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Actual and counterfactual growth incidence and delta Lorenz curves: Estimation and inference

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Summary

Different economic growth episodes display very different distributional characteristics, both across countries and over time. Growth is sometimes accompanied by rising and sometimes by falling inequality. Applied economists have come to rely on the growth incidence curve, which gives the quantile-specific rate of income growth over a certain period, to describe these differences. This paper introduces a mean-independent analogue, the delta Lorenz curve, which gives the cumulative change in income share up to each quantile. We also develop estimation and inference procedures for both functions of quantiles. We establish the limiting null distribution of the test statistics of interest for those functions, and propose resampling methods to implement inference in practice. The proposed methods are used to compare the growth processes in the USA and Brazil during 1995–2007. Although growth in the average real wages was disappointing in both countries, the distribution of that growth was markedly different. In the USA, wage growth was mediocre for the bottom 80% of the sample, but much more rapid for the top 20%. In Brazil, conversely, wage growth was rapid below the median, and negative at the top. Wage shares fell in the USA up to the 83rd percentile, and rose in Brazil up to the 65th percentile.

1 | INTRODUCTION

Growth episodes display very different distributional characteristics across countries and over time. The same rate of growth in average incomes has been accompanied by rising inequality in some cases, and by falling inequality in others. A large literature on "pro-poor growth" and, more generally, on the incidence of economic growth processes has developed, and attracted attention among both researchers and policymakers.

Over time, this literature has come to rely heavily on the growth incidence curve (GIC), which describes the rate of income growth at each quantile $\tau \in (0,1)$ of the distribution (Ravallion & Chen, 2003). It has been used to compare the distributional characteristics of growth processes both across countries and over time (see, e.g., Besley and Cord, 2007). It has also been shown to underlie changes in certain widely used classes of poverty and inequality measures, which can be formally expressed as functionals of the GIC (Ferreira, 2012).

GICs have also featured in a long-standing literature that uses counterfactual income distributions to decompose changes (or differences) in inequality and poverty over time (or between countries), and to attribute such changes to different factors—for example, changes in worker characteristics or in the returns to those characteristics. The original contributions to this literature, including Juhn et al. (1993), Dinardo et al. (1996), and Donald et al. (2000), pre-date the Ravallion and Chen (2003) article that introduced the term GIC, and hence do not use it. Yet, each of those papers sought to account for differences across entire wage or income distributions—which can be formally expressed as GICs—using counterfactual distributions. Ferreira (2012) defines counterfactual growth incidence curves as functionals of counterfactual distributions, and establishes the link to this earlier literature on distributional change.

Although it has been shown that changes in specific inequality measures can be expressed as functionals of the GIC, the general relationship between changes in the Lorenz curve and the GIC has not to our knowledge been previously explored. In addition, despite their conceptual importance and widespread practical use, formal conditions for uniform inference using growth incidence curves—actual or counterfactual—have not been established.

In this paper, we attempt to fill both gaps. We introduce a mean-independent analogue of the GIC which describes the cumulative change in income share up to each quantile $\tau \in (0,1)$ of the distribution, which we call the delta Lorenz curve (DLC). The DLC is itself a functional of GIC and, more specifically, a weighted average of differences between the GIC and the growth rate in average income. Some of its graphical properties, and relationships to changes in specific inequality indices, are discussed.

We then rely on the formal analogy between distributional change and treatment effect heterogeneity to obtain formal conditions for estimation and inference for both the GIC and DLC.[1] A simple three-step semiparametric estimator for both actual and counterfactual growth incidence and DLCs is proposed, which relies on established sample reweighting and quantile regression (QR) techniques. When applied to counterfactual GICs or DLCs, this procedure has the advantage that it requires no assumptions on the structural relationship between income and its covariates, as was the case with most of the previous literature.

We establish the asymptotic properties of these estimators, namely uniform consistency and weak convergence. The provision of uniform results over the set of quantiles is a necessary condition to establish the results for the testing procedures. We also show that the estimator is uniformly efficient, as the asymptotic variance of the estimator coincides with the semiparametric efficiency bound. We then propose suitable test statistics, and discuss inference procedures in practice. For practical inference we compute critical values using resampling methods. We provide sufficient conditions and show the theoretical validity of a bootstrap approach. Moreover, we discuss an algorithm for its practical implementation. We also discuss computation of critical values through a subsampling method. This enables researchers to conduct estimation and inference for the GIC and DLC over the entire set of quantiles.[2]

Finally, we illustrate the proposed procedure by comparing actual and counterfactual growth incidence and DLCs (for real hourly wages) for the two largest countries in the Western Hemisphere, namely the USA and Brazil, in the 12 years prior to the onset of the last great financial crisis: 1995–2007. Although growth rates in average wages were disappointing in both countries (especially in Brazil), there were substantial differences in inequality dynamics. The GIC for the USA was flat until approximately the 8th decile, and sharply upward sloping over the top fifth of the distribution. In Brazil, conversely, the GIC peaked around the first quintile and was downward sloping thereafter. The DLC for the USA was negative everywhere, reaching a minimum on the 83rd percentile. In Brazil, again conversely, the DLC was positive everywhere, reaching a maximum on the 65th percentile. As a result, wage inequality rose sharply in the USA and declined in Brazil.

We use counterfactual GICs and DLCs to examine whether these changes were driven primarily by the composition of the labor force—in terms of observed worker characteristics such as gender, age, education, race, occupation, industry, and spatial location—or by changes in the broader structure of the economy. In both countries, we find that changes in inequality were driven predominantly by changes in economic structure.

The remainder of the paper is organized as follows. Section 2 introduces the GIC and DLC. Section 3 describes the three-step estimator, establishes its asymptotic properties, and discusses inference for the quantile process and its practical implementation. The empirical application to the USA and Brazil is presented in Section 4. Section 5 concludes. We relegate all proofs to the Supplemental Appendix, provided online as Supporting Information.

[1]A similar approach, albeit in different contexts, has been employed by Firpo (2007), Flores (2007), Cattaneo (2010), and Galvao and Wang (2015), among others.

[2]Some of the inference contributions are closely related to the literature on quantile treatment effects, which is a particular functional of the vector formed by the quantiles of the potential outcomes. That literature started with Doksum (1974) and Lehmann (1974) and has expanded recently (see, e.g., Abadie et al., 2002; Bitler et al., 2006; Cattaneo, 2010; Chernozhukov and Hansen, 2005; Donald and Hsu, 2014; Firpo, 2007; Firpo and Pinto, 2015; Galvao and Wang, 2015). The results of this paper are also related to those on inference on the quantile process; see, for example, Belloni et al. (2011) and Qu and Yoon (2015) for the nonparametric case; Gutenbrunner and Jureckova (1992), Koenker and Machado (1999), Koenker and Xiao (2002), Chernozhukov and Fernandez-Val (2005), and Angrist et al. (2006) for the parametric case.

2 | ANALYZING CHANGES IN INCOME DISTRIBUTIONS

This section discusses the main objects of interest when analyzing changes in income distribution. We first discuss the GIC, which lies at the core of our analysis of changes in income distribution. We show that several interesting statistics that describe changes in the distribution of income are functionals of the GIC. One such functional, which we call the delta Lorenz curve (DLC or $\Delta_L(\tau)$), describes changes in the Lorenz curve between times $t-1$ and t at a given τ of the distribution. We then define counterfactual measures of these objects and show that both actual and counterfactual distributions can be written as weighted distributions of the data. Finally, depending on the assumptions one is willing to impose on the time evolution of some variables, we discuss the interpretation of these counterfactual objects.

2.1 | The growth incidence and delta Lorenz curves

Let Y be income, the outcome variable of interest in this paper. There are two time periods T: $T=t-1$ and $T=t$.[3] We assume that income is continuously distributed over the population, and denote its cumulative distribution function (CDF) at time t as $F_{Y|T}(\cdot|t)$, or simply $F_{Y|T=t}(\cdot)$. The τth quantile of income at time t is given by the inverse of the CDF: $q_t(\tau)=F^{-1}_{Y|T}(\tau|t)$. We assume that the support of $F_{Y|T}(\cdot|t)$ is $\mathcal{Y}_t \subset \mathbb{R}$. Finally, throughout the paper we assume availability of a random sample of size n from the joint distribution of (Y,T,X), where X is a vector of length d of covariates; that is, $X \in \mathcal{X} \subset \mathbb{R}^d$.

Our interest is in learning how the income distribution as a whole evolves between periods. The evolution of the income distribution between $t-1$ and t can be characterized by the income distribution at time $t-1$ and the GIC, which was originally introduced by Ravallion and Chen (2003) and defined as the income growth rate for a given quantile τ between two time periods. For any $q_{t-1}(\tau) \neq 0$, it can then be written as

$$\mathrm{GIC}(\tau)=\frac{q_t(\tau)}{q_{t-1}(\tau)}-1. \tag{1}$$

The quantiles involved in the computation of Equation 1 are based on the ranking of individuals in each period-specific distribution. Therefore, unless individual i happens to keep his ranking over time, the GIC will not be an appropriate tool to infer individual movements over time. Equation 1 is thus said to define the *anonymous* GIC, which is suitable for welfare or inequality comparisons over time, but not for the study of individual mobility (see Bourguignon, 2011; Essama-Nssah et al., 2013; Grimm, 2007).

Under anonymity, the GIC and the distribution of income at $t-1$ fully characterize time changes in the income distribution. In fact, it is possible to express any statistic that describes changes in aspects of the income distribution over time as a functional of the GIC. In other words, consider the functional $\nu(\cdot)$ of the pair of the distributions $F_{Y|T=t}$ and $F_{Y|T=t-1}$, $\nu(F_{Y|T=t},F_{Y|T=t-1})$. Then the functional $\nu(\cdot)$ can be written as

$$\nu(F_{Y|T=t},F_{Y|T=t-1})=\xi(\mathrm{GIC},q_{t-1}).$$

Two important examples concern changes over time in the mean and in the Lorenz curve of a distribution. The growth rate in average income is $\gamma=\frac{\mu_t}{\mu_{t-1}}-1$, where $\mu_t=\int_{-\infty}^{+\infty} y dF_{Y|T}(y|t)$ is the mean income level at time t. The parameter γ is a function of $F_{Y|T=t}$ and $F_{Y|T=t-1}$ but can also be written as a functional of GIC and q_{t-1} as follows:

$$\gamma=\frac{\mu_t}{\mu_{t-1}}-1=\frac{\int_{-\infty}^{+\infty} y dF_{Y|T=t}(y)}{\int_{-\infty}^{+\infty} y dF_{Y|T=t-1}(y)}-1=\frac{\int_0^1 q_{t-1}(\tau)\mathrm{GIC}(\tau)d\tau}{\int_0^1 q_{t-1}(s)ds}=\int_0^1 \lambda_{t-1}(\tau)GIC(\tau)d\tau,$$

where $\lambda_t(\tau)=q_t(\tau)/\int_0^1 q_t(s)ds$. Thus, the rate of growth in average income, γ, is a weighted average of GIC using λ_{t-1} as weights. Note, of course, that the rate of growth in average income differs, in general, from the average growth rate across quantiles, $\gamma_{\mathrm{AVG}}=\int_0^1 \mathrm{GIC}(\tau)d\tau$, precisely because of the weights. Whereas the average growth rate across quantiles is a simple average, the growth rate in average income weights the growth rate across quantiles by the quantile's income share. Naturally, $\gamma=\gamma_{\mathrm{AVG}}$ when $\mathrm{GIC}(\tau)=r$, $r \in \mathbb{R}$ for all $\tau \in (0,1)$. As for the Lorenz curve, this can be defined as

$$L_t(\tau)=\int_0^\tau \frac{q_t(s)}{\mu_t}ds. \tag{2}$$

[3]The variable T could be rewritten to be a dummy variable without loss of generality. For example, $D=1\{T=t\}$is a dummy that equals 1 if $T=t$ and 0 if $T=t-1$.

From Equation 2 we are able to define the time difference of the Lorenz curve as

$$\begin{aligned} \mathrm{DLC}(\tau) = \Delta_L(\tau) := L_t(\tau) - L_{t-1}(\tau) &= \int_0^\tau \left(\frac{q_t(s)}{\mu_t} - \frac{q_{t-1}(s)}{\mu_{t-1}} \right) ds \\ &= \int_0^\tau \lambda_{t-1}(s) \left(\frac{\mathrm{GIC}(s) - \gamma}{1+\gamma} \right) ds. \end{aligned} \tag{3}$$

Equation 3 defines the DLC, which, like the GIC, is a functional of quantiles $\tau \in (0,1)$. Whereas the GIC gives the quantile-specific growth rate, the DLC shows the change over time in the income share cumulatively appropriated by all quantiles up to τ. A positive $\Delta_L(\tau)$ signifies that the share of the population with ranks lower than τ has increased its income share between the two periods. Differentiation of $\Delta_L(\tau)$ with respect to τ further reveals that its local optima occur at quantiles where $\mathrm{GIC}(\tau) = \gamma$. The second-order condition indicates that the DLC will be at a maximum if the GIC crosses γ from above, and at a minimum if from below. This makes intuitive sense: If growth in quantiles lower than τ is greater (less) than in average income, income shares up to τ are rising (falling). Section 4 empirically illustrates the former case for Brazil, and the latter for the USA.

The DLC is, therefore, a mean- or position-independent analogue of the GIC. Whereas the latter shows the distribution of changes (growth rates) in income *levels*, the DLC shows the changes in income *shares*. Since inequality measures are closely linked to the Lorenz curve (Atkinson, 1970), it follows that changes in inequality will be closely associated with the DLC. The Gini coefficient, for example, can be defined as a function of the Lorenz curve:

$$G_t = 1 - 2\int_0^1 L_t(\tau) d\tau,$$

and therefore the time difference of the Gini coefficient can also be defined as a function of both the DLC and the GIC:[4]

$$\Delta_G = G_t - G_{t-1} = -2\int_0^1 \Delta_L(\tau) d\tau = 2\frac{\int_0^1 \int_0^\tau \lambda_{t-1}(s)\,(\gamma - \mathrm{GIC}(s))\, ds\, d\tau}{(1+\gamma)}.$$

We next turn to decomposition analyses of distributional change, and the use of counterfactual measures of changes in income distributions in that context.

2.2 | Counterfactual changes in income distributions

The GIC, the DLC, and summary statistics such as γ and Δ_G, are useful to *quantify* changes in different aspects of the income distribution. A first step towards *understanding* these changes has typically been to think of $F_{Y|T}$ as the margin of a joint distribution of income and covariates, $F_{Y,X|T}$, and to decompose changes in that marginal distribution into a component due to changes in the conditional distribution of income on the covariates, $F_{Y|X,T}$, and another due to changes in the joint distribution of the covariates, $F_{X|T}$. This immediately leads to the notion of counterfactual income distributions. Consider, for example, the counterfactual income distribution at time t that would have prevailed if the distribution of covariates were unchanged from time $t-1$, but the conditional distribution was that prevailing at time t:

$$F^*_{Y|T}(y|t) = \int F_{Y|X,T}(y|x,t) dF_{X|T}(x|t-1).$$

This counterfactual distribution allows us to conduct counterfactual analysis: Given $F^*_{Y|T}(y|t)$, one can evaluate the functional $\nu(\cdot)$ at the pair of the distributions $F^*_{Y|T=t}$ and $F_{Y|T=t-1}$, $\nu\left(F^*_{Y|T=t}, F_{Y|T=t-1}\right)$, which captures counterfactual time changes between $t-1$ and t, as it fixes the distribution of X at $t-1$. Then, differences between $\nu\left(F_{Y|T=t}, F_{Y|T=t-1}\right)$ and $\nu\left(F^*_{Y|T=t}, F_{Y|T=t-1}\right)$ may be solely attributable to changes in the distribution of covariates. Interestingly, $\nu\left(F^*_{Y|T=t}, F_{Y|T=t-1}\right)$ can be written as

$$\nu\left(F^*_{Y|T=t}, F_{Y|T=t-1}\right) = \xi(\mathrm{GIC}^*, q_{t-1}),$$

[4]Ferreira (2012) notes that changes in a number of other inequality indices can also be expressed as functions of the GIC, but does not introduce the DLC, which generalizes the link between the GIC and changes in inequality.

where GIC* is the counterfactual GIC, the growth incidence curve that would have been observed had the distribution of covariates remained unchanged. It is written as

$$\text{GIC}^*(\tau) = \frac{q_t^*(\tau)}{q_{t-1}(\tau)} - 1, \tag{4}$$

where $q_t^*(\tau) = F_{Y|T}^{*-1}(y|t)$.

For other parameters of interest, such as γ, $\Delta_L(\tau)$, and Δ_G, we can also define their counterfactual counterparts by replacing $F_{Y|T=t}$ by $F^*_{Y|T=t}$ as follows:

$$\begin{aligned}
\gamma^* &= \int_0^1 \lambda_{t-1}(\tau) GIC^*(\tau) d\tau, \quad \Delta_L^*(\tau) = \int_0^\tau \lambda_{t-1}(s) \left(\frac{\text{GIC}^*(s) - \gamma^*}{1+\gamma^*} \right) ds, \\
\Delta_G^* &= 2 \frac{\int_0^1 \int_0^\tau \lambda_{t-1}(s) \left(\gamma^* - \text{GIC}^*(s)\right) ds\, d\tau}{(1+\gamma^*)}.
\end{aligned} \tag{5}$$

With these quantities, we can provide measures of time changes in the income distribution, such as income growth (overall and by percentile) and changes in inequality, that would have attained had the distribution of covariates not changed from $t-1$ to t. While the objects previously presented, such as γ, $\text{GIC}(\tau)$, $\Delta_L(\tau)$, and Δ_G, capture *actual* changes in the income distribution, γ^*, $\text{GIC}^*(\tau)$, $\Delta_L^*(\tau)$ and Δ_G^*, capture *counterfactual* changes in the income distribution. In the next section we rewrite the counterfactual objects to facilitate estimation in practice.

2.3 | Counterfactual distributions as weighted distributions

A straightforward approach to obtaining counterfactual distributions is to apply weights to an actual distribution (see Dinardo et al., 1996). This procedure proves helpful for the estimation in the next section. In our case, we can rewrite

$$\begin{aligned}
F^*_{Y|T}(y|t) &= \int F_{Y|X,T}(y|x,t) dF_{X|T}(x|t-1) \\
&= \int F_{Y|X,T}(y|x,t) \phi(x;t,t-1) dF_{X|T}(x|t) = F_{Y|T}^{\phi_{t,t-1}(X)}(y|t),
\end{aligned}$$

where, for s and $u \in \{t-1, t\}$,

$$F_{Y|T}^{\phi_{s,u}(X)}(y|t) = \mathrm{E}\left[\phi_{s,u}(X) \cdot 1\{Y \leq y\} | T = t\right],$$

and

$$\phi_{s,u}(x) = \frac{dF_{X|T}(x|u)}{dF_{X|T}(x|s)} = \frac{\Pr[T=s]}{\Pr[T=u]} \cdot \frac{\Pr[T=u|X=x]}{\Pr[T=s|X=x]}.$$

Thus, from the above derivation, we see that the construction of counterfactual distribution boils down to the construction of simple weighted distributions at time t. In fact, even actual distributions can be written as weighted distributions. In order to treat all distributions considered in this paper as weighted ones, which will simplify the exposition of our inference results, we rewrite

$$F_Y^{w_{s,u}(X,T)}(y) = \mathrm{E}\left[w_{s,u}(X,T) \cdot 1\{Y \leq y\}\right],$$

where $w_{s,u}(X,T) = \frac{1\{T=s\}}{\Pr[T=s]} \cdot \phi_{s,u}(X)$, for s and $u \in \{t-1, t\}$.

These results imply that all of the actual and the counterfactual changes can be constructed using weighted functionals. This observation has been prevalent in the treatment effects literature. We will take advantage of that for inference.

2.4 | Interpreting counterfactual changes

Consider a single individual i, and represent his income at time t by Y_{it}, where $Y_{it} = g_t(X_{it}, \epsilon_{it})$. In that representation, three standard determinants of income at time t feature: X_{it} ("observables" or covariates), ϵ_{it} ("unobservables") and $g_t(\cdot)$ (the "structural" function). Thus the income distribution may change between $t-1$ and t because: (i) the distribution of covariates changes over time: $F_{X|T}(\cdot|t) \neq F_{X|T}(\cdot|t-1)$; (ii) the distribution of unobservables changes over time: $F_{\epsilon|T}(\cdot|t) \neq F_{\epsilon|T}(\cdot|t-1)$; (iii) the structural function differs over time: $g_t(\cdot) \neq g_{t-1}(\cdot)$; or (iv) any combination of the above.

Although there are three components of the individual income, we can only identify two objects at each time t, which are the two factors of the income distribution: the conditional distribution of income given covariates and the distribution of covariates. Therefore, the conditional distribution of income given covariates conflates the structural function with the distribution of unobservables.

The time change in the distribution of income can then be written as

$$F_{Y|T}(y|t) - F_{Y|T}(y|t-1) = \left(F_{Y|T}(y|t) - F^*_{Y|T}(y|t)\right) + \left(F^*_{Y|T}(y|t) - F_{Y|T}(y|t-1)\right),$$

where the difference $F_{Y|T}(y|t) - F^*_{Y|T}(y|t)$ is a weighted average of time changes in the distribution of covariates. It is often called the "composition" or "endowment" effect in the literature and denoted by

$$\Delta_{F_X}(\cdot) = F_{Y|T}(\cdot|t) - F^*_{Y|T}(\cdot|t) = \int F_{Y|X,T}(\cdot|x,t)\left(dF_{X|T}(x|t) - dF_{X|T}(x|t-1)\right).$$

The difference $F^*_{Y|T}(y|t) - F_{Y|T}(y|t-1)$, on the other hand, is a weighted average of time changes in the conditional income distribution. It is often described as the "structure" effect and written

$$\Delta_{F_{Y|X}}(\cdot) = F^*_{Y|T}(\cdot|t) - F_{Y|T}(\cdot|t-1) = \int \left(F_{Y|X,T}(\cdot|x,t) - F_{Y|X,T}(\cdot|x,t-1)\right) dF_{X|T}(x|t-1).$$

Note that $\Delta_{F_{Y|X}}(\cdot)$ captures two changes that may have occurred over time: changes in the distribution of unobservable components, and changes in the function that combines all observable and unobservable components into income. These two components are not separably identifiable from observed data unless we impose additional assumptions. One natural, and frequently used, assumption is that the conditional distribution of unobservables given covariates does not change over time (Fortin et al., 2011). This so-called "ignorability assumption" can be written as

$$F_{\epsilon|X,T}(\cdot|\cdot,t) = F_{\epsilon|X,T}(\cdot|\cdot,t-1) = F_{\epsilon|X}(\cdot|\cdot). \tag{6}$$

If the above Equation 6 holds, then

$$\Delta_{F_{Y|X}}(y) = \int (\Pr[g_t(X,\epsilon) \le y|x] - \Pr[g_{t-1}(X,\epsilon) \le y|x])\, dF_{X|T}(x|t-1),$$

and it follows that, for any given y, $\Delta_{F_{Y|X}}(y)$ will be different from zero only if $g_t(\cdot,\cdot) \neq g_{t-1}(\cdot,\cdot)$. Therefore, if Equation 6 holds, then $\Delta_{F_{Y|X}}$ will capture only changes in the structural function—that is, in the structural component of the income distribution. In that case, the only driver of counterfactual changes γ^*, $\mathrm{GIC}^*(\cdot)$, $\Delta^*_L(\cdot)$ and Δ^*_G are changes over time in the economic structure.

Although the ignorability assumption is eminently plausible in a number of settings, such as comparisons between randomly assigned treatment and control groups within groups formed by same values of X (the so-called stratified random assignment mechanism), it is clearly more demanding in the present setting of over-time distributional changes. In what follows, we are agnostic and do not necessarily impose it. Readers should interpret $\Delta_{F_{Y|X}}(y)$ as comprising effects arising from both changes in the structural function and in the distribution of unobservable determinants of income, unless they are prepared to impose Equation 6.

3 | ESTIMATION AND ASYMPTOTIC INFERENCE

This section discusses estimation and inference procedures. Because several statistics of interest, as for example, $\mathrm{GIC}(\tau)$, $\mathrm{GIC}^*(\tau)$, $\Delta_L(\tau)$, and $\Delta^*_L(\tau)$, can be written as functions of the vectors of quantiles (q_t, q^*_t, q_{t-1}), we first show how to estimate these vectors and then obtain the uniform convergence results for the joint distribution of these estimators. An application of the delta method allows one to proceed with inference for both GIC and GIC* processes. For Hadamard-differentiable functionals of (GIC, q_{t-1}) and $(\mathrm{GIC}^*, q_{t-1})$, such as the DLC, we obtain uniform inference results. Finally, we suggest resampling procedures for practical inference.

3.1 | Estimation

In this paper, we impose restrictions on the conditional probability of being observed at time t given X. We define it as $p(X)$ and the unconditional probability as p. Since we only have two time periods, the conditional probability of being observed at time $t-1$ given X is $1-p(X)$ and the unconditional probability of being observed at $t-1$ is $1-p$.

In what follows, it is useful to define the function m as $m(a,b;\tau) = \tau - 1\{a < b\}$.

Assumption 1.

(a) For each $\tau \in \mathcal{T}$, and for s and $u \in \{t-1, t\}$, $q_{s,u}(\tau)$ uniquely solves $E[w_{s,u}(X,T) \cdot m(Y, q_{s,u}(\tau); \tau)] = 0$. Thus $q_{t,t} = q_t$, $q_{t-1,t-1} = q_{t-1}$ and $q_{t,t-1} = q_t^*$.

(b) For some $c > 0$, $c < p(X) < 1 - c$, a.e. in X.

Assumptions 1(a) and 1(b) are standard in the literature. Condition 1(a) is a uniqueness condition on the parameters of interest. Condition 1(b) states that for almost all values of X both time period assignment levels have a positive probability of occurrence; that is, there is no specific value of X that is observed in one period only. For estimation, we define the following quantile vectors:

$$Q(\tau,\tau') = \begin{bmatrix} q_t(\tau) \\ q_{t-1}(\tau') \end{bmatrix} \quad \text{and} \quad Q^*(\tau,\tau') = \begin{bmatrix} q_t^*(\tau) \\ q_{t-1}(\tau') \end{bmatrix}.$$

We are interested in estimation and inference for the GIC(τ), GIC$^*(\tau)$, $\Delta_L(\tau)$, and $\Delta_L^*(\tau)$. Equations 1 and 4 show that both GIC and GIC* can be written as a function of the quantiles. Thus we estimate each component of the vectors $Q(\tau,\tau')$, and $Q^*(\tau,\tau')$ to construct estimators for the GIC(τ) and GIC$^*(\tau)$. Functionals of (GIC($\cdot$), $q_{t-1}(\cdot)$) and (GIC$^*(\cdot)$, $q_{t-1}(\cdot)$), such as $\Delta_L(\tau)$, and $\Delta_L^*(\tau)$, which appear in Equations 3 and 5, will therefore be functionals of $Q(\cdot,\cdot)$, and $Q^*(\cdot,\cdot)$.

Let $\mathbb{E}$ denote the sample average; that is, $\mathbb{E}[X] = \frac{1}{n}\sum_{i=1}^n X_i$. We estimate the parameters of interest using a three-step estimator as follows:

- *Step 1.* Estimate $p(X)$ parametrically or nonparametrically and obtain an estimator $\hat{p}(X)$.[5] The estimator of p is the sample average of $D = 1\{T = t\}$; that is, $\hat{p} = \mathbb{E}D = n^{-1}\sum_{i=1}^n D_i$.
- *Step 2.* For each $(\tau,\tau') \in \mathcal{T} \times \mathcal{T}$, obtain

$$\hat{Q}(\tau,\tau') = \begin{bmatrix} \hat{q}_t(\tau) \\ \hat{q}_{t-1}(\tau') \end{bmatrix} \quad \text{and} \quad \hat{Q}^*(\tau,\tau') := \begin{bmatrix} \hat{q}_t^*(\tau) \\ \hat{q}_{t-1}(\tau') \end{bmatrix},$$

where $\hat{q}_t(\tau)$, $\hat{q}_{t-1}(\tau)$ and $\hat{q}_t^*(\tau)$ satisfying the following conditions:

$$\begin{aligned}\mathbb{E}[\hat{w}_{t-1,t-1}(X,T) \cdot m(Y,\hat{q}_{t-1}(\tau);\tau)] &= \mathbb{E}[\hat{w}_{t,t}(X,T) \cdot m(Y,\hat{q}_t(\tau);\tau)] \\ &= \mathbb{E}[\hat{w}_{t,t-1}(X,T) \cdot m(Y,\hat{q}_t^*(\tau);\tau)] = 0,\end{aligned}$$

where, for s and $u \in \{t-1, t\}$,

$$\hat{w}_{s,u,i} = \hat{w}_{s,u}(X_i,T_i) = \frac{1\{T_i = s\}}{\mathbb{E}[1\{T = u\}]} \cdot \frac{\widehat{\Pr}[T = u|X_i]}{\widehat{\Pr}[T = s|X_i]},$$

such that $\widehat{\Pr}[T = t|X_i] = \hat{p}(X_i)$ and $\widehat{\Pr}[T = t-1|X_i] = 1 - \hat{p}(X_i)$. Finally, one can easily check that $\hat{w}_{t,t,i} = D_i/\hat{p}$, $\hat{w}_{t-1,t-1,i} = (1-D_i)/(1-\hat{p})$, and $\hat{w}_{t,t-1,i} = [D_i/(1-\hat{p})]\,[(1-\hat{p}(X_i))/\hat{p}(X_i)]$. In practice, estimators of $q_t(\tau)$, $q_{t-1}(\tau)$ and $q_t^*(\tau)$ can be obtained by weighted quantile regressions:

$$\hat{q}_t(\tau) = \arg\min_q \mathbb{E}\left[\hat{w}_{t,t}\rho_\tau(Y-q)\right], \tag{7}$$

$$\hat{q}_{t-1}(\tau) = \arg\min_q \mathbb{E}\left[\hat{w}_{t-1,t-1}\rho_\tau(Y-q)\right], \tag{8}$$

$$\hat{q}_t^*(\tau) = \arg\min_q \mathbb{E}\left[\hat{w}_{t,t-1}\rho_\tau(Y-q)\right], \tag{9}$$

where $\rho_\tau(u) := u(\tau - 1\{u < 0\})$ is the check function as in Koenker and Bassett (1978).
- *Step 3.* Finally, we can plug estimates of the desired quantiles into the expressions to estimate GIC in Equation 1 as follows:

$$\widehat{\text{GIC}}(\tau) = \frac{\hat{q}_t(\tau) - \hat{q}_{t-1}(\tau)}{\hat{q}_{t-1}(\tau)} = \frac{[1\ 0]\,\hat{Q}(\tau,\tau)}{[0\ 1]\,\hat{Q}(\tau,\tau)} - 1,$$

where we estimate $\hat{q}_t(\tau)$ and $\hat{q}_{t-1}(\tau)$ as in Equations 7 and 8, respectively.

Analogously, we can also estimate the counterfactual GIC* in Equation 4 as

$$\widehat{\text{GIC}}^*(\tau) = \frac{\hat{q}_t^*(\tau) - \hat{q}_{t-1}(\tau)}{\hat{q}_{t-1}(\tau)} = \frac{[1\ 0]\,\hat{Q}^*(\tau,\tau)}{[0\ 1]\,\hat{Q}^*(\tau,\tau)} - 1,$$

[5] The Supporting Information Appendix discusses the practical estimation of $p(X)$.

which, as described previously, is the growth incidence curve for quantile τ if the distribution of observed income covariates had remained fixed from period $t-1$ to t.

One can also easily use the three-step estimator defined above to obtain estimates for other functionals of interest as described in Section 2 above; for instance, $\Delta_L(\tau)$, and $\Delta_L^*(\tau)$.

There are other alternative estimators available in the literature for the quantile objects of interest defined in step 2 above. Donald and Hsu (2014) discussed a three-step estimator that made use of the inverse of the CDF. Nevertheless, the estimator discussed in this paper has several practical advantages. First, the quantile estimator is obtained without having to invert the CDF. This is possible because of the second advantage of our method: QR has a linear program representation, which makes practical computation simple and allows using weights directly into the objective function. Finally, if one is interested in quantiles and its transformations, the proposed estimator is attractive because of its computational efficiency and accuracy in finite samples.[6]

3.2 | Asymptotic properties

In this section, we derive the asymptotic properties of the multistep estimator for the quantile process. The asymptotic properties of the $\widehat{\text{GIC}}(\tau)$, $\widehat{\text{GIC}}^*(\tau)$, $\hat{\Delta}_L(\tau)$, $\hat{\Delta}_L^*(\tau)$, and other functionals follow from these results. We relegate formal regularity conditions and proofs to the Supporting Information Appendix, which also discusses the practical estimation of the weights $w_{s,u}(\cdot)$, for s and $u \in \{t-1, t\}$.

The following result establishes asymptotic properties of the $\hat{q}_{s,u}(\tau)$ estimator over the set of quantiles, under some mild regularity conditions, which are formally stated in the Supporting Information Appendix.

Theorem 1. *For s and $u \in \{t-1, t\}$: (a) Suppose that $E[w_{s,u}(X,T)m(Y, q_{s,u}(\tau);\tau)] = 0$, and that conditions QC.I–QC.III in the Supporting Information Appendix are satisfied. Then, as $n \to \infty$,*

$$\sup_{\tau \in \mathcal{T}} |\hat{q}_{s,u}(\tau) - q_{s,u}(\tau)| = o_{p^*}(1).$$

(b) In addition, suppose that $\left|\mathrm{E}\left[w_{s,u}(X,T)m(Y,q_{s,u}(\tau);\tau)\right]\right|_\infty = 0$, that $|\hat{q}_{s,u} - q_{s,u}|_\infty = o_{p^*}(1)$, and that conditions QC.I–QC.II, QG.I–QG.III in the Supporting Information Appendix are satisfied. Then, as $n \to \infty$, in $\ell^\infty(\mathcal{T})$,

$$\sqrt{n}(\hat{q}_{s,u} - q_{s,u}) \rightsquigarrow \mathbb{G}_{s,u},$$

where $\mathbb{G}_{s,u}$ is a mean zero Gaussian process with covariance function

$$\mathrm{E}\left[\mathbb{G}_{s,u}(\tau)\mathbb{G}_{s,u}(\tau')^\top\right] = D_{s,u}^{-1}(\tau)S_{s,u}(\tau,\tau')\left[D_{s,u}^{-1}(\tau')\right]^\top,$$

with

$$D_{s,u}(\tau) = \frac{\partial \mathrm{E}[w_{s,u}(X,T)m(Y,q;\tau)]}{\partial q}\Big|_{q=q_{s,u}(\tau)}$$

and

$$\begin{aligned} S_{s,u}(\tau,\tau') = \mathrm{E}\Big[&\Big(w_{s,u}(X,T)m(Y,q_{s,u}(\tau);\tau) - \big(w_{s,u}(X,T) - w_{u,u}(X,T)\big)\,\mathrm{E}\left[m(Y,q_{s,u}(\tau);\tau)|X,T=s\right]\Big) \\ &\cdot\Big(w_{s,u}(X,T)m(Y,q_{s,u}(\tau');\tau') - \big(w_{s,u}(X,T) - w_{u,u}(X,T)\big)\,\mathrm{E}\left[m(Y,q_{s,u}(\tau');\tau')|X,T=s\right]\Big)\Big]. \end{aligned}$$

The results in Theorem 1 show that the limiting distribution of the estimator is a Gaussian process.[7] Thus, if one fixes a quantile at $\bar{\tau}$, the limiting distribution collapses to a simple normal distribution.[8,9]

Given the result in Theorem 1, it is simple to establish the weak convergence to the vectors $\hat{Q}(\tau,\tau')$ and $\hat{Q}^*(\tau,\tau')$. Corollary 2 in the Supporting Information Appendix formalizes the result. Moreover, in order to perform inference on functions of the $Q(\tau,\tau')$ and $Q^*(\tau,\tau')$, we impose a differentiability condition on such functions and state a functional delta method result. The formal differentiability condition and details are collected in the Appendix (see Lemma 1). As

[6]See Koenker et al. (2013) for a discussion and comparison on the statistical properties of the distribution regression and the QR approaches.
[7]An alternative estimator for $q_{s,u}(\tau)$, where $s$and $u \in \{t, t-1\}$, emerges from the fact that it solves for $q$the equation: $E[w_{s,u}(X,T)m(Y,q;\tau)]$=$\mathrm{E}\left[(w_{s,u}(X,T) - w_{u,u}(X,T))\mathrm{E}[m(Y,q;\tau)|X,T=s]\right]$. The double-robust (DR) estimator $\hat{q}_{s,u}^{\mathrm{DR}}(\tau)$ discussed in Cattaneo (2010) solves the sample analog of that equation. The main advantage of using a DR estimator in this case would be in terms of accuracy of the asymptotic approximation, as discussed in Rothe and Firpo (2017). The disadvantage is that one has to estimate not only the weights but also $\mathrm{E}\left[m(Y,q;\tau)|X,T=s\right]$.
[8]Firpo and Pinto (2015) presented a similar result to Theorem 1. Nevertheless, our proof technique is different on the treatment of both infinite-dimension parameters. In addition, we do not require compactness of the support of $X$and impose weaker assumptions on $\hat{w}$.
[9]Donald and Hsu (2014) establish a weak convergence result for an estimator that makes use of the inverse of the CDF. Their result (Theorem 3.8) is similar to that in Theorem 1. Nevertheless, as mentioned previously, the quantile estimators are different.

an example of application of Theorem 1, we derive the asymptotic distribution for $\widehat{\text{GIC}}(\tau)$ and $\widehat{\text{GIC}}^*(\tau)$. Corollary 2, in the Appendix, establishes that

$$\sqrt{n}(\hat{Q} - Q) \rightsquigarrow \mathbb{G}, \quad \text{and} \quad \sqrt{n}(\hat{Q}^* - Q^*) \rightsquigarrow \mathbb{G}^*,$$

where $\mathbb{G}(\tau, \tau')$ and $\mathbb{G}^*(\tau, \tau')$ are Gaussian processes with given variance–covariance functions.

Recall that $\text{GIC}(\tau) = \frac{[1 \quad 0]Q(\tau,\tau)}{[0 \quad 1]Q(\tau,\tau)} - 1$, and $\text{GIC}^*(\tau) = \frac{[1 \quad 0]Q^*(\tau,\tau)}{[0 \quad 1]Q^*(\tau,\tau)} - 1$. These functionals are differentiable at (Q, Q^*), as long as $q_{t-1} \neq 0$. Therefore, from a functional delta method, we have the following results.

Corollary 1. *Assume the conditions of Theorem 1, as $n \to \infty$, in $\ell^\infty(\mathcal{T})$:*

$$\sqrt{n}\left(\widehat{GIC} - GIC\right) \rightsquigarrow GIC(\mathbb{G})', \quad \text{and} \quad \sqrt{n}\left(\widehat{GIC}^* - GIC^*\right) \rightsquigarrow GIC^*(\mathbb{G}^*)'.$$

The asymptotic distribution of $\hat{\Delta}_L(\tau)$ and $\hat{\Delta}_L^*(\tau)$ are similar to those in Corollary 1.

3.3 | Inference procedures

In this section, we turn our attention to inference procedures. We seek to develop inference for functionals of $(q_t(\tau), q_t^*(\tau), q_{t-1}(\tau))$, such as GICs and DLCs, over the set of quantiles $\tau \in \mathcal{T}$.

3.3.1 | Test statistics

Important questions posed in the econometric and statistical literature concern the nature of the impact of a policy intervention or treatment on the outcome distributions of interest. Questions for the GIC are, for example, whether there is significant income growth at any quantile ($\text{GIC}(\tau) = 0$ for all τ); or whether growth is uniform or heterogeneous ($\text{GIC}(\tau)$ equals the average growth rate, for all τ). One could also ask whether growth is nondecreasing in τ ($\text{GIC}(\tau)' \geq 0$ for all τ). Other questions may be related to the time difference of the Lorenz curve, as for example whether $\Delta_L(\tau) = 0, \forall\tau$ or $\Delta_L^*(\tau) = 0, \forall\tau$.

Let $\beta(\tau)$ be a functional of Q and Q^*; that is, $\beta(\tau) = h(Q(\tau, \tau))$. In particular, one may be interested in $\beta(\tau) = \text{GIC}(\tau)$, the corresponding counterfactual $\beta(\tau) = \text{GIC}^*(\tau)$, or $\beta(\tau) = \Delta_L(\tau)$ and $\beta(\tau) = \Delta_L^*(\tau)$. In what follows we concentrate the presentation on the $\beta(\tau) = \text{GIC}(\tau)$, but the development for $\beta(\tau) = \Delta_L(\tau)$ and other functionals is analogous.

We discuss testing two main hypotheses of interest. First, we consider the following standard null hypothesis:

$$H_0: \ \beta(\tau) - r(\tau) = 0, \quad \tau \in \mathcal{T}, \tag{10}$$

uniformly, where the vector $r(\tau)$ is assumed to be known, continuous in τ over $\mathcal{T}$, and $r \in \ell^\infty(\mathcal{T})$. More generally, the hypothesis in Equation 10 embeds several interesting hypotheses about the quantile process. Consider the following example.

Example 1. (Static distribution). A basic hypothesis is that the growth incidence curve, $\text{GIC}(\tau)$, is equal to zero for all $\tau \in \mathcal{T}$. The alternative is that it differs from zero at least for some $\tau \in \mathcal{T}$. In this case, $r(\tau) = 0$, implying that $\gamma = \Delta_L = \Delta_G = 0$.

A basic inference process to test the null hypothesis (Equation 10) is

$$W_n(\tau) := \hat{\beta}(\tau) - r(\tau), \quad \tau \in \mathcal{T}.$$

To derive the asymptotic properties of the above statistic, we need to compute the estimator $\hat{\beta}(\tau)$, which is given by $\hat{\beta} = h(\hat{Q})$. The $\text{GIC}(\tau)$ estimate is $\hat{\beta}(\tau) = \widehat{\text{GIC}}(\tau)$, and the estimate for $\text{GIC}^*(\tau)$ is $\hat{\beta}(\tau) = \widehat{\text{GIC}}^*(\tau)$, which for a fixed quantile τ has an asymptotic normal distribution as given in Corollary 1.

General hypotheses about $\beta(\tau)$ can be accommodated through functions of $W_n(\cdot)$. We consider the Kolmogorov–Smirnov and Cramér–von Mises type test statistics, $V_n = f(W_n(\cdot))$, where $f(\cdot)$ is a general functional of the process $W_n(\cdot)$. In particular, we consider the following functionals that lead to different test statistics, such as

$$V_{1n} := \sqrt{n} \sup_{\tau \in \mathcal{T}} |W_n(\tau)|, \quad V_{2n} := \sqrt{n} \int_{\tau \in \mathcal{T}} |W_n(\tau)| d\tau.$$

There are many alternative possible statistics, such as $V_{3n} := \sqrt{n} \sup_{\tau \in \mathcal{T}} W_n(\tau)^2$ and $V_{4n} := \sqrt{n} \int_{\tau \in \mathcal{T}} W_n(\tau)^2 d\tau$, among others. In this paper we concentrate on V_{1n} and V_{2n}. These statistics and their associated limiting theory provide a natural foundation for testing.

The limiting distributions of the test statistics are given in the Supporting Information Appendix. From Corollary 2 and Lemma 1, in the Appendix, under the null hypothesis ($H_0 : \beta = h(Q) = r$), it follows that $\sqrt{n}(h(\hat{Q}) - h(Q)) \rightsquigarrow h(\mathbb{G})'$. Thus Lemma 2 in the Appendix shows that V_{1n} and V_{2n} converge weakly to functionals of Gaussian processes. Finally, under the null hypothesis of interest (H_0 : GIC(τ) = $r(\tau)$), it follows that $\sqrt{n}(\widehat{\mathrm{GIC}}(\tau) - r(\tau)) \rightsquigarrow \mathrm{GIC}(\mathbb{G})'$. The result for H_0 : GIC$^*(\tau) = r(\tau)$ is analogous. The results are given in Corollary 3.

The second hypothesis of interest concerns an unknown $r(\tau)$, which needs to be estimated. In many practical examples, the component $r(\tau)$ in the null hypothesis (Equation 10) is unknown or defined as a function of the conditional distribution and thus needs to be estimated (see, e.g., Chernozhukov and Fernandez-Val, 2005; Koenker and Xiao, 2002). The term $r(\tau)$ might, for example, be GIC(τ) for a different country, or period. Or it might be GIC$^*(\tau)$. Consider the following example.

Example 2. (Distribution-neutral growth)**.** A basic hypothesis is that the GIC(τ) is statistically equal to mean growth rate for all $\tau \in \mathcal{T}$; that is, growth has no distributional heterogeneity. The alternative is that GIC(τ) differs from the mean at least for some $\tau \in \mathcal{T}$. In this case, $\hat{r}(\tau) = \hat{\gamma}_{\mathrm{AVG}} = \hat{\gamma}$.

The natural expedient of replacing the unknown r in the test statistic by its estimate introduces some fundamental difficulties. The estimate will be denoted by $\hat{r}(\tau)$. Let

$$\bar{W}_n(t) := \hat{\beta}(\tau) - \hat{r}(\tau), \quad \tau \in \mathcal{T}.$$

In this framework, we follow Chernozhukov and Fernandez-Val (2005) and assume that the quantile and nuisance parameter estimates satisfy the following: $\sqrt{n}$ -consistent estimators for $\hat{\beta}(\cdot)$ and $\hat{r}(\cdot)$, such that $\sqrt{n}(\hat{\beta}(\cdot) - \beta(\cdot)) \rightsquigarrow h(\mathbb{G}(\cdot))'$ and $\sqrt{n}(\hat{r}(\cdot) - r(\cdot)) \rightsquigarrow \mathbb{G}_r(\cdot)$ jointly in $\ell^\infty(\mathcal{T})$, where $(h(\mathbb{G}(\cdot))', \mathbb{G}_r(\cdot))$ is a zero mean continuous function of a Gaussian process with a nondegenerate covariance kernel. Thus we have that $\sqrt{n}(\hat{\beta}(\tau) - \hat{r}(\tau)) \rightsquigarrow h(\mathbb{G}(\tau))' - \mathbb{G}_r(\tau)$. The process remains asymptotically Gaussian; however, the estimation of $r(\tau)$ introduces a new drift component that additionally complicates the covariance kernel of the process.

Under the null hypothesis $H_0 : \beta(\tau) = r(\tau)$, the test statistics become

$$\bar{V}_{1n} := \sqrt{n} \sup_{\tau \in \mathcal{T}} |\bar{W}_n(\tau)|, \quad \bar{V}_{2n} := \sqrt{n} \int_{\tau \in \mathcal{T}} |\bar{W}_n(\tau)| \, d\tau.$$

The limiting distributions of these test statistics under the null hypothesis are provided in Lemma 3 in the Supporting Information Appendix. This result can be applied to test the GIC. Under the null (H_0 : GIC(τ) = $r(\tau)$), it follows that $\sqrt{n}\left(\widehat{\mathrm{GIC}}(\tau) - r(\tau)\right) \rightsquigarrow \mathrm{GIC}(\mathbb{G})'$, and $\sqrt{n}(\hat{r}(\tau) - r(\tau)) \rightsquigarrow \mathbb{G}_r(\tau)$. The result for H_0 : GIC$^*(\tau) = r(\tau)$ is analogous. Corollary 4 in the Appendix summarizes the results.

To perform practical inference in the tests described above we suggest the use of resampling techniques to approximate the limiting distributions and obtain critical values.

3.3.2 | Practical implementation of testing procedures

Implementation of the proposed tests in practice is simple. First, we discuss the test H_0 in Equation 10. To implement the tests one needs to compute the statistics of test V_{1n} or V_{2n}. Analogously, when $r(\tau)$ is unknown, one computes $\bar{V}_{1n}$ or $\bar{V}_{2n}$. We suggest the use of a recentered bootstrap procedure to calculate critical values. The steps for practical implementation are the following.

First, the estimates $\hat{\beta}(\tau)$ are computed by solving the problems in Equations 7–9 and taking the appropriate function. Second, W_n is calculated by centralizing $\hat{\beta}(\tau)$ at $r(\tau)$, and V_{1n} or V_{2n} is computed by taking the maximum over τ (V_{1n}) or summing over τ (V_{2n}). For the general case with unknown $r(\tau)$, the tests are computed in the same fashion. The only adjustment is the use of $\hat{r}(\tau)$ to compute $\bar{W}_n$. Third, after obtaining the test statistic, it is necessary to compute the critical values. We propose the following scheme. We use the test statistic V_{1n} as an example, but the procedure is the same for the other cases. Take B as a large integer. For each $b = 1, \ldots, B$:

(i) Obtain the resampled data $\{\left(Y_i^b, T_i^b, X_i^b\right), i = 1, \cdots, n\}$.
(ii) Estimate $\hat{\beta}^b(\tau)$ and set $W_n^b(\tau) := \left(\hat{\beta}^b(\tau) - \hat{\beta}(\tau)\right)$.
(iii) Compute the test statistic of interest $\hat{V}_{1n}^b = \max_{\tau \in \mathcal{T}} \sqrt{n} |W_n^b(\tau)|$.

Let $\hat{c}^{B}_{1-\alpha}$ denote the empirical $(1-\alpha)$ -quantile of the simulated sample $\{\hat{V}^{1}_{1n}, \cdots, \hat{V}^{B}_{1n}\}$, where $\alpha \in (0,1)$ is the nominal size. Reject the null hypothesis if V_{1n} is larger than $\hat{c}^{B}_{1-\alpha}$. In practice, the maximum in step (iii) is taken over a discretized subset of $\mathcal{T}$. A formal justification the simulation method is stated in Theorem 2 in the Supporting Information Appendix.

The Supporting Information Appendix collects Monte Carlo simulations conducted to evaluate the finite-sample properties of the tests. The results provide evidence that the empirical levels approximate well the nominal levels. The tests possess large power against selected alternatives. The results improve as sample size increases, but are not very sensitive to the numbers of bootstraps.

4 | WAGE DISTRIBUTION DYNAMICS IN THE USA AND BRAZIL, 1995–2007

This section provides an empirical application of the methods developed in Sections 2 and 3 above. We compute and compare the $\mathrm{GIC}(\tau)$, $\mathrm{GIC}^{*}(\tau)$, $\Delta_L(\tau)$, and $\Delta^{*}_L(\tau)$ for the two most populous nations in the Western Hemisphere, namely the USA and Brazil, for the 1995–2007 period. We find considerable differences in the shapes of the growth incidence and DLCs across these countries, and that these differences are quite informative of the nature of distributional change in each.

Our reweighting method allows for the direct construction of the counterfactual GIC and Δ_L, with no need to postulate a structural relationship between wages, covariates and unobserved terms, as was required by the earlier literature that followed Juhn et al. (1993).[10] Under that approach, economists would typically estimate ordinary least squares regressions for the two time periods separately and then construct a counterfactual wage distribution using estimated parameters and residuals from time t but covariates from time $t-1$. This would yield a counterfactual distribution of wages at time t, with a distribution of covariates that was fixed at time $t-1$ (see, e.g., Bourguignon et al., 2008). In addition to requiring strong functional form assumptions, however, it is not clear how one would perform statistical inference on the counterfactual GIC using that method.

In this section we report the estimates for GIC and Δ_L and their counterfactual counterparts GIC^{*} and Δ^{*}_L; that is, $\widehat{\mathrm{GIC}}(\tau)$, $\widehat{\mathrm{GIC}}^{*}(\tau)$, $\hat{\Delta}_L(\tau)$, and $\hat{\Delta}^{*}_L(\tau)$ respectively, over $\tau \in \mathcal{T}$. We also report the actual and counterfactual growth rates γ, γ^{*}, γ_{AVG} and $\gamma_{\mathrm{AVG}^{*}}$, for comparison. Moreover, using the techniques developed in the previous section, we perform inference on both sets of curves. Specifically, we apply the uniform tests, Kolmogorov–Smirnov (KS) and Cramér–von Mises (CVM), to test the following six hypotheses:

(i) Static distribution: (H_0 : $\mathrm{GIC}(\tau) = 0$ vs. H_A : $\mathrm{GIC}(\tau) \neq 0 \forall\tau$).
(ii) Distribution-neutral growth (H_0 : $\mathrm{GIC}(\tau) = \gamma_{\mathrm{AVG}}$ vs. H_A : $\mathrm{GIC}(\tau) \neq \gamma_{\mathrm{AVG}} \forall\tau$).
(iii) Constant inequality: (H_0 : $\Delta_L(\tau) = 0$ vs. H_A : $\Delta_L(\tau) \neq 0 \forall\tau$).
(iv) Static distribution, conditional on covariates, (H_0 : $\mathrm{GIC}^{*}(\tau) = 0$ vs. H_A : $\mathrm{GIC}^{*}(\tau) \neq 0 \quad \forall\tau$).
(v) Distribution-neutral growth, conditional on covariates (H_0 : $\mathrm{GIC}^{*}(\tau) = \gamma_{\mathrm{AVG}^{*}}$ vs. H_A : $\mathrm{GIC}^{*}(\tau) \neq \gamma_{\mathrm{AVG}^{*}} \quad \forall\tau$).
(vi) Constant inequality, conditional on covariates, (H_0 : $\Delta^{*}_L(\tau) = 0$ vs. H_A : $\Delta^{*}_L(\tau) \neq 0 \quad \forall\tau$).

Hypotheses (i) and (ii) correspond to the two examples discussed in Section 3.3.1. Hypothesis (iii) is formally analogous to (i) but applied to the DLC rather than to the GIC. Because the DLC is mean independent, it is consistent with equiproportional changes along the distribution, which preserve the Lorenz curve and hence is an alternative test of the distribution-neutral growth hypothesis (ii). Hypotheses (iv)–(vi) are analogous to (i)–(iii), for the counterfactual objects that correspond to unchanged distributions of observed covariates. Before we test these hypotheses, we briefly discuss the data sets we use for both countries.

4.1 | Data

4.1.1 | CPS—USA

Data for the USA come from the March Supplement to the Current Population Surveys (CPS) for 1995 and 2007.[11] The dataset provides the distribution of labor earnings in the USA in 1995 and 2007 for full-time workers of both genders. We use the following variables for our analysis. Y denotes real hourly labor earnings (sum of annual pretax wages, salaries, tips, and bonuses, divided by the number of hours worked annually). The vector X consists of 10 covariates, namely: (i) the

[10] In other words, the structural function $g_t(X_{it}, \epsilon_{it})$ need not be specified.
[11] We use data provided by Center for Economic and Policy Research (CEPR), 2016, March CPS Uniform Extracts, Version 1.0, Washington, DC.

TABLE 1 Summary statistics: USA and Brazil

	CPS		PNAD	
	1995	**2007**	**1995**	**2007**
Hourly work earnings	16.733	19.627	5.558	5.617
Age	37.074	39.313	34.971	36.385
Female	0.476	0.477	0.374	0.415
Married	0.569	0.552	0.619	0.602
Public sector	0.160	0.151	0.130	0.124
Urban	0.567	0.587	0.496	0.553
Metropolitan	0.227	0.266	0.336	0.331
Rural	0.210	0.147	0.168	0.116
Education				
Primary	0.036	0.033	0.703	0.493
Some high school	0.093	0.075	0.061	0.077
High school	0.333	0.305	0.137	0.263
Some college	0.294	0.290	0.032	0.067
College	0.166	0.199	0.067	0.099
Postgraduate	0.078	0.098	0.002	0.006
Race				
White	0.762	0.683	0.567	0.517
Black	0.111	0.111	0.054	0.082
Hispanic/Mixed	0.093	0.141	0.372	0.392
Asian	0.025	0.050	0.005	0.005
Others	0.008	0.011	0.001	0.002
Observations	68,918	95,212	117,768	153,946

Note. We only report means. All variables apart from real hourly wage and age are binary; thus standard deviations and maximum and minimum values are straightforward. Standard deviations for real hourly wages are, respectively: 11.41, 16.76, 7.33, and 6.95. Standard deviations for age are: 12.11, 12.81, 11.95, and 12.03.

worker's age in years; (ii) a gender dummy; (iii) a categorical variable for highest educational level attained (six categories: "primary," "some high school," "high school," "some college," "college," and "postgraduate"); (iv) a categorical variable for race (five categories: "white," "black," "Hispanic," "Asian," and "others"); (v) a set of state dummies; (vi) a categorical variable for location (rural and metropolitan, with urban as base category); (vii) a dummy for marital status; (viii) a dummy for public sector; (ix) a set of dummies comprising 13 occupations;[12] and (x) a set of dummies comprising 13 industries.[13]

We restrict the sample to individuals aged 16–65 that report a positive value for real hourly earnings. Individuals with missing values for any of the variables in Y or X were excluded from the sample. After applying these filters, we trimmed the sample by dropping the top and bottom 0.5% of the distribution of hourly wages in each year, to eliminate outliers. This step also eliminates observations subject to CPS top-coding. Hourly wages are in US dollars of March 2007. The final sample contains a total of 68,918 observations for 1995 and 95,212 observations for 2007. Summary statistics are presented in Table 1.

4.1.2 PNAD—Brazil

The Brazilian data come from the Pesquisa Nacional por Amostra de Domicílios (PNAD), an annual Brazilian household survey that samples households across (almost) the entire country.[14] It collects information on various household characteristics, as well as individual incomes and education levels. We use PNAD data for 1995 and 2007. For comparability, we use the same set of variables as for the CPS: real hourly labor earnings; age; gender; a categorical educational attain-

[12]Executive, administrative, and managerial occupations are the base category. We follow Acemoglu and Autor (2011) to classify each occupation and do the crosswalk from the 2007 to the 1995 classification.

[13]Agriculture, forestry, fishing, and hunting is the base category. We follow the crosswalk provided by the Census to classify 1995 industries as in the 2007 categorization. Document available at www.census.gov/topics/employment/industry-occupation/guidance/code-lists.html (last accessed March 11, 2018).

[14]In 1995, the PNAD did not survey households in the rural areas of Acre, Amapa, Amazonas, Para, Rondonia or Roraima—six states in the Amazon region. Although PNAD samples cover these rural areas from 2004 onwards, we exclude them from the 2007 sample in order to preserve comparability across years.

TABLE 2 Inequality measures hourly real wages (HRW): USA and Brazil

	USA			Brazil		
	Factual			Factual		
	1995	2007	Counterfactual	1995	2007	Counterfactual
Gini	0.355	0.383	0.378	0.539	0.490	0.476
Theil entropy	0.205	0.261	0.255	0.538	0.458	0.446
Theil mean log deviation	0.219	0.254	0.247	0.512	0.411	0.390
Standard deviation of logs	0.683	0.711	0.700	0.963	0.850	0.823
Growth of mean wage (γ)	0.173		0.113	0.011		−0.118
Mean GIC (γ_{AVG})	0.127		0.063	0.138		−0.016

ment variable; race, now following the Brazilian racial classification (white; black; *pardo* or mixed race; Asian; or other); state and location variables; marital status; public sector; occupation;[15] and industry. Except for earnings, these are the variables that constitute the largest set containing the most used wage-setting determinants that could also be found in the CPS.

As for the USA, we restrict the sample to individuals aged 16–65 that report positive labor earnings. Individuals with missing values for income or any covariate were excluded from the sample. The top and bottom 0.5% of the distribution of hourly wages in each year were trimmed, as in the CPS. Hourly wages are in Brazilian reals (BRL) of September 2007. The final sample contained a total of 117,768 observations for 1995 and 153,946 observations for 2007. Summary statistics are presented in Table 1.

By examining Table 1 we observe considerable differences between the two labor forces. US full-time workers are on average almost 3 years older than their Brazilian counterparts, and earn much higher wages: the nominal exchange rate in September 2007 was 1.90 BRL to the USD, so average wages in 2007 in this sample were more than 6 times higher in the US than in Brazil. US workers are also much more educated, and the female share of the labor force is higher in the USA. Over the 12 years between 1995 and 2007, both labor forces became a little older and more educated. Educational attainment rose in both countries but more markedly in Brazil, which started from a much lower level. Completion of high school in Brazil almost doubled over the period, and the college-educated share also rose from 7% to 10%. The female share of the labor force was essentially stable at 48% in the USA, but rose from 37% to 42% in Brazil, driven primarily by a higher rate of female labor force participation (Ferreira et al., 2016).

4.2 | Results

Before we present results for the GIC and DLC, Table 2 presents some standard inequality indices for hourly real wages for both countries. The first panel presents four common measures of relative wage inequality for the two countries in 1995 and 2007, as well as for the counterfactual wage distribution $F^*_{Y|T}(y|t)$.[16] The inequality measures are the Gini coefficient, the Theil T index (i.e., the generalized entropy measure with parameter = 1), the mean log deviation (also known as Theil L, or GE(0)), and the standard deviation of logarithms. The second panel gives actual and counterfactual growth rates in mean hourly wages (γ) and the average of quantile-specific growth rates across quantiles, γ_{AVG}.

Below, we discuss the findings for each country separately, including the KS and CVM tests of the four hypotheses listed earlier, before briefly comparing results across countries.[17]

4.2.1 | USA

Figure 1 presents the estimates for the GIC and the DLC for the USA. The left-hand panel displays for GIC, GIC*, γ_{AVG}, and γ_{AVG^*}. The solid line displays the GIC, and the straight horizontal dash/dot black line represents the corresponding

[15]We follow Firpo et al. (2016) for the crosswalks between 2007 and 1995. Brazil's occupational classification is not identical to that used for the USA.
[16]For both countries, the estimator of $\Pr[T = t|X]$ is based on a logistic regression using the full set of dummy variables as regressors. The set of dummies was built upon the values of original categorical variables. This is a practical way to guarantee flexibility for the model and to preserve comparability across countries while being parsimonious, as we have avoided including interaction terms.
[17]In reality, both CPS and PNAD data sets have more complex sampling schemes and survey design structures, and our natural i.i.d. assumption may not necessarily hold for these data. Nevertheless, our empirical results partially account for this feature of the data, by using the appropriate sampling weights.

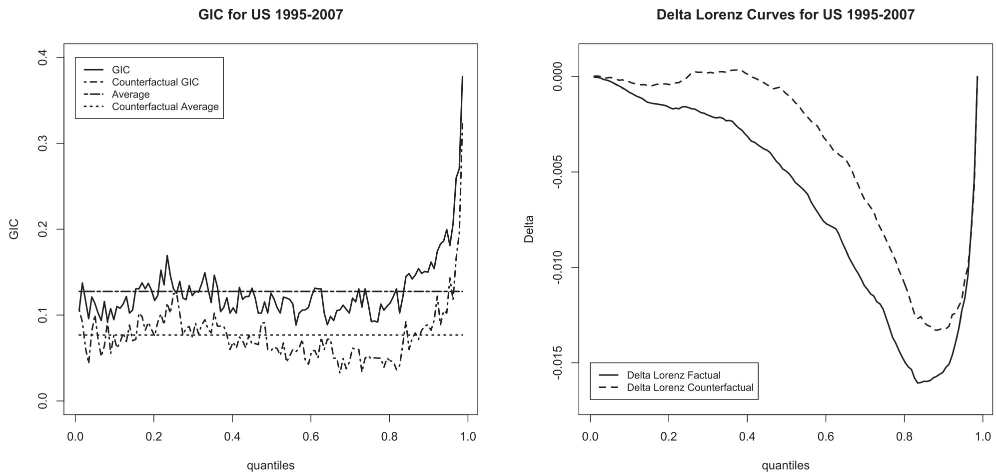

FIGURE 1 Box plots, USA: left, GICs; right, DLCs

TABLE 3 Kolmogorov–Smirnov (KS) and Cramér–von Mises (CVM) tests: USA and Brazil

	USA				Brazil			
Null hypothesis	**KS**	**1% CV**	**CVM**	**1% CV**	**KS**	**1% CV**	**CVM**	**1% CV**
$\text{GIC}(\tau) = 0$	0.378	0.079	15.686	2.501	0.521	0.108	20.052	3.122
$\text{GIC}(\tau) = \gamma_{\text{AVG}}$	0.250	0.078	2.711	1.369	0.389	0.103	18.181	3.107
$\Delta_L(\tau) = 0$	0.016	0.004	0.792	0.208	0.045	0.005	3.357	0.361
$\text{GIC}^*(\tau) = 0$	0.322	0.083	9.442	1.780	0.383	0.095	19.615	2.791
$\text{GIC}^*(\tau) = \gamma_{\text{AVG}^*}$	0.246	0.075	2.621	1.405	0.368	0.085	19.704	2.360
$\Delta_L^*(\tau) = 0$	0.013	0.005	0.477	0.303	0.061	0.006	4.380	0.239

Note. This table shows the results for the Kolmogorov–Smirnov (KS) and Cramér–von Mises (CVM) tests for USA and Brazil. Column 1 shows the null hypotheses. The KS and CVM columns display the corresponding KS and CVM test statistics. Columns 1% CV report the 1% level of significance critical values for the corresponding KS and CVM tests.

average growth rate, γ_{AVG}. The dashed curve displays GIC*, and the dashed horizontal line shows its corresponding mean effect, γ_{AVG^*}. The right-hand panel shows the DLC and its corresponding counterfactual.[18]

The growth incidence curve for the USA is essentially flat, oscillating around a cumulative growth rate of approximately 10% for the first eight deciles of the distribution. From $\tau = 0.8$ onwards it begins to slope upwards, and the slope increases sharply for the uppermost decile. A growth rate of 10% over 12 years translates into an average annual wage growth rate of less then 1% over the period, supporting earlier descriptions of wage stagnation for most US workers, even during the "Goldilocks"economy that preceded the great financial crisis of 2008–09 (see, e.g., Kopczuk et al., 2010; Mishel et al., 2012). The fact that growth in the average wage was considerably higher, at 17.3%, reflects the much better performance of the top quintile. The average quantile-specific growth rate across quantiles was 12.7%.

The more rapid growth of wages among the top fifth of full-time workers naturally translated into rising inequality, as shown by all four inequality measures in Table 2. The commonly used Gini coefficient rose by almost three percentage points. The generality of these results is confirmed by the actual DLC in panel 2, which is negative everywhere, with a global minimum at $\tau = 0.834$. This means that the bottom 83% of the US wage distribution lost income share between 1995 and 2007. That share was captured by the top 17%.

The basic findings that there was positive but heterogeneous wage growth in the USA, and that wage inequality increased, are found to be statistically significant by the inference results for the formal hypotheses formulated earlier. These results are presented in Table 3, which reports the Kolmogorov–Smirnov (KS) and Cramér–von Mises tests (CVM) (V_{1n} and V_{2n}, respectively) for both countries. First, we test the static distribution hypothesis for the GIC uniformly over quantiles (H_0 : $\text{GIC}(\tau) = 0$), which is rejected at the 1% level of significance for both tests. Thus we reject the hypothesis that the US wage distribution did not change at all. Second, we test whether growth was distribution neutral over the

[18]Confidence intervals, constructed using 500 bootstrap replications, are omitted from the figure for visual clarity, but are available on request.

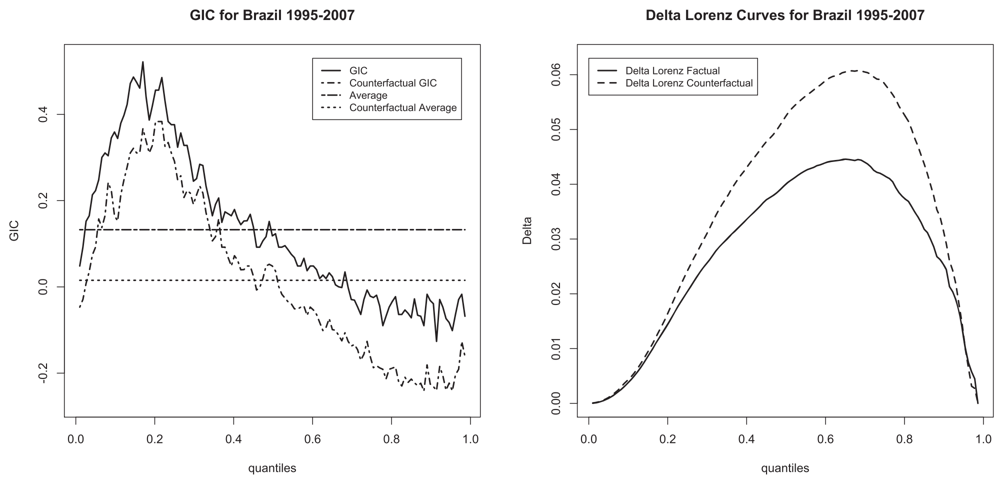

FIGURE 2 Box plots, Brazil: left, GICs; right, DLCs

period—that is, whether GIC$(\tau) = \gamma_{\text{AVG}}$. In this test we have an estimated $(\hat{r})$ under the null hypothesis and apply the $\bar{V}_{1n}$ and $\bar{V}_{2n}$ tests. Again, we strongly reject the null hypothesis, which is in line with the heterogeneity observed across quantiles in Figure 1. As implied by the latter finding, we also reject hypothesis (iii) of no change in the Lorenz curve and hence constant inequality ($H_0 : \Delta_L(\tau) = 0$). Since the DLC is negative everywhere, we can conclude that the increase in US inequality is unambiguous, and holds for any relative inequality measure that satisfies anonymity and the Pigou–Dalton transfer axiom.

The second interesting finding from our analysis is that the counterfactual growth incidence curve, GIC*, lies everywhere between the no-growth line at zero and the actual GIC, and its shape is very similar to that of the latter. This implies that both changes in (broadly defined) economic structure—encompassing changes in returns to observed worker attributes, as well as changes in the distribution of unobserved characteristics—and changes in the joint distribution of age, gender, and education, contributed to the modest increase in US wages during the study period. Since the GIC* is also flat until $\tau = 0.8$ or thereabouts, and then sharply increasing, we can conclude that the rise in wage inequality is not driven primarily by changes in the composition of the workforce (in terms of gender, race, age, educational attainment, marital status, geographic location, occupation or industry). It is more likely to be driven by changes in economic structure (including unobservables) and by their impact on the remuneration structure of various worker attributes. The DLCs depicted in panel 2 confirm the primacy of the structure effect, but also suggest that compositional changes played a supporting role in increasing inequality (visible in the gap between DLC* and DLC).

This finding is confirmed by inspection of the wage inequality measures for the US counterfactual distribution, $F^*_{Y|T}(y|t)$, in Table 2. All four measures lie strictly between the actual wage inequality levels in 1995 and 2007, but are all much closer to the higher 2007 levels. Taking the mean log deviation as an example, the decomposition indicates that changes in economic structure between 1995 and 2007 shifted the measure from 0.219 in 1995 to 0.247. Changes in the joint distribution of covariates—that is, the age, gender, and educational make-up of the full-time labor force—account only for the residual change from 0.247 to 0.254. As expected, then, hypotheses (iv)–(vi) are also comfortably rejected at the 1% level of significance for both the KS and CVM tests, which are also presented in Table 3. They confirm that the GIC$^*(\tau)$ was neither constant nor distribution neutral over the period, and constancy of the $\Delta^*_L(\tau)$ is also comfortably rejected.

4.2.2 | Brazil

The results for the Brazilian GIC and GIC$^*(\tau)$ for 1995–2007 are displayed in Figure 2. As before, the solid curve denotes the actual GIC, and the straight horizontal dash/dot black line represents the corresponding average growth rate, γ_{AVG}. The dashed curve displays the counterfactual growth incidence curve, GIC*, and the dashed horizontal line shows its corresponding mean effect, γ_{AVG^*}.

Remarkably, there was even less growth in average wages for full-time workers in Brazil than in the USA over this period. Cumulative growth in real wages was a paltry 1.1%—a tenth of the US rate.[19] However, the distribution of that

[19]It is quite likely that this dismal performance is due, at least in part, to a composition effect. Ferreira et al. (2016) report that formal employment in Brazil rose by a fifth, from 48% to 58% of the labor force, between 1995 and 2012. While not strictly the same, formal employment is highly correlated

growth was completely different from the US case. Brazil's GIC is positive and rises sharply up until the first quintile, at which quantile wages grew by 40% or more over the period. The GIC is then downward sloping from $\tau = 0.2$ to $\tau = 1.0$. It crosses the x-axis near the 7th decile, and is negative thereafter. This growth pattern is consistent with a substantial decline in wage inequality among full-time workers, as shown in Table 2. Whereas all four inequality indices reported rose for the USA, all four declined for Brazil. The Gini coefficient fell by almost five points, and the mean log deviation, which is more sensitive to income gaps at the bottom of the distribution, lost almost 20% of its initial value.

More generally, the actual DLC in panel 2 is positive everywhere, indicating a higher Lorenz curve at every quantile.[20] Its global maximum is at $\tau = 0.65$, indicating that the bottom 65% of the Brazilian population gained income share over the period, at the expense of the top 35%.

This pro-poor pattern is also evident in the fact that the average growth rate across quantiles was 13.8%—higher than in the USA—despite a near-stagnant average wage. Unsurprisingly, then, hypotheses (i)–(iii) are resoundingly rejected at 1% level of significance for Brazil as well, in both the Kolmogorov–Smirnov and Cramér–von Mises tests (Table 3).

As in the US case, the GIC* lies everywhere below the GIC, and has a similar shape. This quasi-parallelism suggests that the main drivers of distributional heterogeneity—which in this case were highly equalizing—belong to the realm of changes in economic structure, affecting remuneration patterns and unobserved worker characteristics. One plausible such candidate driver was the sustained rise in Brazil's minimum wage over this period, which is both consistent with the shape of the GIC and with earlier findings in the literature (e.g., Engbom and Moser, 2017). Changes in the joint distribution of observed attributes—for example, gender, age, race, education, geographic location, occupation, industry—on the other hand, had roughly equi-proportional effects across the distribution. These effects were generally positive—that is, wage increasing—as one would expect from rising experience and educational levels.

Once again, this finding is consistent with the inequality measures for the Brazilian counterfactual distribution, reported in the last column of Table 2. These are all lower than the actual inequality values in both 1995 and 2007, suggesting that the observed decline in inequality was due entirely to changes in economic structure (including unobservables). This may well reflect both the effects of a rising minimum wage and the decline in the economy-wide skill premium, as discussed earlier in the literature (see, e.g., Barros et al., 2010; Wang et al., 2016). The effect of changes in the observed composition of the labor force was actually countervailing: Composition effects worked to partly offset the inequality decline, through a mildly unequalizing effect of the second term of the decomposition. This is confirmed by the fact that Brazil's $\Delta_L^*(\tau)$ lies everywhere above $\Delta_L(\tau)$, in marked contrast to the US case.[21] In terms of formal inference, as should be expected from Figure 2 and the above discussion, hypotheses (iv)–(vi) are rejected at the 1% level of significance for both KS and CVM tests (see Table 3).

A comparison of results suggests that the 1995–2007 period saw very different distributional dynamics for real hourly wages among full-time employees across the two countries. Growth in average wages was muted in both countries; and almost zero in Brazil. But such an aggregated description misses important differences in the distribution of that growth: whereas wages were growing at less than 1% per year in the USA for all but the top fifth of workers (who experienced much faster increases), Brazil saw relatively rapid wage growth for the bottom half of the distribution, while wages fell for the top quarter. As a result, wage inequality rose in the USA and fell markedly in Brazil, as shown by the sharply contrasting DLCs for both countries. While the poorest 83% of US full-time workers lost income share to the benefit of the top 17%, the bottom 65% of Brazilian workers gained income share at the expense of the top 35%.

In both cases, changes in the distribution of wages conditional on observables—that is, the effects captured by GIC* and DLC*—were the main drivers of distributional change. These changes—due to changes in returns to observable characteristics, and changes in unobservables—were unequalizing in the USA and inequality reducing in Brazil. Changes in the observed composition of the labor force (notably higher levels of education and experience), on the other hand, were unequalizing in both countries. Graphically, this effect is captured by the differences GIC − GIC* and DLC − DLC* in Figures 1 and 2. In the USA, this effect reinforced the structure effect. In Brazil, it partially offset the structure effect, thereby dampening the extent of inequality reduction.

with full-time status. The same authors also report that the formalization of labor contracts was more common among lower earners. Such a process is likely to lower average earnings in that sample through a composition effect.

[20] Actual and counterfactual Lorenz curves (and CDFs) for Brazil and the USA are shown in the Supporting Information Appendix.

[21] The unequalizing effect of educational expansions when returns are (artificially) held constant is not a novel finding. Bourguignon et al. (2005) referred to this as the "paradox of progress" and explained that it reflected the generally observed convexity of returns to schooling. As workers became more educated, mass in the schooling distribution shifted to ranges where returns were steeper and inequality rose.

5 | CONCLUSION

Growth incidence curves have proved to be very useful tools for the analysis of the heterogeneity of economic growth processes along the distribution of income. In this paper we first introduce the delta Lorenz curve and show that it is a functional of the growth incidence curve which gives the change in the income share cumulatively appropriated by all quantiles up to τ, over a particular period. We then define counterfactual growth incidence and delta Lorenz curves using weighted distributions, and discuss how they should be interpreted under different assumptions. A simple semiparametric procedure that allows for the estimation of the GIC with no need for restrictive functional form assumptions on the relationship between income and its covariates is proposed. We establish the asymptotic properties of these estimators and develop statistical inference procedures uniformly over the set of quantiles $\mathcal{T}$.

The methods are applied in the estimation of the actual and counterfactual growth incidence and delta Lorenz curves for the USA and Brazil during 1995–2007. The results document important heterogeneity across the quantiles of the income distribution in both growth processes. Neither country had a static income distribution over that period, and neither growth process was distribution neutral.

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REFERENCES

Abadie, A., Angrist, J., & Imbens, G. (2002). Instrumental variables estimates of the effect of subsidized training on the quantiles of trainee earnings. *Econometrica*, *70*, 91–117.

Acemoglu, D., & Autor, D. (2011). Skills, tasks and technologies: Implications for employment and earnings, *Handbook of labor economics*, (Vol. *4*, pp. 1043–1171) Amsterdam, Netherlands: Elsevier.

Angrist, J., Chernozhukov, V., & Fernandez-Val, I. (2006). Quantile regression under misspecification, with an application to the U.S. wage structure. *Econometrica*, *74*, 539–563.

Atkinson, A. (1970). On the measurement of inequality. *Journal of Economic Theory*, *2*, 244–263.

Barros, R., de Carvalho, M., Franco, S., & Mendonca, R. (2010). Markets, the state, and the dynamics of inequality in Brazil, *Declining Inequality in Latin America: A Decade of Progress?* (pp. 134–174) Washington, DC: Brookings Institution Press.

Belloni, A., Chernozhukov, V., & Fernandez-Val, I. (2011). Conditional quantile processes based on series or many regressors. (*Working paper*). Boston, MA: Boston University.

Besley, T., & Cord, L. J. (2007). *Delivering on the Promise of Pro-Poor Growth*. Washington, DC: World Bank and Palgrave MacMillan.

Bitler, M. P., Gelbach, J. B., & Hoynes, H. W. (2006). What mean impacts miss: Distributional effects of welfare reform experiments. *American Economic Review*, *96*, 988–1012.

Bourguignon, F. (2011). Non-anonymous growth incidence curves, income mobility and social welfare dominance. *Journal of Economic Inequality*, *9*, 605–627.

Bourguignon, F., Ferreira, F. H. G., & Leite, P. G. (2008). Beyond Oaxaca-Blinder: Accounting for differences in household income distributions. *Journal of Economic Inequality*, *6*, 117–148.

Bourguignon, F., Ferreira, F. H. G., & Lustig, N. (2005). *The Microeconomics of Income Distribution Dynamics in East Asia and Latin America*. Washington, DC: World Bank and Oxford University Press.

Cattaneo, M. (2010). Efficient semiparametric estimation of multi-valued treatment effects under ignorability. *Journal of Econometrics*, *155*, 138–154.

Chernozhukov, V., & Fernandez-Val, I. (2005). Subsampling inference on quantile regression processes. *Sankhya*, *67*, 253–276.

Chernozhukov, V., & Hansen, C. (2005). An IV model of quantile treatment effects. *Econometrica*, *73*, 245–261.

Dinardo, J., Fortin, N. M., & Lemieux, T. (1996). Labor market institutions and the distribution of wages, 1973–1992: A semiparametric approach. *Econometrica*, *64*, 1001–1044.

Doksum, K. (1974). Empirical probability plots and statistical inference for nonlinear models in the two-sample case. *Annals of Statistics*, *2*, 267–277.

Donald, S. G., Green, D. A., & Paarsch, H. J. (2000). Differences in wage distributions between Canada and the United States: An application of a flexible estimator of distribution functions in the presence of covariates. *Review of Economic Studies*, *67*, 609–633.

Donald, S. G., & Hsu, Y.-C. (2014). Estimation and inference for distribution functions and quantile functions in treatment effect models. *Journal of Econometrics*, *178*, 383–397.

Engbom, N., & Moser, C. (2017). Earnings inequality and the minimum wage: Evidence from Brazil. (*CESifo Working Paper Series No. 6393*). Munich, Germany: CESifo Group.

Essama-Nssah, B., Paul, S., & Bassole, L. (2013). Accounting for heterogeneity in growth incidence in Cameroon using recentered influence function regression. (*ECINEQ Working Paper No. 289*). Rome, Italy: Society for the Study of Economic Inequality.

Ferreira, F. H. G. (2012). Distributions in motion: Economic growth, inequality, and poverty dynamics, *Oxford Handbook of the Economics of Poverty*. (pp. 427–462). Oxford: Oxford University Press.

Ferreira, F. H. G., Firpo, S., & Messina, J. (2016). Understanding recent earnings inequality dynamics in Brazil, *New order and progress: Development and democracy in Brazil*. (pp. 187–211). New York, NY: Oxford University Press.

Firpo, S. (2007). Efficient semiparametric estimation of quantile treatment effects. *Econometrica*, *75*, 259–276.

Firpo, S., Carvalho, S., & Pieri, R. (2016). Using occupational structure to measure employ- ability with an application to the Brazilian labor market. *Journal of Economic Inequality*, *14*, 1–19.

Firpo, S., & Pinto, C. (2015). Identification and estimation of distributional impacts of interventions using changes in inequality measures. *Journal of Applied Econometrics*, *31*, 457–486.

Flores, C. A. (2007). Estimation of dose–response functions and optimal doses with a continuous treatment. (*Working Papers 0707*). Miami, FL: University of Miami, Department of Economics.

Fortin, N., Lemieux, T., & Firpo, S. (2011). Decomposition methods in economics, *Handbook of labor economics*. (pp. 1–102). Amsterdam, Netherlands: Elsevier.

Galvao, A. F., & Wang, L. (2015). Uniformly semiparametric efficient estimation of treatment effects with a continuous treatment. *Journal of the American Statistical Association*, *110*, 1528–1542.

Grimm, M. (2007). Removing the anonymity axiom in assessing pro-poor growth. *Journal of Economic Inequality*, *5*, 179–197.

Gutenbrunner, C., & Jureckova, J. (1992). Regression rank scores and regression quantiles. *Annals of Statistics*, *20*, 305–330.

Juhn, C., Murphy, K. M., & Pierce, B. (1993). Wage inequality and the rise in returns to skill. *Journal of Political Economy*, *101*, 410–442.

Koenker, R., & Bassett, G. W. (1978). Regression quantiles. *Econometrica*, *46*, 33–49.

Koenker, R., Leorato, S., & Peracchi, F. (2013). Distributional vs. quantile regression. (*CEIS Tor Vergata, Research Paper Series*). Rome, Italy: Centre for Economic and International Studies.

Koenker, R., & Machado, J. A. F. (1999). Goodness of fit and related inference processes for quantile regression. *Journal of the American Statistical Association*, *94*, 1296–1310.

Koenker, R., & Xiao, Z. (2002). Inference on the quantile regression process. *Econometrica*, *70*, 1583–1612.

Kopczuk, W., Saez, E., & Song, J. (2010). Earnings inequality and mobility in the United States: Evidence from social security data since 1937. *Quarterly Journal of Economics*, *125*, 91–128.

Lehmann, E. L. (1974). *Nonparametrics: Statistical Methods Based on Ranks*. San Francisco, CA: Holden-Day.

Mishel, L., Bivens, J., Gould, E., & Shierholz, H. (2012). *The State of Working America (12th ed.)*. Ithaca, NY: Cornell University Press.

Qu, Z., & Yoon, J. (2015). Nonparametric estimation and inference on conditional quantile processes. *Journal of Econometrics*, *185*, 1–19.

Ravallion, M., & Chen, S. (2003). Measuring pro-poor growth. *Economics Letters*, *78*, 93–99.

Rothe, C., & Firpo, S. (2017). Properties of doubly robust estimators when nuisance functions are estimated nonparametrically, *Econometric Theory*, forthcoming.

Wang, Y., Lustig, N., & Bartalotti, O. (2016). Decomposing changes in male wage distribution in Brazil. *Research in Labor Economics*, *44*, 49–78.

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

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