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Improving futures hedging performance using option information: Evidence from the S&P 500 index[☆]

Yujuan Bai[a], Zhiyuan Pan[a,b], Li Liu[*,c]

[a] Institute of Chinese Financial Studies, Southwestern University of Finance and Economics, China
[b] Collaborative Innovation Center of Financial Security, China
[c] School of Finance, Nanjing Audit University, West Yushan Road 86, Pukou District, Nanjing 211815 China

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ABSTRACT

Option prices contain important information about risk preferences. This study proposes an option-based model to estimate the optimal dynamic hedging ratio. Using a sample of S&P 500 index, we find that the option-implied hedging ratio has the best performance both in-sample and out-of-sample due to its relative risk aversion. This finding will help risk managers reduce their hedging risk.

1. Introduction

Derivative securities have useful applications. Futures contracts mitigate the price risks caused by spot price fluctuations for investors holding the relevant spot. In practice, the most widely used hedging strategy is based on the minimum-variance (MV) approach. The simplest methods for estimating the static optimal hedge ratio (OHR) are the ordinary-least-square (OLS) model and vector error correction model (VECM). However, neither method considers the time-varying distribution of the spot and futures price (Park and Switzer, 1995; Choudhry, 2003). Other methods for determining the dynamic OHR are the random coefficient autoregressive model (RCAM) and the generalized autoregressive conditional heteroscedasticity (GARCH) model, but studies have shown that the conditional hedging process does not always perform better than the unconditional hedging process (see, e.g., Lien and Shrestha, 2005; Wang et al., 2015).

In this study, we contribute to the literature in using the forward-looking moments contained in the option prices of spot-return distributions to estimate the spot volatility. The motivation is from DeMiguel et al. (2013) in which the portfolio selection performance is improved using option-implied volatility and skewness. Since futures hedging is a special case of portfolio allocation, we suspect that the hedging performance may be also likely to improved using forward-looking option data. We determine the spot model-free implied volatility (MFIV) and calculate the correlation between the spot and futures returns using the spot MFIV and futures volatility. The predictive content of option implied volatility for future spot volatility has been well documented in early literature for stock markets (e.g., Canina and Figlewski, 1993; Jiang and Tian, 2005) and in recent literature for commodity markets (e.g., Luo and Ye, 2015; Luo et al., 2016). Then the estimated OHR needed is computed for the MV approach.

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* Corresponding author.
E-mail address: liuli840821@126.com (L. Liu).

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We use variance reduction (VR), value at risk at the 1% quantile (VaR 1%), VaR 5%, expected shortfall at the 1% quantile (ES 1%), ES 5%, Sharp ratio (SR), and the mean-variance utility to evaluate the hedging performances of various hedging models. We find that for most measure indicators, the option-implied hedging models have better in-sample and out-of-sample performances than non-option-implied hedging models. We further perform the analysis by considering the transaction costs, and find that the empirical results are similar.

The remainder of this paper is organized as follows: Section 2 briefly describes the data. Section 3 shows econometric models for futures hedging. We report the empirical results in Section 4 and conclude the paper in the last section.

2. Data

Our sample covers the period from September 11, 2006 to April 29, 2016. The daily spot and futures price data are collected from Datastream. Each series contains 2425 observations. The option data are downloaded from IvyDB OptionMetrics. We use the out-of-money call and put options with the standard maturity of 30 days.[1] Here, we have 13 implied volatility points every day. In addition, we use the risk-free interest rate from the zero-coupon bond yields.

3. Methodology

To determine OHR, it is necessary to minimize the variance of the spot and futures portfolio. Therefore, the hedge ratio is defined as

$$h_t = \frac{Cov(\Delta s_t, \Delta f_t)}{Var(\Delta f_t)} \tag{1}$$

where the spot and futures returns are represented by Δs_t and Δf_t.

In this study, we use several methods to obtain the OHR. For static OHRs, we use the OLS technique and VECM. To realize the dynamic OHRs, we use the RCAM and multivariate GARCH models. For the RACM, we estimate the parameters using the nonlinear maximum-likelihood method to obtain the OHRs. There are three multivariate GARCH models that can compute the dynamic OHRs. The general specification of GARCH is

$$\begin{bmatrix} \Delta s_t \\ \Delta f_t \end{bmatrix} = \mu_t + \epsilon_t, \tag{2}$$

where

$$\epsilon_t | I_{t-1} \sim \mathcal{N}(0, H_t), \tag{3}$$

$$H_t = D_t R_t D_t = \begin{bmatrix} H_{ss,t} & H_{sf,t} \\ H_{fs,t} & H_{ff,t} \end{bmatrix}, \tag{4}$$

$\epsilon_t = [\epsilon_{s,t}, \epsilon_{f,t}]'$ and $\mu_t = [\mu_{s,t}, \mu_{f,t}]'$ denotes a vector of conditional mean value and H_t is the covariance matrix. In this model, the time-varying hedge ratio can be computed by

$$h_{t|t-1} = \frac{H_{sf,t}}{H_{ff,t}}. \tag{5}$$

We consider three models accounting for TVP hedge ratio. The first model is the constant conditional correlation GARCH (CCC-GARCH) of Bollerslev (1990). The content can be expressed as

$$D_t = diag(\sqrt{d_{ss,t}}, \sqrt{d_{ff,t}}), \quad \text{where} \quad d_{ii,t} = \omega_i + \beta_i d_{ii,t-1} + \alpha_i \epsilon_{i,t-1}^2, \ i \in (s, f), \tag{6}$$

and

$$R_t = R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad \text{where} \quad \rho = \frac{Cov(\epsilon_{s,t}, \epsilon_{f,t})}{std(\epsilon_{s,t}) std(\epsilon_{f,t})} \tag{7}$$

The second model is the dynamic conditional correlation GARCH (DCC-GARCH) of Engle (2002). It allows the conditional correlation to change over time. The specification of DCC is given by

$$R_t = diag(Q_t)^{-1/2} \cdot Q_t \cdot diag(Q_t)^{-1/2}, \tag{8}$$

where

$$Q_t = (1 - \delta_1 - \delta_2)\overline{Q} + \delta_1 u_{t-1} u_{t-1}' + \delta_2 Q_{t-1}, \tag{9}$$

[1] For call options implied volatilities, the relevant deltas are smaller than or equal to 0.5. For the call options implied volatilities, the relevant deltas are bigger than −0.5.

and $u_{i,t} = \epsilon_{i,t}/\sqrt{d_{ii,t}}$ for $i = s, f$.

The third model is the asymmetric DCC GARCH (ADCC-GARCH) of Cappiello et al. (2006). It adds asymmetry to the dynamic process of covariance matrix, as follows:

$$Q_t = (1 - \delta_1 - \delta_2)\overline{Q} - \delta_3\overline{\Gamma} + \delta_1 u_{t-1}u'_{t-1} + \delta_2 Q_{t-1} + \delta_3 n_{t-1}n'_{t-1}, \tag{10}$$

where $n_t = I(u_t < 0) \otimes u_t$ shows the models asymmetry.

In the next subsection, we describe in detail the option-implied hedging process that uses the implied volatility information from the option data. We adopt the MFIV put forward by Bakshi et al. (2003) to represent stock index volatility. We use MFIV rather than the CBOE's volatility index (VIX) based on SPX because of the latters biases. First, truncation errors are introduced by the finite availability of strike prices, especially very low and very high strike prices. Second, discretization errors are introduced by the numerical integration method used to approximate the coarse integral prices of options from a limited set of discrete strike prices. Third, the Taylor series expansion of the log function that is used to calculate the VIX introduces errors. Just as in Jiang and Tian (2005), to offset these problems, an interpolation-extrapolation approach can be used to construct the implied volatility.

The detailed steps for MFIV are discussed below. $S(t)$ is the stock index price at time t, and $R(t, \Delta t)$ denotes the Δt-period log return given by the log price series relative. The Δt-period fair value of the variance contract of the stock index can be calculated by

$$MFIV(t, \Delta t) = \sqrt{V(t, \Delta t)} \tag{11}$$

and

$$V(t, \Delta t) = E_t^Q[e^{-r\Delta t}R(t, \Delta t)^2], \tag{12}$$

where $E_t^Q[\cdot]$ is the expectation operator under risk-neutral density.

The first step is to standardize the strike prices, that is, the moneyness, given by

$$m_t = \frac{\log(S(t)e^{r\Delta t}) - \log(K(t))}{\sqrt{\Delta t}} \tag{13}$$

The second is to obtain the continuum implied volatility using the interpolation-extrapolation method. At each time t, we interpolate only 13 implied volatilities and strike prices inside the available moneyness range using cubic splines. We use the last known value to fill in 1001 grid points[2] in the moneyness range from the minimum available moneyness to the maximum available moneyness.

Next, we calculate the call and put option prices from the 1001 interpolated volatilities and moneynesses. For $K(t) < S(t)$,

$$P(t, \Delta t, K(t)) = K(t)e^{-r\Delta t}[1 - \mathcal{N}(d_2)] - S(t)[1 - \mathcal{N}(d_1)], \tag{14}$$

where

$$K(t) = S(t)e^{r\Delta t - m_t\sqrt{\Delta t}}, \tag{15}$$

$$d_1 = \frac{\log(S(t)/K(t)) + (r + \sigma^2/2)\Delta t}{\sigma\sqrt{\Delta}} = \frac{m_t}{\sigma} + \frac{\sigma}{2}\sqrt{\Delta t}, \tag{16}$$

and

$$d_2 = d_1 - \sigma\sqrt{\Delta t}. \tag{17}$$

For $K(t) > S(t)$,

$$C(t, \Delta t, K(t)) = S(t)\mathcal{N}(d_1) - K(t)e^{-r\Delta t}\mathcal{N}(d_2). \tag{18}$$

Then, we use option prices to compute the MFIV by discretizing the respective integrals. We can recap the MFIV as follows:

$$\begin{aligned} V(t, \Delta t) = &\int_{S(t)}^{\infty} \frac{2(1 - \log(K(t)/S(t)))}{K(t)^2} C(t, \Delta t, K(t))dK \\ &+ \int_0^{S(t)} \frac{2(1 + \log(S(t)/K(t)))}{K(t)^2} P(t, \Delta t, K(t))dK. \end{aligned} \tag{19}$$

On this basis, we still use a GARCH model to determine the OHRs. The option-implied hedging strategies are different from the previously discussed hedging strategies because the variance of spot return and the correlation between the spot and futures returns are interrelated through the option-implied moments. We no longer assume that spot variance is a standard GARCH process. Instead, it is an option-implied variance, that is, $V(t, \Delta t)$. The correlation matrix is also related to the $V(t, \Delta t)$. We define these altered models as option-implied GARCH processes (OCCC-GARCH, ODCC-GARCH, and OADCC-GARCH).

[2] We choose 1001 grid points because a wider grid gives a similar result and a narrower grid leads to a deterioration in accuracy.

Table 1
In-sample hedging performance without transaction cost.

	Hedge ratio	VR	VaR (1%)	VaR (5%)	ES(1%)	ES(5%)	SR	Utility(%)
OLS	0.9213	0.8178	0.4889	0.3450	0.5595	0.4330	−0.8331	−0.0108
VECM	0.7984	0.8032	0.5528	0.3896	0.6329	0.4894	−0.7980	−0.0096
RACM	0.8847	0.8586	0.4536	0.3188	0.5197	0.4013	−0.9386	−0.0036
CCC	0.9227	0.8077	0.5180	0.3655	0.5928	0.4588	−0.8117	−0.0112
DCC	0.8462	0.8227	0.5138	0.3644	0.5790	0.4478	−0.8428	−0.0086
ADCC	0.8466	0.8228	0.5130	0.3639	0.5782	0.4472	−0.8430	−0.0087
OCCC	1.1516	0.4154	0.4807	0.3390	0.5502	0.4257	−0.4651	−0.0386
ODCC	0.9512	0.5797	0.4746	0.3291	0.5460	0.4181	−0.5480	−0.0265
OADCC	0.9796	0.5419	0.5425	0.3848	0.6115	0.4729	−0.5241	−0.0285

Note: The hedging evaluation measures are described below. VR is defined as $VR = 1 - \frac{Var(r_p)}{Var(r_s)}$.
VaR is described as $VaR_\alpha(r_p) = -\inf_{VaR}[Prob(r_p \geq VaR) \leq \alpha] = -Q_{r_p}(\alpha)$, where α denotes probability and $Q_{r_p}(\alpha)$ is the quantile function of r_p. As in Fan et al. (2016), the expected shortfall (ES) is calculated using the Riemann sum of the estimated $Q_{r_p}(\alpha)$, that is, $ES_p(r_p) \approx -\frac{1}{\alpha}\sum_{\alpha_i \leq \alpha} \hat{Q}_{rp}(\alpha_i)[\alpha_i - \alpha_{i-1}]$, where $[\alpha_i - \alpha_{i-1}]$ is an interval of $(0, \alpha]$, and α is equal to 1% and 5%. In addition, SR_{r_p} is the sharp ratio of the hedged ratio. We also use the mean-variance utility function, $U = E(r_p) - \gamma Var(r_p)$, to measure the hedging performance. As a rule of thumb, we make the coefficient γ equal to 4.

Table 2
In-Sample Hedging Performance with transaction cost.

	Hedge ratio	VR	VaR (1%)	VaR (5%)	ES(1%)	ES(5%)	SR	Utility(%)
OLS	0.9213	0.8178	0.4889	0.3450	0.5595	0.4330	−0.8331	−0.0108
VECM	0.7984	0.8032	0.5528	0.3896	0.6329	0.4894	−0.7980	−0.0096
RACM	0.8847	0.8586	0.4540	0.3194	0.5201	0.4018	−0.9403	−0.0045
CCC	0.9227	0.8077	0.5188	0.3663	0.5936	0.4596	−0.8131	−0.0120
DCC	0.8462	0.8228	0.5117	0.3625	0.5801	0.4490	−0.8458	−0.0102
ADCC	0.8466	0.8228	0.5116	0.3624	0.5800	0.4489	−0.8459	−0.0102
OCCC	1.1516	0.4153	0.4825	0.3408	0.5520	0.4275	−0.4669	−0.0405
ODCC	0.9512	0.5797	0.4768	0.3311	0.5483	0.4202	−0.5498	−0.0280
OADCC	0.9796	0.5419	0.5410	0.3833	0.6136	0.4750	−0.5261	−0.0302

Note: The hedging evaluation measures are described below. VR is defined as $VR = 1 - \frac{Var(r_p)}{Var(r_s)}$.
VaR is described as $VaR_\alpha(r_p) = -\inf_{VaR}[Prob(r_p \geq VaR) \leq \alpha] = -Q_{r_p}(\alpha)$, where α denotes probability and $Q_{r_p}(\alpha)$ is the quantile function of r_p. As in Fan et al. (2016), the expected shortfall (ES) is calculated using the Riemann sum of the estimated $Q_{r_p}(\alpha)$, that is, $ES_p(r_p) \approx -\frac{1}{\alpha}\sum_{\alpha_i \leq \alpha} \hat{Q}_{rp}(\alpha_i)[\alpha_i - \alpha_{i-1}]$, where $[\alpha_i - \alpha_{i-1}]$ is an interval of $(0, \alpha]$, and α is equal to 1% and 5%. In addition, SR_{r_p} is the sharp ratio of the hedged ratio. We also use the mean-variance utility function, $U = E(r_p) - \gamma Var(r_p)$, to measure the hedging performance. As a rule of thumb, we make the coefficient γ equal to 4.

4. Results

We calculate the returns of the hedged portfolio, $r_{p,t} = r_{s,t} - \hat{h}_{t|t-1} r_{f,t}$, and the net hedged portfolio, $r_{np,t} = r_{s,t} - \hat{h}_{t|t-1} r_{f,t} - c_{t-1}$, where $r_{s,\ t}$ and $r_{f,\ t}$ are the returns on the spot and the futures at time t, respectively. Here, $c_t = c^*|h_{t|t-1} - h_{t-1|t-2}|$, where c_t is the transaction cost and c is a constant cost for each dollar of SPX futures trading.[3]

Table 1 reports the in-sample hedging performance of various hedging strategies for SPX. The OLS performs better than both the VECM and historical GARCH hedging strategies, but it does not outperform the option-implied GARCH hedging processes. We compare the performances of the historical GARCH processes and option-implied GARCH models. The OCCC-GARCH model and ODCC-GARCH model provide better performances than similar common GARCH models in terms of VaR 1%, VaR 5%, ES 1%, ES 5%, and SR. The only exception is that the OADCC-GARCH model has slightly worse performance than the ADCC-GARCH model. The historical GARCH models perform better than the option-implied GARCH models in terms of VR and utility. This may be attributed to the indeterminate nature of the option information.

We also consider trade cost in our study. Table 2 reports the in-sample hedging performance for SPX when trade cost is considered. The results in Table 2 are similar to those in Table 1. The main difference between the results in Tables 1 and 2 is that the absolute values of all of the measures in Table 2 are slightly larger than those in Table 1.

We use the moving window method to measure the out-of-sample hedging performance. In our study, the in-sample size is 1675 and the out-of-sample size is 750. Table 3 summarizes the different models' out-of-sample hedging performance. For the constant hedge ratio, the hedge ratios of the OLS and VECM perform worse than most of the GARCH models for all of the measures. The results

[3] Following the Futures Trading Regulations, we choose c to be 3 basis points.

Table 3
Out-of-sample hedging performance without transaction cost.

	Hedge ratio	VR	VaR (1%)	VaR (5%)	ES(1%)	ES(5%)	SR	Utility(%)
OLS	0.9082	0.8971	0.5173	0.3650	0.5921	0.4582	−0.3143	−0.0003
VECM	0.7898	0.8757	0.5219	0.3669	0.5979	0.4617	−0.2706	0.0037
RACM	0.8924	0.8946	0.5179	0.3655	0.5927	0.4588	−0.3116	−0.0006
CCC	0.8981	0.8838	0.5374	0.3788	0.6152	0.4758	−0.2909	0.0007
DCC	0.9180	0.8907	0.4260	0.3017	0.4870	0.3778	−0.3204	−0.0048
ADCC	0.9180	0.8907	0.4362	0.3089	0.4987	0.3868	−0.3204	−0.0048
OCCC	0.6842	0.7742	0.4538	0.3110	0.5238	0.3983	−0.1354	0.0271
ODCC	0.6869	0.7860	0.4252	0.2909	0.4911	0.3731	−0.1402	0.0269
OADCC	0.6828	0.7838	0.4041	0.2759	0.4670	0.3543	−0.1381	0.0275

Note: The hedging evaluation measures are described below. VR is defined as $VR = 1 - \dfrac{Var(r_p)}{Var(r_s)}$.
VaR is described as $VaR_\alpha(r_p) = -\inf_{VaR}[Prob(r_p \geq VaR) \leq \alpha] = -Q_{r_p}(\alpha)$, where α denotes probability and $Q_{r_p}(\alpha)$ is the quantile function of r_p. As in Fan et al. (2016), the expected shortfall (ES) is calculated using the Riemann sum of the estimated $Q_{r_p}(\alpha)$, that is, $ES_p(r_p) \approx -\frac{1}{\alpha}\sum_{\alpha_i \leq \alpha} \widehat{Q}_{r_p}(\alpha_i)[\alpha_i - \alpha_{i-1}]$, where $[\alpha_i - \alpha_{i-1}]$ is an interval of (0, α], and α is equal to 1% and 5%. In addition, SR_{r_p} is the sharp ratio of the hedged ratio. We also use the mean-variance utility function, $U = E(r_p) - \gamma Var(r_p)$, to measure the hedging performance. As a rule of thumb, we make the coefficient γ equal to 4.

Table 4
Out-of-Sample Hedging Performance with transaction cost.

	Hedge ratio	VR	VaR (1%)	VaR (5%)	ES(1%)	ES(5%)	SR	Utility(%)
OLS	0.9082	0.8971	0.5107	0.3603	0.5845	0.4523	−0.3144	−0.0003
VECM	0.7898	0.8757	0.5221	0.3670	0.5981	0.4619	−0.2708	0.0036
RACM	0.8924	0.8946	0.5183	0.3659	0.5931	0.4591	−0.3129	−0.0010
CCC	0.8981	0.8838	0.5384	0.3797	0.6162	0.4768	−0.2934	−0.00002
DCC	0.9180	0.8906	0.4268	0.3026	0.4878	0.3786	−0.3238	−0.0058
ADCC	0.9180	0.8906	0.4363	0.3093	0.4986	0.3870	−0.3238	−0.0058
OCCC	0.6842	0.7741	0.4543	0.3117	0.5242	0.3989	−0.1383	0.0259
ODCC	0.6869	0.7858	0.4261	0.2919	0.4920	0.3740	−0.1432	0.0258
OADCC	0.6828	0.7836	0.4000	0.2733	0.4622	0.3509	−0.1409	0.0263

Note: The hedging evaluation measures are described below. VR is defined as $VR = 1 - \dfrac{Var(r_p)}{Var(r_s)}$.
VaR is described as $VaR_\alpha(r_p) = -\inf_{VaR}[Prob(r_p \geq VaR) \leq \alpha] = -Q_{r_p}(\alpha)$, where α denotes probability and $Q_{r_p}(\alpha)$ is the quantile function of r_p. As in Fan et al. (2016), the expected shortfall (ES) is calculated using the Riemann sum of the estimated $Q_{r_p}(\alpha)$, that is, $ES_p(r_p) \approx -\frac{1}{\alpha}\sum_{\alpha_i \leq \alpha} \widehat{Q}_{r_p}(\alpha_i)[\alpha_i - \alpha_{i-1}]$, where $[\alpha_i - \alpha_{i-1}]$ is an interval of (0, α], and α is equal to 1% and 5%. In addition, SR_{r_p} is the sharp ratio of the hedged ratio. We also use the mean-variance utility function, $U = E(r_p) - \gamma Var(r_p)$, to measure the hedging performance. As a rule of thumb, we make the coefficient γ equal to 4.

of the analysis of the option-implied hedging strategies are particularly important. First, the OCCC-GARCH and OADCC-GARCH models dominate, respectively, the CCC-GARCH and ADCC-GARCH models for all of the indicators. For all of the evaluation measures except ES 1%, the ODCC-GARCH model performs better than the DCC-GARCH model. In all of the hedging cases, the option-implied GARCH process has better performance than the other hedging processes. Finally, all of the option-implied GARCH hedging techniques have positive utility, whereas the other models have negative utility. When trade cost is considered, we find similar results, as reported in Table 4.

5. Conclusion

In our study, we use VR, VaR 1%, VaR 5%, ES 1%, ES 5%, SR, and mean-variance utility measures to evaluate the hedging performances of unconditional hedging models, historical conditional hedging models, and option-implied conditional hedging models. Our results show that option-implied conditional hedging processes generate better in-sample performance than the relevant conditional hedging processes. In this study, we are more concerned with the out-of-sample hedging processes. For the vast majority of hedging strategies, the option-implied conditional hedging models have better out-of-sample performance than the conditional hedging models, RACM hedging model, and unconditional hedging models for all of the measures except variance reduction. Option-implied hedging strategies continue to perform when transaction costs are considered.

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