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Chebyshev Polynomial Functions based Locally Recurrent Neuro-Fuzzy Information System for Prediction of Financial and Energy Market Data

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Abstract:

In this paper Chebyshev Polynomial Functions based locally recurrent neuro-fuzzy information system is presented for the prediction and analysis of financial and electrical energy market data. The normally used TSK-type feedforward fuzzy neural network is unable to take the full advantage of the use of the linear fuzzy rule base in accurate input-output mapping and hence the consequent part of the rule base is made nonlinear using polynomial or arithmetic basis functions. Further the Chebyshev polynomial functions provide an expanded nonlinear transformation to the input space thereby increasing its dimension for capturing the nonlinearities and chaotic variations in financial or energy market data streams. Also the locally recurrent neuro-fuzzy information system (LRNFIS) includes feedback loops both at the firing strength layer and the output layer to allow signal flow both in forward and backward directions, thereby making the LRNFIS mimic a dynamic system that provides fast convergence and accuracy in predicting time series fluctuations. Instead of using forward and backward least mean square (FBLMS) learning algorithm, an improved firefly-harmony search (IFFHS) learning algorithm is used to estimate the parameters of the consequent part and feedback loop parameters for better stability and convergence. Several real world financial and energy market time series databases are used for performance validation of the proposed LRNFIS model.

Key words: Locally recurrent neuro-fuzzy network, Chebyshev polynomials, TSK fuzzy rules, Firefly optimization, Harmony search algorithm, electricity price forecasting, currency exchange rate prediction, stock price prediction.

1. Introduction

Most of the time series data that occur in financial and engineering fields are chaotic in nature and exhibit nonstationary behavior amounting to chaotic fluctuations. Significant research efforts have been undertaken to mine these highly chaotic time series databases in the way of either forecasting or classifying patterns in the database. With the introduction of restructuring and deregulation of electric power industry, electricity price forecasting has become a very valuable tool in energy trading. Also, it has become an important paradigm for the market operators as well as electricity consumers either to bid or hedge against price volatility. Due to its nonlinearity, unusual high volatility, and chaotic nature; the electricity price market shows a very complex behavior as compared to other commodity markets. When bidding in an electricity pool system the market operators are asked to express their bids in terms of price and the quantity and, therefore, the company that is able to forecast hourly pool prices accurately will be able to adjust its price/power production schedules. At the same time the consumers can derive a plan to optimize their consumption of electricity and self production facilities if any to safeguard themselves against sudden surge in electricity prices. It is well known that the electricity price volatility is influenced by various factors such as sudden changes in electricity demand, fluctuations in hydroelectricity generation, transmission congestion, forced outages of generators and transmission lines, behavior of market participants and market power fluctuations, etc.

Financial time series data belonging to stock and currency exchange markets are more complicated than other statistical data due to their long term trends, cyclical and seasonal variations and irregular movements. Predicting such highly fluctuating and irregular data is usually subject to large errors. So developing more realistic models for predicting accurately financial time series data bases to extract meaningful statistics from them is of great interest in financial data mining research. As the time series stock prices are highly dynamic and volatile in nature, efficient prediction of such dynamic time series values can be useful for the investors from accruing profits in the ways of buying, selling or holding the stocks. In a similar way the Foreign Exchange (FX) market is the largest and most lucrative one amongst other financial markets. Prediction of foreign exchange (FX) rates is one of the most important issues in financial market as FX rate is an important economic index in the global monetary markets. The motivation behind exchange rate prediction is to get financial benefit. FX rates are affected by many economical, psychological, and political factors and thus reflect complex behavior. It carries limited information about the data and therefore, it has been a great challenge for researchers and practitioners in the global economy

Many techniques have been employed over the years for forecasting the financial and energy market time series databases, which include either statistical or intelligent system models or a hybridization of both. Amongst the statistical models most familiar once are the Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models [1-5], which are used to forecast both financial and energy market data on a short or long term basis. On the other hand there has been a significant number of intelligence system techniques that include artificial neural networks (ANNs) [6-11], fuzzy inference system (FIS) [12-15], and support vector machines (SVMs) [16-20], etc. Although time series models like ARMA, ARIMA, and GARCH, are found to be effective in the producing reasonable forecast in financial and electricity markets their accuracy deteriorates due to the presence of outliers, and spikes during high market uncertainties. However, for short-term forecasts, Artificial neural networks (ANNs) are flexible computing frameworks and universal approximators that can be applied to a wide range of time series forecasting problems with a high degree of accuracy. According to a recent study, the machine learning methods (ANNs, SVMs) outperform other time series methods (ARMA, GARCH) due to their superior handling capacity of the chaotic behavior of the time series databases. Due to the generalization capabilities and robustness, the ANN is extensively used in various time series forecasting.

The most widely used neural network models for both financial and energy market forecasting include Radial Basis Function Neural Network (RBFNN) [21-24], Wavelet Neural Network (WNN) [25], Recurrent Neural Network (RNN) [26,27], and single layer feed forward neural network based extreme learning machine (ELM) [28], etc. Further to meet the increasing needs for better forecasting models; several nonlinear models have been developed that include Adaptive Neural Network based Fuzzy Information Systems (ANFIS) [29-37] and Fuzzy Neural Networks (FNN), self evolving fuzzy neural network [33], etc. Further it has been observed by the researchers that a single model may not be able to capture all the uncertainties in the nonlinear time series and therefore may not be able to predict its futuristic behavior accurately and thus the hybridization of statistical models like ARMA, ARIMA or GARCH with feedforward neural networks [38-41] is used widely for time series forecasting especially in financial and energy price time series databases. In recent years, more hybrid forecasting models like the Fuzzy information systems with ARIMA [40,41], Fuzzy wavelet neural networks with GARCH , etc. have been successfully applied in time series prediction.

Although numerous studies involving neuro-fuzzy inference systems for financial and electricity market time series prediction, there is hardly any substantial research in the application of recurrent neuro-fuzzy inference systems for predicting highly chaotic nonlinear times series like financial or electricity market data. Normally the recurrent structure in the network is introduced either in the firing strength layer or in output and input layers and so on. Thus in this research, a new locally recurrent neuro-fuzzy inference System (LRNFIS) is presented that includes a feedback loop in the firing strength layer to memorize the temporal nature of the fuzzy rule base and another one in the output layer to provide a dynamic nature to the time series forecasting problem. In the widely used neuro-fuzzy information systems, the consequent part of the fuzzy rule base is of the TSK-type that has more free parameters to adjust providing a better input-output mapping. However, the TSK-type feedforward recurrent fuzzy neural network does not take the full advantage of the use of the fuzzy rule base in approximating functions and therefore a hybridization procedure is adopted by using Chebyshev polynomial functions to construct the consequent part. The Chebyshev polynomial function model [42] will provide an expanded nonlinear transformation to the input space and will be adequate to capture the nonlinearities and chaotic variations in time series prediction problems. It is well known that the weight parameters of the fuzzy neural information systems are normally trained by gradient descent, recursive least squares, extended and unscented Kalman filters, etc. and most of these algorithms suffer from slow convergence and high computational overhead. To circumvent these problems metaheuristic evolutionary algorithms like Particle swarm optimization (PSO), Differential evolution (DE), Firefly [43-45], and Harmony search [46-49] are used to estimate the parameters of the weights associated with TSK Fuzzy rule base and locally recurrent feedback loops. Further to improve the accuracy of forecasting the conventional Firefly algorithm is hybridized with Harmony search algorithm and the improved hybrid version (IFFHS) provides better diversity, and speed of convergence. The proposed LRNFIS model is compared with the well known radial basis function neural network (RBFNN) and adaptive neuro-fuzzy information system (ANFIS) models trained by the same IFFHS metaheuristic evolutionary algorithm for the prediction of electricity market prices time series [50-52], which exhibits severe fluctuations and spikes in prices. Further, financial time series databases like currency exchange rate time series and stock time series are taken for prediction over a time frame varying from one day to several days ahead to prove the validity of the proposed LRNFIS model.

The paper is organized in 7 sections. Section-2 presents the TSK fuzzy model of the LRNFIS and its training algorithm comprising the IFFHS algorithm is presented in section 3. The performance metrics are presented in section 4 whereas section 5 presents the applications to predict the PJM electricity market price, four major currency exchange rates, and two stock market time series databases. Concluding remarks are given in section 6.

2. TSK fuzzy model used for Financial and Energy market Time Series prediction

The architecture of LRNFIS is presented in this section where the firing strength of each fuzzy rule is locally fed back to itself creating a locally recurrent fuzzy system that captures the dynamic behavior of the highly fluctuating temporal databases. The consequent part of each recurrent fuzzy rule has a first order TSK-type built around Chebyshev or Legendre polynomial basis functions. Fig.1 depicts the seven layer architecture that comprises the LRNFIS, each layer of which performs a specific mathematical function summarized below:

Layer 1 (Input Layer):

In this layer the crisp values of the past time series data are used as inputs which are passed directly to the next layer known as the fuzzification layer. Thus there is no computation to perform in this layer as there are no weights to be updated.

Layer 2 (Fuzzification Layer):

Each node in this layer is associated with a fuzzy membership function that performs fuzzification of the input data. For the *i* th fuzzy set $A_{i,j}$ the input data x_j is fuzzified using the membership function of the form

$$\mu_{i,j}(x_j) = u_{j,2} = \exp\left\{-\frac{1}{2}\frac{(x_j - c_{i,j})^2}{\sigma_{i,j}^2}\right\} \tag{1}$$

$i = 1,2,................, m,\ j = 1,2,............, n$

where $c_{i,j}$ and $\sigma_{i,j}$ are the centre and spread of the *i*th membership function for the *j*th input, respectively. The output from this layer is obtained as

Layer 3 (Spatial Firing Layer):

In this layer each node functions as a spatial rule node that represents a TSK type fuzzy rule, which is used to evaluate the output by a T-norm (product or minimum) of the antecedent fuzzy set operation. The total number of nodes in this layer is m and the aggregated spatial firing strength is obtained as

$$u_{j,3} = \prod_{i=1}^{n} \frac{\exp\left\{-.5(x_i - c_{i,j})^2\right\}}{2\sigma_{i,j}^2} \tag{2}$$

Layer 4 (Normalization and locally recurrent layer)

Each node output in this layer is a normalized temporal firing strength, which is obtained from the ratio of the j^{th} rule's temporal firing strength to the sum of all rule outputs temporal firing strengths as follows:

$$u_{j,4} = u_{j,3} \Big/ \sum_{i=1}^{m} u_{j,3} \tag{3}$$

where m is the total number of fuzzy rules used in the rule base. This layer contains the locally recurrent rule node providing dynamic nature to the network structure and its output is a temporal firing strength $O_{j,4}$ which depends not only on the current spatial firing strength $u_{j,4}$ but also on the one step delayed temporal firing strength $O_{j,4}(k-1)$. At any instant '*k*' the temporal firing strength of the recurrent rule is described as a linear combination of the current spatial firing strength and the one step delayed temporal firing strength in the following way:

$$O_{j,4}(k) = \lambda_j \times u_{j,4} + (1-\lambda_j) \times O_{j,4}(k-1) \tag{4}$$

where $0 \le \lambda_j \le 1$ represents the internal recurrent feedback parameters. These parameters control the contribution ratio of current spatial and delayed temporal firing strength to the final output from this layer towards the network outputs.

Layer 5 (Consequent Layer):

The consequent layer is a layer of adaptive nodes which comprises the TSK rule base using Chebyshev polynomial basis function and the rules are obtained as

$$\text{Rule: If } x_1 \text{ is } A_1, \wedge \ldots\ldots \wedge x_n \text{ is } A_n \quad \text{Then } y = f(x_1, x_2, \ldots., x_n) \tag{5}$$

where $x_1, x_2, \ldots., x_n$ are the input variables and $A_1, A_2, \ldots., A_n$ are the fuzzy sets represented by the membership functions $\mu_{A_j}(x_i)$ and y is output from the consequent part. The function *f*(.) is chosen to comprise a nonlinear functional expansion of the input variables and therefore, the fuzzy rule base takes the form

$$\begin{aligned} &R_j : \text{If } x_1 \text{ is } A_{1,j}, \wedge x_2 \text{ is } A_{2,j}, \ldots.., x_i \text{ is } A_{i,j}, \ldots. x_n \text{ is } A_{n,j} \\ &\text{Then } y_j = \beta_j f_j(x_1, x_2, \ldots., x_n) \end{aligned} \tag{6}$$

where the A_{ij} is the *i*th fuzzy set associated with the *i*th input.

The nonlinear functional output from the consequent part of the rule base is obtained as

$$f_j = \tanh\left(\omega_{j0}\phi_0 + \omega_{j1}\phi_1 + \ldots\ldots\ldots + \omega_{jM}\phi_M\right)$$

and $M = 3n$. Further the lower and higher order Chebyshev Polynomials are given in equations (8) and (9) as

$$c_1(x_i) = x, \qquad c_2(x_i) = 2x^2 - 1$$
$$c_3(x_i) = 4x^3 - 3x, \quad c_4(x_i) = 8x^4 - 8x^2 + 1 \qquad (7)$$

The higher order Chebyshev Polynomials can be generated using the generalized recursive formula given in equation (9)

$$c_{r+1} = 2 * x * c_r(x) - c_{r-1}(x_i) \qquad (8)$$

Thus comparing equations (7) and (8) the following equation is obtained as

$$\phi_0 = 1,\ \phi_1 = x_1,\ \phi_2 = 2x_1^2 - 1,\ \phi_3 = 4x_1^3 - 3x_1\ \phi_{M-2} = x_n,\ \phi_{M-1} = 2x_n^2 - 1,\ \phi_M = 4x_n^3 - 3x_n \qquad (9)$$

Layer 6 (Output Layer):

In this layer, the recurrent weighted one step delayed past outputs are added to the rule consequent output to provide an overall output that helps to increase the ability of the network in capturing the dynamic behavior of financial and energy time series data. Thus the overall output of the LRNFIS is calculated as the summation of the rule consequences and the one step delayed past estimated outputs as follows:

$$\hat{y}(k) = \sum_{j=1}^{m} O_{j,4}(k) f_j + \sum_{i=1}^{r} \eta_i\, y(\hat{k} - i) \qquad (10)$$

Where α_i represents the weight corresponding to the i^{th} delayed estimated output, $y(\hat{k} - i)$ represents the i^{th} delayed estimated output for *i=1, 2,…, r*. The performance of the proposed LRNFIS model mainly depends on fine tuning of the antecedent, consequence, internal and external feedback weights for highly fluctuating financial and energy market databases.

3. Training Algorithms for the LRNFIS Network

In this paper, a hybrid learning paradigm comprising a structure learning for the centre and spread (c_{ij} and σ_{ij}) of the fuzzy membership values and weight parameter learning for the recurrent and consequent parts of the LRNFIS network is used. In the structure learning step the fuzzy membership function parameters are updated using a gradient descent rule as outlined below. In the second step, the weights associated with the local recurrent outputs and the consequent parts of the TSK Fuzzy rules are updated by a meta-heuristic hybrid Firefly- Harmony search evolutionary algorithm.

Step 1: Structure Learning

In this step it is important to determine the number of fuzzy rules by obtaining a fuzzy rule from each cluster of input data points [33]. Using simple clustering approach the centre and width of each fuzzy set A_{ij} are obtained as

$$\mu_{i,1} = \sum_{p=1}^{P} \frac{x_{p,1}}{P}, \sigma_{i,1} = \sum_{p=1}^{P} \frac{(x_{p,1} - \mu_{i,1})^2}{P} \tag{11}$$

where P=number of patterns; and $i = 1,2,3,..........,n$.

The total number of fuzzy rules can be obtained by examining the following value $\rho_{ij} = \min_i \max_j$ (firing strength of the jth rule, $j = 1,2,3,..........,m$, and $i = 1,2,3,..........,n$. If $\rho_{i,j} < \xi$, and ξ lies between 0 and 1, a new fuzzy rule is added to the existing m rules. Thus the centre and spread of the Fuzzy set used in LRNFIS is obtained as

$$\mu_{i,j} = \sum_{p=1}^{P} \frac{x_{p,j}}{P}, \sigma_{i,j} = \sum_{p=1}^{P} \frac{(x_{p,j} - \mu_{i,j})^2}{P} \tag{12}$$

After obtaining the centre and spread of fuzzy sets , it is required to tune these parameters in the following way:

An error cost function $E(k)$ is formulated to obtain the updated fuzzy membership parameters as

$$E(k) = \frac{1}{2}(y_d(k) - \hat{y}(k))^2 \tag{13}$$

Using the error cost function depicted in eq. (11), the gradient descent rule for parameter adjustment is obtained as :

$$c_{ij}(k+1) = c_{ij}(k) - \eta_l e(k) \frac{\partial E}{\partial c_{ij}} \tag{14}$$

$$\sigma_{ij}(k+1) = \sigma_{ij}(k) - \eta_l e(k) \frac{\partial E}{\partial \sigma_{ij}} \tag{15}$$

where $$\frac{\partial E}{\partial c_{ij}} = (\hat{y} - y_d)\frac{(y_j - \hat{y})}{\sum_j^m \mu_j}\mu_j(x_i)\frac{2(x_i - c_{ij})}{\sigma_{ij}^2} \tag{16}$$

and $$\frac{\partial E}{\partial c_{ij}} = (\hat{y} - y_d)\frac{(y_j - \hat{y})}{\sum_j^m \mu_j}\mu_j(x_i)\frac{2(x_i - c_{ij})^2}{\sigma_{ij}^3} \tag{17}$$

η_l = learning rate used for updating the fuzzy membership function parameters.

After obtaining the membership values of the fuzzy sets used in layer 2 of the LRNFIS, the next step is to estimate the unknown parameters of the antecedent, consequent parts of the fuzzy if-then rules and the weights of the internal and external feedback loops, minimizing the root mean square error (RMS) of prediction.

Step 2: **Hybrid Firefly-Differential Harmony search Algorithm for Weight Parameters learning**

Thus the objective function for optimization is written as:

$$E(p) = \sqrt{\sum_{k=1}^{N}\left(y_d(k) - \hat{y}(k)\right)^2} \tag{18}$$

where p is the parameter set that includes the weights of the feedback loops used in the network; $y_d(k)$ and $\hat{y}(k)$ are the desired and estimated values of the financial or energy market data sample at the kth instant, and N is the total number of data points used during the learning stage. In this paper the parameters of the proposed model have been estimated through a hybrid Firefly-Differential Harmony search algorithm. Although the Firefly algorithm (FA) has similarities with other swarm intelligence techniques like Particle Swarm Optimization (PSO), Artificial Bee Colony optimization (ABC), Bacteria Foraging algorithm (BFA), etc., it is conceptually much simpler and easier to implement. Firefly algorithm (FA) originally proposed by Xin-She Yang [44] is a metaheuristic algorithm which is based on the flashing behavior of the fireflies. As fireflies are unisex one firefly can be attracted to the other one depending on its brightness, which is measure of higher value of the objective function. Despite being an effective optimization tool, FA has difficulty of randomization between global and local search and thus an improved Harmony search (HS) is hybridized with conventional FA to overcome this drawback by increasing the diversity in firefly population. The HS algorithm is conceptualized from musical instruments that depends on the operators like the harmony memory (HM) and its size (HMS), the harmony memory consideration rate (HMCR), the pitch adjustment rate (PAR), and the pitch adjustment bandwidth (BW).

Let *D* represents the dimension of the total number of weights of LRNFIS network to be estimated and for each dimension *M* is the number of fireflies which are chosen randomly. Thus the total number of decision variables is $M \times D$ and their range is chosen randomly; *G*=total number of generation, and *Ub* and *Lb* are upper and lower limits of decision variables which are weights of the LRNFIS. The final expression of the solution vector (*X*) is:

$$[X]_{M \times D} = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1D} \\ X_{21} & X_{22} & \cdots & X_{2D} \\ \vdots & \vdots & & \vdots \\ X_{M1} & X_{M2} & \cdots & X_{MD} \end{bmatrix} \tag{19}$$

The attractiveness value β between any two fireflies *i* and *j* varies with the Cartesian distance r_{ij} between them and is obtained as

$$\beta = \beta_0 e^{-\gamma r_{ij}^2} \tag{20}$$

where $r_{ij} = \|X_i - X_j\| = \sqrt{\sum_{k=1}^{D} (X_{ik} - X_{jk})^2}$ (21)

and $\beta_0 \in (0,1)$ is the initial attractiveness at distance between any two fireflies $r_{ij} = 0$. Here $\gamma \in (0,1)$ is the absorption coefficient that controls the decrease of light intensity; X_{ik} is the kth component of the spatial coordinate of the X_i of the ith firefly. For conventional FF algorithm the movement of one firefly '*i*' towards another brighter firefly '*j*' is given by using the following expression:

$$X_{i,k} = X_{i,k} + \beta(X_{j,k} - X_{i,k}) + \alpha(rand_{i,k} - 0.5) \tag{22}$$

where $\alpha \in (0,1)$ is a random movement is factor and $rand_{i,k}$ is a random number generated uniformly distributed between 0 and 1. Further a higher value of α points to global search, whereas its lower value points to local search operation. However, in this paper α is varied in each iteration as

$$\alpha^{k+1} = \left(\frac{1}{2k_{\max}}\right)^{1/k_{\max}} \alpha^k \tag{23}$$

According to the concept of global optima, the fireflies with minimum light intensity are regarded as the best fireflies (X_{gbest}) for that particular generation. Instead of comparing each firefly with other fireflies, it is more convenient to compare it with the best firefly to reduce computational complexity. Thus the Eq.(21) is modified by using the global optimization concept as:

$$X_{i,k} = X_{i,k} + \beta_{best} * (X_{gbest,k} - X_{i,k}) + \alpha * (rand_{i,k} - 0.5) \tag{24}$$

where the best attractiveness parameter (β_{best}) is expressed as:

$$\beta_{best} = (\beta_{\max} - \beta_{\min})e^{-\gamma r_{igbest}^{2}} + \beta_{\min} \tag{25}$$

Here $\beta_{\max}$ and $\beta_{\min}$ are maximum and minimum attractiveness for that generation. Thus the best distance $(r_{ij,best})$ is obtained as

$$r_{igbest} = \sqrt{\sum_{k=1}^{D}\left(X_{i,k} - X_{gbest,k}\right)} \tag{26}$$

The light intensity value of the ith firefly is equal to the functional value which can be described as:

$$I_{i,k} \text{ is proportional to } f(X_{i,k}) \Rightarrow I_{i,k} = f(X_{i,k}) \tag{27}$$

and for the brightest firefly $f(X_{gbest,k}) \Rightarrow I_{gbest,k} = f(X_{gbest,k})$ (28)

However, the conventional Firefly Algorithm has some randomness and may be trapped at any local point having lesser light intensity but may not be the minimum one. Also FF takes high run time (i.e. runs M^2 times where M=no of fireflies) to find new solutions as compared with proposed improved FF. The basic reason of such behavior is its nested iterative loop structure in which each firefly will be compared with every other firefly [43,45]. As time complexity increases the performance of the standard FF deteriorates. In the proposed hybrid optimization technique the global minima concept and mutation process using the improved Harmony search (IHS) algorithm are also added to the conventional firefly algorithm. Here the mutation process improves the features of the brighter firefly to achieve better fitness result and higher speed of convergence. In this paper mainly the following equations are used for the mutation and they are:

If $I_{gbest,k} < I_{i,k}$

$$X_{i,k} = X_{i,k} + (2 \times rand(0,1) - 1) \times BW \tag{29}$$

However, the mutation process also depends on the two other parameters namely *PAR* and *HMCR*. The *PAR* is linearly varied by using the equation:

$$PAR(k) = PAR_{\min} + \left(PAR_{\max} - PAR_{\min}\right) \times (1 - k / K) \tag{30}$$

In a similar way the HMCR is varied as

$$HMCR(k) = HMCR_{\min}(HMCR_{\max} - HMCR_{\min}) \times (k / K) \tag{31}$$

and the band width is exponentially decreased by using the equation:

$$BW(k) = BW_{\max} \times \exp\left(\frac{\log\left(\frac{BW_{\min}}{BW_{\max}}\right)}{K}\right) \times k \tag{32}$$

where k is the current iteration number and K is the maximum value of iterations; PAR_{min} and PAR_{max} are the minimum and maximum pitch adjustment rate, and BW_{min} and BW_{max} are the minimum and maximum pitch adjustment bandwidth, respectively. The reason behind such improvisation is existing HS is because, larger *BW* value at initial generations ensures considerable diversity in solution vector, where during final generations small *BW* value provides fine tuning of solution vector. Further the harmony memory consideration rate (*HMCR*) varies within 0 <*HMCR*<1. If HMCR is too small the selection of a few best harmonies results in a slow convergence of the process, however, if it is too high then the remaining harmonies will not be explored well. Pitch adjustment band width (*BW*) and pitch adjustment rate (*PAR*) play vital roles in deciding the convergence speed of the algorithm. The PAR expresses the probability of moving from the existing harmony to a new harmony and a high value of PAR results in a solution to scatter around an optimum solution. Typical values of PAR lie in arrange of 0.4-0.75. The proposed improved harmony search has exponentially decreased '*BW*' and linearly increased '*PAR*' whereas in conventional harmony search algorithm '*BW*' and '*PAR*' are considered as fixed valued. A Flow chart for implementing the hybrid FF-HS algorithm is shown in Fig.2 presents the Flow chart for implementing the hybridized Firefly and Harmony search algorithm.

4. Performance Metrics for Forecasting Accuracy Evaluation

The Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) are used to compare the performance of the models for predicting the time series datasets twenty four hours in advance using LRNFIS and IFFHS learning algorithm. To evaluate the performance of the proposed recurrent fuzzy neural system the following error metrics are defined as:

$$\begin{aligned} RMSE &= \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(y_k - \hat{y}_k\right)^2} \\ MAPE &= \frac{1}{N}\sum_{k=1}^{N}\left|\frac{y_k - \hat{y}_k}{y_k}\right| \times 100 \\ MSE &= \frac{1}{N}\sum_{k=1}^{N}\left(y_k - \hat{y}_k\right)^2 \end{aligned} \tag{33}$$

where y_k = actual closing price on *k*th day

$\hat{y}_k$ = predicted closing price on *k*th day

N= number of test data.

Further for choosing the best model for improving wind speed and wind power forecasting performance the correlation coefficient (CC^2) is calculated. The CC^2 gives the amount of correlation of the measured values with the targeted values and defined as follows (N =total number of patterns or samples of the time series data):

$$CC^2 = \frac{\left(N\sum_{i=1}^{N} \hat{y}_i \, y_i - \sum_{i=1}^{N} \hat{y}_i \sum_{i=1}^{N} y_i \right)^2}{\left(N\sum_{i=1}^{N} (\hat{y}_i)^2 - (\sum_{i=1}^{N} \hat{y}_i)^2 \right)\left(N\sum_{i=1}^{N} y_i^{\,2} - (\sum_{i=1}^{N} y_i)^2 \right)} \tag{34}$$

Another statistic with values closer to zero indicates good forecasting accuracy and is known as Theil statistic expressed in the form as:

$$U = \frac{\sqrt{\sum_{i=1}^{N} (y_i - \hat{y})^2}}{\sqrt{\sum_{i=1}^{N} y_i^2} + \sqrt{\sum_{i=1}^{N} (\hat{y}_i)^2}} \tag{35}$$

5. Applications of LRNFIS to Real World Time Series Databases

For implementing the proposed LRNFIS to forecast the day-ahead values of the financial and energy market time series, the entire data set belonging to each category is normalized to lie within the interval [0, 1]. Further both the energy market and the financial market data are also divided into three sets namely the training set, validation set, and testing set. Normally 50% of the total data set is used for training, 20% for validation, and the rest 30% for testing. Also for comparison the widely used radial basis function neural network (RBFNN) and adaptive neuro-fuzzy information system (ANFIS) using IFFHS evolutionary algorithm are considered for providing the day-ahead forecast. implementing the IFFHS algorithm, the following parameters are selected:

G=100, M=20, $\gamma = 0.5$, $\beta_0 = 0.5, \beta_{\max} = 0.95, \beta_{\min} = 0.3, PAR_{\max} = 0.9, PAR_{\min} = 0.3, HMCR = 0.8$, $BW_{\max} = 0.5, BW_{\min} = 0.2.$ For validating the forecasting performance the highly fluctuating electricity price data in a deregulated power pool that exhibits a number of sudden spikes in the data is considered first.

Case Study-1: PJM Electricity Price Forecasting

The experiment is performed on the PJM price series that contains 8760 hourly integrated day-ahead LMP (locational marginal price) data from the most recent year 2015 (from January 1, 2015 to

December 31, 2015). Keeping in view the seasonal variations in PJM market, the day-ahead weekly forecasting is conducted on each first week (day 1-7) of the months of February, May, August, and November in the year 2015. The electricity price series is divided into training validation, and testing sets, where 500 data samples are used from the first day of January 2015, and 200 data samples for validation and the rest for testing. For processing the electricity price time series data on a day-ahead basis , the return price series is considered as input to the LRNFIS as it exhibits efficient statistical properties with bounded fluctuations. The single-period continuously compounded return series known as log-return series is defined in eq.(33) as:

$$R_t = \ln(P_t) - \ln(P_{t-1}) = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{36}$$

where R_t is the single-period log return at time t, P_t is the electricity price at time t.

The original price of the PJM market is shown in Fig.3 (a) and the corresponding return series is shown in Fig.3 (b). The past return series data are considered in the following manner defined in eq.(37) for day-ahead electricity price forecasting. The electricity price to forecast at time '*t*' , depends on the past return data prices of R_{t-1}, R_{t-2}, R_{t-23}, R_{t-24}, R_{t-47}, R_{t-48},….., R_{t-167}, R_{t-168} (up to one week or 168 hours) and these data samples and is given by

$$P_t = f\ (R_{t-1}, R_{t-2}, R_{t-23}, R_{t-24}, R_{t-47}, R_{t-48}, \ldots\ldots\ldots\ldots R_{t-167}, R_{t-168}) \tag{37}$$

where *f* represents a nonlinear function of the price return series inputs. For highly fluctuating time series the number of lag terms in eq. (36) is determined by taking the autocorrelation and partial autocorrelation of lag *k* for the return price series (*d* signifies the day of forecast):

$$ACF_k = \frac{\sum_{t=1}^{T-k}(P_{t,d} - \overline{P_d})(P_{t+d,k} - \overline{P_d})}{\sum_{t=1}^{T-k}(P_{t,d} - \overline{P_d})^2} \tag{38}$$

Using the autocorrelation function (ACF) and partial autocorrelation function (PACF), the order of the return price series order is thus identified. In the return series the ACF is observed to die out more quickly with 95% confidence interval. With single period compounded log return series, it is seen that the both the ACF and PACF die out after lag 2 with a damping sine-cosine curve. Therefore, keeping in view of stationary relationship, the price time series is processed with inputs from return series. As the evolutionary algorithms are based on randomness, the performance metrics are observed from ten independent executions of the algorithm with same training and testing sets. The stability of prediction is ascertained from the convergence characteristics shown in Fig.4 for both the FF and IFFHS algorithms. The LRNFIS trained by conventional FF algorithm that takes nearly 60 iterations to converge, whereas the IFFHS algorithm takes nearly 40 iterations with less error in the objective function.

The weekly forecast results for the first weeks of February, May, August, and November are depicted in Figs. 5(a) to 5(d), respectively using the proposed LRNFIS-IFFHS methods and three other methods such as LRNFIS optimized with FF algorithm (LRNFIS-FF), ANFIS optimized with IFFHS (ANFIS-IFFHS) and finally RBFNN optimized with IFFHS (RBFNN-FFHS). The evaluated MAPE values from experiments for the considered weeks during testing are shown in Table-1. Also from the graphical results, it is clearly noticed that the actual values of electricity prices in the considered week of February, August, and November 2015 are captured more accurately for day ahead forecasting experiment using the LRNFIS model optimized with IFFHS algorithm (LRNFIS-IFFHS). The model has been experimented using FF algorithm without mutation with HS algorithm (LRNFIS-FF) and verified MAPEs are given in Table-1. The weekly forecasted MAPE and RMSE for the period February 1-7 using the LRNFIS-FF and LRNFIS-IFFHS methods are 6.82% and 6.03% and 0.0065 and 0.0024, respectively. On the contrary both ANFIS-IFFHS and RBFNN-IFFHS models exhibit MAPE of 7.96% and 8.02% and RMSE of 0.0055 and 0.0054, respectively, for the same period of February 1-7. Thus comparing the performances of the four proposed prediction models for the first weeks of the months of February, May, August, and November presented in Table-1 it is observed that the proposed LRNFIS-IFFHS model depicts more robust forecasting performance in comparison to other three models.

Two performance metrics provided in eqs. (34) and (35) are used to compare the forecasting efficiency of the four depicted prediction models. The correlation parameter CC^2 and Theil statistic U are 0.995, 0.986, 0.974, 0.961, and 0.015, 0.232, 0.253, 0.266 for the four prediction models namely LRNFIS-IFFHS, LRNFIS-FF, ANFIS-IFFHS, RBFNN-IFFHS, respectively. These two parameters clearly indicate that the optimized neural models exhibit good forecasting accuracy in predicting the electricity market day-ahead prices. However, the LRNFIS prediction model outperforms all the other three models in producing excellent performance metrics.

Case Study-2: Currency Exchange Rate Forecasting

Once the LRNFIS model is established with superior performance results for the day-ahead electricity price prediction, another important financial time series known as currency exchange rates are considered for day-ahead prediction using the same model in a different noisy environment. In this case study, daily historical currency exchange rates of Australian Dollar (AUD) / US Dollar (USD), Swiss Francs (CHF) / US Dollar (USD), Mexican Pesos (MXN) / US Dollar (USD), and Brazilian Dollar (BRL) / US Dollar (USD) which are in short known as AUD/USD, CHF/USD, MXN/USD, and BRL/USD are chosen from recent years spanning January 2011 to December 2014 and each time series contains 800 data samples. For training the model, first 500 data from January 2011 to December 2012 are reserved and the rest 300 data from January 2013 to December 2014 are kept for validation and testing the model. The currency exchange rates versus the days of AUD/USD, CHF/USD, MXN/USD, and BRL/USD time series are presented in Figs. 6(a), 6(b), 6(c), and 6(d), respectively, using the proposed LRNFIS model. From the figure, it is very prominent that all the four time series are highly fluctuating in nature and show unnatural behavior throughout the years 2011 to 2014.

The performances of the day-ahead currency exchange rates of the four major currencies are measured in terms of Mean Absolute Percentage Error (MAPE), Mean Square Error (MSE), Root Mean Square Error (RMSE) as these are the well known performance evaluations measures existing in the literature. The measured experimental outcomes of the four currencies in terms of MAPE, MSE, and RMSE are depicted in Table-3 and graphical results are shown in Figs. 7 to 10, respectively, for AUD/USD, CHF/USD, MXN/USD, and BRL/USD time series. Further a comparative performance of the four models namely LRNFIS-IFFHS, LRNFIS-FF, ANFIS-IFFHS, RBFNN-IFFHS are presented in Tables 2(a) and 2(b) for currency exchange rates using prediction time horizon varying from one day-ahead to 5, 7 and 10-days ahead. From the graphical results it is quite clear that the LRNFIS shows more robust performance predicting accurately the currency exchange for the basket of currencies against USA dollars in comparison to other three models depicted earlier. However, the performance of the RBFNN is found to be the least in comparison to other two models namely the LRNFIS-FF and ANFIS-IFFHS. Further multistep ahead (5, 7, and 10-days) results for different currency markets are displayed graphically in Figs. 11(a) to 11(d) clearly validating the superior currency prediction performance of the proposed LRNFIS model using the IFFHS evolutionary algorithm. Since the single-step performance of other three algorithms is inferior to LRNFIS, their graphical comparisons are not displayed. The correlation parameter CC^2 and Theil statistic U exhibit similar values as given in the preceding section for the electricity price forecasting.

Case Study-3: Stock Price Forecasting

The LRNFIS model further proves its efficiency using two other stock market time series datasets: Standard & Poor's 500 stock market abbreviated as S&P 500, an American stock market and Nikkei

225 commonly known as Nikkei Index from Tokyo stock exchange. Each of the datasets contains 800 data covering the time span from 04 January 2011 to 10 March 2014 out of which the first 500 data from 04 January 2011 to 27 December 2012 are chosen for training the model and the rest 300 data from 28 December 2012 to 10 March 2014 are taken for validation and testing to prove the efficiency of the model. Normally for one day-ahead stock market prediction, the input data to the LRNFIS comprises past three days closing price indices along with two to three technical indicators. It is well known from stock market analysis that the technical indicators are parameters that predict the future stock market indices in a given set of market conditions. In this paper three stock market technical indicators namely the simple moving average indicator (SMA) for 5 days, its rate of change, and the relative strength indicator (RSI) are used as inputs. They are defined in the following way:

$SMA(k) == \frac{1}{5}\sum_{k=1}^{5} CP(k)$, $CP(k)$ = stock closing price for the kth day

$$\text{Moving Average rate} = \frac{SMA(k) - SMA(k-1)}{SMA(k-1)} \quad (39)$$

The relative strength index is a very popular indicator used for stock market analysis that signifies the average price changes of up days and down days. The mathematical expression for RSI is obtained as:

$$RSI(k) = \frac{100}{100 + UP/DN} \quad (40)$$

where UP represents the average price changes for up days, and DN denotes the average price change during down days over a period of k days. The graphical representation of the testing results for the models LRNFIS, RBFNN, and ANFIS with IFFHS optimization algorithm is shown in Figs. 12(a) and 12(b), respectively, for the S&P 500 and Nikkei 225. These results clearly validate the performance of the LRNFIS-IFFHS over the other models depicted earlier in case studies 1 and 2. The S&P 500 testing performance shows a MAPE of 0.6453, while that of Nikkei 225 shows a MAPE of 0.7016, respectively using the LRNFIS-IFFHS algorithm for one day-ahead prediction as shown Table-3. Further prediction performance of 5,7, and 10 days ahead are also depicted in Table-3, producing the maximum MAPE of 1.25, which is reasonably very accurate. The day-ahead forecasting metrics like the MAPE, MSE and RMSE values are displayed in Table-4 for all the hybrid neural models clearly validating the superior predictive ability of the LRNFIS-IFFHS model in comparison to other depicted models. Similar to case studies 1 and 2, the correlation parameter and Theil statistic show the highest and lowest values for LRNFIS-IFFHS model for day-ahead stock index forecasting.

6. Conclusion

The paper presents a new locally recurrent neuro-fuzzy information system (LRNFIS) for the prediction of day-ahead energy and financial market data sets. In order to provide an expanded nonlinear transformation of the inputs to handle the non-stationary time series data , the consequent part of the TSK type fuzzy rule base of the LRNFIS comprises Chebyshev polynomial functions. Further dynamic memory is provided by using locally recurrent nodes after the fuzzy output normalization layer and at the output layer allowing past and present inputs to be used for a robust prediction. A two-step learning algorithm is used for the LRNFIS, where the parameters of the fuzzy membership functions are learnt using clustering and gradient descent algorithm in the first step and an improved Firefly-Harmony search algorithm for the network weights in the second step. Three real world time series data sets namely the PJM electricity market, currency exchange market, and stock indices of New York and Tokyo stock markets are used to validate the superior performance of the proposed LRNFIS hybrid model in comparison to other three well known neural models described in the paper. Further two prediction statistics are used to compare the performance of all the models with a view to choose the best model for handling accurately the highly chaotic electricity price, and financial time series data sets.

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Table- 1 Measured RMSE and MAPE (%) of PJM Electricity Market in four different models

Period	LRNFIS-FF		LRNFIS-IFFHS		ANFIS-IFFHS		RBFNN-IFFHS	
	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
Feb 1-7	0.0074	6.82	0.0065	6.03	0.0055	7.96	0.0054	8.02
May 1-7	0.0043	4.98	0.0024	4.78	0.0054	5.92	0.0079	6.66
Aug 1-7	0.0063	5.04	0.0045	4.51	0.0079	6.28	0.0087	7.06
Nov 1-7	0.0081	6.21	0.0052	5.23	0.0076	7.06	0.057	7.95

Table-2 Measured MAPE, MSE, and RMSE for AUD/USD, CHF/USD, MXN/USD, and BRL/USD datasets in four different prediction models

(a)

LRNFIS-IFFHS			RBFNN-IFFHS			
1-day ahead prediction			1-day ahead prediction			
Currency Exchange rates against US Dollar	MAPE	MSE	RMSE	MAPE	MSE	RMSE
AUD/USD	0.5374	4.2582e-05	0.0065	0.8374	6.5032e-05	0.0181
CHF/USD	0.4532	2.9674e-05	0.0053	0.7853	6.0674e-05	0.0053
MXN/USD	0.5585	6.1859e-05	0.0079	0.9855	4.2204e-04	0.0079
BRL/USD	0.5915	6.2444e-05	0.0103	0.8815	6.5411e-05	0.0103
5-days ahead prediction			5-days ahead prediction			
AUD/USD	0.9405	7.9347e-05	0.0104	1.5503	8.1323e-05	0.0104
CHF/USD	0.8411	4.3713e-04	0.0247	1.8989	0.6512	0.0099
MXN/USD	0.8560	1.4455e-04	0.1125	1.7622	6.8085e-04	0.1125
BRL/USD	0.7992	4.5823e-04	0.0201	1.0972	6.7821e-05	0.0201
7-days ahead prediction			7-days ahead prediction			
AUD/USD	1.5065	1.0391e-04	0.0534	2.0507	0.0035	0.0304
CHF/USD	1.3963	1.0234e-04	0.0111	2.1876	0.0082	0.0415
MXN/USD	1.1312	0.0333	0.2099	1.8813	0.0333	0.1971
BRL/USD	1.4588	7.3018e-04	0.0435	1.4588	7.3018e-04	0.0642
10-days ahead prediction			10-days ahead prediction			
AUD/USD	2..0252	1.5540e-04	0.0372	3.0244	1.9237e-04	0.0336
CHF/USD	1.9305	1.49942e-04	0.0094	2.1705	1.5818e-04	0.0238
MXN/USD	2.2242	0.0597	0.2852	3.3512	0.0454	0.1859
BRL/USD	2.0266	6.7488e-04	0.1312	3.1705	0.0306	0.1307

(b)

LRNFIS-FF				ANFIS-IFFHS		
1-day ahead prediction				1-day ahead prediction		
Currency Exchange rates against US Dollar	MAPE	MSE	RMSE	MAPE	MSE	RMSE
AUD/USD	0.6343	4.3593e-05	0.0106	0.7845	6.7834e-05	0.0281
CHF/USD	0.5891	2.8377e-05	0.0053	0.6978	5.8634e-05	0.0085
MXN/USD	0.7108	7.1025e-05	0.0226	0.8896	5.0124e-04	0.0095
BRL/USD	0.7760	0.0230	0.0103	0.8018	7..9421e-05	0.0308
5-days ahead prediction				5-days ahead prediction		
AUD/USD	1.0233	1.5638e-04	0.0160	1.4651	0.5433	0.0104
CHF/USD	0.9948	4.8833e-04	0.0243	1.6158	3.1323e-04	0.0093
MXN/USD	1.0128	0.0412	0.1118	1.4123	5.5673e-04	0.2026
BRL/USD	1.0102	1.4037e-04	0.0313	1.0746	1.5912e-04	0.0423
7-days ahead prediction				7-days ahead prediction		
AUD/USD	1.8523	2.075e-04	0.0581	1.9933	0.0105	0.0315
CHF/USD	1.31.61	1.1937e-04	0.0231	1.7947	4.2426e-04	0.0714
MXN/USD	1.6674	0.1318	0.3408	1.8219	0.2233	0.2188
BRL/USD	1.8953	6.5331e-04	0.0519	1.4482	6.3995e-04	0.0667
10-days ahead prediction				10-days ahead prediction		
AUD/USD	2..6212	0.5213	0.0444	2.9793	2.0567e-04	0.0417
CHF/USD	2.2433	3.1018e-04	0.0173	2.0139	3.1453e-04	0.0441
MXN/USD	2.8563	0.0613	0.3221	3.0015	0.0658	0.1905
BRL/USD	2.8206	8.7485e-04	0.1318	2.8854	0.1522	0.0923

Table-3 Performance Evaluation in terms of MAPE, MSE, and RMSE for stocks S&P 500 and Nikkei 225 using LRNFIS-IFFHS forecasting algorithm

1-day ahead prediction			
Stock market	MAPE	MSE	RMSE
S&P 500	0.6453	2.6452e-04	0.0159
Nikkei 225	0.7016	2.6285e-05	0.0054
5-day ahead prediction			
S&P 500	0.9024	1.1327e-04	0.0105
Nikkei 225	0.9736	1.4816e-04	0.0093
7-day ahead prediction			
S&P 500	1.0254	1.5022e-04	0.0113
Nikkei 225	1.0751	0.0236	0.1032
10-day ahead prediction			
S&P 500	1.2520	1.5118e-04	0.0105
Nikkei 225	1.2065	0.0317	0.2013

Table-4 Performance Comparison among LRNFIS-IFFHS, LRNFIS-FF, RBFNN-IFFHS, ANFIS-IFFHS for S&P 500 and Nikkei 225

1-day ahead prediction						
LRNFIS-IFFHS				RBFNN-IFFHS		
Stock market	MAPE	MSE	RMSE	MAPE	MSE	RMSE
S&P 500	0.6453	2.6452e-04	0.0159	1.2648	0.0516	0.0347
Nikkei 225	0.7016	2.6285e-05	0.0054	1.6013	4.4229e-04	0.0165
LRNFIS-FF			ANFIS-IFFHS			
S&P 500	0.9424	2.3619-04	0.1416	1.0216	2..3315e-04	0.0235
Nikkei 225	1.0328	0.0315	0.2099	1.0671	3.7616e-04	0.0411

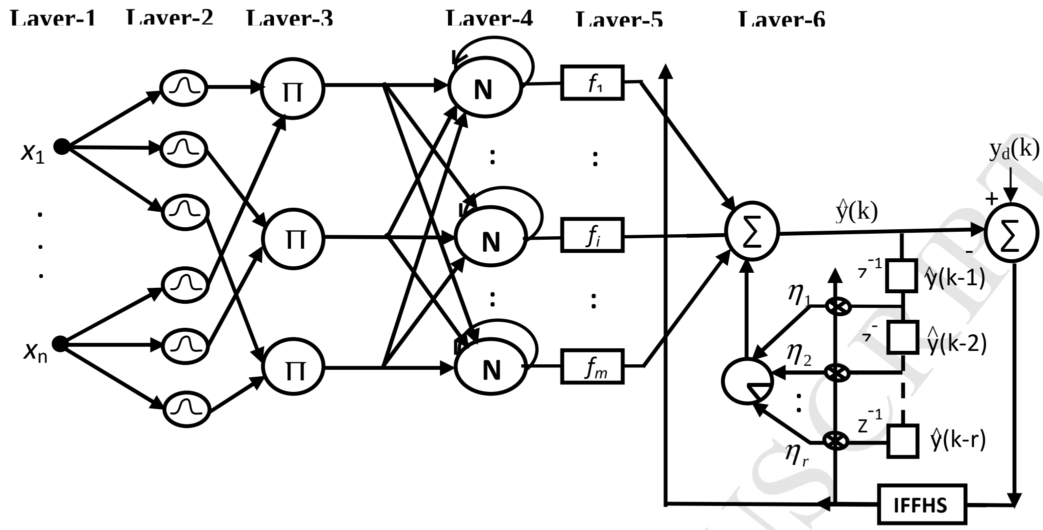


Fig.1. Locally recurrent neuro-fuzzy information system (LRNFIS)

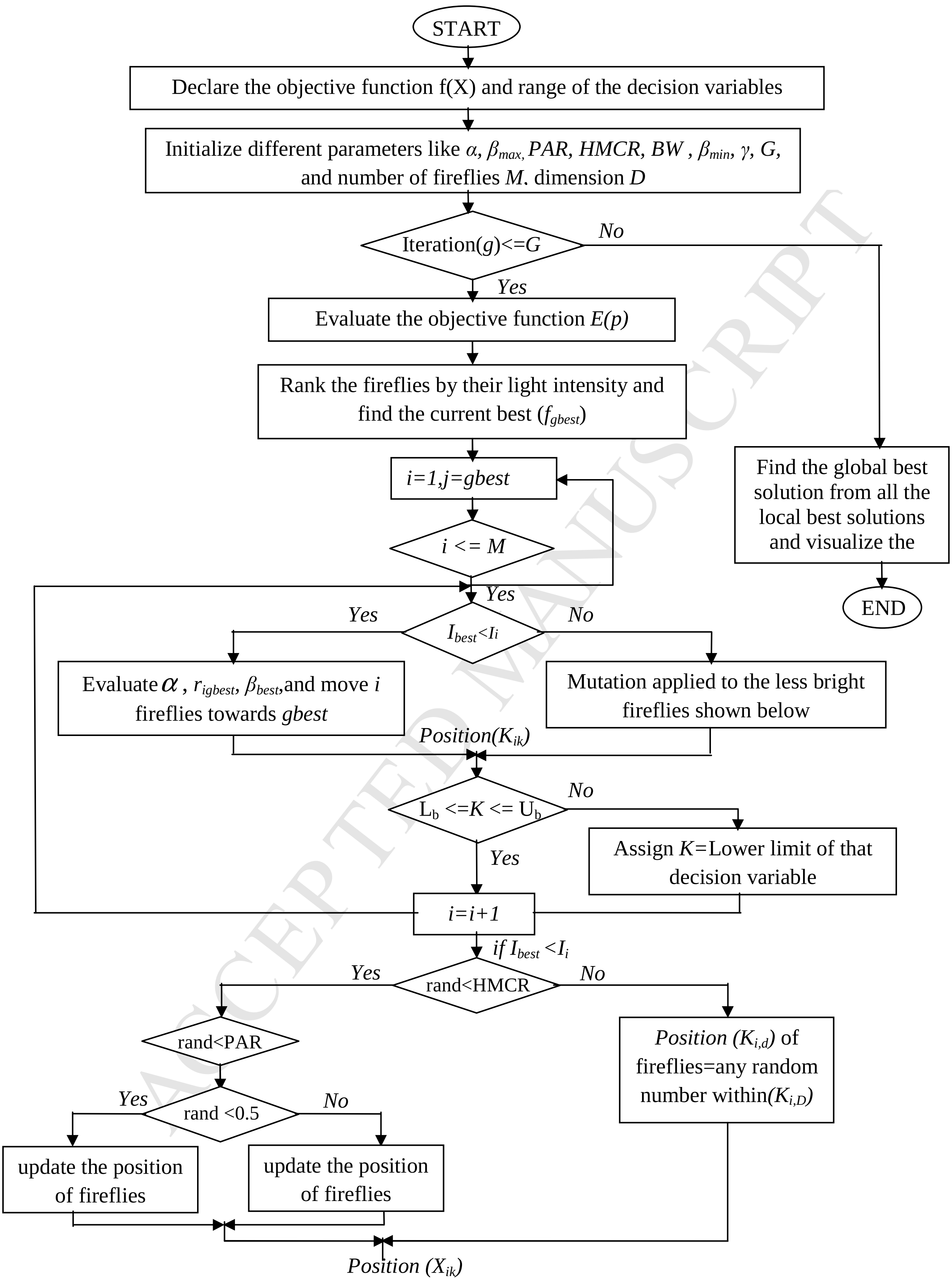


Fig.2 Flow chart for implementing the IFFHS optimization algorithm

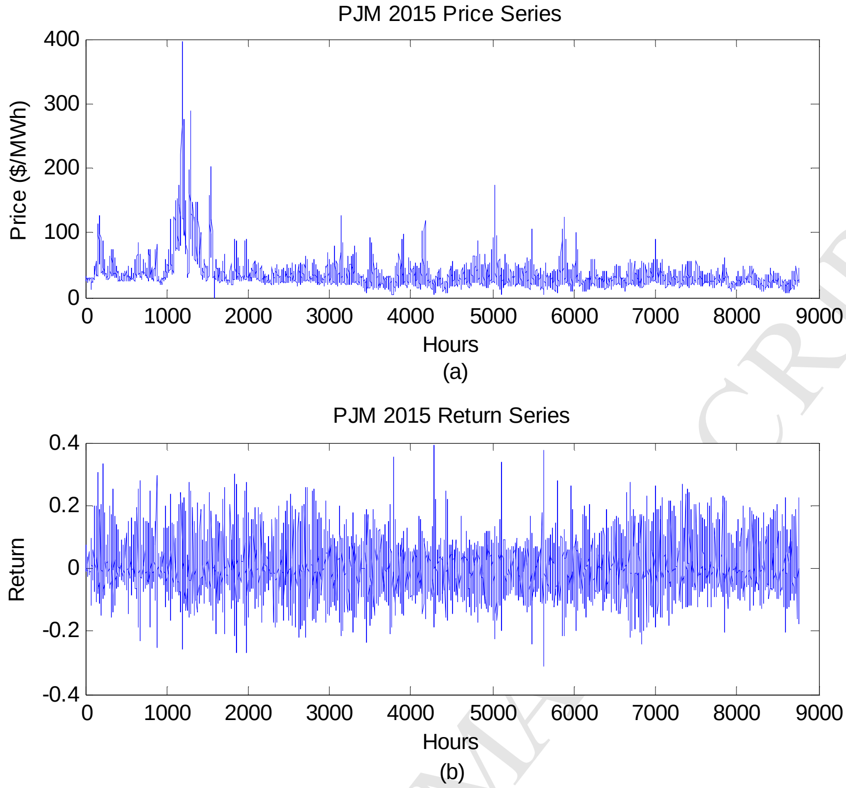
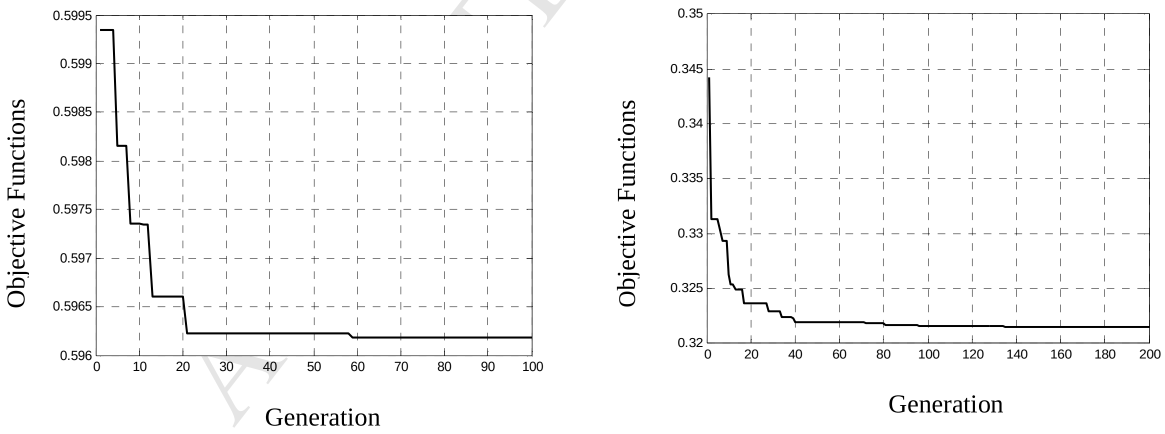


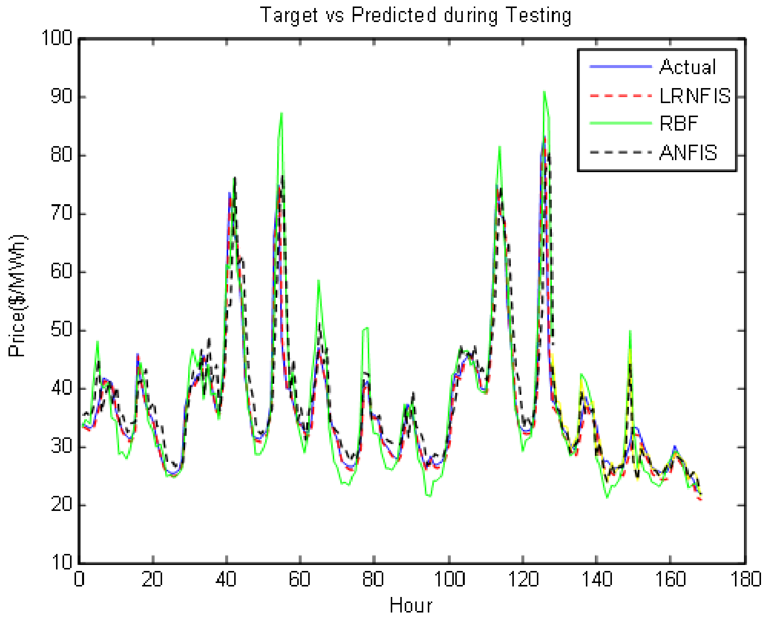
Fig.3. (a) PJM Price Series (b) PJM Return Series



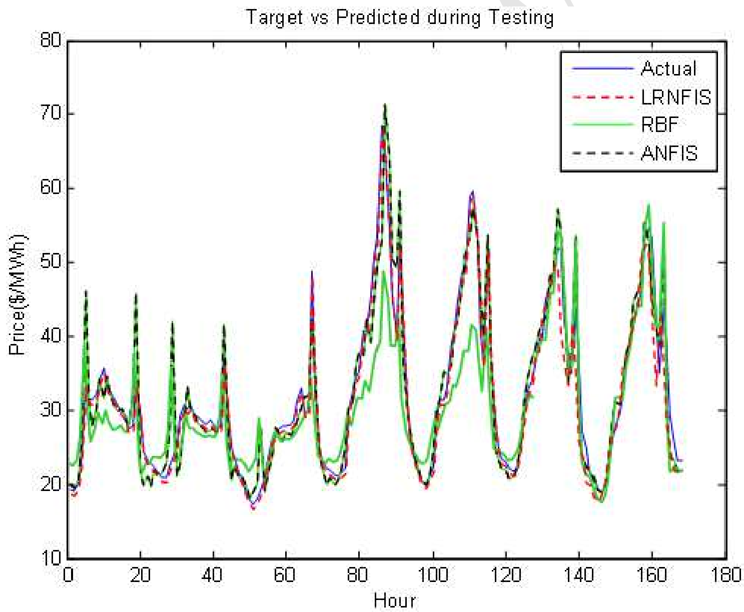
(a) With FF algorithm

(b) With IFFHS algorithm

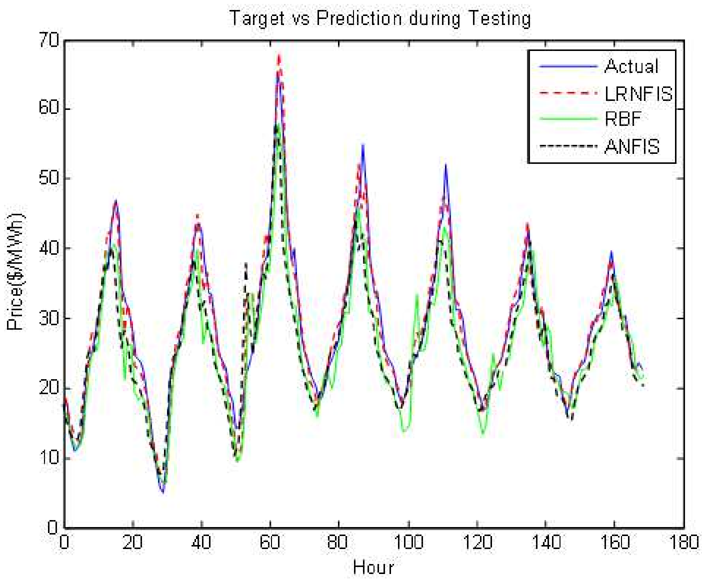
Fig. 4 Convergence characteristics of the FF and IFFHS algorithms



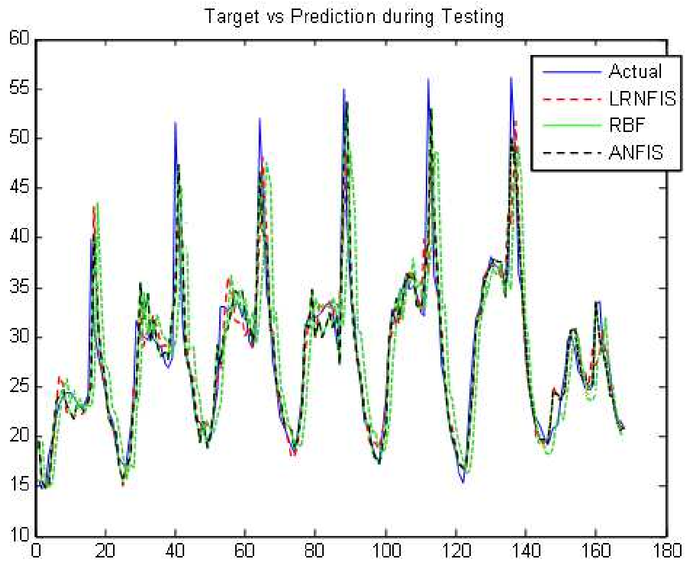
(a) Feb. 1-7



(b) May 1-7



(c) August 1-7



(d) November 1-7

Fig. 5 Forecasting comparison among LRNFIS, RBF, ANFIS optimized with IFFHS algorithm

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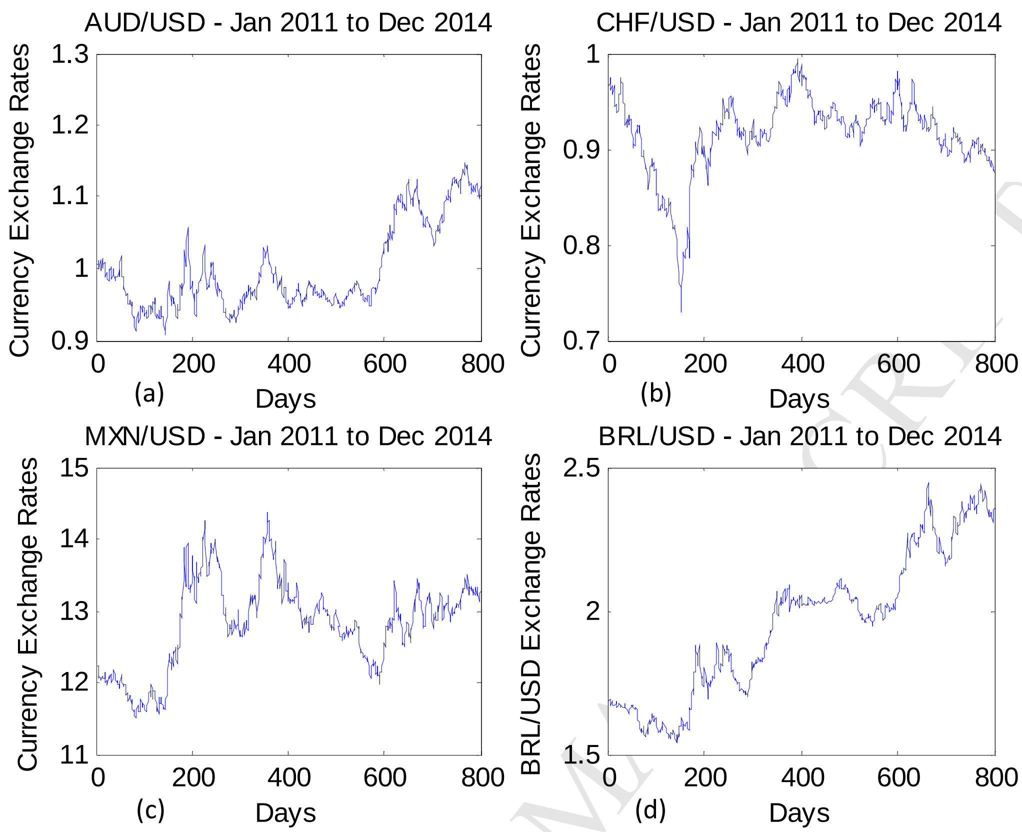


Fig. 6 AUD/USD, CHF/USD, MXN/USD, and BRL/USD time series from January 2011 to December 2014

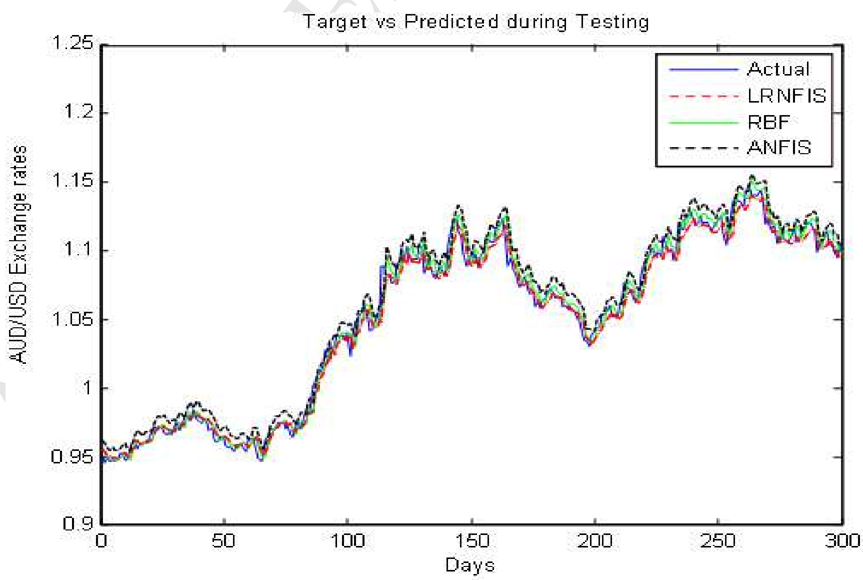


Fig.7 Comparative figures of AUD/USD using LRNFIS, RBF, ANFIS, optimized with IFFHS algorithm

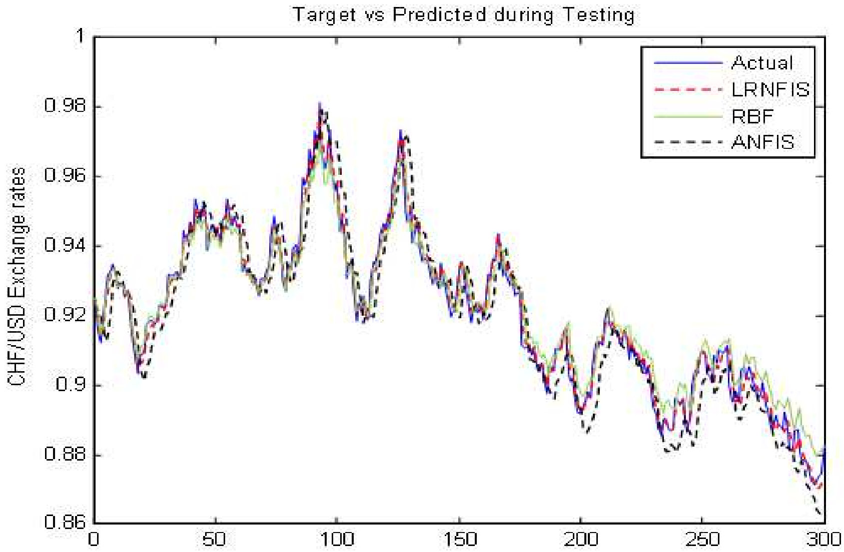


Fig. 8 Comparative figures of CHF/USD using LRNFIS, RBF, ANFIS, FLANN optimized with IFFS algorithm

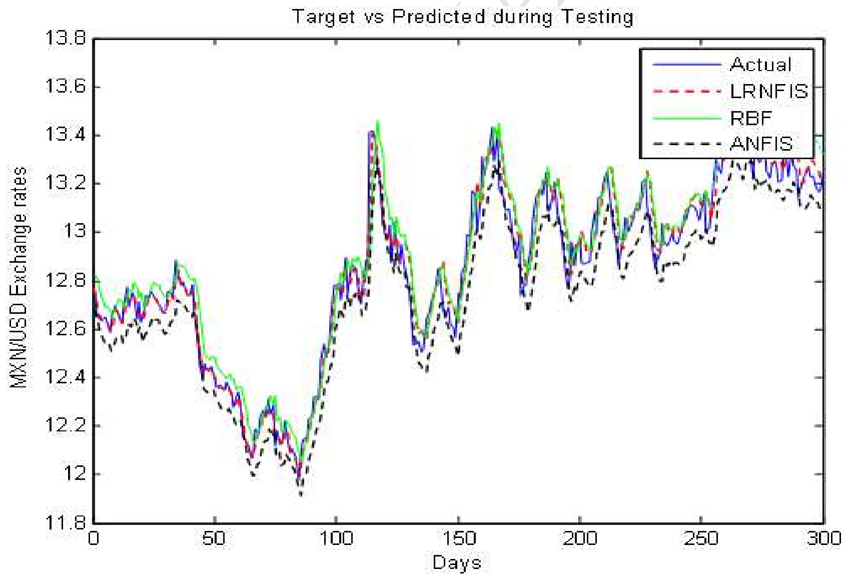


Fig. 9 Comparative figures of MXN/USD using LRNFIS, RBF, ANFIS, optimized with IFFHS algorithm

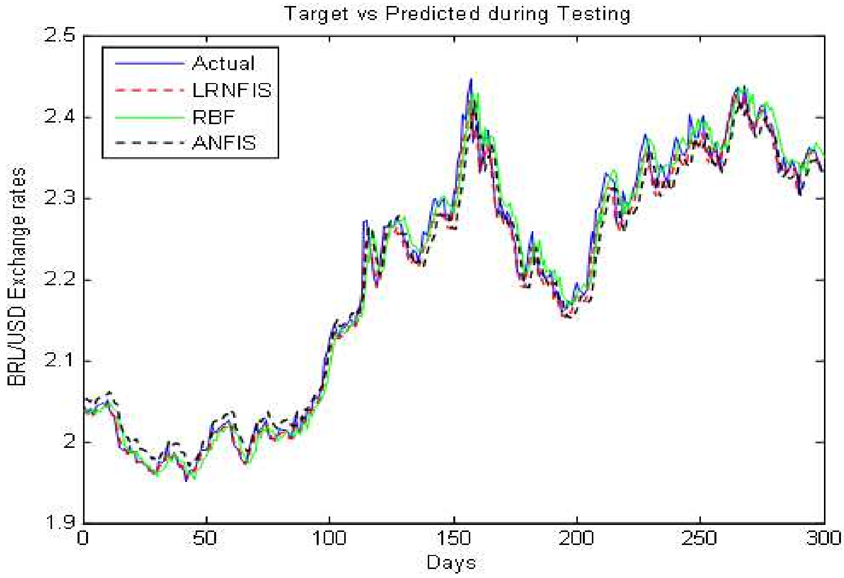
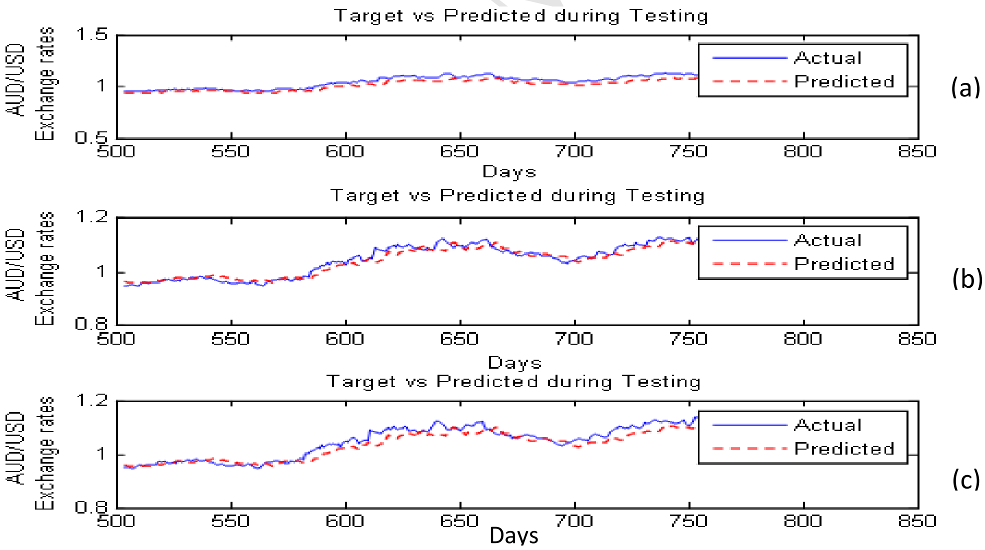
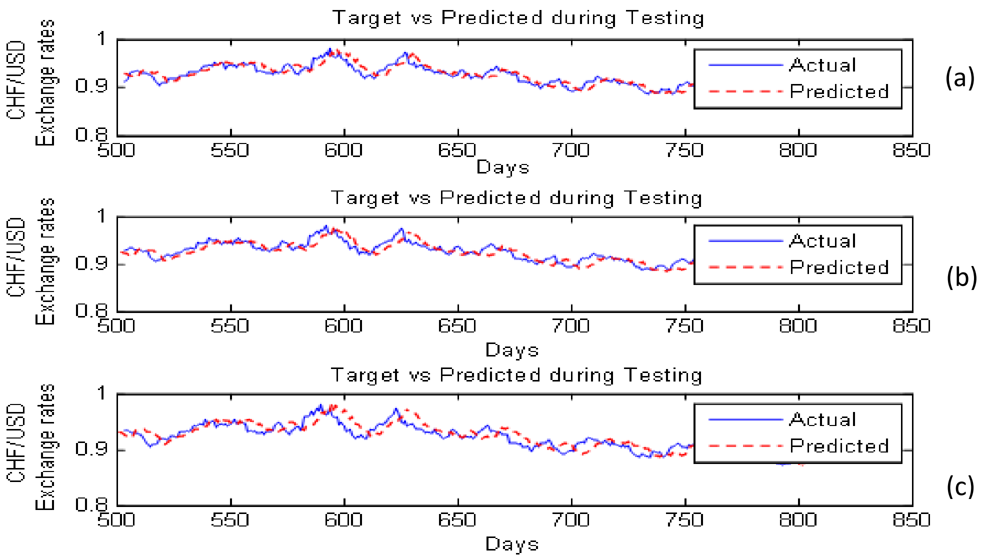


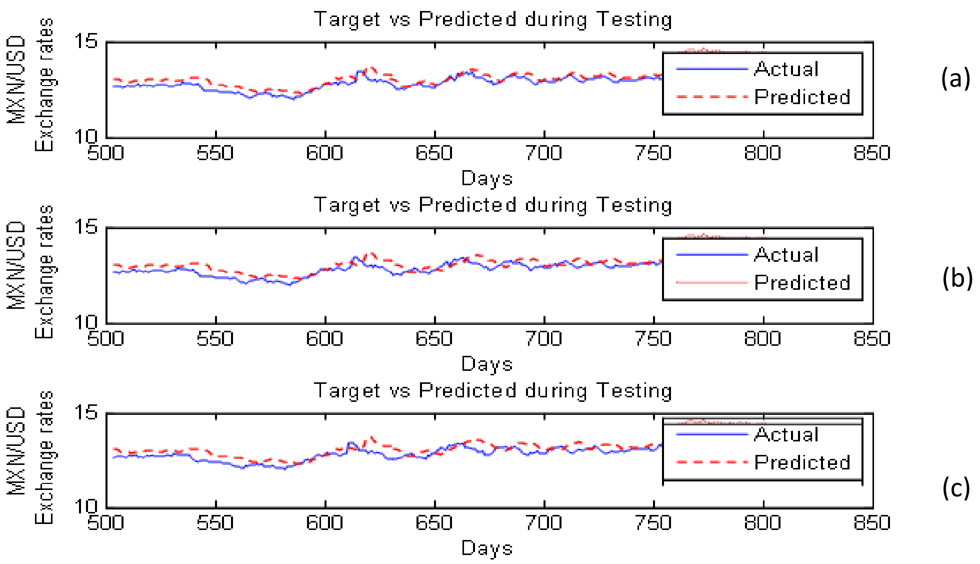
Fig. 10 Comparative figures of BRL/USD using LRNFIS, RBF, ANFIS, optimized with IFFHS algorithm



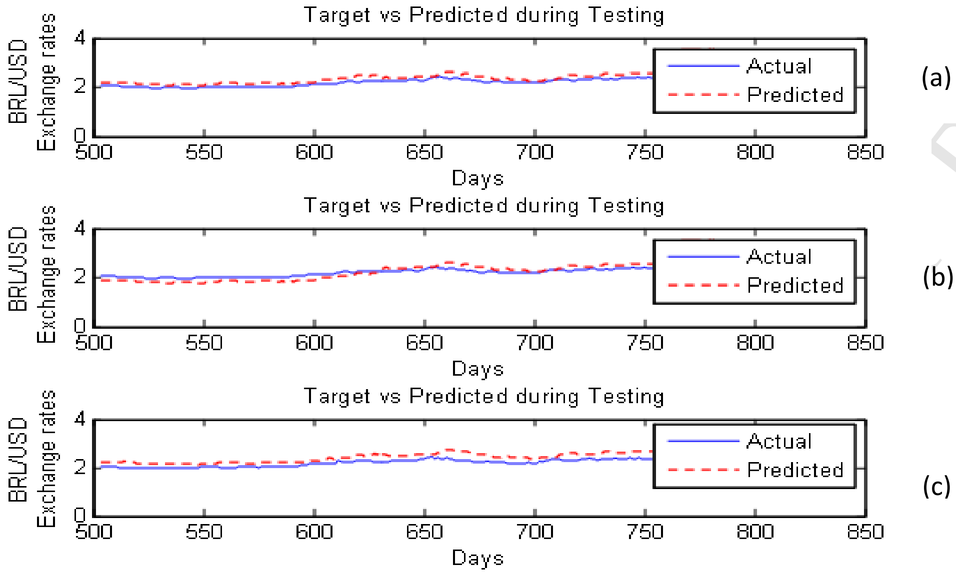
(a) 5, 7, and 10- days ahead prediction results of AUD/USD Currency Exchange Rate Series



(b) 5, 7, and 10- days ahead prediction results of CHF/USD Currency Exchange Rate Series

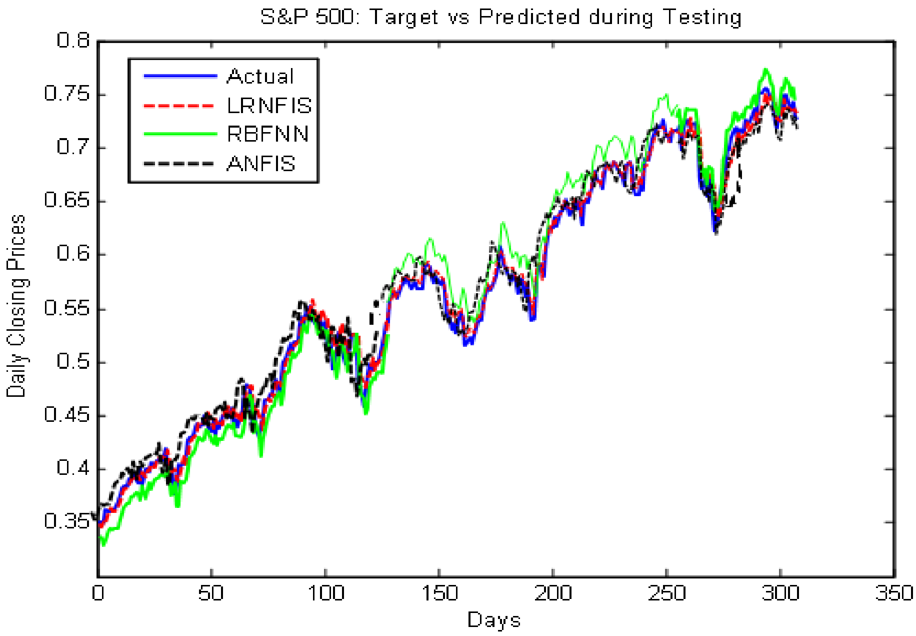


(c) 5, 7, and 10- days ahead prediction results of MXN/USD Currency Exchange Rate Series

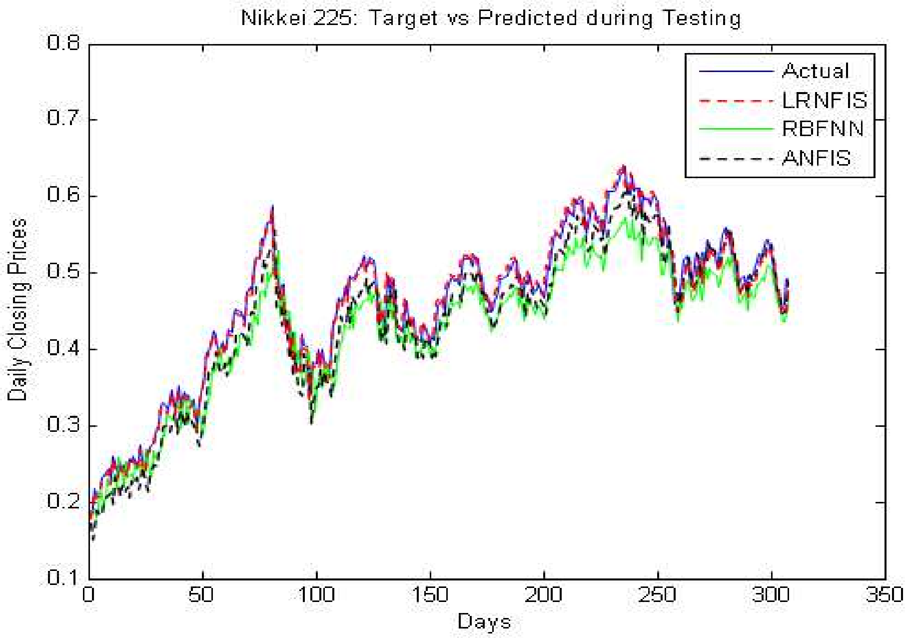


(d) 5, 7, and 10- day ahead prediction results of BRL/USD Currency Exchange Rate Series

Fig.11 Multi-step prediction of currency exchange time series using LRNFIS-IFFHS model



(a) Comparative figures of S&P 500 using LRNFIS, RBF, ANFIS, optimized with IFFHS algorithm



(b) Comparative figures of Nikkei 225 using LRNFIS, RBF, ANFIS, optimized with IFFHS algorithm

Fig.11 Forecasting performance of Different optimized Neural models