high risk and MVP's low returns, thereby achieving better portfolio performance. The combination coefficient reflects the extent to which investors trust the estimated MVP since 1-ϖ represents the weight allocated to the MVP in the combination strategy. The smaller the estimation errors, the more the MVP should be trusted, and the higher the proportion of funds should be assigned to the MVP. Therefore, all factors that affect estimation errors should have an impact on the combination coefficient. These factors, among others, include the sample size and method used to estimate the covariance matrix, as well as the presence of the no-short-selling restriction in the optimization problem.

The contributions of this study are in the following respects. First, we propose to combine both the MVP and the EWP to achieve better out-of-sample performance than holding either of the MVP or EWP alone. The idea of combining portfolio strategies can be seen in many recent studies (Brandt et al., 2009; DeMiguel et al., 2009a; Kan and Zhou, 2007). Our study is particularly motivated by Tu and Zhou (2011), who evaluate the combination of the EWP with various Markowitz-type rules. However, their study focuses on the optimal combination obtained by minimizing the expected loss function. In contrast, we empirically assess the out-of-sample performance of the combined portfolio relative to the single MVP or EWP under various circumstances. Further, Tu and Zhou (2011) consider combinations of the EWP and tangency portfolio implied by sophisticated strategies, while our combination includes the EWP and MVP. As is known, the tangency portfolio is affected by estimation errors in both the means and covariance matrix, whereas the MVP is affected only by the estimation of the covariance matrix. Our analysis does not depend on estimates of asset returns.

Second, we empirically obtain the optimal combination coefficient for the combination strategy and further examine the optimal strategy's out-of-sample performance. To empirically obtain the optimal combination coefficient for the combination strategy, we examine the factors that impact estimation errors in the MVP, and establish a relationship between the optimal combination coefficient and those factors, including sample size, estimation method, no-short-selling restriction, and among others. In Tu and Zhou (2011), the optimal combination coefficient is obtained by minimizing the expected loss function. Applying the optimization technique introduces additional estimation errors and does not allow for an analysis of the factors that determine the optimal combination. Our method can avoid the estimation of means of asset returns, and can even consider the scenarios where short-selling is not allowed, which is more practically relevant in portfolio management.

Our results indicate that combining the MVP and EWP can enhance the Sharpe ratio, and also reduce risk if short-selling is allowed. However, the combination strategy is not able to generate a lower risk level than the MVP when the no-short-selling restriction is imposed. We find that the need for the MVP to combine EWP decreases as the sample size increases, the shrinkage estimation method is adopted, or the no-short-selling restriction is imposed, while the need increases with length of the out-of-sample period under consideration.

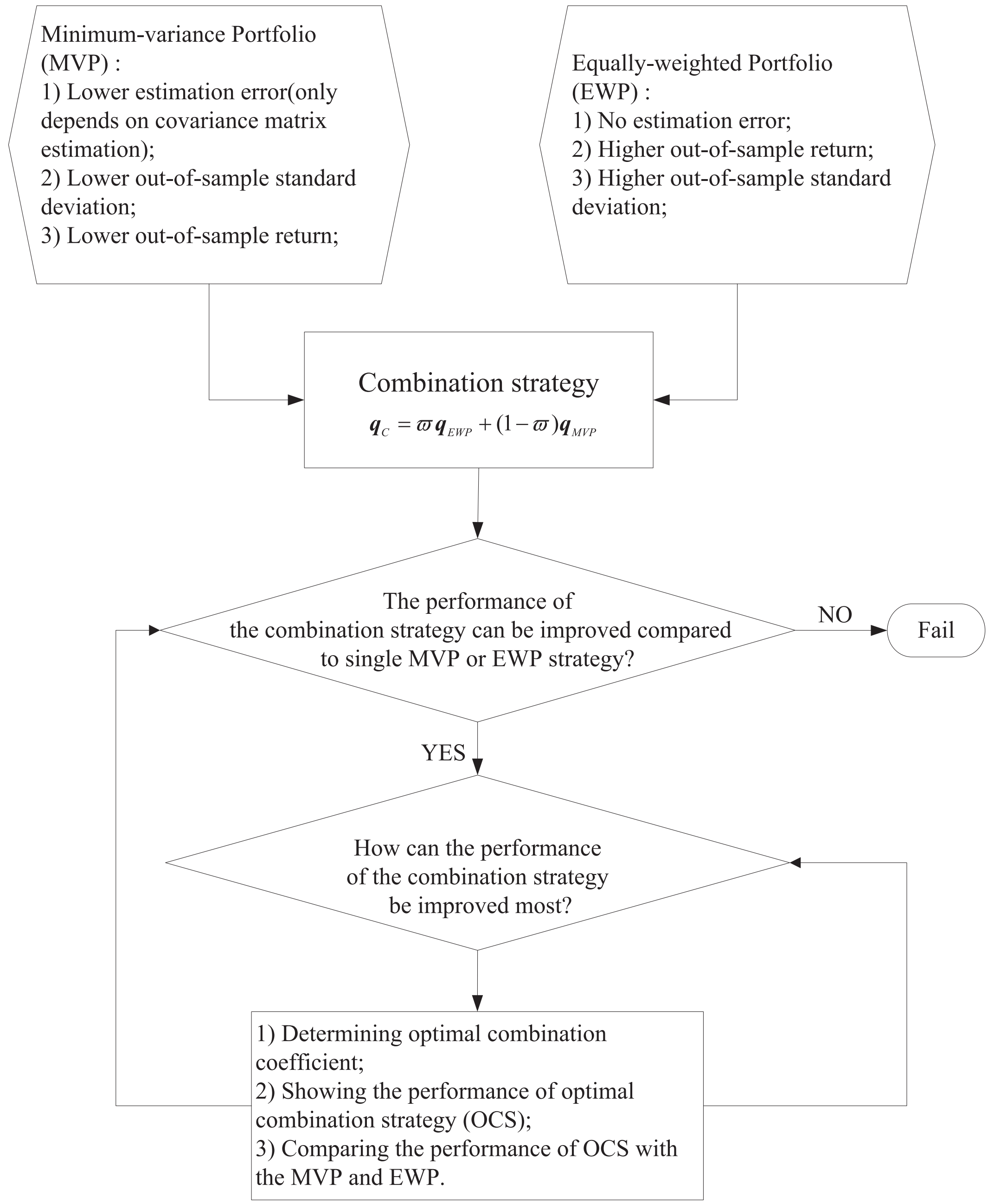
The remainder of this paper is organized as follows. Section 2 describes the research methodology. Section 3 presents the data used in the analysis. Section 4 provides an empirical analysis. Section 5 examines the robustness of the findings, while Section 6 concludes the paper.

2. Research methodology

2.1. Minimum-variance portfolio and equally-weighted portfolio

We assume that there are N different risky assets available in the market with a column return vector $\mathbf{r}$. A portfolio is a vector $\boldsymbol{q} = (q_1, q_2, \ldots, q_N)^T$, where q_i is the proportion of the portfolio invested in asset i and the superscript T represents the transpose operation. Thus, the portfolio return is $r_p = \mathbf{q}^T\mathbf{r}$. The minimum-variance portfolio (MVP) is the portfolio with the lowest possible variance:

[1] The combination coefficient is the weight assigned to the EWP in the combination strategy, which is defined in Equation (7).



The flowchart shows the research motivation and two main research questions of this study. Considering the advantages and disadvantages of the EWP and MVP, we propose a combination strategy based on certain complementarities between advantages and disadvantages of the two portfolios. We mainly examine the following two questions: (1) whether combining EWP and MVP can improve portfolio performance compared with either single strategy? (2) How to determine the best way to combine these two strategies to improve portfolio performance?

Fig. 1. Motivations and main questions examined in this work.

$$\min \ \mathbf{q}^T\mathbf{V}\mathbf{q} \tag{1}$$

$$\text{s.t.} \ \ \mathbf{q}^T\mathbf{1} = 1$$

where $\mathbf{V}$ stands for the covariance matrix of risky asset returns, and is non-singular. $\mathbf{1}$ is an N-column vector with all elements equal to one. If short-selling is not allowed, we need to add one additional constraint $\mathbf{q} \geq \mathbf{0}$ to the optimization problem, where $\mathbf{0}$ is an N-column vector with all elements equal to zero. In the following empirical analysis, we consider the combination of the EWP and MVP, where the MVP is constructed with and without no-short-selling constraint.

The solution to Equation (1) or the MVP is given by:

$$\mathbf{q}_{\text{MVP}} = \frac{\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}}. \tag{2}$$

Apparently, construction of the MVP requires an accurate estimate of

$\mathbf{V}$, regardless of whether or not the no-short-selling constraint is imposed.

The equally-weighted portfolio (EWP) is the portfolio with the same weight in each asset, or $q_i = 1/N$ $(i = 1, 2\cdots, N)$. Unlike the MVP, this portfolio does not require estimation of any unknown variables and does not rely on any optimization techniques.

2.2. The covariance matrix estimation

Estimation of the covariance matrix is most difficult in the portfolio selection problem, especially when the number of assets is large relative to the number of return observations. Traditionally, we calculate the sample covariance matrix $\widehat{\mathbf{V}}_{\text{samp}}$ as follows:

$$\widehat{\mathbf{V}}_{\text{samp}} = \frac{1}{N-1}\sum_{i=1}^{N}(\mathbf{r}_i - \bar{\mathbf{r}})(\mathbf{r}_i - \bar{\mathbf{r}})^T, \tag{3}$$

where $\bar{\mathbf{r}}$ is the vector of the arithmetic average of observed returns $\mathbf{r}_i$ $(i = 1, 2, \cdots, N)$. The sample covariance matrix is easy to compute and is an unbiased estimator of the true covariance matrix. However, $\widehat{\mathbf{V}}_{\text{samp}}$ contains much estimation error, and efficient portfolios based on this estimator perform poorly (Jobson and Korkie, 1980). To improve the accuracy of covariance matrix estimation, researchers propose shrinking $\widehat{\mathbf{V}}_{\text{samp}}$ toward a lower-variance target, thereby reducing estimation errors.[2] It is documented that shrinkage estimators are better than those obtained using other methods (Disatnik and Benninga, 2007; Fletcher, 2009). For this reason, we also consider two shrinkage estimators and a combination of various estimators in addition to the sample covariance matrix.

The first shrinkage estimator considered is the one based on the single-factor model (Ledoit and Wolf, 2003). If asset returns are assumed to follow the single-factor model, then the estimated covariance matrix $\widehat{\mathbf{F}}_{\text{sing}}$ contains relatively little estimation error, given the relatively few parameters that need to be estimated in the model. However, the single-factor model can be misspecified, resulting in a very biased estimator $\widehat{\mathbf{F}}_{\text{sing}}$. Ledoit and Wolf (2003) propose shrinking $\widehat{\mathbf{V}}_{\text{samp}}$ toward $\widehat{\mathbf{F}}_{\text{sing}}$ to reduce the variance of the estimate:

$$\widehat{\mathbf{V}}_{\text{sing}} = \alpha\widehat{\mathbf{F}}_{\text{sing}} + (1-\alpha)\widehat{\mathbf{V}}_{\text{samp}}, \tag{4}$$

where $\alpha \in [0, 1]$ is called the shrinkage intensity, and $\widehat{\mathbf{F}}_{\text{sing}}$ the shrinkage target.

The second shrinkage estimator, denoted as $\widehat{\mathbf{V}}_{\text{corr}}$, is obtained by shrinking the sample covariance matrix toward the constant correlation matrix (Ledoit and Wolf, 2004):

$$\widehat{\mathbf{V}}_{\text{corr}} = \alpha\widehat{\mathbf{F}}_{\text{ccm}} + (1-\alpha)\widehat{\mathbf{V}}_{\text{samp}}, \tag{5}$$

In this model, the shrinkage target of the covariance matrix is $\widehat{\mathbf{F}}_{\text{ccm}}$, which is the estimated covariance matrix by assuming that the correlation coefficients of any two risky asset returns are equal. This equal correlation coefficient is called common constant correlation. Thus, the shrinkage target in this case can be computed using estimates of sample variances of all assets and the common constant correlation. α is the shrinkage intensity similar to that defined in Equation (4). In the following empirical analysis, we take the mean of all the correlation coefficients of any two risky asset returns as the common constant correlation.

The final estimator $\widehat{\mathbf{V}}_{\text{port}}$ considered is the simple average of the above three estimators, i.e.,

$$\widehat{\mathbf{V}}_{\text{port}} = \frac{1}{3}\left(\widehat{\mathbf{V}}_{\text{samp}} + \widehat{\mathbf{V}}_{\text{sing}} + \widehat{\mathbf{V}}_{\text{corr}}\right). \tag{6}$$

By averaging, this estimator can cancel out certain estimation errors in each component estimator resulting from different model assumptions, thereby further reducing estimation errors.

2.3. Combination of the MVP and EWP

Denote the EWP as $\mathbf{q}_{EWP}$, then the combination of $\mathbf{q}_{EWP}$ and $\mathbf{q}_{MVP}$ is written as:

$$\mathbf{q}_C = \varpi\,\mathbf{q}_{EWP} + (1-\varpi)\mathbf{q}_{MVP}, \tag{7}$$

where ϖ is the combination coefficient, and $0 \le \varpi \le 1$.[3] The optimal combination coefficient is obtained by maximizing the expected utility function in DeMiguel et al. (2009a) and Tu and Zhou (2011). In implementing the optimization technique, estimates of asset returns must be obtained. In addition, estimation of the optimal combination coefficient itself further introduces estimation errors.

In this paper, we first derive a theoretical optimal combination based on the investment objective. To a large extent, this method is similar to those used in DeMiguel et al. (2009a) and Tu and Zhou (2011), and therefore the optimal combination coefficient can be largely affected by estimation errors in asset returns and co-variances. To avoid estimation errors, we propose a factor approach to obtain an empirically optimal combination coefficient. To this end, we consider combinations with ϖ ranging from 0, 0.01, 0.02, …, 0.99, to 1, and then analyze their performance under various circumstances to gauge the major factors that impact the optimal combination coefficient. We will show that our method outperforms both the MVP and EWP in terms of out-of-sample performance.

2.4. Performance measures

The performance of the combined portfolio is assessed based on the out-of-sample standard deviation and Sharpe ratio. The Sharpe ratio is defined as the ratio of the return in excess of the risk-free rate to the standard deviation of the portfolio. More specifically, if the portfolio return is r_p and the risk-free rate is r_f, then the Sharpe ratio S_p is given by:

$$S_p = \frac{E\left(r_p - r_f\right)}{\sigma\left(r_p - r_f\right)}. \tag{8}$$

The Sharpe ratio measures the trade-off between risk and return, which reflects the mean-variance efficiency of the portfolio under consideration.

3. Data and empirical design

The data used for the numerical analysis include the value-weighted average monthly returns on 40 industry portfolios in the US from July

[2] The target is the estimated covariance matrix, assuming asset returns follow a particular model, such as the single-factor model (Sharpe, 1963), or the three-factor model (Fama and French, 1993).

[3] Note that the purpose of this paper is to improve the out-of-sample performance of the portfolio by reducing the impacts of estimation errors. If $\varpi < 0$ is allowed, this means that the proceeds of short-selling EWP are invested in the MVP, which will exacerbate the problem of estimation errors. If $\varpi > 1$ is allowed, this means that the short-selling of the MVP will finance investors to hold a large position in EWP. This strategy tends to be more risky than the EWP, and farther away from an asymptotically efficient portfolio. Thus, setting $\varpi \in [0, 1]$ reflects a compromise between the MVP and EWP. It is noteworthy that when short-selling is allowed in the MVP, we are not sure whether there are any short positions in the combination strategy even if $\varpi \in [0, 1]$ is set. However, when short-selling is not allowed in the MVP, there will be no short positions in the combination strategy if $\varpi \in [0, 1]$ is set.

1926 to December 2017 obtained from Kenneth R. French's personal website.[4] There are 1098 observations in each data series. The risk-free rate for the same time period is also downloaded from French's website. During this time period, the Aero (aircraft) portfolio generated the highest annualized average return of 17.1%, while the Other[5] portfolio produced the lowest average return of 8.72%. In terms of risk, the Telcm (telephone communications) portfolio was least risky with a standard deviation of 15.9%, and the Coal (recreation) portfolio was most risky with a standard deviation of 37.5%. The correlation coefficient between any pair of portfolios ranged from 0.35 to 0.88.

To compute the out-of-sample standard deviation and Sharpe ratio of a combined portfolio, we need to generate the out-of-sample return series of the MVP and EWP. To this end, we estimate the covariance matrix using the most recent 60 (five years), 120 (10 years), and 180 (15 years) monthly returns at a particular month, and solve for the MVP weights. Then, we build the MVP at the same time. Finally, the out-of-sample one-month, six-month, and one-year returns of the MVP are computed according to Equations (9)–(11), respectively. In the following month, we move the estimation window one month forward to re-estimate the MVP and calculate the out-of-sample returns for the MVP until the end of the sample period. In this way, we can get a series of out-of-sample returns for the MVP. The same procedure is applied to calculate the out-of-sample returns for the EWP.

The out-of-sample one-month return on a portfolio is defined as:

$$r_{p1} = \sum_{i=1}^{N} q_i r_{i1}, \tag{9}$$

The out-of-sample six-month return on a portfolio is defined as:

$$r_{p6} = \sum_{i=1}^{N} q_i \left(\prod_{k=1}^{6} (1 + r_{ik}) - 1 \right), \tag{10}$$

The out-of-sample one-year return on a portfolio is defined as:

$$r_{p12} = \sum_{i=1}^{N} q_i \left(\prod_{k=1}^{12} (1 + r_{ik}) - 1 \right). \tag{11}$$

When Equations (9)–(11) are applied to calculate the out-of-sample returns for the MVP, q_i is the weight of the ith asset in the MVP, which is estimated based on the data of past 5 years (or 10 years, or 15 years). When Equations (9)–(11) are applied to calculate the out-of-sample returns for the EWP, q_i equals 1/N. In addition, r_{i1} in Equation (9) is the one-month return of the ith asset post estimation window, and r_{p1} represents the one-month return of the portfolio (MVP or EWP) post estimation window, i.e., the out-of-sample one-month return. In Equation (10), $\prod_{k=1}^{6}(1 + r_{ik}) - 1$ represents the six-month return of the ith asset post estimation window, while r_{p6} represents the six-month return of the portfolio (MVP or EWP) post estimation window, i.e., the out-of-sample six-month return. In Equation (11), $\prod_{k=1}^{12}(1 + r_{ik}) - 1$ represents the 12-month return of the ith asset post estimation window, and r_{p12} represents the 12-month return of the portfolio (MVP or EWP) post estimation window, i.e., the out-of-sample one-year return. Accordingly, the excess returns are given as:

$$er_{p1} = \sum_{i=1}^{N} q_i r_{i1} - r_{f1}, \tag{12}$$

$$er_{p6} = \sum_{i=1}^{N} q_i \prod_{k=1}^{6} (1 + r_{ik}) - \prod_{j=1}^{6} (1 + r_{fj}), \tag{13}$$

$$er_{p12} = \sum_{i=1}^{N} q_i \prod_{k=1}^{12} (1 + r_{ik}) - \prod_{j=1}^{12} (1 + r_{fj}). \tag{14}$$

Equations (12)–(14) represent the excess returns of one month, six months, and one year, respectively. Based on the calculation of out-of-sample excess returns for the MVP and EWP, we can also calculate a series of out-of-sample excess returns for the combination strategy according to the specified combination coefficient ϖ. The out-of-sample performance of various strategies is determined based on these out-of-sample excess return data. In particular, the calculation of out-of-sample standard deviation and Sharpe ratio for each strategy is based on the one-month, six-month, and one-year out-of-sample returns.

To make these measures comparable under different scenarios, we start constructing portfolios from July 1941 based on the estimated inputs using different estimation methods and data sets, and reconstruct the portfolio every month until the end of the sample period. As a result, we obtain 917 one-month, 912 six-month, and 906 one-year out-of-sample returns for the MVP and EWP. These returns are recorded and used for our analysis.

4. Empirical results

4.1. The performance of the MVP and EWP and their combinations

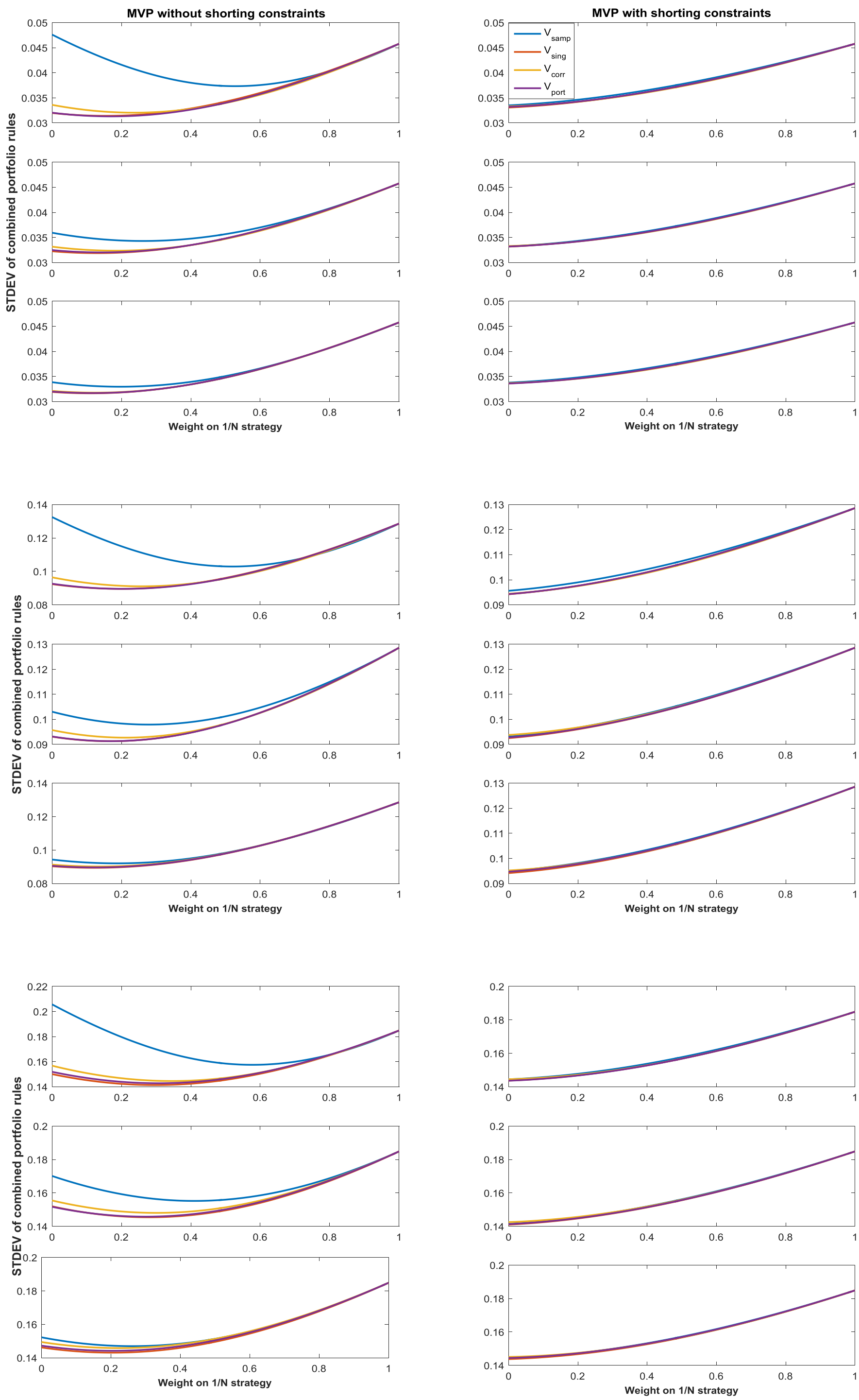
To assess the benefits of combining both the MVP and EWP in terms of risk reduction and Sharpe ratio enhancement, we plot the standard deviation and Sharpe ratio of the combined portfolio against the combination coefficient ϖ in Figs. 2 and 3, respectively. Equation (7) indicates that the corresponding portfolio is the MVP if $\varpi = 0$, and the EWP if $\varpi = 1$.

4.1.1. The MVP versus EWP

We now compare the performance of the MVP with that of the EWP. First, we note from Fig. 2 that, when short-selling is allowed, the standard deviation of the MVP is lower when its estimation is based on shrinkage covariance estimators than when it is based on the sample covariance matrix. This is particularly pronounced when the estimation period is only five years. This is because shrinkage estimators or long estimation periods help reduce estimation errors. However, these differences disappear when the no-short-selling constraint is present. Imposing the no-short-selling constraint is equivalent to shrinking the sample covariance matrix toward a particular target, thereby also reducing estimation errors in the sample covariance matrix (Jagannathan and Ma, 2003). This is because estimating the MVP with the no-short-selling constraint based on $\widehat{\mathbf{V}}_{\text{samp}}$ is equivalent to estimating the MVP without the no-short-selling constraint based on $\widehat{\mathbf{V}}_{\text{samp}} - (\boldsymbol{\lambda}\mathbf{1}^T + \mathbf{1}\boldsymbol{\lambda}^T)$, which can be explained as a shrinkage estimator of the covariance matrix. At the same time, estimating the MVP with the no-short-selling constraint based on a shrinkage estimator has the same effect of shrinking the shrinkage estimator toward a new target, which increases estimation errors in the MVP. These two effects explain why the MVPs based on the sample covariance matrix and various shrinkage estimators have very close standard deviations in the presence of the no-short-selling constraint. Fig. 2 also shows that the standard deviation of the MVP with the no-short-selling constraint is lower than the standard deviation of the EWP in all cases, regardless of the estimation method used and the out-of-sample period considered. This is also true if short-selling is allowed except for cases in which when the sample matrix is used to estimate the MVP and the estimation period

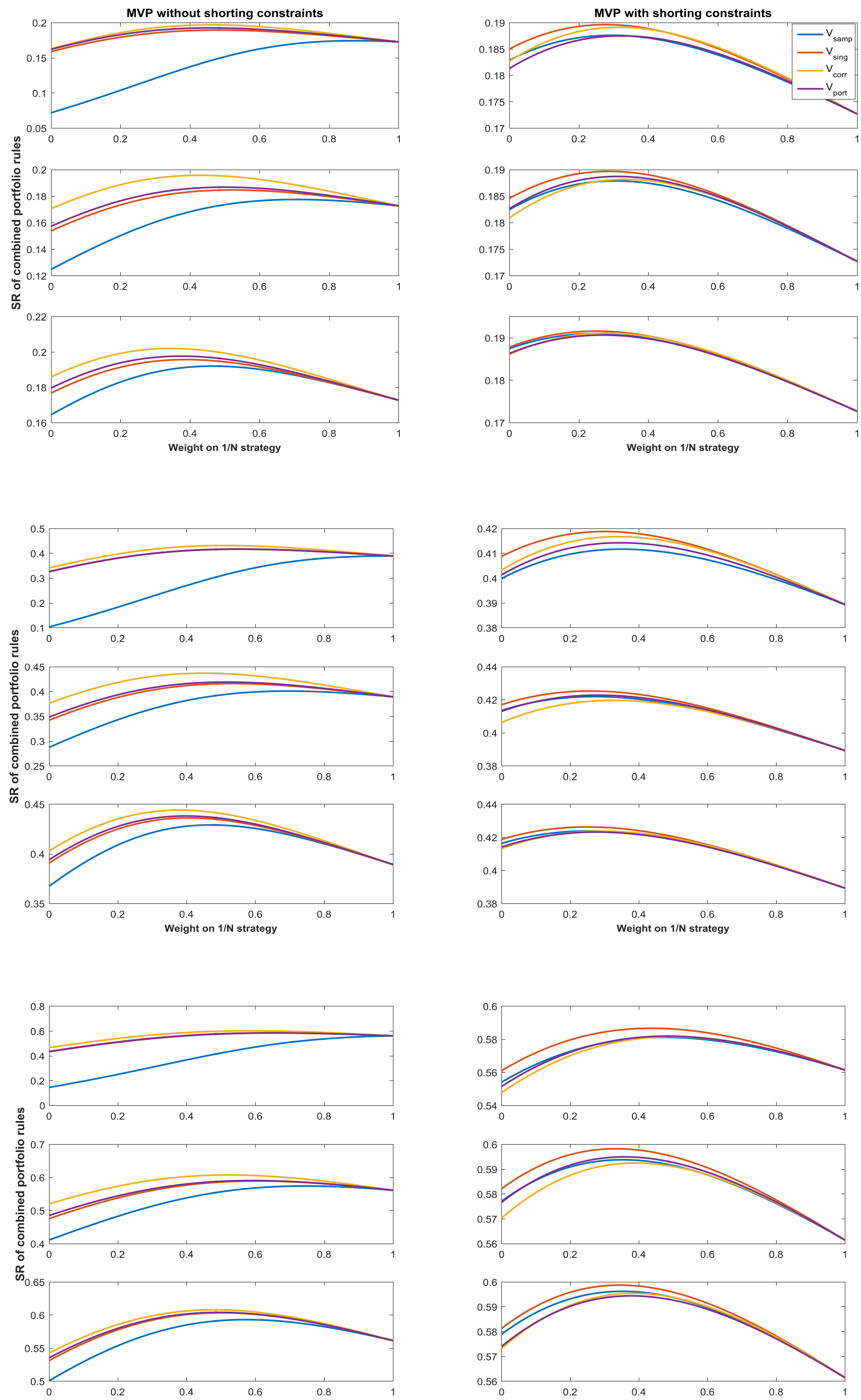
[4] The original data sets include 48 industry indices. Since many data are missing in the indices for Soda, Hlth, Rubbr, FabPr, Guns, Gold, PerSv, and Paper industries, we exclude these indices in our analysis.

[5] The Other portfolio is formed by stocks from Sanitary services, Steam, air conditioning supplies, Irrigation systems, and Cogeneration - SM power producer.



This figure displays the standard deviation of the combined portfolio against the combination coefficient in various cases. The left figures are for the case in which short-selling is allowed, while the right figures are for the case in which short-selling is not allowed. The top, middle, and bottom six figures are for the cases in which one-month, six-month, and one-year out-of-sample periods are considered, respectively. The top, middle, and bottom two figures in each of these three groups of figures are for the cases when the estimation periods are 5, 10, and 15 years, respectively.

Fig. 2. The standard deviation of the combination of MVP and EWP.



This figure plots the Shape ratio of the combined portfolio against the combination coefficient in various cases. The left figures are for the case in which short-selling is allowed, while the right figures are for the case in which short-selling is not allowed. The top, middle, and bottom six figures are for the cases in which one-month, six-month, and one-year out-of-sample periods are considered, respectively. The top, middle, and bottom two figures in each of these three groups of figures are for the cases when the estimation periods are 5, 10, and 15 years, respectively.

Fig. 3. The Sharpe ratio of the combination of MVP and EWP.

is five years. Our finding implies that the MVP strategy outperforms the EWP in terms of risk reduction, as long as the estimation risk in the covariance matrix is well controlled.

From Fig. 3, we observe that the shrinkage estimators help enhance the Sharpe ratio of the MVP relative to the sample covariance matrix if there is no no-short-selling constraint, especially when the estimation period is short. However, the out-of-sample Sharpe ratio for the MVP is lower than that of the EWP in 34 out of 72 cases, regardless of the estimation method. Even in the cases where the MVP generates a higher Sharpe ratio than does the EWP, the differences in Sharpe ratios are not significant.

4.1.2. Combinations of the MVP and EWP versus the MVP or EWP

The performance of combined portfolios can also be seen from Figs. 2 and 3 for $0 < \varpi < 1$. First, Fig. 2 shows that the standard deviation of combined portfolios increases with the combination coefficient if the no-short-selling constraint is imposed, indicating that no benefits are obtained by combining the MVP and EWP. The MVP generates the lowest risk. Moreover, all 4 curves seem to be very close to one another, suggesting that using shrinkage estimation methods does not help reduce the risk of the combined portfolio when short-selling is not allowed. There are two reasons for this finding. First, with the no-short-selling constraint, standard deviations of the MVPs based on the sample covariance matrix are very close to those of the MVPs based on various shrinkage estimators. Second, in this case, the correlation coefficients between the EWP and MVPs obtained using different methods are also similar, ranging from 0.77 to 0.83, as can be seen in Table 1.

Second, if the no-short-selling constraint is absent, the U-shaped figure demonstrates that there is an optimal $0 < \varpi < 1$ such that the risk of the combined portfolio is minimized. The reason behind this result is that in the first case (with the no-short-selling constraint) returns on the MVP and returns on the EWP are highly correlated, with a correlation ranging from 0.77 to 0.83, while in this case (without the no-short-selling constraint) these returns have correlations from 0.24 to 0.64. Apparently, in the case where the no-short-selling constraint is absent, the diversification effect of combining both portfolios is more prominent. Further, the MVP based on the sample covariance matrix seems to benefit more in risk reduction from combining with the EWP than do the MVPs based on other estimation methods. This is primarily because the MVP based on the sample covariance matrix has a higher standard deviation, and is less correlated with the EWP than are the MVPs based on other estimation methods.

Fig. 3 shows that an optimal combination coefficient exists such that the Sharpe ratio of the combined portfolio is maximized, regardless of whether or not the no-short-selling constraint is imposed, and that this Sharpe ratio is higher than that of either the MVP or EWP. Intuitively, the return of the combined portfolio is simply the weighted average of the returns of the MVP and EWP, while the standard deviation of the combined portfolio is always less than the weighted average of the standard deviations of the MVP and EWP. Thus, there is a great chance that the combined portfolio has a higher Sharpe ratio.

We also notice that the curve corresponding to the combinations of the MVP based on the sample covariance matrix and EWP is always below other curves in the case when the no-short-selling constraint is absent, while this is not the case when the constraint is present. If short-selling is allowed, the MVPs based on the shrinkage estimators have a lower standard deviation than the MVP based on the sample covariance matrix, and thereby have a higher Sharpe ratio. On the other hand, if the no-short-selling constraint is imposed, the estimation errors in the MVP based on the covariance matrix can be effectively reduced, and therefore, the differences in Sharpe ratios among the MVPs based on different estimated covariance matrix diminish.

4.2. Combination of the MVP and EWP

4.2.1. Optimal combination coefficient

We now focus on how the MVP and EWP should be combined to achieve the best possible performance. To this end, we analyze the optimal combination coefficients under various scenarios such that the standard deviation is minimized or the Sharpe ratio is maximized. Table 2 reports these optimal coefficients that minimize the standard deviation (Panel A) or maximize the Sharpe ratio (Panel B).

A number of observations can be derived from Panel A of Table 2. First, when short-selling is allowed, the optimal combination ratios are generally less than 0.50 except when the estimation period is five years and when the sample covariance matrix is used to estimate the MVP. This suggests that the MVP should carry a higher weight than the EWP to minimize portfolio risk. In addition, the combination coefficient decreases with the estimation period, and also falls significantly when the estimated covariance matrix is switched from the sample covariance matrix to shrinkage estimators. Intuitively, the optimal combination coefficient reflects how much investors trust the estimated MVP. The more accurate the estimated MVP, the higher the weight of the MVP in the combined portfolio. Long estimation periods and shrinkage estimators of the covariance matrix help reduce estimation errors in the MVP relative to short estimation periods and the sample covariance matrix. We also find that the optimal combination coefficient increases with the out-of-sample return period under consideration for any given estimation method and sample period. This is because returns used to estimate the MVP and to construct the portfolio are historical, monthly returns. Thus, the performance of such estimated MVPs deteriorates with the length of the out-of-sample period.

Table 1
The correlation coefficients of the out-of-sample returns of MVP and EWP.

Estimation period	Without no-short-selling constraint				With no-short-selling constraint			
	V_samp	V_sing	V_corr	V_port	V_samp	V_sing	V_corr	V_port
One month out-of-sample								
60	0.2799	0.5369	0.4519	0.5126	0.8084	0.7956	0.7741	0.7890
120	0.5169	0.5794	0.5346	0.5693	0.7966	0.7882	0.7706	0.7818
180	0.5415	0.5915	0.5804	0.5876	0.8036	0.7961	0.7799	0.7932
Six months out-of-sample								
60	0.2438	0.4990	0.4366	0.4893	0.8345	0.8200	0.8066	0.8187
120	0.5215	0.5580	0.5258	0.5560	0.8203	0.8104	0.7968	0.8080
180	0.5464	0.5862	0.5807	0.5861	0.8203	0.8130	0.8001	0.8119
One year out-of-sample								
60	0.3152	0.5038	0.4794	0.5091	0.8323	0.8187	0.8105	0.8190
120	0.5436	0.5738	0.5798	0.5883	0.8305	0.8213	0.8131	0.8208
180	0.6089	0.6274	0.6353	0.6364	0.8222	0.8151	0.8054	0.8150

This table reports the correlation coefficients between the minimum-variance portfolio (MVP) and equally-weighted portfolio (EWP) out-of-sample returns in various cases. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Table 2
The combination coefficients.

Estimation period	One-month out-of-sample			Six-month out-of-sample			One-year out-of-sample		
	60	120	180	60	120	180	60	120	180
Panel A: Minimizing the standard deviation									
without no-short-selling constraint									
V_samp	0.53	0.26	0.20	0.52	0.28	0.19	0.58	0.41	0.26
V_sing	0.15	0.13	0.11	0.20	0.17	0.12	0.30	0.28	0.20
V_corr	0.24	0.18	0.13	0.26	0.21	0.13	0.35	0.30	0.22
V_port	0.17	0.14	0.12	0.20	0.17	0.12	0.31	0.27	0.21
with no-short-selling constraint									
V_samp	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_sing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_corr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_port	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel B: Maximizing the Sharpe ratio									
without no-short-selling constraint									
V_samp	0.87	0.71	0.47	0.99	0.70	0.47	1.00	0.75	0.57
V_sing	0.48	0.52	0.39	0.55	0.53	0.41	0.65	0.61	0.50
V_corr	0.46	0.43	0.35	0.51	0.45	0.38	0.59	0.52	0.48
V_port	0.46	0.50	0.37	0.55	0.52	0.40	0.66	0.59	0.49
with no-short-selling constraint									
V_samp	0.29	0.30	0.25	0.35	0.27	0.26	0.47	0.35	0.35
V_sing	0.28	0.29	0.25	0.30	0.26	0.25	0.44	0.33	0.34
V_corr	0.32	0.33	0.28	0.34	0.33	0.29	0.50	0.39	0.38
V_port	0.32	0.31	0.27	0.35	0.28	0.28	0.48	0.36	0.38

This table reports the combination coefficients such that the standard deviation of the combined portfolio is minimized or the Sharpe ratio is maximized in various cases. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Second, if there is a no-short-selling constraint, the optimal coefficient is always zero, regardless of the scenario under consideration. This suggests that the MVP can always achieve the lowest possible variance with the no-short-selling constraint.

The results in Panel B of Table 2 show that it is beneficial to combine the MVP and EWP in terms of Sharpe ratio improvement, regardless of whether or not the no-short-selling constraint is imposed. These optimal coefficients exhibit a similar pattern to that observed in Panel A under different scenarios. In addition, the combination coefficients are lower when the no-short-selling constraint is present than when it is absent. Therefore, the presence of the no-short-selling constraint makes the MVP a more reliable component to improve the Sharpe ratio of the combined portfolio.

A comparison of the results in Panels A and B of Table 2 indicates that the combination coefficients are lower when the objective for the combined portfolio is risk reduction rather than Sharpe ratio enhancement. The MVP can better help reduce portfolio risk than the EWP, while the EWP can better help increase the Sharpe ratio.

4.2.2. A new approach to estimation of the optimal combination coefficient

In theory, after both the MVP and EWP are constructed, we then combine them to achieve the lowest variance or the highest Sharpe ratio. However, this process requires estimation of asset returns. To illustrate, we consider the case in which the optimal combination maximizes the Sharpe ratio. Denote the MVP as $\mathbf{q}_{MVP} = \frac{\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}}$ and the EWP $\mathbf{q}_{EWP} = \frac{1}{N}\mathbf{1}$. The returns on the MVP and EWP are, respectively, given as:

$$r_{MVP} = \frac{\mathbf{r}^T\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}}, \tag{15}$$

$$r_{EWP} = \frac{\mathbf{r}^T\mathbf{1}}{N}. \tag{16}$$

Thus, the return on the combination of the MVP and EWP is:

$$r_p = \varpi\, r_{EWP} + (1-\varpi)\, r_{MVP} = \varpi\frac{\mathbf{r}^T\mathbf{1}}{N} + (1-\varpi)\frac{\mathbf{r}^T\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}}. \tag{17}$$

The expected return in excess of the risk-free rate can be written as:

$$E(r_p) - r_f = \varpi\frac{E(\mathbf{r})^T\mathbf{1}}{N} + (1-\varpi)\frac{E(\mathbf{r})^T\mathbf{V}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}} - r_f = \varpi\frac{a}{N} + (1-\varpi)\frac{b}{c} - r_f, \tag{18}$$

where $a = E(\mathbf{r})^T\mathbf{1}$, $b = E(\mathbf{r})^T\mathbf{V}^{-1}\mathbf{1}$, and $c = \mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}$.

The variance of the excess return on the combined portfolio is:

$$\sigma^2(r_p - r_f) = \varpi^2\frac{\mathbf{1}^T\mathbf{V}\mathbf{1}}{N^2} + (1-\varpi)^2\frac{\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1}}{(\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1})^2} + 2\varpi(1-\varpi)\frac{N}{N(\mathbf{1}^T\mathbf{V}^{-1}\mathbf{1})}$$

$$= \varpi^2\frac{d}{N^2} + (1-\varpi)^2\frac{1}{c} + 2\varpi(1-\varpi)\frac{1}{c}, \tag{19}$$

where $d = \mathbf{1}^T\mathbf{V}\mathbf{1}$.

Thus, the Sharpe ratio is given by:

$$SR = \frac{\varpi\frac{a}{N} + (1-\varpi)\frac{b}{c} - r_f}{\sqrt{\varpi^2\frac{d}{N^2} + (1-\varpi)^2\frac{1}{c} + 2\varpi(1-\varpi)\frac{1}{c}}}. \tag{20}$$

Maximizing the Sharpe ratio yields the optimal combination coefficient:

$$\varpi^* = \frac{\frac{1}{c}\left(\frac{a}{N} - \frac{b}{c}\right)}{\left(\frac{b}{c} - r_f\right)\left(\frac{d}{N^2} - \frac{1}{c}\right)} = \frac{\frac{E(r_{EWP}) - E(r_{MVP})}{E(r_{MVP}) - r_f}}{\frac{\sigma^2(r_{EWP}) - \sigma^2(r_{MVP})}{\sigma^2(r_{MVP})}}. \tag{21}$$

Equation (21) shows that the optimal combination coefficient depends greatly on estimation of expected returns of risky assets. In light of the fact that expected returns are most difficult to estimate, the optimal combination coefficient determined by this method may not be able to deliver the best performance.

To avoid estimation of expected returns and covariances of risky assets, we use a regression analysis to establish the relationship between the optimal combination coefficient and various factors such as the sample, estimation method, short-selling constraint, and length of the out-of-sample period considered. Specifically, we run the following regression:

$$CE_i = \alpha_i + \beta_{i1}(T/N) + \beta_{i2}D_1 + \beta_{i3}D_2 + \beta_{i4}D_1 \times D_2 + \beta_{i5}out\_term + \zeta_i, \tag{22}$$

where CE_i ($i = 1, 2$) is the optimal combination coefficient such that the standard deviation of the combined portfolio is minimized ($i = 1$) or the Sharpe ratio is maximized ($i = 2$). T is the number of sample periods, and N is the number of assets. Thus, T/N is intended to proxy the estimation errors attributable to the sample used. D_1 is a dummy variable equal to one if the covariance matrix estimator is $\widehat{V}_{\text{sing}}$, $\widehat{V}_{\text{corr}}$, or $\widehat{V}_{\text{port}}$, and equal to zero otherwise. The coefficient of this dummy variable is therefore to measure the incremental effect on the optimal combination coefficient attributable to using $\widehat{V}_{\text{sing}}$, $\widehat{V}_{\text{corr}}$, or $\widehat{V}_{\text{port}}$ as the covariance matrix estimator instead of using the sample covariance matrix as the estimator. D_2 is a dummy variable equal to one if the no-short-selling constraint is imposed, and equal to zero otherwise. The coefficient of D_2 represents the incremental effect on the optimal combination coefficient of imposing the no-short-selling constraint in estimation of the MVP. $D_1 \times D_2$ describes the case in which the shrinkage or portfolio estimator is used and at the same time the no-short-selling constraint is imposed. Finally, to control for the effect of the length of the out-of-sample period, we include *out_term* in the regression. *out_term* is equal to sqrt(1/12), sqrt(1/2), or sqrt(1) if the out-of-sample period is one month, six months, or one year.

The estimation results in Table 3 show that all the factors in the regression model have a significant impact on the optimal combination coefficient. On the one hand, the adjusted R-square is more than 77%, demonstrating that the combination coefficient can be well explained by the factors considered. On the other hand, all estimated coefficients are significant at the 1% level.

The coefficients of variables T/N, D_1, and D_2 are all negative, regardless of whether the standard deviation is minimized or the Sharpe ratio is maximized. This implies that using more sample observations, employing the shrinkage estimators or portfolio estimators, or imposing the no-short-selling constraint in estimation of the MVP can significantly reduce the combination coefficient. Under these circumstances, the need for the MVP to combine with the EWP is reduced, compared with the other cases. We also notice that the magnitude of the coefficient for D_2 is larger than that for D_1 in the case when the Sharpe ratio is maximized. Therefore, the effect of imposing the no-short-selling constraint in estimation of the MVP is more pronounced than the effect of using shrinkage/portfolio estimators.

However, the coefficient of $D_1 \times D_2$ is positive when the objective is to maximize the Sharpe ratio. The size of the coefficient of $D_1 \times D_2$ is larger than the size of the coefficient of D_1, but is much lower than the size of D_2. It follows that the effect of imposing the no-short-selling constraint when shrinkage/portfolio estimators are used is lower than when the sample covariance matrix is used. This also suggests that when the no-short-selling constraint is present in the estimation of the MVP, using shrinkage/portfolio estimators may increase the need for the MVP to combine with the EWP. This is an interesting finding. Given that the no-short-selling constraint is usually imposed in practice, this finding simply implies that combining the MVP with EWP can partially correct the estimation errors in the sample covariance matrix.

We expect that these observations can be explained by how the performance of the MVP is related to these factors. To confirm this, we consider a similar regression:

$$U_i = \alpha_i + \beta_{i1}(T/N) + \beta_{i2}D_1 + \beta_{i3}D_2 + \beta_{i4}D_1 \times D_2 + \beta_{i5}out\_term + \zeta_i, \tag{23}$$

where U_i ($i = 1, 2, 3$) represents the out-of-sample MVP performance measured by standard deviation ($i = 1$), Sharpe ratio ($i = 2$), and cumulative return ($i = 3$).

The estimation results are reported in Table 4. Similar to what we observe in Table 3, the adjusted R-square is more than 90%, demonstrating that the MVP performance can be well explained by the factors in this model.

The coefficients of variables T/N, D_1, and D_2 are all negative when performance is measured by the MVP's standard deviation, while these coefficients are all positive when performance is measured by the Sharpe ratio or the total return of the portfolio. This shows that increasing the sample period, using the shrinkage estimators or portfolio estimators, or imposing the no-short-selling constraint, can significantly reduce the MVP's risk and enhance its Sharpe ratio. This can be explained by the fact that estimation errors in the MVP are smaller if the MVP is estimated using more sample observations, using the shrinkage estimators or

Table 3
Factors that affect the combination coefficients.

Independent Variables	Dependent Variables			
	CE_1		CE_2	
	Coefficients	P-value	Coefficients	P-value
Constant	0.4104	0.0000	0.7655	0.0000
T/N	−0.0500	0.0000	−0.0465	0.0000
D_1	−0.1593	0.0000	−0.2311	0.0000
D_2			−0.4044	0.0000
D_1 *D_2			0.2407	0.0000
Out_term	0.1480	0.0001	0.1498	0.0000
Adj R Square	0.7773		0.8554	
Observations	36		72	

This table reports the regression results of Equation (21). CE_1 and CE_2 represent the combination coefficient if the standard deviation of the combined portfolio is minimized and the coefficient if the Sharpe ratio is maximized, respectively. T is the length of the sample period, and N is the number of assets. D_1 is a dummy variable equal to one if the covariance matrix estimator is $\widehat{V}_{\text{sing}}$, $\widehat{V}_{\text{corr}}$, or $\widehat{V}_{\text{port}}$, and equal to zero otherwise; D_2 is a dummy variable equal to one if the no-short-selling constraint is imposed, and equal to zero otherwise. *out_term* is equal to sqrt(1/12), sqrt(1/2), or sqrt(1) if the out-of-sample period is one month, six months, or one year.

Table 4
Factors that affect MVP performance.

Independent Variables	Dependent Variables					
	Standard Deviation		Sharpe Ratio		Total Return	
	Coefficients	P-value	Coefficients	P-value	Coefficients	P-value
Constant	0.0054	0.1782	−0.1370	0.0000	−0.0532	0.0000
T/N	−0.0018	0.0181	0.0194	0.0000	0.0019	0.0528
D_1	−0.0162	0.0000	0.0981	0.0000	0.0079	0.0451
D_2	−0.0179	0.0000	0.1461	0.0000	0.0129	0.0084
D_1*D_2	0.0160	0.0003	−0.0984	0.0001	−0.0081	0.1451
Out_term	0.1629	0.0000	0.4824	0.0000	0.1463	0.0000
Adj R Square	0.9769		0.9292		0.9522	
Observations	72		72		72	

This table reports the regression results of Equation (22). T is the length of the sample period, and N is the number of assets. D_1 is a dummy variable equal to one if the covariance matrix estimator is $\widehat{V}_{\text{sing}}$, $\widehat{V}_{\text{corr}}$, or $\widehat{V}_{\text{port}}$, and equal to zero otherwise; D_2 is a dummy variable equal to one if the no-short-selling constraint is imposed, and equal to zero otherwise. *out_term* is equal to sqrt(1/12), sqrt(1/2), or sqrt(1) if the out-of-sample period is one month, six months, or one year.

portfolio estimators instead of the sample covariance matrix, or imposing the no-short-selling constraint rather than allowing short-selling. As a result, the combined portfolio places a relatively high weight in the MVP when shrinkage estimators are used or the no-short-selling constraint is imposed, as observed from Table 3, and this will improve the combined portfolio's out-of-sample performance. We also notice that, measured by risk reduction or Sharpe ratio enhancement, the effect of imposing the no-short-selling constraint in the estimation of the MVP is more pronounced than that of using shrinkage/portfolio estimators. This explains why the no-short-selling constraint has a higher marginal effect on the optimal combination coefficient than do the advanced estimators of the covariance matrix.

The coefficient of $D_1 \times D_2$ is positive when performance is measured by the standard deviation, and is negative when performance is measured by the Sharpe ratio or total return. Thus, once a shrinkage estimator is used or the no-short-selling constraint is present, the marginal contribution to performance improvement of adding the no-short-selling constraint or using shrinkage estimators is reduced.

As the relationship between the optimal combination coefficient and various factors is primarily due to the performance of the MVP relative to the EWP, we can use model (22) to empirically estimate the optimal combination coefficient. Interestingly, we find that this regression method outperforms either the MVP or the EWP in terms of out-of-sample performance.

Table 5 reports the Sharpe ratios of the combined portfolios when the optimal combination coefficients are determined by Equations (22). We observe that when the combination ratios are estimated using the regression method, the Sharpe ratios are higher than those of either the EWP or MVP under various scenarios, meaning that the combination of MVP and EWP can further increase Sharpe ratio.

Table 6 presents the standard deviations of the EWP, MVP, and the optimal combined portfolios when risk minimization is the objective in various cases. Consistent with our previous observations, the standard deviations of the combined portfolio determined by the regression methods are the same as those of the MVP in the case where the no-short-selling constraint is present, indicating that there is essentially no need to combine the MVP with EWP to reduce risk under this scenario. However, the optimal combined portfolio generates lower standard deviations than those of EWP or MVP in the case where short-selling is allowed. As a result, it is still empirically beneficial in terms of standard deviation reduction to combine the MVP and EWP when estimation errors in the MVP estimation are large.

5. Robustness tests

Our sample period spans from July 1926 to December 2017, and there are 40 risky assets considered in the sample for the major tests. To be sure that our results are not due to the sample period selection and the number of assets considered, we also consider additional four datasets to test the robustness of the main findings. Table 7 describes the detailed information on these datasets. Datasets #1 and #2 contain data for the most recent period from July 1963 to December 2017 with similar number of risky assets as that in the datasets used in the previous analysis. Thus, these two datasets are mainly applied to test whether the sample period influences the empirical results. In addition, the rationale for using this recent data is that most relevant research on this subject uses data from July 1963 (Clarke et al., 2006; DeMiguel et al., 2009a, 2009b). Datasets #3 and #4 include fewer risky assets with the same sample period as that in the main test. Thus, these two datasets are used to test the effect of the number of assets on the empirical results. Intuitively, when constructing portfolios with fewer risky assets, the number of parameters to be estimated is also fewer, which entails low estimation errors, and therefore this may reduce the need for the MVP to combine with the EWP for these two cases. All these data are at a monthly frequency and downloaded from Kenneth R. French's personal website.

5.1. The performance of the combined portfolio

Using the same method as in the previous section, we plot the standard deviation and Sharpe ratio of the combined portfolio against the combination ratio under various scenarios for the four datasets. To save space, we do not include these figures in the paper. We observe a pattern similar to that observed in Figs. 2 and 3.

Table 5
Sharpe ratios of the EWP, MVP, and their combinations.

Estimation period	One-month out-of-sample			six-month out-of-sample			One-year out-of-sample		
	60	120	180	60	120	180	60	120	180
Panel A: Sharpe ratios of the EWP and MVP									
EWP	0.1727	0.1727	0.1727	0.3894	0.3894	0.3894	0.5614	0.5614	0.5614
MVP without no-short-selling constraint									
V_samp	0.0720	0.1248	0.1645	0.1037	0.2876	0.3674	0.1462	0.4112	0.5007
V_sing	0.1589	0.1538	0.1767	0.3269	0.3422	0.3908	0.4357	0.4757	0.5312
V_corr	0.1629	0.1706	0.1859	0.3414	0.3764	0.4030	0.4665	0.5203	0.5426
V_port	0.1621	0.1572	0.1796	0.3259	0.3486	0.3942	0.4337	0.4853	0.5354
MVP with no-short-selling constraint									
V_samp	0.1829	0.1825	0.1875	0.3999	0.4136	0.4164	0.5541	0.5772	0.5790
V_sing	0.1850	0.1846	0.1878	0.4088	0.4170	0.4188	0.5610	0.5822	0.5813
V_corr	0.1827	0.1809	0.1861	0.4033	0.4063	0.4132	0.5477	0.5703	0.5732
V_port	0.1813	0.1826	0.1863	0.4014	0.4131	0.4142	0.5515	0.5767	0.5741
Panel B: Sharpe ratios of the combined portfolios: the regression method									
Optimal Combination when MVP is estimated without no-short-selling constraint									
V_samp	0.1744	0.1775	0.1920	0.3894	0.4010	0.4292	0.5614	0.5743	0.5931
V_sing	0.1895	0.1847	0.1957	0.4167	0.4161	0.4363	0.5858	0.5895	0.6035
V_corr	0.1969	0.1958	0.2020	0.4317	0.4370	0.4442	0.6021	0.6077	0.6080
V_port	0.1925	0.1868	0.1977	0.4174	0.4191	0.4382	0.5844	0.5908	0.6041
Optimal Combination when MVP is estimated with no-short-selling constraint									
V_samp	0.1876	0.1879	0.1910	0.4117	0.4220	0.4239	0.5812	0.5938	0.5962
V_sing	0.1896	0.1897	0.1916	0.4188	0.4253	0.4263	0.5867	0.5983	0.5987
V_corr	0.1891	0.1882	0.1911	0.4167	0.4197	0.4238	0.5815	0.5925	0.5954
V_port	0.1875	0.1887	0.1907	0.4143	0.4228	0.4232	0.5819	0.5950	0.5944

This table reports the Sharpe ratios of the equally-weighted portfolio (EWP), minimum-variance portfolio (MVP), and their optimal combinations determined by the regression methods. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Table 6
Standard deviations of the EWP, MVP, and their combinations.

Estimation period	One-month out-of-sample			Six-month out-of-sample			One-year out-of-sample		
	60	120	180	60	120	180	60	120	180
Panel A: Standard deviations of the EWP and MVP									
EWP	0.0458	0.0458	0.0458	0.1286	0.1286	0.1286	0.1848	0.1848	0.1848
MVP without no-short-selling constraint									
V_samp	0.0476	0.0359	0.0339	0.1325	0.1031	0.0943	0.2057	0.1701	0.1522
V_sing	0.0320	0.0323	0.0319	0.0924	0.0931	0.0903	0.1500	0.1520	0.1461
V_corr	0.0336	0.0332	0.0321	0.0964	0.0957	0.0913	0.1568	0.1555	0.1494
V_port	0.0320	0.0325	0.0320	0.0926	0.0932	0.0905	0.1519	0.1517	0.1472
MVP with no-short-selling constraint									
V_samp	0.0335	0.0332	0.0338	0.0956	0.0931	0.0949	0.1444	0.1414	0.1442
V_sing	0.0331	0.0330	0.0335	0.0942	0.0925	0.0941	0.1437	0.1410	0.1438
V_corr	0.0333	0.0333	0.0337	0.0944	0.0938	0.0952	0.1445	0.1425	0.1450
V_port	0.0332	0.0332	0.0336	0.0943	0.0928	0.0947	0.1436	0.1413	0.1444
Panel B: Standard deviations of the combined portfolios: the regression method									
Optimal Combination when MVP is estimated without no-short-selling constraint									
V_samp	0.0373	0.0343	0.0330	0.1029	0.0979	0.0920	0.1575	0.1552	0.1469
V_sing	0.0314	0.0319	0.0316	0.0896	0.0913	0.0894	0.1413	0.1454	0.1430
V_corr	0.0320	0.0324	0.0318	0.0911	0.0927	0.0901	0.1445	0.1480	0.1458
V_port	0.0313	0.0320	0.0317	0.0895	0.0913	0.0896	0.1428	0.1458	0.1441
Optimal Combination when MVP is estimated with no-short-selling constraint									
V_samp	0.0335	0.0332	0.0338	0.0956	0.0931	0.0949	0.1444	0.1414	0.1442
V_sing	0.0331	0.0330	0.0335	0.0942	0.0925	0.0941	0.1437	0.1410	0.1438
V_corr	0.0333	0.0333	0.0337	0.0944	0.0938	0.0952	0.1445	0.1425	0.1450
V_port	0.0332	0.0332	0.0336	0.0943	0.0928	0.0947	0.1436	0.1413	0.1444

This table reports the standard deviations of the equally-weighted portfolio (EWP), minimum-variance portfolio (MVP), and their optimal combinations determined by the theoretical and regression methods. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Table 7
Dataset description for robustness tests.

#	Dataset Description	N	Sample period	Observations
1	40 industry portfolios	40	from July 1963 to December 2017	654
2	32 Size/BM/INV portfolios	32	from July 1963 to December 2017	654
3	25 Size/BM portfolios	25	from July 1926 to December 2017	1098
4	10 industry portfolios	10	from July 1926 to December 2017	1098

This table lists the 4 datasets used in the robustness tests. Each dataset contains N risky assets with different sample period. All these datasets could be downloaded from the Ken French's Website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/).

5.2. The optimal combined portfolio

Tables 8 and 9 report the optimal combination coefficients under various scenarios for four datasets. From the results, we observe that the combination coefficients have a similar pattern to that observed in Table 2 in various cases, regardless of whether the standard deviation is minimized (Table 8) or the Sharpe ratio is maximized (Table 9). For datasets # 1 and # 2, when the standard deviation is minimized without the no-short-selling constraint, the combination coefficients are basically larger than those in Table 2, specifically, for the subsamples of 40 industry portfolios, the combination coefficients are all larger than those in Table 2. This shows that it becomes more important for the MVP to combine with the EWP to reduce risk in the recent time period. This is true when the Sharpe ratio is maximized only in the cases where the MVP is estimated with relatively large errors. It seems that the need for the MVP to combine with the EWP to achieve a higher Sharpe ratio is reduced in the recent sample period, as long as the no-short-selling constraint is imposed, and as long as more samples and shrinkage estimators are used in estimation of the MVP.

For datasets # 3 and # 4, the combination coefficients are generally smaller than those in the corresponding scenarios in Table 2, regardless of whether the out-of-sample standard deviation is minimized or the out-of-sample Sharpe ratio is maximized. This is because the number of parameters needs to be estimated in this cases (25 size/BM portfolios, 10 industry portfolios) is less than that in the former case (40 industry portfolios). Therefore, the impacts of estimation errors are less pronounced for this case than they are for the former case even if the sample length is the same. Consequently, there is less need for the MVP to combine with the EWP for the case of 10 industry portfolios, and 25 size/BM portfolios than for the case of 40 industry portfolios.

To see whether the factors considered in Equation (22) still have a similar impact on the optimal combination coefficients for the four datasets, we re-run the regression; the results are presented in Table 10. These results re-confirm our findings in Table 3.

Finally, we examine the out-of-sample performance of the combination strategy whose optimal combination coefficient is estimated using regression models, and then compare the results with those of the MVP and EWP. The conclusions are consistent with those in Tables 5 and 6. To save space, we do not report the results.

6. Conclusions

Due to estimation errors, mean-variance efficient portfolios determined by portfolio selection models usually perform poorly in practice. This paper empirically investigates the benefits of combining the MVP and EWP, in terms of out-of-sample risk reduction and Sharpe ratio improvement. The MVP component in the combined portfolio is asymptotically unbiased but affected by estimation errors, while the EWP component is not subject to any estimation errors but is biased. Thus, the combination of the MVP and EWP represents a trade-off between bias and variance.

We show that when short-selling is allowed, combining the MVP and EWP will further reduce total risk. However, when short-selling is not allowed, the combination strategy cannot achieve a lower risk than the MVP. We also illustrate that the combined portfolio yields an even higher Sharpe ratio than can the MVP or EWP alone, regardless of whether or not the no-short-selling constraint is present. The out-of-sample performance enhancement depends on the combination coefficient. We find

Table 8
Optimal combination coefficients for four datasets: the objective is to minimize the standard deviation.

	one month out-of-sample			six months out-of-sample			one year out-of-sample		
Estimation period	60	120	180	60	120	180	60	120	180
Panel A: Minimizing the standard deviation (40 industry portfolios: from July 1963 to December 2017)									
without no-short-selling constraint									
V_samp	0.55	0.28	0.21	0.52	0.33	0.21	0.60	0.49	0.30
V_sing	0.17	0.14	0.12	0.19	0.19	0.14	0.28	0.33	0.23
V_corr	0.28	0.21	0.14	0.30	0.27	0.15	0.40	0.39	0.26
V_port	0.18	0.15	0.12	0.19	0.18	0.13	0.31	0.31	0.23
with no-short-selling constraint									
V_samp	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_sing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_corr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_port	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel B: Minimizing the standard deviation (32 Size/BM/INV portfolios: from July 1963 to December 2017)									
without no-short-selling constraint									
V_samp	0.43	0.28	0.23	0.47	0.38	0.37	0.61	0.53	0.50
V_sing	0.19	0.18	0.15	0.33	0.30	0.20	0.47	0.45	0.38
V_corr	0.41	0.37	0.32	0.50	0.41	0.35	0.57	0.49	0.44
V_port	0.15	0.14	0.10	0.30	0.26	0.17	0.43	0.40	0.34
with no-short-selling constraint									
V_samp	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_sing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_corr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_port	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel C: Minimizing the standard deviation (25 size/BM portfolios: from July 1926 to December 2017)									
without no-short-selling constraint									
V_samp	0.40	0.33	0.27	0.47	0.35	0.31	0.53	0.44	0.38
V_sing	0.35	0.19	0.15	0.38	0.14	0.09	0.41	0.26	0.23
V_corr	0.16	0.12	0.11	0.18	0.08	0.07	0.25	0.21	0.20
V_port	0.14	0.12	0.10	0.16	0.12	0.09	0.25	0.20	0.16
with no-short-selling constraint									
V_samp	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_sing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_corr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_port	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel D: Minimizing the standard deviation (10 industry portfolios: from July 1926 to December 2017)									
without no-short-selling constraint									
V_samp	0.29	0.22	0.20	0.29	0.21	0.19	0.42	0.32	0.30
V_sing	0.24	0.20	0.19	0.25	0.20	0.18	0.37	0.31	0.29
V_corr	0.28	0.21	0.20	0.29	0.24	0.22	0.37	0.35	0.33
V_port	0.21	0.19	0.18	0.22	0.19	0.18	0.34	0.30	0.28
with no-short-selling constraint									
V_samp	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_sing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_corr	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
V_port	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

This table presents the combination coefficients such that the standard deviation of the combined portfolio is minimized in various cases for four datasets: 1) 40industry portfolios for the sub-period from July 1963 to December 2017; 2) 32 Size/BM/INV portfolios for the period from July 1963 to December 2017; 3) 25 size/BM portfolios for the period from July 1926 to December 2017; 4) 10 industry portfolios for the period from July 1926 to December 2017. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Table 9
Optimal combination coefficients for four datasets: the objective is to maximize the Sharpe ratio.

Estimation period	one month out-of-sample			six months out-of-sample			one year out-of-sample		
	60	120	180	60	120	180	60	120	180
Panel A: Maximizing the Sharpe ratio (40 industry portfolios: from July 1963 to December 2017)									
without no-short-selling constraint									
V_samp	1.00	0.73	0.40	1.00	0.70	0.40	1.00	0.70	0.46
V_sing	0.52	0.44	0.28	0.54	0.42	0.27	0.65	0.46	0.32
V_corr	0.39	0.30	0.19	0.42	0.31	0.20	0.49	0.34	0.25
V_port	0.43	0.39	0.24	0.51	0.39	0.24	0.64	0.42	0.28
with no-short-selling constraint									
V_samp	0.19	0.17	0.04	0.22	0.12	0.02	0.33	0.17	0.07
V_sing	0.12	0.13	0.04	0.10	0.08	0.00	0.21	0.13	0.05
V_corr	0.18	0.14	0.08	0.17	0.10	0.06	0.21	0.22	0.12
V_port	0.16	0.17	0.06	0.13	0.13	0.04	0.25	0.18	0.10
Panel B: Maximizing the Sharpe ratio (32 Size/BM/INV portfolios: from July 1963 to December 2017)									
without no-short-selling constraint									
V_samp	0.46	0.35	0.30	0.53	0.43	0.32	0.60	0.48	0.33
V_sing	0.40	0.12	0.00	0.39	0.20	0.14	0.49	0.23	0.20

(*continued on next page*)

Table 9 (*continued*)

Estimation period	one month out-of-sample			six months out-of-sample			one year out-of-sample		
	60	120	180	60	120	180	60	120	180
V_corr	0.05	0.03	0.00	0.12	0.11	0.09	0.18	0.17	0.16
V_port	0.03	0.02	0.00	0.13	0.12	0.11	0.22	0.20	0.18
with no-short-selling constraint									
V_samp	0.37	0.21	0.09	0.18	0.14	0.20	0.30	0.27	0.27
V_sing	0.08	0.15	0.03	0.19	0.11	0.00	0.41	0.28	0.19
V_corr	0.10	0.15	0.05	0.18	0.13	0.01	0.38	0.29	0.20
V_port	0.18	0.15	0.08	0.17	0.16	0.04	0.35	0.32	0.22
Panel C: Maximizing the Sharpe ratio (25 size/BM portfolios: from July 1926 to December 2017)									
without no-short-selling constraint									
V_samp	0.60	0.39	0.37	0.65	0.47	0.38	0.68	0.53	0.43
V_sing	0.36	0.17	0.16	0.37	0.11	0.11	0.37	0.23	0.21
V_corr	0.21	0.19	0.16	0.24	0.16	0.12	0.30	0.27	0.24
V_port	0.22	0.19	0.17	0.25	0.19	0.14	0.32	0.29	0.25
with no-short-selling constraint									
V_samp	0.28	0.22	0.20	0.34	0.28	0.27	0.41	0.39	0.26
V_sing	0.21	0.20	0.17	0.29	0.26	0.25	0.39	0.33	0.20
V_corr	0.24	0.23	0.19	0.30	0.28	0.27	0.40	0.35	0.24
V_port	0.23	0.25	0.16	0.29	0.30	0.27	0.40	0.37	0.24
Panel D: Maximizing the Sharpe ratio (10 industry portfolios: from July 1926 to December 2017)									
without no-short-selling constraint									
V_samp	0.58	0.56	0.53	0.66	0.64	0.62	0.77	0.72	0.69
V_sing	0.58	0.56	0.42	0.63	0.62	0.46	0.74	0.69	0.56
V_corr	0.54	0.50	0.41	0.59	0.57	0.46	0.70	0.67	0.56
V_port	0.55	0.50	0.44	0.58	0.61	0.50	0.71	0.69	0.59
with no-short-selling constraint									
V_samp	0.51	0.46	0.50	0.59	0.57	0.53	0.75	0.63	0.60
V_sing	0.49	0.44	0.42	0.54	0.51	0.50	0.64	0.61	0.59
V_corr	0.49	0.44	0.41	0.53	0.50	0.50	0.63	0.60	0.58
V_port	0.49	0.46	0.44	0.55	0.53	0.52	0.68	0.61	0.59

This table presents the combination coefficients such that the standard deviation of the combined portfolio is minimized in various cases for four datasets: 1) 40 industry portfolios for the sub-period from July 1963 to December 2017; 2) 32 Size/BM/INV portfolios for the period from July 1963 to December 2017; 3) 25 size/BM portfolios for the period from July 1926 to December 2017; 4) 10 industry portfolios for the period from July 1926 to December 2017. V_samp, V_sing, V_corr, and V_port stand for the sample covariance matrix, shrinkage covariance matrix estimator with the covariance matrix implied by the single-factor model as the target, shrinkage covariance matrix estimator with the constant correlation covariance matrix as the target, and a simple average of the above three estimators, respectively.

Table 10
Impacts of various factors on the optimal combination coefficients for the four datasets.

Independent Variables	Dataset #1				Dataset #2				Dataset #3				Dataset #4			
	CE_1		CE_2		CE_1		CE_2		CE_1		CE_2		CE_1		CE_2	
	Coef	P-val	Coef	P-val	Coef	P-val	Coef	P-val	Coef	P-val	Coef	P-val	Coef	P-val	Coef	P-val
Constant	0.415	0.000	0.901	0.000	0.328	0.000	0.428	0.000	0.435	0.000	0.532	0.000	0.240	0.000	0.581	0.000
T/N	−0.048	0.000	−0.078	0.000	−0.029	0.002	−0.036	0.000	−0.026	0.000	−0.040	0.000	−0.006	0.000	−0.007	0.000
D_1	−0.166	0.000	−0.327	0.000	−0.096	0.003	−0.271	0.000	−0.204	0.000	−0.278	0.000	−0.019	0.217	−0.070	0.000
D_2			−0.562	0.000			−0.197	0.000			−0.206	0.000			−0.070	0.000
$D_1 * D_2$			0.304	0.000			0.216	0.000			0.254	0.000			0.028	0.114
Out_term	0.176	0.000	0.067	0.035	0.305	0.000	0.198	0.000	0.118	0.002	0.133	0.000	0.150	0.000	0.223	0.000
Adj R Square	0.745		0.896		0.683		0.718		0.773		0.849		0.673		0.878	
Observations	36		72		36		72		36		72		36		72	

This table reports the estimation results from the regression of the optimal combination coefficient on various factors for four datasets: 1) 40industry portfolios for the sub-period from July 1963 to December 2017; 2) 32 Size/BM/INV portfolios for the period from July 1963 to December 2017; 3) 25 size/BM portfolios for the period from July 1926 to December 2017; 4) 10 industry portfolios for the period from July 1926 to December 2017. CE_1 and CE_2 represent the combination coefficient if the standard deviation of the combined portfolio is minimized and the coefficient if the Sharpe ratio is maximized, respectively. T is the length of the sample period, and N is the number of assets. D_1 is a dummy variable equal to one if the covariance matrix estimator is $\widehat{V}_{\text{sing}}$, $\widehat{V}_{\text{corr}}$, or $\widehat{V}_{\text{port}}$, and equal to zero otherwise; D_2 is a dummy variable equal to one if the no-short-selling constraint is imposed, and equal to zero otherwise. *out_term* is equal to sqrt(1/12), sqrt(1/2), or sqrt(1) if the out-of-sample period is one month, six months, or one year.

that the more accurate the estimated MVP, the higher weight should be placed on it in the combined portfolio. Specifically, increasing the estimation period, using the shrinkage estimation method, or imposing the no-short-selling constraint can effectively reduce estimation errors in the MVP, and thereby can reduce the need for the MVP to combine with the EWP. Additionally, the longer the out-of-sample period considered, the more necessary it is for the MVP to combine with the EWP.

Acknowledgements

We thank the editor and two anonymous reviewers for their helpful comments. Jiang and Du acknowledge the support from the National Natural Science Foundation of China (# 71471029; # 71101019). An acknowledges the support from the National Natural Science Foundation of China (# 71561011).

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2018.11.012.

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