

Optimal Timing and Tilting of Equity Factors

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Aiming to optimally harvest global equity factor premiums, we investigated the benefits of parametric portfolio policies for *timing* factors conditioned on time-series predictors and *tilting* factors based on cross-sectional factor characteristics. We discovered that equity factors are predictably related to fundamental and technical time-series indicators and to such characteristics as factor momentum and crowding. We found that such predictability is hard to benefit from after transaction costs. Advancing the timing and tilting policies to smooth factor allocation turnover slightly improved the evidence for factor timing but not for factor tilting, which renders our analysis a cautionary tale on dynamic factor allocation.

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CE Credits: 1

Since the financial crisis of 2008–2009, factor investing has gained traction in the asset management industry. A key milestone was a study for the Norwegian Government Pension Fund by Ang, Goetzmann, and Schaefer (2009), which demonstrated that a large fraction of the fund's active returns can be explained by well-known factor risk premiums. The authors concluded that the fund should directly engage in systematic factor investing. Moreover, Ilmanen and Kizer (2012) argued that the alleged failure of diversification during the 2008 financial crisis was mostly a “user error”; that is, diversification based on factors would have been more effective than diversification based on traditional asset classes. However, although the value added by factor investing is now widely recognized in the literature (Clarke, de Silva, and Murdock 2005; Dimson, Marsh, and Staunton 2017; Kim, Kim, and Fabozzi 2017; Bergeron, Kritzman, and Sivitsky 2018), optimal management of factor exposure is still the subject of debate.

Given the benefits of factor diversification (Ilmanen and Kizer 2012; Bass, Gladstone, and Ang 2017), an open question is whether a forecasting-based factor allocation can add value over and above a

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diversified passive factor allocation. Notably, the quest for forecasting factor returns is almost as old as the factor literature.¹ To this day, however, researchers are divided into skeptics (Asness 2016; Asness, Chandra, Ilmanen, and Israel 2017; Lee 2017) and optimists (Arnott, Beck, Kalesnik, and West 2016; Bender, Sun, Thomas, and Zdorovtsov 2018; Hodges, Hogan, Peterson, and Ang 2017).

The skeptics argue that factor timing is difficult because factor diversification easily outperforms the potential of factor timing. Factor-timing strategies are considered too correlated with the underlying factor strategies to have a substantial impact on strategic factor allocation. Moreover, timing factors is rather costly because turning over the factor allocation, as well as rebalancing the underlying factors, involves trading a large number of long and short positions in individual securities.

The optimists acknowledge that factor prediction based on market, sentiment, and macroeconomic indicators is difficult. Yet, they believe that an investor with a long horizon and a good understanding of the investment rationale behind the factor returns will be able to overcome these obstacles.

The main contribution of our study is an exploration of the benefits of active factor allocation, which is often generally referred to as “factor timing.” Our analysis distinguishes, however, between factor timing, which seeks to exploit time-series information, and “factor tilting,” which seeks to exploit cross-sectional information. Specifically, factor timing attempts to establish factor return predictability based on the fundamental macroeconomic environment or factor-specific technical predictors. In contrast, factor tilting is essentially used to identify “factors within factors” when explaining the cross-section of factor returns based on factor characteristics such as valuation and momentum.

To arrive at meaningful factor-timing and factor-tilting allocations, we applied the parametric portfolio policy (PPP) frameworks of Brandt and Santa-Clara (2006) and Brandt, Santa-Clara, and Valkanov (2009), which naturally lend themselves to, respectively, factor timing and factor tilting. Both approaches avoid estimating the joint distribution of factor returns and instead directly determine factor allocation weights based on a set of information variables. Our analysis thus allows for a fresh take on dynamic factor allocation by assessing the joint relevance of indicators or characteristics for

an investor's portfolio utility (as opposed to simply tying a variable's importance to its accuracy in forecasting equity factor returns). The parametric factor-timing and -tilting policies can favor return-enhancing variables for a mean-variance investor but may also consider variables that predominantly help reduce overall portfolio variance.²

The PPP approach has generally been implemented on stock-level data. For instance, the original work of Brandt et al. (2009) accommodated the traditional size, value, and momentum factors, and follow-up studies have “rediscovered” established equity factors (e.g., see Hand and Green 2011 for evidence on accounting-based characteristics, such as accruals, change in earnings, and asset growth). Mimicking a classic quantitative multifactor equity approach, Ammann, Coqueret, and Schade (2016) used a mean-variance PPP under leverage constraints to exploit a set of 12 company characteristics.

Many equity factors have been put forward in the literature to explain the cross-section of equity returns, but this mounting “factor zoo” can be effectively navigated by using the PPP. DeMiguel, Martin-Utrera, Nogales, and Uppal (forthcoming) investigated up to 100 company characteristics, of which only a handful proved relevant. Such applications of the PPP are mainly geared toward jointly exploiting a large number of factors in a diversified equity portfolio; they are not aimed at explicitly timing or tilting the equity factors themselves. In this sense, our innovative application of PPPs at the factor level may help uncover new sources of equity factor return predictability both in the time series and the cross-section of equity factor returns.

Given the debate surrounding the feasibility of factor timing and tilting, accounting for all transaction costs that could inhibit capitalization of any informational advantage is crucial. Therefore, our study is not based on pure academic factors. Rather, we compiled a representative set of 20 investable, global, long-short equity factor portfolios and computed their returns net of transaction costs. Of course, dynamically allocating across these factors would create additional costs, but the use of parametric factor-timing and -tilting policies allowed us to optimally manage the trade-off between a potential factor allocation's alpha and its implementation cost.

Overall, our article provides a cautionary tale concerning the use of dynamic factor allocation. When transaction costs are ignored, optimal factor

timing with fundamental and technical time-series predictor variables produces returns that are statistically significant and more economically relevant than the returns from simply holding an equally weighted equity factor portfolio. Similarly, optimal factor tilting favors factors with positive short-term momentum but avoids factors that exhibit crowding. In both cases, however, the resulting turnover is high and transaction costs tend to erode much of the value added by factor predictability. Therefore, the trade-off between the factor allocation alpha and the cost of the associated turnover is important in our optimization of parametric timing and tilting. Taming the factor allocations by using constraints on weights, Black-Litterman (1992) shrinkage, and transaction cost penalties can salvage part of the post-transaction-cost performance for optimal factor-timing allocations but not for optimal factor-tilting allocations.

Building Investable Global Equity Factors

In this section, we discuss how we chose the equity universe and constructed the equity factors. Then, we delve into how we measured transaction costs, because they are crucial to the ultimate performance of an active strategy.

Global Equity Universe and Construction of Equity Factors. To allow for a holistic investigation of factor timing and tilting, we assembled a representative and investable set of global equity factors. These factors came from a global universe that encompassed the constituents of global and regional equity indexes from MSCI, FTSE, S&P, and STOXX. Company-specific data, such as financial statement data, came from the Worldscope database. To allow for a reasonably broad universe, even in the regional subsets, we started the sample of monthly factor returns in January 1997. The sample ends in December 2016, giving us two decades of global equity factor returns. The overall investable universe comprised roughly 4,500 stocks in December 1996; the number had increased to 5,000 stocks in December 2016. In a nutshell, our analysis is built on a global large- and midcap stock universe that is characterized by an ample degree of regional diversification and investability.³

We built on factors that are widely used and well documented in academic research. Appendix A provides a concise definition of each factor and

highlights the papers that discuss it. The factors can be roughly assigned to the following categories:

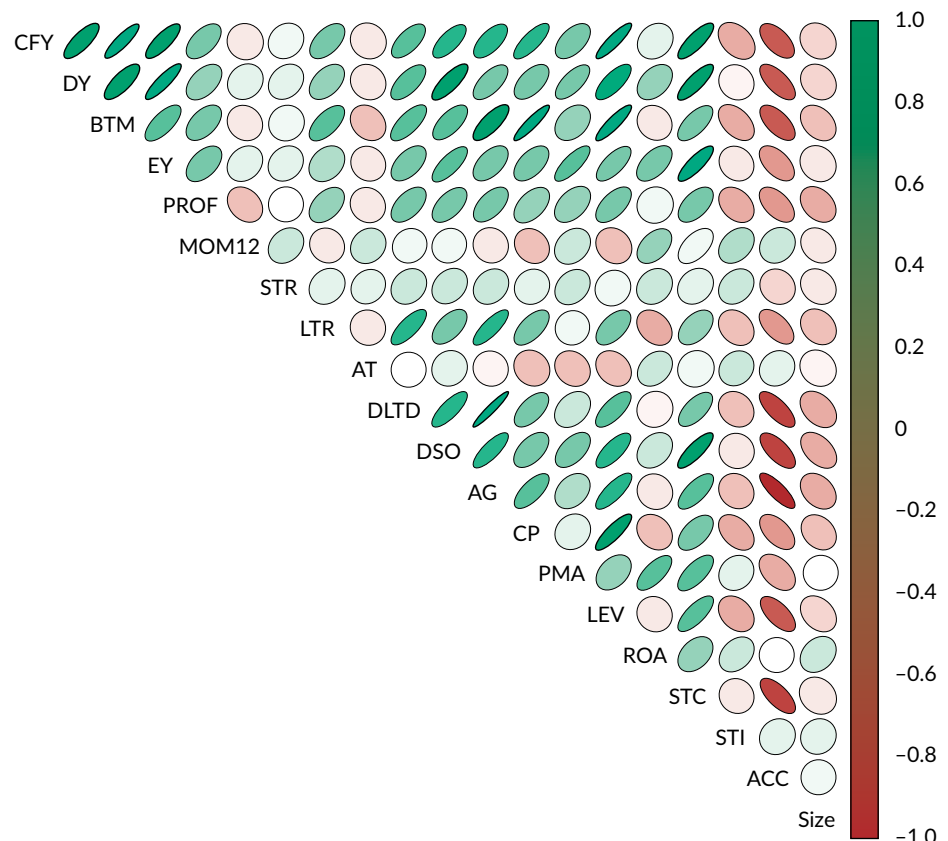
- **Value:** cash flow yield (CFY), dividend yield (DY), book-to-market ratio (BTM), earnings yield (EY), and profitability (PROF)
- **Momentum:** 12-month price momentum (MOM12), short-term reversal (STR), and long-term reversal (LTR)
- **Quality:** asset turnover (AT), change in long-term debt (DLTD), change in shares outstanding (DSO), asset growth (AG), cash productivity (CP), profit margin (PMA), leverage (LEV), return on assets (ROA), sales-to-cash ratio (STC), sales-to-inventory ratio (STI), and accruals (ACC)
- **Size:** Size

To construct a given equity factor, we sorted the global stock universe according to the factor characteristic on a monthly basis and computed quintile portfolio returns as the mean of the subsequent one-month local return of each quintile. The long-short factor return is the difference between the top-quintile portfolio's return and the bottom-quintile portfolio's return.⁴ The correlation chart in **Figure 1** shows this set of global equity factors to be fairly heterogeneous. The average equity factor correlation is 0.21, albeit factors trying to harvest the value premium display higher correlations. The pairwise correlations among CFY, DY, BTM, and EY range from 0.7 to 0.9. The Size and ACC factors add diversification potential to the factor set by exhibiting a negative correlation with most other factors.⁵

Descriptive statistics for the equity factors are presented in **Table 1**. The best-performing factor is price momentum, MOM12, with an average annualized return of 12.05%, but it was also the most volatile. ACC, STR, and STI had modest annual returns. All the factors offered a positive premium during the sample period. ACC, AT, and STI were the least volatile. The Sharpe ratios of the equity factors range from 0.05 for ACC to 1.15 for PROF.

Transaction Costs of Strategies. In evaluating the performance of active factor allocation strategies, accounting for any transaction costs incurred in their implementation is essential. In our analysis, every equity factor strategy needed to be rebalanced monthly to conform to the underlying factor characteristic. We took these factor-specific transaction costs into account when computing the equity factor returns presented in Table 1.

Figure 1. Equity Factor Correlation Matrix, January 1997–December 2016



Notes: The chart transforms positive correlations into upward-sloping ellipsoids (in green) and negative correlations into downward-sloping ellipsoids (in red). The stronger the correlation, the stronger the color shading. For correlations equal to ± 1 , the ellipsoids equate a straight line.

We assumed transaction costs of 75 bps for 100% turnover on the long side. On the short side, we added 40 bps as shorting costs. So, we assumed 115 bps for 100% turnover.⁶ Moreover, to gain exposure to a balanced portfolio of long–short equity factors, an equity swap is the typical instrument of choice. We assumed $12 \times 8 \text{ bps} = 96 \text{ bps}$ to be the annual cost of holding such a swap. (Note that these costs affect the static $1/N$ factor benchmark as well as the parametric timing and tilting strategies.) Finally, changing factor weights is costly and the major impediment to exploiting factor-allocation alpha. Therefore, as did Miller, Li, Zhou, and Giamouridis (2015), we assumed transaction costs of 20 bps for turning over 100% of the factor allocation.⁷

To summarize, we compiled a broad set of global equity factors, all of which are based on a strong economic rationale and are widely documented in the literature. A well-known phenomenon is that factor returns tend to weaken following publication of the factor strategy.⁸ Our sample period

corresponds to post-publication periods for most factors, but we refrained from further cherry-picking factors and considered the whole set of factors in our empirical analysis. Even weak factors may exhibit time variation in returns, which might be successfully navigated through active management of factor exposures.

Factor Timing

To improve performance over an equally weighted factor allocation benchmark, we implemented factor timing by relating factor returns to a variety of fundamental variables and technical indicators commonly used for predicting the equity risk premium. The idea is that the identification of good and bad times for a given factor helps to improve the risk–return profile of an equally weighted benchmark strategy.⁹ Methodologically, we operationalized the potential predictive power of time-series predictors in the PPP framework of Brandt and Santa-Clara (2006).

Table 1. Descriptive Statistics for Equity Factors, January 1997–December 2016

Factor	Return	Volatility	Min. Return	Max. Return	Max. DD	Sharpe Ratio	t-Statistic
CFY	9.69	12.61	-14.00	17.14	46.01	0.76	3.43
DY	5.63	14.08	-14.82	19.25	47.13	0.39	1.79
BTM	3.58	11.66	-14.26	16.95	46.36	0.30	1.37
EY	8.33	11.35	-9.28	14.84	35.38	0.73	3.28
PROF	7.70	6.66	-7.93	9.91	15.80	1.15	5.17
MOM12	12.05	20.21	-33.12	22.45	56.04	0.59	2.67
STR	1.94	14.56	-16.26	15.87	37.14	0.13	0.60
LTR	3.20	12.72	-11.08	16.08	38.41	0.25	1.13
AT	4.55	5.25	-3.66	4.62	12.51	0.86	3.87
DLTD	4.96	7.22	-7.34	11.71	17.23	0.68	3.07
DSO	7.28	9.02	-8.61	12.55	20.83	0.80	3.61
AG	5.96	10.04	-10.51	15.71	25.09	0.59	2.66
CP	4.09	8.19	-7.30	12.18	22.82	0.50	2.24
PMA	3.99	8.63	-8.55	9.22	29.81	0.46	2.07
LEV	3.75	13.75	-18.49	18.20	51.84	0.27	1.22
ROA	5.07	7.12	-6.87	5.49	20.52	0.71	3.19
STC	5.28	11.84	-15.07	15.15	51.29	0.44	1.99
STI	2.48	5.68	-4.20	7.64	23.92	0.43	1.95
ACC	0.29	5.70	-8.80	6.43	30.85	0.05	0.23
Size	2.97	13.56	-11.45	12.84	45.27	0.21	0.98

Notes: Annualized excess returns were calculated from the arithmetic average of simple returns. The standard deviation and Sharpe ratio were annualized through multiplication by $\sqrt{12}$. The minimum return and maximum return denote, respectively, the lowest and highest monthly excess return in the sample period. "Max. DD" is the maximum drawdown. Return, Volatility, Min. Return, Max. Return, and Max. DD are in percentage terms.

Predictor Variables. We have divided the discussion of the predictor variables into fundamental variables and technical indicators. In this subsection, we also consider how to reduce the number of predictor variables.

Fundamental variables. We used fundamental variables that contain information about future states of the economy (see Neely, Rapach, Tu, and Zhou 2014). In particular, we used the following variables from Welch and Goyal (2008):¹⁰ dividend-to-price ratio (*dp*), dividend yield (*dy*), earnings-to-price ratio (*ep*), dividend payout ratio (*de*), stock variance (*svar*), book-to-market ratio (*bm*), net equity expansion (*ntis*), US T-bills (*tbl*), long-term yield (*lty*), long-term rate of return (*ltr*), term spread (*tms*), default

yield spread (*dfy*), default return spread (*dfr*), and inflation (*infl*).

To avoid spurious findings resulting from high autocorrelations, a useful measure is to detrend the variables (see Ferson, Sarkissian, and Simin 2003). We standardized any predictor variable *X* at time *t* by subtracting its arithmetic mean and dividing by its standard deviation. For the calculation of the mean and standard deviation, we used a rolling window covering the 12 months preceding *t*.¹¹ The current observation of *X* was omitted, which allows for stronger innovations in the standardized predictors. Furthermore, we truncated the variables at ± 5 because some standardized fundamental variables might attain extreme values.

Technical indicators. In addition to fundamental variables that capture the state of the economy, we followed Neely et al. (2014) in using factor-specific technical indicators and trading rules derived from past factor returns. Similar to Hammerschmid and Lohre (2018), we included 11 technical indicators based on two sets of trading rules related to the general concepts of momentum and moving averages. These trading rules capture the trend-following idea of technical analysis and are representative of typical rules analyzed in the literature (see Brock, Lakonishok, and LeBaron 1992; Sullivan, Timmermann, and White 1999).

1. **Momentum (MOM_m).** The momentum indicator gives a buy signal if the price at time t , P_t , is higher than the price at time $t - m$, P_{t-m} , and gives a sell signal otherwise:

$$MOM_m = \begin{cases} 1 & \text{if } P_t > P_{t-m} \\ 0 & \text{if } P_t \leq P_{t-m} \end{cases} \quad (1)$$

We computed momentum indicators for five look-back periods: $m = 1, 3, 6, 9$, and 12 months. The conjecture was that factor returns are trending in such a way that recent positive returns are followed by subsequent positive returns.

2. **Moving average (MA_{s-l}).** The moving-average indicator is based on the comparison of a short-term and a long-term moving average, which was calculated as

$$MA_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} P_{t-i} \quad \text{for } j = s, l, \quad (2)$$

where s and l are the lengths in months of the look-back periods for the short- and long-term moving averages, with $s = 1, 2, 3$ and $l = 9, 12$. The indicator gives a buy signal if the short-term moving average is greater than the long-term moving average:

$$MA_{s-l} = \begin{cases} 1 & \text{if } MA_{s,t} > MA_{l,t} \\ 0 & \text{if } MA_{s,t} \leq MA_{l,t} \end{cases} \quad (3)$$

which provided us with six moving-average indicators. A crossing of the long-term moving average from below (above) by the short-term moving average was considered an upward (downward) shift in trend.

Given that the technical indicators used up to 12 months of price data, all the technical predictors could be used only from January 1998 onward. Subsequent analyses, therefore, apply to the sample period January 1998 to December 2016.

Reducing the number of predictor variables.

Regardless of the model used to generate signals from the preceding predictors, dealing with the sheer number of predictors and the associated multicollinearity is a struggle. Indeed, we found high correlations within but not across the two sets of predictors, suggesting, at best, complementary predictive ability of technical and fundamental predictors. Therefore, we sought to reduce the number of independent predictor variables while preserving their embedded information. To this end, we separately applied principal components analysis (PCA) to the fundamental variables and the technical indicators in the spirit of Neely et al. (2014), Ludvigson and Ng (2007, 2009), and Hammerschmid and Lohre (2018). The objective was to find a reduced number of predictive factors that synthesized the relevant information of the 25 predictor variables and to eliminate the noise within the predictors. The PCA also generated orthogonal predictors so that we could avoid multicollinearity problems. In our main analysis, we used the first principal component of the fundamental variables (denoted FUN1) and the first principal component of the technical indicators (denoted TECH1). Both captured a significant proportion of variation in the underlying variables and indicators (27% and 93%, respectively).¹²

Optimal Factor Timing. Next, we examined whether a risk-averse investor could profit from timing the equity factors with respect to fundamental variables and technical indicators. To this end, we used the PPP of Brandt and Santa-Clara (2006), which relates predictive variables and investor utility in a portfolio-theoretic framework. Their approach directly translates any predictive power embedded in the PCA factors into optimal portfolio weights.

Methodology of Brandt and Santa-Clara (2006).

Brandt and Santa-Clara (2006) considered the dynamic maximization problem of a mean-variance investor who is risk averse according to a risk aversion parameter, γ , and solved

$$\max_{\mathbf{w}_t} E_t \left[\mathbf{w}_t' \mathbf{r}_{t+1} - \frac{\gamma}{2} \mathbf{w}_t' \mathbf{r}_{t+1} \mathbf{r}_{t+1}' \mathbf{w}_t \right], \quad (4)$$

where \mathbf{r}_{t+1} is the vector of future excess returns of the N equity factors and \mathbf{w}_t denotes the vector of factor portfolio weights. A key ingredient of the Brandt and Santa-Clara approach is an assumption that the optimal portfolio strategy, \mathbf{w}_t , is linear in the vector \mathbf{z}_t of the K conditioning variables (of which the first element is simply a constant):

$$\mathbf{w}_t = \boldsymbol{\theta} \mathbf{z}_t, \quad (5)$$

where $\boldsymbol{\theta}$ is an $N \times K$ matrix of parameters. Plugging the linear portfolio policy (Equation 5) into Equation 4, Brandt and Santa-Clara (2006) showed that the original optimization problem is equivalent to running the following optimization:

$$\max_{\tilde{\mathbf{w}}} E \left[\tilde{\mathbf{w}}' \tilde{\mathbf{r}}_{t+1} - \frac{\gamma}{2} \tilde{\mathbf{w}}' \tilde{\mathbf{r}}_{t+1} \tilde{\mathbf{r}}_{t+1}' \tilde{\mathbf{w}} \right], \quad (6)$$

where $\tilde{\mathbf{w}} := \text{vec}(\boldsymbol{\theta})$ and $\tilde{\mathbf{r}}_{t+1} := \mathbf{z}_t \otimes \mathbf{r}_{t+1}$.¹³ Therefore, the original dynamic optimization problem can be restated as a static (unconditional) Markowitz optimization applied to an augmented set of equity factors that includes not only the original equity factors but also synthetic or “managed” ones. Each of these managed equity factors invests in a single equity factor according to the realization of one of the conditioning variables.

In our case, the conditioning variables were the first fundamental PCA factor and the first technical PCA factor.¹⁴ We computed the first optimal portfolio weights over a 60-month window, which expanded over time, so we obtained the first portfolio for January 2003. For the risk-aversion parameter, γ , governing the quadratic utility function, we chose a value of 5, implying moderate risk aversion. The coefficients in $\boldsymbol{\theta}$ pertaining to the original equity factors were constrained in such a way that they equaled the weights of an equally weighted benchmark portfolio. Therefore, we implemented a benchmark-relative portfolio allocation where deviations from the equally weighted benchmark resulted only from changes in the conditioning variables.¹⁵ To avoid extreme allocations, we rescaled the ensuing timing portfolio weights so that they obey a maximum *ex ante* annualized tracking error of 2.5%.¹⁶

Empirical results. Table 2 shows the estimates and significance levels of the $\boldsymbol{\theta}$ coefficients for factor timing. Statistical significance was assessed in terms of confidence bands calculated as $\hat{\theta}_i \pm 1.96 \times SE_i$, where SE denotes standard error and i ranged from

1 to 40 (that is, 20 factors \times 2 conditioning variables). Because the PPP expresses the portfolio problem in an estimation framework, we followed Brandt and Santa-Clara (2006) and calculated standard errors from the covariance matrix of $\tilde{\mathbf{w}}$ as follows:

$$\frac{1}{\gamma^2} \frac{1}{T - N \times K} (\mathbf{1}_T - \tilde{\mathbf{r}} \tilde{\mathbf{w}})' (\mathbf{1}_T - \tilde{\mathbf{r}} \tilde{\mathbf{w}}) (\tilde{\mathbf{r}}' \tilde{\mathbf{r}})^{-1}, \quad (7)$$

where $\mathbf{1}_T$ denotes a $T \times 1$ vector of 1s.

Table 2 shows that of the 40 $\boldsymbol{\theta}$ coefficients that represent the factor-timing strategy, only 11 are statistically significant at the 5% level, with an even share for fundamental indicators (6) and technical indicators (5).

Although the overall statistical significance of the $\boldsymbol{\theta}$ coefficients is moderate, we assessed how this evidence translates into economic performance. Inspecting the corresponding factor allocations, we found the biggest average weights over the sample period for CFY (12.0%), PROF (11.0%), AT (11.0%), and MOM12 (7.4%), whereas the factors with the lowest average weights were ROA (2.0%), BTM (0.1%), and STC (−0.1%). A factor-timing allocation that followed these weights might be rewarded in performance. In particular, the overweighting of PROF and MOM12 and the underweighting of ACC and BTM should contribute positively to active performance, and the underweighting of AG and the overweighting of CP would be expected to detract from active performance. Confirming that notion, Table 3 documents that the factor-timing strategy's gross return (4.17% per year) exceeded that of the equally weighted benchmark by 95 bps in the sample period.¹⁷ Given an *ex post* tracking error of 1.13%, this abnormal return equates to a gross information ratio (IR) of 0.84.

These gross performance figures have to be taken with a pinch of salt because of the considerable average two-way turnover: 480% a year. Accounting for transaction costs, as laid out in the introductory section, brings down the 1/ N benchmark performance from 3.22% to 2.21% a year. Holding the benchmark in an equity factor swap is assumed to cost 96 bps, and monthly reallocation to the 1/ N benchmark weights accounts for 5 bps per year. Given the high turnover of the factor-timing strategy, the performance drag resulting from transaction costs is substantial; most of the factor-timing alpha is eroded by these costs, and the net figure over the 1/ N benchmark is only 4 bps (2.25% – 2.21%) per year.

Table 2. Factor-Timing Coefficients, January 1998–December 2016

Factor	Principal Component FUN1		Principal Component TECH1	
	FUN1	SE	TECH1	SE
CFY	0.81	0.67	0.68	0.93
DY	−0.94*	0.43	−0.30	0.64
BTM	−0.17	0.78	−0.67	0.96
EY	2.40*	0.77	1.28	0.85
PROF	0.53	0.48	3.02*	1.04
MOM12	0.30*	0.17	0.73*	0.27
STR	−0.03	0.14	−0.94*	0.34
LTR	−0.12	0.36	−0.51	0.44
AT	0.27	0.72	2.91*	0.94
DLTD	0.40	1.06	−0.11	1.82
DSO	−1.85*	0.63	−1.14	1.17
AG	0.16	0.99	−0.53	1.36
CP	−1.28	0.77	1.10	1.14
PMA	−1.31*	0.67	−0.40	0.94
LEV	0.39	0.56	0.85	0.60
ROA	−0.77	0.97	−0.27	0.97
STC	−0.16	0.62	−1.43*	0.68
STI	−0.69	0.62	0.82	1.00
ACC	−2.96*	0.70	2.08	1.22
Size	−0.01	0.26	0.58	0.33

Note: SE denotes standard error.

*Significant at the 5% level.

Although these results appear sobering initially, we will revisit potential remedies for this mean–variance allocation outcome in the section “Smoothing Mean–Variance Factor Allocations.”

Factor Tilting

A complementary method of equity factor investing exploits cross-sectional differences in factor characteristics by tilting the factor allocation according to those characteristics. Using the cross-sectional PPP developed by Brandt et al. (2009), we exploited cross-sectional factor characteristics, such as valuation and momentum, to derive optimal factor allocations.

Cross-Sectional Factor Characteristics.

In this subsection, we discuss the equity factor characteristics to be fed to the cross-sectional PPP.

Factor valuation. A value strategy invests in relatively cheap assets while avoiding (or even shorting) securities that are relatively expensive. Applying this rationale at the factor level, we focused on the equity factors’ aggregate valuation levels—overweighting factors that were relatively cheap and underweighting those that were relatively expensive. We determined a factor’s relative value by taking the difference between the average valuation (as measured by the book-to-market ratio) of the factor’s top quintile and that of its bottom quintile. Given a

Table 3. Performance Statistics: Factor Timing, January 2003–December 2016

Statistic	Benchmark: 1/N		Factor Timing	
	Gross	Net	Gross	Net
Excess return (%)	3.22	2.21	4.17	2.25
Standard deviation (%)	3.73	2.72	2.83	2.85
Sharpe ratio	1.18	0.81	1.47	0.79
Tracking error (%)	—	—	1.13	1.13
Information ratio	—	—	0.84	0.04
Turnover	0.26	0.26	4.80	4.80

Notes: Annualized excess returns were calculated from the arithmetic average of simple returns. The standard deviation and Sharpe ratio were annualized through multiplication by $\sqrt{12}$. The information ratio used arithmetic active returns of factor timing over the 1/N benchmark. Annualized turnover is stated as two-way turnover.

pervasive value premium among stocks, one would expect valuation levels to work similarly at the factor level (Arnott et al. 2016). Bear in mind, however, that any factor will trade on its own norm. By definition, value-type equity factors will be cheap compared with growth factors. Moreover, when the book-to-market ratio is used as a proxy for valuation, whether there is any additional benefit to using a valuation indicator for factor tilting is *a priori* not clear when the overall equity factor allocation model contains a book-to-market value factor.

Factor spread. A factor spread measures the difference in a characteristic between the top and bottom quintiles. If the mean factor spread between top and bottom quintiles is large, the factor is relatively cheap in terms of the factor-defining characteristic and one can readily distinguish between attractive and unattractive stocks. In this sense, a factor spread might proxy for the factor's potential future return dispersion: If a given factor spread is wide, the corresponding factor return opportunity is expected to be large (Huang, Liu, Ma, and Osiol 2010). Given the diversity of the factors' defining characteristics, we standardized factor spreads over an expanding window of each factor's history.

Factor momentum. We used one-month price momentum to capture short-term factor momentum. Avramov, Cheng, Schreiber, and Shemer (2017) documented that a naive active one-month factor momentum strategy applied to a set of 15 equity factors consistently outperforms a 1/N benchmark. The momentum measure for a given equity factor

was calculated simply as its return over the previous month.

Factor volatility. As far back as the 1970s, low-volatility stocks have been found to outperform high-volatility stocks on a risk-adjusted basis (Haugen and Heins 1975; Jensen, Black, and Scholes 1972; Haugen and Baker 1991). We tested for a volatility effect among equity factors and calculated the factor volatility of each equity factor in a given month based on the daily returns of the underlying stocks in the two long-short legs.

Crowding. Crowding measures the risk that different investors are exposed to and suffer from the same shocks because they hold the same securities in their portfolios. Factor portfolios based on the same fundamental characteristics can be prone to crowding as investors seek to exploit the return potential associated with those characteristics. To capture crowding, we applied the mean pairwise correlation method proposed by Gustafson and Halper (2010), Cahan and Luo (2013), and Lou and Polk (2012). Because many investors operate under a long-only constraint, we conjectured that crowding in a factor's long leg might differ from crowding in the short leg. Therefore, we distinguished between the mean pairwise correlation of the long leg and the short leg by using a month's daily returns of leg constituents to calculate the mean pairwise correlation across each leg.

Methodology. We incorporated the cross-sectional characteristics into the PPP of Brandt et al.

(2009), which allowed us to exploit the information content in a utility-based portfolio optimization. An application of the mean–variance approach of Markowitz (1952) requires estimating first and second moments of all asset returns, but Brandt et al. (2009) proposed a more parsimonious optimization problem that leads to a tremendous reduction in dimensionality. Specifically, they considered an investor seeking to maximize conditional expected utility over portfolio return $r_{p,t+1}$:

$$\max_{\{w_{i,t}\}_{i=1}^{N_t}} E_t \left[u(r_{p,t+1}) \right] = E_t \left[u \left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} \right) \right], \quad (8)$$

where $w_{i,t}$ denotes the portfolio weight for asset i and N_t denotes the number of assets at time t . Brandt et al. proposed modeling the portfolio weight as a linear function of asset characteristics $\mathbf{x}_{i,t}$:

$$\begin{aligned} w_{i,t} &= f(\mathbf{x}_{i,t}; \boldsymbol{\phi}) \\ &= w_{b,i,t} + \frac{1}{N_t} \boldsymbol{\phi}' \hat{\mathbf{x}}_{i,t}, \end{aligned} \quad (9)$$

where $w_{b,i,t}$ denotes the benchmark weight, $\boldsymbol{\phi}$ is the vector of coefficients to be estimated through utility maximization, and $\hat{\mathbf{x}}_{i,t}$ denotes the standardized factor characteristics. The parameterization in Equation 9 implicitly assumes that the characteristics fully capture the joint distribution of asset returns.

The characteristics are cross-sectionally standardized at time t across all factors, rendering their cross-sectional distribution stationary over time. Because the cross-sectional mean for each standardized characteristic is zero, deviations from the benchmark are equivalent to a zero-investment portfolio. As a result, the weights of the resulting portfolio always add up to 100%. The optimization problem is further simplified because the coefficients that maximize the conditional expected utility of the investor at a specific time t are constant over time and across assets in such a way that the optimization problem can be written in terms of the $\boldsymbol{\phi}$ coefficients. The parametric portfolio return,

$$r_{p,t+1}(\boldsymbol{\phi}) = \mathbf{w}'_{b,t} \mathbf{r}_{t+1} + \boldsymbol{\phi}' \frac{\mathbf{X}'_{t+1} \mathbf{r}_{t+1}}{N_t}, \quad (10)$$

is driven by the benchmark portfolio return, $\mathbf{w}'_{b,t} \mathbf{r}_{t+1}$, and the characteristic return $\mathbf{X}'_{t+1} \mathbf{r}_{t+1} / N_t$ (both at time $t + 1$). For a mean–variance utility function, the

original optimization problem (Equation 8) can be written as

$$\max_{\boldsymbol{\phi}} E[r_{p,t+1}(\boldsymbol{\phi})] - \frac{\gamma}{2} \text{var}[r_{p,t+1}(\boldsymbol{\phi})], \quad (11)$$

which can be restated as

$$\max_{\boldsymbol{\phi}} \boldsymbol{\phi}' \hat{\boldsymbol{\mu}}_c - \left(\frac{\gamma}{2} \boldsymbol{\phi}' \hat{\boldsymbol{\Sigma}}_c \boldsymbol{\phi} + \gamma \boldsymbol{\phi}' \hat{\boldsymbol{\sigma}}_{bc} \right), \quad (12)$$

where $\hat{\boldsymbol{\Sigma}}_c$ is the sample covariance matrix, $\hat{\boldsymbol{\mu}}_c$ is the mean of the characteristic return vector, and $\hat{\boldsymbol{\sigma}}_{bc}$ is the sample vector of covariances between the benchmark portfolio return and the characteristic return vector (DeMiguel et al., forthcoming).

Empirical Results. Panel B of Table 4 presents estimation results and performance statistics for the six factor-tilting allocations based on a univariate PPP. Across the univariate models, the only significant coefficient suggests a short-term price momentum effect among equity factors. Therefore, factors with positive price momentum would be overweighted relative to the benchmark and factors with negative price momentum would be underweighted. The annualized gross return of the corresponding PPP based on momentum is 65 bps (i.e., 3.87% – 3.22%) higher than that of the equally weighted benchmark, but volatility increased by 27 bps (i.e., 2.99% – 2.72%). These figures correspond to a gross IR of 0.90 for momentum. Although the remaining factor characteristics are weaker than momentum in a univariate setting, some have positive gross IRs also—in particular, volatility and crowding (short).

Instead of relying on momentum only, Panel C of Table 4 considers all six characteristics jointly in a multivariate PPP. The coefficient for momentum remains statistically significant in a multivariate framework; those for crowding (long) and crowding (short) now become significant also. As in the univariate case, momentum still has a positive coefficient, leading to an overweighting of factors with positive price momentum and an underweighting of those with negative price momentum. Crowding (long) and crowding (short) exhibit negative coefficients, prompting an underweighting of crowded factors. Relative to the univariate momentum strategy, the gross return for the multivariate case decreases by 10 bps, and the factor-tilting approach delivers a gross alpha of 55 bps (i.e., 3.77% – 3.22%)

Table 4. Performance Statistics: Factor Tilting, January 2003–December 2016

Characteristic	ϕ	Excess Return (per year)		Volatility (per year)	Sharpe Ratio	Information Ratio		Turnover (per year, two way)
		Gross	Net			Gross	Net	
A. Benchmark model								
1/N		3.22	2.21	2.72	0.81	—	—	0.26
B. Univariate model								
Momentum	2.070*	3.87	2.35	2.99	0.78	0.90	0.20	2.81
Volatility	0.690	3.40	1.86	3.00	0.62	0.25	−0.45	2.91
Valuation	0.134	3.01	1.91	3.05	0.63	−0.25	−0.35	0.69
Spread	−1.155	2.91	1.61	2.99	0.54	−0.35	−0.67	1.73
Crowding (long)	0.525	2.90	1.12	2.75	0.41	−0.32	−1.10	4.09
Crowding (short)	−1.634	3.24	1.65	2.61	0.63	0.03	−0.74	3.14
C. Multivariate model								
Momentum	4.595*	3.77	2.01	2.99	0.67	0.70	−0.23	3.99
Volatility	1.770							
Valuation	−0.600							
Spread	−1.821							
Crowding (long)	−4.497*							
Crowding (short)	−8.159*							

Notes: All performance statistics are based on the out-of-sample period, January 2003–December 2016. Statistical significance for the tilting coefficients is assessed over the sample period January 1998–December 2016. Excess return and volatility are given in percentage terms.

*Significant at the 5% level.

a year while the volatility increases by 27 bps (i.e., 2.99% – 2.72%) compared with the benchmark.¹⁸ The strategy's gross IR is 0.70, confirming the usefulness of the chosen characteristics for tilting equity factors. Yet, this positive IR comes with a two-way turnover of 399% per year, indicating a high degree of factor-tilting activity. When transaction costs are accounted for, the strategy's net return is only 2.01%, compared with a net return of 2.21% for the equally weighted benchmark. This reduction in return due to transaction costs translates to a Sharpe ratio of 0.67 and a negative IR, −0.23.

Because transaction costs completely consume the benefits of the full set of tilting characteristics, one may wonder whether a cherry-picked set of tilting characteristics would lead to a more favorable outcome. When we focused on the three characteristics that proved statistically significant over the whole sample period, however, the factor-tilting policy outperformed the 1/N benchmark by a mere

7 bps (i.e., 2.28% – 2.21%) after transaction costs. Despite the benefit of hindsight bias, one would have achieved a net IR as low as 0.09.¹⁹ So, the timing and tilting PPP allocations rarely preserve any added value when transaction costs are accounted for.

Smoothing Mean–Variance Factor Allocations

Because our analysis was rooted in a mean–variance framework, we investigated three ways to smooth the PPP allocations for factor timing and factor tilting that might lead to improved performance after transaction costs. The first remedy added constraints to the optimization to avoid overly concentrated portfolios and limited the effect of error maximization in the mean–variance framework. The second remedy was to apply Black–Litterman (1992) shrinkage to the optimization inputs, which led to more stable factor allocations. The third remedy was to

impose a transaction cost penalty to slow down the change in portfolio weights from one optimization to the next.

Applying Constraints, Black–Litterman Shrinkage, and Transaction Cost Penalties.

In the following subsections, we describe how we implemented each of the three smoothing methods.

Constraints. Introducing portfolio constraints is an effective means of reducing the sampling error of the inputs (Jagannathan and Ma 2003). In our context, we applied a long-only constraint together with a 10% cap on the factor weights.

Black–Litterman shrinkage. To mitigate the sensitivity of classical mean–variance optimization to the input data, the Black–Litterman approach combines the original subjective view on portfolio assets with an objective view related to some prior market equilibrium (Black and Litterman 1990, 1992; He and Litterman 2002). Depending on the degree of confidence in the subjective view, the resulting Black–Litterman mean–variance allocation governs the amount of deviation from the market equilibrium view.

Translating this notion to the PPP framework is straightforward. We simply used the standard Black–Litterman master formulas for estimating returns and the variance–covariance matrix. We set the equilibrium view for all associated θ and ϕ entries (see Equation 5 and Equation 9) equal to zero to anchor our portfolio optimization in the equally weighted benchmark factor allocation. The subjective views on θ and ϕ are the historical means of the respective (synthetic) factor portfolio returns (for factor timing) or the characteristic returns (for factor tilting). The implicit shrinkage in the Black–Litterman approach not only scales down the original mean–variance allocation to a lower tracking-error level but also affects the relative sizes of θ and ϕ . In particular, the shrinkage method reassesses the relative importance of the time-series indicators and the cross-sectional characteristics, eventually leading to less extreme compositions of the optimal factor allocation portfolios.

Transaction cost penalties. Given the quadratic nature of the mean–variance objective function underlying our PPPs, we considered adding a quadratic transaction cost penalty term that would be

effective at the level of the timing and tilting factor allocations:

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} - \frac{\gamma}{2} \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} - \lambda_{TC} \boldsymbol{\Gamma} |\Delta\mathbf{w}|^2, \quad (13)$$

where $\boldsymbol{\Gamma}$ denotes the transaction cost matrix and $\Delta\mathbf{w} = \mathbf{w} - \mathbf{w}_0$ is the weight change relative to the initial factor allocation weights, \mathbf{w}_0 . Rearranging terms produces

$$\max_{\mathbf{w}} \mathbf{w}'(\boldsymbol{\mu} + 2\lambda_{TC} \boldsymbol{\Gamma} \mathbf{w}_0) - \mathbf{w}'\left(\frac{\gamma}{2} \boldsymbol{\Sigma} + \lambda_{TC} \boldsymbol{\Gamma}\right) \mathbf{w}, \quad (14)$$

where we set the expected returns on the factor level, $\boldsymbol{\mu}$, equal to the implied returns of the current factor-timing or -tilting allocation.

Adding a transaction cost penalty has two effects that make the optimizer less willing to deviate from the current allocation. First, the transaction cost penalty implicitly leads to an increase in the expected return for the current portfolio holdings. Second, the transaction cost penalty artificially increases the perceived volatility of all the equity factors, thereby diminishing the general attractiveness of any factor in terms of risk. The degree of these two effects is ultimately governed by the choice of the underlying transaction cost matrix: Entries with very high transaction costs ultimately lead to freezing the current allocation.²⁰

The Impact of Smoothing Factor Timing and Tilting.

Smoothing the factor-timing and -tilting allocations reduced factor allocation turnover and transaction costs, but the resulting changes in the portfolio allocations might have been detrimental to the gross alpha potential of the original dynamic mean–variance factor weights. In fact, as shown in Panel A of **Table 5**, going from plain-vanilla mean–variance optimization (MVO) to the smoothed allocation that jointly used all three smoothing approaches subtracted a gross alpha potential of 15 bps in factor timing and 35 bps in factor tilting.²¹ This reduction is consistent with the fact that high-turnover characteristics, such as the one-month factor momentum, were driving most of the gross PPP performance.

The reduction in gross alpha potential might be compensated for, however, by the observed reductions in turnover. The two-way turnover for factor timing is considerably reduced by the smoothing—from

Table 5. Smoothing Factor Allocations (t-statistics in parentheses)

Performance Statistic	1/N	Factor Timing		Factor Tilting	
		MVO	Smooth	MVO	Smooth
A. Gross of transaction costs					
Excess return (%)	3.22	4.17	4.02	3.77	3.42
Standard deviation (%)	2.73	2.83	3.12	2.99	2.94
Sharpe ratio	1.18	1.47	1.29	1.26	1.16
	(4.43)	(5.52)	(4.83)	(4.73)	(4.37)
Tracking error (%)	—	1.13	1.07	0.79	0.52
IR	—	0.84	0.75	0.70	0.40
	—	(3.16)	(2.82)	(2.63)	(1.49)
Turnover	0.26	4.80	1.76	3.99	1.38
B. Net of transaction costs					
Excess return (%)	2.21	2.25	2.71	2.01	2.19
Standard deviation (%)	2.72	2.83	3.12	2.99	2.94
Sharpe ratio	0.81	0.79	0.87	0.67	0.75
	(3.04)	(2.96)	(3.26)	(2.52)	(2.80)
Tracking error (%)	—	1.13	1.07	0.79	0.52
IR	—	0.04	0.47	−0.23	−0.04
	—	(0.15)	(1.76)	(−0.88)	(−0.13)

Notes: MVO denotes the application of mean–variance optimization. The “Smooth” columns represent the factor allocation that jointly implemented weight constraints, Black–Litterman adjustments, and a transaction cost penalty. Annualized excess returns were calculated from the arithmetic average of simple returns. The standard deviation and Sharpe ratio were annualized through multiplication by $\sqrt{12}$. The underlying out-of-sample period is January 2003–December 2016.

480% to 176%—and for factor tilting, the reduction is from 399% to 138%. Indeed, even when all three smoothing remedies were applied, as shown in Panel B of Table 5, factor timing still provided a net excess return over the 1/N benchmark of 50 bps (i.e., 2.71% – 2.21%). For factor tilting, the smoothing of factor allocations increased the net strategy return only to the level of the 1/N benchmark, with a net active return of –2 bps (i.e., 2.19% – 2.21%). The tracking error is 1.07% for factor timing, corresponding to an IR net of transaction costs of 0.47. For factor tilting, the smoothing remedies led to a tracking error of 0.52% and a net IR of only –0.04.

Finally, although the factor-tilting strategy based on the full set of tilting characteristics can slightly benefit from taming the mean–variance factor allocation, unreported results document a flattish pattern for the cherry-picking model (the model that used only those characteristics that proved statistically

significant over the whole sample period). Its net active return of 7 bps (MVO) was slightly reduced, by 5 bps down to 2 bps (i.e., 2.23% – 2.21% per year), so its IR fell from 0.09 to 0.06.

Conclusion

This article contributes to the ongoing debate about whether a forecasting-based factor allocation can add value over and above a diversified passive factor allocation. Given the well-known drawbacks of the classical Markowitz model, we applied the parametric portfolio policies introduced by Brandt and Santa-Clara (2006) and Brandt et al. (2009). Using a comprehensive set of global equity factors, we incorporated time-series and cross-sectional information into the models and tested whether the out-of-sample performance of an equally weighted factor allocation can be improved by using a PPP model.

Ignoring transaction costs associated with factor allocation turnover, we found that factor timing using fundamental and technical time-series predictors generates statistically significant and economically relevant results. Similarly, a factor-tilting strategy that exploits cross-sectional information favors factors with positive short-term momentum but avoids factors that exhibit crowding. The resulting high portfolio turnover entails considerable transaction costs, however, which tend to erode much of the potential benefit of factor predictability and raise questions about its practical relevance. Taming portfolio allocations by imposing transaction cost penalties and Black-Litterman shrinkage helped to preserve part of the performance after transaction costs for the time-series-based factor-timing approach but not for factor tilting.

From a practitioner's perspective, this article tells a cautionary tale about the benefits of dynamic factor allocation vis-à-vis the diversification benefits of simply holding an equally weighted equity factor portfolio. Still, the parametric approach to optimal factor timing and tilting provides an effective base approach for an investor to test for the relevance of additional insight or information. Our study leaves open the possibility of adding value in dynamic factor allocation at a comfortably high level after transaction costs.

Appendix A. Definitions Factors

- *Accruals (ACC)* is long stocks with low accruals and short stocks with high accruals, where accruals are measured as the change in working capital per share divided by the book value per share; see Sloan (1996).
- *Asset growth (AG)* is based on research by Fairfield, Whisenant, and Yohn (2003); Richardson, Sloan, Soliman, and Tuna (2005); Titman, Wei, and Xie (2004); Fama and French (2006); and Cooper, Gulen, and Schill (2008). All of these papers documented a negative relationship between investment activity and returns. The factor is long stocks with a low asset-growth ratio and short stocks with a high asset-growth ratio. Asset growth is measured by the year-on-year change in total assets divided by the total assets in $t - 2$.
- *Asset turnover (AT)* measures asset utilization and efficiency; it is defined as sales divided by average net operating assets. Following Soliman (2008), we considered companies with high asset turnover to be associated with future positive returns because those companies are managing their inventory efficiently.
- *Book to market (BTM)* is long stocks with a high book-to-market ratio and short stocks with a low book-to-market ratio. This factor builds on the finding of Basu (1977); Rosenberg, Reid, and Lanstein (1985); Jaffe, Keim, and Westerfield (1989); Chan, Hamao, and Lakonishok (1991); and Fama and French (1992) that value stocks outperform growth stocks in the long run.
- *Cash flow yield (CFY)* captures the excess return of going long stocks with a high ratio of cash flow to price and shorting stocks with a low ratio of cash flow to price; see Sloan (1996); Da and Warachka (2009); and Hou, Karolyi, and Kho (2011). Cash flows are measured as the sum of funds from operations, extraordinary items, and funds from other operating activities.
- *Cash productivity (CP)* is a negative predictor of returns; see Chandrashekar and Rao (2009). Companies with high cash productivity have low subsequent stock returns, and companies with low cash productivity have high future returns. The factor is defined as market value plus long-term debt minus total assets divided by cash.
- *Change in long-term debt (DLTD)* is defined as year-on-year changes divided by the long-term debt in $t - 2$. An increase in long-term debt may indicate managerial empire-building behavior, which is associated with negative future returns; see Richardson et al. (2005).
- *Change in shares outstanding (DSO)* is based on the idea that companies with a large change in shares outstanding underperform relative to nonissuing companies, as documented by Ritter (1991) and Loughran and Ritter (1995). See also Daniel and Titman (2006) and Pontiff and Woodgate (2008). We measured change in shares outstanding as the year-on-year change in shares outstanding divided by outstanding shares in $t - 2$.
- *Dividend yield (DY)* is long stocks with a high dividend-to-price ratio and short stocks with a low dividend-to-price ratio. Dividends include all extra dividends declared during the year. See Litzenberger and Ramaswamy (1979), Blume (1980), Fama and French (1988), and Campbell and Shiller (1988).

- *Earnings yield (EY)* is long stocks with a high earnings-to-price ratio and short stocks with a low earnings-to-price ratio; see Basu (1977).
- *Leverage (LEV)* is defined as total liabilities divided by the market value of the company; see Bhandari (1988). The factor is long stocks with a low ratio of liability to market value and short stocks with a high one.
- *Long-term reversal (LTR)* exploits the reversal patterns in long-term past performance documented by De Bondt and Thaler (1985). Following DeMiguel et al. (forthcoming), we chose the horizon to be 36 months. To control for the momentum effect, we excluded the most recent year from our three-year horizon of past performance. This factor goes long stocks with weak long-term past performance and short stocks with strong long-term past performance.
- *Profitability (PROF)* is long stocks with robust operating profitability and short stocks with weak operating profitability. Profitability is calculated as annual revenues less cost of goods sold, interest, and other expenses divided by book value for the last fiscal year-end. The factor is based on Haugen and Baker (1996); Cohen, Gompers, and Vuolteenaho (2002); Novy-Marx (2013); and Fama and French (2006, 2016).
- *Profit margin (PMA)* is defined as operating income divided by sales. Companies with high profit margins are often associated with a first-mover advantage or high brand recognition, which translates into high pricing power; see Soliman (2008). The factor is long stocks with a high profit margin and short stocks with a lower profit margin.
- *Return on assets (ROA)* is long stocks with a high return on assets and short stocks with a low return on assets. A high return on assets is an indicator of a successful company; see Balakrishnan, Bartov, and Faurel (2010).
- *Sales to cash (STC)* is based on research by Ou and Penman (1989), who found a positive relationship between the sales-to-cash ratio and future returns. The factor is long stocks with a high sales-to-cash ratio and short stocks with a low sales-to-cash ratio.
- *Sales to inventory (STI)* measures the effective use of the company's assets. A high sales-to-inventory ratio indicates company effectiveness and is associated with high future returns; see Ou and Penman (1989). The factor is long stocks with

a high sales-to-inventory ratio and short stocks with a low sales-to-inventory ratio.

- *Short-term reversal (STR)* is long stocks with a weak previous month's performance and short stocks with a strong previous month's performance; see Jegadeesh (1990) and Lehmann (1990).
- *Size* is long stocks with the smallest market cap and short stocks with the largest market cap; see Fama and French (1992). Stocks with larger market cap tend to underperform stocks with smaller market cap; see Banz (1981).
- *Twelve-month price momentum (MOM12)* captures a medium-term continuation effect in returns by buying recent winners and selling recent losers; see Jegadeesh (1990) and Jegadeesh and Titman (1993). We controlled for the short-term reversal effect (see that entry) by excluding the most recent month ($t - 1$) at time t .

Fundamental Predictors

- *Book-to-market ratio (bm)* is the ratio of book value to market value for the Dow Jones Industrial Average.
- *Default return spread (dfr)* is the difference between the return on long-term corporate bonds and the return on long-term government bonds.
- *Default yield spread (dfy)* is the difference between BAA rated and AAA rated corporate bond yields.
- *Dividend payout ratio (de)* is the difference between 12-month moving sums of dividends on the S&P 500 Index and the log of 12-month moving sums of earnings on the S&P 500.
- *Dividend-to-price ratio (dp)* is the difference between the log of 12-month moving sums of dividends paid on the S&P 500 and the log of S&P 500 prices.
- *Dividend yield (dy)* is the difference between the log of 12-month moving sums of dividends paid on the S&P 500 and the log of 1-month lagged S&P 500 prices.
- *Earnings-to-price ratio (ep)* is the difference between 12-month moving sums of earnings on the S&P 500 and the log of S&P 500 prices.
- *Inflation (infl)* is the US Consumer Price Index (all urban consumers), representing the buying

habits of the residents of urban or metropolitan areas.

- *Long-term rate of return (ltr)* is the return on long-term government bonds.
- *Long-term yield (lty)* is the yield on long-term government bonds.
- *Net equity expansion (ntis)* is the ratio of 12-month moving sums of net issues by NYSE-listed stocks divided by the total market cap of NYSE stocks.
- *Stock variance (svar)* is realized variance, calculated as the monthly sum of squared daily returns on the S&P 500.
- *Term spread (tms)* is the difference between the long-term yield on government bonds and the US T-bill rate.
- *Treasury bills (tbl)* denotes the interest rate on a three-month T-bill traded on the secondary market.

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Notes

1. See Arnott, Kelso, Kiscadden, and Macedo (1989) for an early perspective on equity factor return predictability.
2. Anchoring the analysis in a mean-variance-utility framework allows the PPP approach to help alleviate the framework's well-known drawbacks, such as a high degree of input sensitivity (Best and Grauer 1991a, 1991b) and the problem of estimation error maximization (Michaud 1989). Hjalmarsen and Manchev (2012) provided an in-depth discussion of mean-variance parametric portfolio strategies.
3. As for the regional split, the universe, on average, included 1,700 European and 1,300 US stocks, with both figures being fairly constant over time. In December 1996, around 1,000 stocks belonged to the Asia-Pacific region, mostly made up of Japan and Australia. This number rose to 1,400 by the end of the sample period. Additional companies emerged from a "rest of the world" universe, which included Canada, New Zealand, Israel, and Hong Kong.
4. This construction of equity factors does not explicitly control for country or sector exposures and could thus lead to country or sector concentrations at various times. We rarely observed such patterns, however, during the sample period.
5. A controversy is occurring in the literature about whether a reliable small-size premium ever existed. Alquist, Israel, and Moskowitz (2018) found neither strong empirical evidence nor robust theoretical support for a size premium. Although they thus argued that tilting toward small-cap stocks is unlikely to provide much of a premium, they nevertheless concluded that the size factor can be valuable in combination with other factors (e.g., because of potential trading cost diversification).
6. Two caveats are necessary. First, although the present analysis accounts for transaction costs in the computation of single equity factor returns, one could argue that less frequent rebalancing of factors—say, quarterly—could provide a less costly exposure to the factors. Second, rather than assuming average transaction costs for any given stock, one could assume more specific transaction costs.
7. This transaction cost assumption is relatively modest, so assuming transaction costs higher than 20 bps would only strengthen our results.
8. See Schwert (2003); Chordia, Roll, and Subrahmanyam (2011); and McLean and Pontiff (2016).
9. In unreported results, we observed that alternative benchmarks based on minimum-variance or risk parity factor allocations led to active factor allocations similar to the one induced by the 1/N benchmark. Yet, these risk-based allocations tended to be concentrated in a few equity factors. We, therefore, anchored our analysis of timing and tilting policies on the equally weighted equity factor allocation.
10. The dataset is available at www.hec.unil.ch/agoyal/. For a more detailed description of the variables, see Appendix A or Welch and Goyal (2008). Note that these variables are based on US fundamental data. Because US data are predictive for other developed countries' stock market returns (Rapach, Strauss, and Zhou 2013), applying the data in a global setting seems appropriate.
11. Inflation potentially requires a longer look-back horizon for standardization than 12 months. Our baseline findings for optimal factor timing proved to be robust, however, when longer look-back periods (36, 60, and 120 months) were chosen for standardizing inflation. To alleviate any concerns about cherry-picking and to be consistent with the remaining fundamental variables, we used a look-back period of 12 months to standardize inflation throughout the article.
12. A robustness check of a PPP using three fundamental principal components (jointly capturing 56% of fundamental predictor variation) led to similar conclusions. Because a smaller number of predictors allows for a longer out-of-sample backtesting window, our main analysis is based on the first fundamental principal component only.

13. Note that “vec” is a linear transformation that converts the matrix into a column vector and \otimes denotes the Kronecker product of two matrices.
14. Any given equity factor could also interact with a factor-specific set of conditioning variables; that is, one could select only those factor-conditioning variables that were deemed meaningful. We refrained from pursuing such a cherry-picking exercise, however, and included the maximum amount of information incorporated in the two PCA factors.
15. The original approach of Brandt and Santa-Clara (2006) allowed the policy to shift the portfolio into a risk-free investment when equity factors were expected to perform poorly. Imposing a full investment constraint could thus prevent the strategy from fully exploiting the information content embedded in the PCA factors. In unreported tests, we observed, however, that relaxing the full investment constraint rarely led the PPP to divest and the overall performance was similar to the full investment case.
16. Such rescaling ensures that the strategy is not forced into more extreme allocations when the signals from the PCA factors are deemed weak. In such cases, the strategy will naturally resort to the equally weighted benchmark.
17. In unreported tests, we regressed the gross factor-timing returns (returns in excess of the risk-free rate) on standard global asset pricing factors (sourced from Kenneth French’s website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) to test whether this outperformance could be explained by constant factor overweights implicit in the factor-timing policy. Indeed, the latent overweight of MOM12, explaining some 40% of the factor-timing returns’ cross-sectional variation, is a main contributor to the gross alpha. Relative to the Fama–French three-factor intercept, the consideration of MOM12 leads to halving the factor-timing alpha, which, nevertheless, continues to be significant at the 5% level.
18. Similar to the regression analysis for the factor-timing returns, we also tested (results not shown) whether the factor-tilting returns contain exposures to standard global asset pricing factors. We found that, unlike the factor-timing returns, factor-tilting returns are not significantly exposed to any of the common factors, including momentum.
19. These results are available from the authors upon request (carsten.rother@invesco.com).
20. The transaction cost matrix Γ is often populated on the basis of estimates derived from transaction cost models (Gârleanu and Pedersen 2013). In contrast, we simply related Γ to the variance–covariance matrix and assumed Γ was linear in the diagonal of the variance–covariance matrix, with $\lambda_{TC} = 1/3$.
21. Table 5 presents the overall joint impact of constraints, the Black–Litterman shrinkage, and transaction cost penalties on factor timing and tilting, but we also investigated their incremental benefits when implemented separately. The results are available from the authors upon request.

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