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Forecasting realized variance using asymmetric HAR model with time-varying coefficients

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ABSTRACT

This paper proposes an asymmetric HAR model with time-varying coefficients (TVC-AHAR) for modeling and forecasting realized variance. The TVC-AHAR model includes good and bad volatilities and assumes the associated time-varying coefficients to be driven by a latent Gaussian autoregressive process. The model is easy to estimate and implement by using maximum likelihood based on Kalman filter. Empirical analysis using two stock market indices of China, the Shanghai Stock Exchange Composite Index and Shenzhen Stock Exchange Component Index, shows that our proposed TVC-AHAR model yields more accurate out-of-sample forecasts of realized variance compared with the other models.

1. Introduction

Volatility modeling and forecasting is an important issue of research in financial markets, and it has received much attention of researchers and practitioners over the last three decades. GARCH and stochastic volatility (SV) models (Bollerslev, 1986; Taylor, 1986) are two popular volatility models that typically utilize daily data to model and extract the latent volatility. With the increasing availability of high-frequency intraday data, many researches have concentrated on employing realized variance to measuring and modeling volatility (Andersen and Bollerslev, 1998; Andersen et al., 2001; Barndorff-Nielsen and Shephard, 2002). The realized variance constructed from the high-frequency data contains rich information regarding the current level of volatility, which yields significantly better forecasts of future volatility compared to the traditional GARCH and SV models (Andersen et al., 2003).

One of the most important characteristics in the realized variance is the long memory feature (strong persistence in their autocorrelations). To account for this stylized fact of realized variance, Corsi (2009) develops a simple and parsimonious heterogeneous autoregressive (HAR) model that builds on the Heterogeneous Market Hypothesis. Corsi (2009) shows that the model provides more accurate out-of-sample forecasts of realized variance than the alternatives. Further, Bollerslev et al. (2016) propose a time-varying coefficient HAR (TVC-HAR) model by letting the autoregressive coefficient of the HAR model be driven by the realized quarticity to account for the heteroskedasticity in the measurement error of realized variance. They show that their TVC-HAR model improves on the standard HAR model. Subsequently, Bekierman and Manner (2018) propose an alternative TVC-HAR model in which the autoregressive coefficient is driven by a latent Gaussian autoregressive process as a more flexible and robust alternative. Bekierman and Manner (2018) show that this model yields more accurate forecasts than the other models, including the TVC-HAR model of Bollerslev et al. (2016).

The realized variance is based on even functions of high-frequency intraday returns (squared intraday returns), which ignores any information contained in the sign of these returns. To address this issue, Barndorff-Nielsen et al. (2010) decompose the realized

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variance into upside and downside realized semivariance measures, or "good" and "bad" volatilities, that relate to positive and negative intraday returns, respectively. The good and bad volatilities has recently been used to explain the cross-section of expected stock returns (Bollerslev et al., 2018) and to improve option pricing performance (Feunou and Okou, 2018). In particular, Patton and Sheppard (2015) propose an asymmetric HAR (AHAR) model that includes good and bad volatilities to capture the asymmetric effect of positive and negative returns. They show that the AHAR model can improve forecasts of realized variance.

Building on the work of Patton and Sheppard (2015) and Bekierman and Manner (2018), this paper proposes a model combination of the AHAR and TVC-HAR models that combines good and bad volatilities and time-varying coefficients. We refer to the model as the asymmetric HAR with time-varying coefficients, or TVC-AHAR model for short. The model features an asymmetric volatility reaction to positive and negative returns and time-varying coefficients driven by a latent Gaussian autoregressive process. We apply the proposed TVC-AHAR model to two stock market indices of China, the Shanghai Stock Exchange Composite Index and the Shenzhen Stock Exchange Component Index. The results show that the TVC-AHAR model yields more accurate out-of-sample forecasts of realized variance than the HAR, AHAR and TVC-HAR models.

The remainder of the paper is organized as follows. In Section 2, we present the methodology, including the realized variance and semivariance measures and the TVC-AHAR model. Empirical results from two stock market indices of China are presented in Section 3. Section 4 concludes.

2. Methodology

2.1. Realized variance and semivariance measures

The realized variance introduced by Andersen and Bollerslev (1998) for day t, RV_t, is defined as

$$RV_t = \sum_{i=1}^{n} r_{t,i}^2 \tag{1}$$

where $r_{t,i}$ is the ith intraday return at day t defined as $r_{t,i} = 100(p_{t,i} - p_{t,i-1})$, where $p_{t,i}$ is the ith intraday logarithmic price at day t, and n is the total number of intraday returns for the day. Andersen et al. (2003) show that under suitable conditions, including the absence of the jumps, RV_t is a consistent estimator of the true integrated variance IV_t:

$$RV_t \to IV_t = \int_{t-1}^{t} \sigma_s^2 ds \tag{2}$$

where σ_s is the instantaneous volatility.

Barndorff-Nielsen et al. (2010) introduce the upside and downside realized semivariance measures, or good and bad volatilities, that relate to the positive and negative high-frequency intraday returns, respectively,

$$RS_t^+ = \sum_{i=1}^{n} r_{t,i}^2 I\{r_{t,i} > 0\} \tag{3}$$

$$RS_t^- = \sum_{i=1}^{n} r_{t,i}^2 I\{r_{t,i} < 0\} \tag{4}$$

where $I\{\cdot\}$ is an indicator function. Note that $RV_t = RS_t^+ + RS_t^-$.

2.2. TVC-AHAR model

The TVC-AHAR model proposed in this paper is given by

$$\begin{aligned} RV_{t+1} = c &+ (\beta_d^+ + \lambda_{t+1}^+)RS_t^+ + (\beta_d^- + \lambda_{t+1}^-)RS_t^- \\ &+ \beta_w RV_t^w + \beta_m RV_t^m + \varepsilon_{t+1} \end{aligned} \tag{5}$$

$$\lambda_{t+1}^+ = \phi^+\lambda_t^+ + \eta_{t+1}^+, \quad \eta_{t+1}^+ \sim N(0, (\sigma_\eta^+)^2) \tag{6}$$

$$\lambda_{t+1}^- = \phi^-\lambda_t^- + \eta_{t+1}^-, \quad \eta_{t+1}^- \sim N(0, (\sigma_\eta^-)^2) \tag{7}$$

where ε_{t+1} is a error term with mean 0, and RV_t^w and RV_t^m are the weekly and monthly realized variances, respectively, so that $RV_t^w = \frac{1}{5}(RV_t + RV_{t-1} + \ldots + RV_{t-4})$ and $RV_t^m = \frac{1}{22}(RV_t + RV_{t-1} + \ldots + RV_{t-21})$. It is obvious that the model combines the good and bad volatilities and the time-varying coefficients. And the time variation in the coefficients associated with the good and bad volatilities is driven by a latent Gaussian autoregressive process. By introducing the good and bad volatilities and the time-varying coefficients, the TVC-AHAR model is able to capture the asymmetric effect of positive and negative returns on future volatility and the effect of heteroscedastic measurement errors of realized semivariances.

It is worth noting that the TVC-AHAR model encompasses the HAR model of Corsi (2009), the AHAR model of Patton and Sheppard (2015) and the TVC-HAR model of Bekierman and Manner (2018). This is obtained by setting $\beta_d^+ = \beta_d^-$ and $\phi^+ = \phi^- = \sigma_\eta^+ = \sigma_\eta^- = 0$ (HAR), or by $\phi^+ = \phi^- = \sigma_\eta^+ = \sigma_\eta^- = 0$ (AHAR) or by $\beta_d^+ = \beta_d^-$, $\phi^+ = \phi^-$ and $\sigma_\eta^+ = \sigma_\eta^-$ (TVC-HAR). Although more

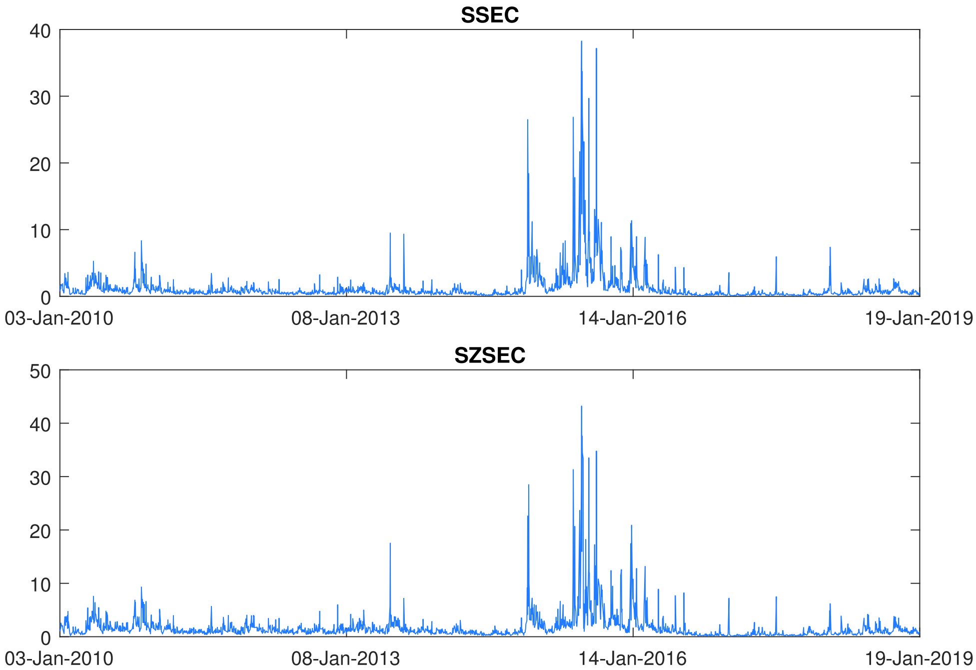

Fig. 1. Time series plots of realized variances for SSEC and SZSEC from January 4, 2010 to January 18, 2019.

general, our TVC-AHAR model can be conveniently estimated and implemented through the maximum likelihood based on Kalman filter.

3. Empirical analysis

In contrast to many previous studies that have focused mainly on developed stock markets (e.g., U.S. stock market), we consider in this paper the problem of volatility (realized variance) forecasting by focusing on emerging stock markets of China. Despite its rapid growth, China's stock market is found to be more volatile than developed stock markets and has experienced large fluctuations in the recent years, e.g., the 2015-2016 Chinese stock market turbulence. Hence, understanding and forecasting the volatility of Chinese stock market is very important, as it is crucial for risk management, asset allocation and option pricing.

3.1. Data

We use the proposed TVC-AHAR model to forecast the realized variances of two stock market indices of China, the Shanghai Stock Exchange Composite Index (SSEC) and Shenzhen Stock Exchange Component Index (SZSEC). Our calculation of the realized variance and semivariance measures is based on 5-minute high-frequency prices data, which are obtained from the Wind Database of China. The sample period for the data is from January 4, 2010 to January 18, 2019, which results in a total of 2200 daily observations for the two indices. The time series plots of the realized variances for the SSEC and SZSEC are presented in Fig. 1. It can be seen from the figure that the realized variance series exhibit large fluctuations in the recent years.

Table 1 presents summary statistics of the realized variances for the SSEC and SZSEC. In particular, the Ljung-Box Q statistic for serial correlation up to 20 lags shows each realized variance series exhibits highly significant autocorrelation that is consistent with long memory behavior.

3.2. Estimation results

Table 2 reports estimation results of the TVC-AHAR model for the SSEC and SZSEC. For the purpose of comparison, we also report

Table 1
Summary statistics of realized variances for SSEC and SZSEC from January 4, 2010 to January 18, 2019.

	Mean	Median	Min.	Max.	Std. Dev.	Q(20)
SSEC	1.2947	0.6348	0.0599	38.2684	2.6223	8578.3970 (0.000)
SZSEC	1.7708	1.0557	0.0720	43.2198	2.9270	6470.6175 (0.000)

Note: $Q(20)$ is the Ljung-Box Q statistic for autocorrelation up to 20 lags for realized variances. The number in parenthesis is the P-value.

Table 2
Estimation results.

	HAR	AHAR	TVC-HAR	TVC-AHAR
		SSEC		
c	0.1210	0.1148	0.0498	0.0508
	(0.0844)	(0.0791)	(0.0156)	(0.0156)
β_d^+ (β_d)	0.2892	−0.2781	0.7039	0.4430
	(0.0060)	(0.0184)	(0.0537)	(0.1050)
β_d^-	–	0.7939	–	0.9567
		(0.0129)		(0.1018)
β_w	0.4457	0.4764	0.1360	0.1436
	(0.0094)	(0.0100)	(0.0303)	(0.0280)
β_m	0.1698	0.1784	0.1569	0.1578
	(0.0120)	(0.0127)	(0.0189)	(0.0174)
σ_ε^2	3.2711	3.0696	0.0083	0.0103
	(0.0220)	(0.0194)	(0.0018)	(0.0017)
ϕ^+ (ϕ)	–	–	−0.0843	−0.1693
			(0.0220)	(0.0600)
$(\sigma_\eta^+)^2$ (σ_η^2)	–	–	0.5205	0.8144
			(0.0127)	(0.0408)
ϕ^-	–	–	–	−0.0626
				(0.0517)
$(\sigma_\eta^-)^2$	–	–	–	0.9957
				(0.0452)
		SZSEC		
c	0.1798	0.1925	0.0575	0.0560
	(0.0979)	(0.0952)	(0.0259)	(0.0253)
β_d^+ (β_d)	0.3602	−0.0255	0.7131	0.3762
	(0.0069)	(0.0183)	(0.0473)	(0.0965)
β_d^-	–	0.6978	–	1.0690
		(0.0126)		(0.0934)
β_w	0.2633	0.2567	0.0994	0.1082
	(0.0101)	(0.0104)	(0.0290)	(0.0285)
β_m	0.2729	0.2960	0.2066	0.2003
	(0.0147)	(0.0153)	(0.0199)	(0.0203)
σ_ε^2	4.6254	4.4734	0.0424	0.0431
	(0.0333)	(0.0313)	(0.0026)	(0.0026)
ϕ^+ (ϕ)	–	–	−0.1095	−0.2372
			(0.0224)	(0.0501)
$(\sigma_\eta^+)^2$ (σ_η^2)	–	–	0.4729	0.6898
			(0.0094)	(0.0327)
ϕ^-	–	–	–	−0.0975
				(0.0444)
$(\sigma_\eta^-)^2$	–	–	–	1.0070
				(0.0401)

Note: The number in parenthesis is the standard error.

the estimation results of the HAR, AHAR and TVC-HAR models. From the parameter estimates of the HAR model, we find that the coefficients on daily, weekly and monthly realized variances are all significantly positive, and the sum of those coefficients ($\beta_d + \beta_w + \beta_m$) is close to 1. The results are consistent with previous findings in the literature.

In the AHAR model, we find that the coefficient on downside realized semivariance is significantly larger than the coefficients on upside realized semivariance, weekly realized variance and monthly realized variance for both the SSEC and SZSEC. And the coefficient on upside realized semivariance is significantly negative for SSEC, while it is small and statistically insignificant for SZSEC. The parameter estimates for the TVC-AHAR model also show that the downside realized semivaiance has a larger impact on future volatility. But the estimated coefficients on upside realized semivariance for the AHAR and TVC-AHAR models are quite different. In fact, the coefficient on upside realized semivariance in the TVC-AHAR model is significantly positive.

In the time-varying coefficient TVC-HAR and TVC-AHAR models, the coefficients σ_η (TVC-HAR) and (σ_η^+, σ_η^-) (TVC-AHAR) are all statistically significant. In addition, all of the coefficients ϕ (TVC-HAR) and (ϕ^+, ϕ^-) (TVC-AHAR) are negative. And in the TVC-AHAR model, ϕ^+ coefficient is more negative and significant than the ϕ^- coefficient. Also note that, by including time-varying coefficients, the TVC-HAR and TVC-AHAR models allow for far stronger persistence than the constant coefficient HAR and AHAR models, particularly in terms of the coefficient on the daily realized variance or semivariance, indicating that large measurement errors in the realized variance result in a significant decrease in the persistence of the models, which is consistent with the findings of Bollerslev et al. (2016) and Bekierman and Manner (2018).

3.3. Out-of-sample forecasting

In this section, we investigate the out-of-sample forecasting performance of the proposed TVC-AHAR model. A rolling window of 1500 observations is employed for the one-day-ahead forecasts. To be specific, we first estimate the model using the first 1500 observations, and use the estimated model to make the first one-day-ahead forecast. When a new observation is added to the sample, we delete the first observation and re-estimate the parameters of the model. The re-estimated model is then used to forecast the realized variance. This process is repeated until we reach the last observation in the sample.

For the out-of-sample comparison of the competing models, we consider four loss functions, namely the mean absolute error (MAE), mean absolute percentage error (MAPE), mean squared error (MSE) and quasi-likelihood (QLIKE). Among these, the MSE and QLIKE are robust loss functions that provide consistent ranking (Patton, 2011). The four loss functions are defined as:

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |RV_t - \hat{RV}_t| \tag{8}$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T} \left|\frac{RV_t - \hat{RV}_t}{RV_t}\right| \tag{9}$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} (RV_t - \hat{RV}_t)^2 \tag{10}$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T} \left(\frac{RV_t}{\hat{RV}_t} - \log\left(\frac{RV_t}{\hat{RV}_t}\right) - 1\right) \tag{11}$$

where RV_t is the actual realized variance, $\hat{RV}_t$ is the forecasted realized variance, and T is the number of out-of-sample forecasts.

Table 3 reports the results of forecast evaluation for the HAR, AHAR, TVC-HAR and TVC-AHAR models. It can be seen from the table that, for the SSEC and SZSEC based on the four loss functions, the AHAR and TVC-HAR models are generally preferred over the standard HAR model. This result suggests that the inclusion of both asymmetric volatility (good and bad volatilities) and time-varying coefficient can improve realized variance forecasts. Comparing the AHAR model with the TVC-HAR model, we find that the TVC-HAR model outperforms the AHAR model, implying that the inclusion of time-varying coefficient is more important for forecasting realized variance than that of asymmetric volatility. Overall, our TVC-AHAR model taking into account both the asymmetric volatility and time-varying coefficients seems to offer the best out-of-sample performance with smallest loss function values for the two indices (except for the SZSEC on QLIKE), followed by the TVC-HAR model.

To gain further insight into the difference of the competing models, we employ the Mincer–Zarnowitz regression:

$$RV_t = a + b\hat{RV}_t + \nu_t \tag{12}$$

The unbiasedness of a model's forecasts can be tested by performing a joint test of H_0: $a = 0$, $b = 1$. In particular, the R^2 coefficient of the Mincer–Zarnowitz regression indicates the explanatory power or information content of the model's forecasts, irrespective of any bias.

Table 3
Out-of-sample forecast results.

	HAR	AHAR	TVC-HAR	TVC-AHAR
		SSEC		
MAE	0.3052	0.2996	0.2792	**0.2707**
MAPE	0.7122	0.6843	0.5676	**0.5554**
MSE	0.3558	0.3788	0.3448	**0.3277**
QLIKE	0.2054	0.2032	0.2144	**0.2009**
		SZSEC		
MAE	0.4611	0.4544	0.4261	**0.4085**
MAPE	0.7496	0.7503	0.5910	**0.5736**
MSE	0.7375	0.7523	0.7273	**0.6988**
QLIKE	0.2231	0.2158	**0.2101**	0.2129

Note: MAE is the mean absolute error, MAPE is the mean absolute percentage error, MSE is the mean squared error and QLIKE is the quasi-likelihood.

Table 4
Mincer–Zarnowitz regression results.

Intercept	HAR	AHAR	TVC-HAR	TVC-AHAR	R^2
			SSEC		
0.1170 (0.0533)	0.6968 (0.0852)				0.1868
0.1854 (0.0638)		0.5964 (0.1002)			0.1729
0.1498 (0.0491)			0.7005 (0.0797)		0.2020
0.1116 (0.0375)				0.7720 (0.0612)	**0.2254**
			SZSEC		
0.1621 (0.0713)	0.7189 (0.0630)				0.2006
0.1998 (0.0905)		0.6744 (0.0850)			0.2024
0.2309 (0.0638)			0.6956 (0.0579)		0.2103
0.1975 (0.0640)				0.7337 (0.0616)	**0.2331**

Note: The number in parenthesis is the heteroskedasticity and autocorrelation consistent standard error, which is computed using the Newey-West procedure.

The estimation results of the Mincer–Zarnowitz regressions are presented in Table 4. Consistent with the results presented in Table 3, the highest R^2 value is obtained by the TVC-AHAR model, which indicates higher information content about the true realized variance.

To determine the relative information content of competing models, we also run an encompassing regression:

$$RV_t = a + b_1 \hat{RV_t}(m) + b_2 \hat{RV_t}(\text{TVC-AHAR}) + \nu_t \tag{13}$$

where m stands for model HAR, AHAR or TVC-HAR. If model TVC-AHAR dominates model m, then it is expected that b_2 is statistically significant while b_1 is not.

Table 5presents the encompassing regression results. It can be seen that the TVC-AHAR model dominates the other models. The inclusion of the other models only slightly increases the R^2 of the encompassing regression. Indeed, once the TVC-AHAR forecasts are included, the forecasts from the other models become insignificant or with wrong signs. In summary, our TVC-AHAR model is the most accurate in forecasting realized variance compared to the HAR, AHAR and TVC-HAR models.

4. Conclusion

This paper proposes a TVC-AHAR model for forecasting realized variance. The model includes the good and bad volatilities and the time-varying coefficients driven by a latent Gaussian autoregressive process, which aims to capture the asymmetric effect of positive and negative returns on future volatility and the effect of heteroscedastic measurement errors of realized semivariances. Although more general, it is convenient to estimate and implement via maximum likelihood based on Kalman filter. An empirical application to two stock market indices of China indicates that both good and bad volatilities and time-varying coefficients are important for realized variance forecast, and our TVC-AHAR model yields more accurate out-of-sample forecasts of realized variance than the other models. Assessing the usefulness of our model in different applications, such as risk measurement and derivative pricing, is a interesting area for future research. Also, it would be meaningful to consider the forecasting performance of a variation of our model that is based on the logarithm of the realized variance.

Table 5
Encompassing regression results.

Intercept	HAR	AHAR	TVC-HAR	TVC-AHAR	R^2
			SSEC		
0.1052 (0.0405)	0.0569 (0.1732)			0.7205 (0.1684)	0.2245
0.1053 (0.0398)		0.0819 (0.1373)		0.6937 (0.1411)	0.2253
0.1111 (0.0370)			−0.2861 (0.2030)	1.0616 (0.2153)	0.2263
			SZSEC		
0.1732 (0.0602)	0.1203 (0.1263)			0.6313 (0.1388)	0.2331
0.1625 (0.0689)		0.1897 (0.0993)		0.5670 (0.1016)	0.2360
0.2060 (0.0631)			−0.4044 (0.2631)	1.1294 (0.2868)	0.2354

Note: The number in parenthesis is the heteroskedasticity and autocorrelation consistent standard error which is computed using the Newey-West procedure.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.frl.2019.04.006.

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