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Mixed-frequency models with moving-average components

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Summary

Temporal aggregation in general introduces a moving-average (MA) component in the aggregated model. A similar feature emerges when not all but only a few variables are aggregated, which generates a mixed-frequency (MF) model. The MA component is generally neglected, likely to preserve the possibility of ordinary least squares estimation, but the consequences have never been properly studied in the MF context. In this paper we show, analytically, in Monte Carlo simulations and in a forecasting application on US macroeconomic variables, the relevance of considering the MA component in MF mixed-data sampling (MIDAS) and unrestricted MIDAS models (MIDAS–autoregressive moving average (ARMA) and UMIDAS-ARMA). Specifically, the simulation results indicate that the short-term forecasting performance of MIDAS-ARMA and UMIDAS-ARMA are better than that of, respectively, MIDAS and UMIDAS. The empirical applications on nowcasting US gross domestic product (GDP) growth, investment growth, and GDP deflator inflation confirm this ranking. Moreover, in both simulation and empirical results, MIDAS-ARMA is better than UMIDAS-ARMA.

1 | INTRODUCTION

The use of mixed-frequency (MF) models has become increasingly popular among academics and practitioners. It is, in fact, by now well recognized that a good nowcast or short-term forecast for a low-frequency (LF) variable, such as gross domestic product GDP growth and its components, requires one to exploit the timely information contained in higher frequency macroeconomic or financial indicators, such as surveys or spreads. A growing literature has flourished proposing different methods to deal with the MF feature. In particular, models cast in state-space form, such as vector autoregressions (VARs) and factor models, can deal with MF data, taking advantage of the Kalman filter to interpolate the missing observations of the series only available at LF; see, among many others, Mariano and Murasawa (2010) and Giannone, Reichlin, and Small (2008) in a classical context, and Eraker, Chiu, Foerster, Kim, and Seoane (2015) and Schorfheide and Song (2015) in a Bayesian context. A second approach has been proposed by Ghysels (2016). He introduces a different class of MF VAR models, in which the vector of endogenous variables includes both high-frequency (HF) and LF variables, with the former stacked according to the timing of the data releases. A third approach is the mixed-data sampling (MIDAS) regression, introduced by Ghysels, Santa-Clara, and Valkanov (2006), and its unrestricted version (UMIDAS) by Foroni, Marcellino, and Schumacher (2015). MIDAS models are tightly parametrized, parsimonious models, which allow for the inclusion of many lags of the explanatory variables. Given their nonlinear form, MIDAS models are typically

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estimated by nonlinear least squares (NLS).[1] UMIDAS models are the unrestricted counterpart of MIDAS models, which can be estimated by simple OLS, but work well only when the frequency mismatch is small.[2]

In this paper, we start from the observation that temporal aggregation generally introduces a moving-average (MA) component in the model for the aggregate variable (see, e.g., Marcellino, 1999, and references therein). A similar feature should be present in the MF models, and indeed we show formally that this is in general the case.[3] The MA component is often neglected, both in same-frequency and in MF models, likely to preserve the possibility of OLS estimation and on the grounds that it can be approximated by a sufficiently long autoregressive (AR) component.

The effects of neglecting the MA component have been rarely explicitly considered. In a single-frequency context, Lutkepohl (2006) showed that VARMA models are especially appropriate in forecasting, since they can capture the dynamic relations between time series with a small number of parameters. Further, Dufour and Stevanovic (2013) showed that a VARMA instead of VAR model for the factors provides better forecasts for several key macroeconomic aggregates relative to standard factor models, as well as producing a more precise representation of the effects and transmission of monetary policy. Leroux, Kotchoni, and Stevanovic (2017) found that ARMA(1, 1) models predict well the inflation change and outperform many data-rich models, confirming the evidence on forecasting inflation by Stock and Watson (2007); Faust and Wright (2013), and Marcellino, Stock, and Watson (2006). Finally, VARMA models are often the correct reduced-form representation of dynamic stochastic general equilibrium models (see, e.g., Ravenna, 2007). For MF models, there are no results available.

We close this gap and analyze the relevance of the inclusion of an MA component in MIDAS and UMIDAS models, with the resulting specifications labeled, respectively, MIDAS-ARMA and UMIDAS-ARMA. We first compare the forecasting performance of the MF models with and without the MA component in a set of Monte Carlo experiments, using a variety of data-generating processes (DGPs). It turns out that the short-term forecasting performance is better when including the MA component, and the gains are higher the more persistent is the series. Moreover, in general, the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor. This pattern is in contrast to the findings in Foroni et al. (2015), and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.

Next, we carry out an empirical investigation, where we predict several quarterly macroeconomic variables using timely monthly indicators. In particular, we forecast three relevant quarterly US macroeconomic variables: real GDP growth, real private nonresidential fixed investment (PNFI) growth, and GDP deflator inflation. The latter variable is particularly relevant, as Stock and Watson (2007) show that the MA component for US inflation is important, especially after 1984. In fact, while during the 1970s the inflation process could be very well approximated by a low-order AR, after the 1980s this has become less accurate and the inclusion of an MA component more relevant. Evidence on the importance of the MA component for US inflation is also found by Ng and Perron (2001); Perron and Ng (1996). As monthly explanatory variables, we consider industrial production and employment for real GDP growth and PNFI growth, consumer price index (CPI) inflation and personal consumption expenditures (PCE) inflation for the GDP deflator. The inclusion of an MA component generally improves the forecasting performance substantially. In particular, adding the MA part to forecast GDP growth 1 year ahead ameliorates the mean square prediction error (MSE) up to 10%, while for PNFI we obtain even bigger gains: up to 30% 1 year ahead. Also in the case of GDP deflator we obtain robust improvements, which go up to 15%. For all variables, and in line with the simulation results, MIDAS-ARMA is better that UMIDAS-ARMA. Lastly, full-sample estimates of MA coefficients are significant and important in most MIDAS-ARMA and UMIDAS-ARMA specifications.

The remainder of the paper proceeds as follows. In Section 2 we show that temporal aggregation generally creates an MA component also in MF models. In Section 3 we describe parameter estimators for the MIDAS-ARMA and UMIDAS-ARMA models. In Section 4 we present the design and results of the simulation exercises. In Section 5 we develop the empirical applications on forecasting US quarterly variables with monthly indicators. In Section 6 we summarize the main results and conclude. Robustness analysis on the Monte Carlo simulations and the empirical applications are reported in the Supporting Information Appendix.

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[1] In a recent paper Ghysels and Qian (2019) proposed to use ordinary least squares (OLS) estimation of the MIDAS regression slope and intercept combined with profiling the polynomial weighting scheme parameters.

[2] The literature on MF approaches is vast. The papers cited in the text are a nonexhaustive list of key contributions to the field. For a review of the MF literature, see Bai, Ghysels, and Wright (2013) and Foroni and Marcellino (2013), among many others.

[3] An analysis of identifiability on autoregressive moving-average (ARMA) processes with MF observations is provided by Anderson, Deistler, Felsenstein, and Koelbl (2016), and on vector autoregressive moving-average (VARMA) processes by Deistler, Koelbl, and Anderson (2017).

2 THE RATIONALE FOR AN MA COMPONENT IN MF MODELS

The UMIDAS regression approach can be derived by aggregation of a general dynamic linear model at HF, as shown by Foroni et al. (2015), while the MIDAS model imposes specific restrictions on the UMIDAS coefficients in order to reduce their number, which is particularly relevant when the frequency mismatch is large (e.g., with daily and quarterly series). In Section 2.1, we briefly review the derivation of the UMIDAS model, highlighting that, in general, there should be an MA component, even though it is generally disregarded. In Section 2.2, we provide two simple analytical examples in which, starting from an HF model without an MA term, we end up with an MF model in which the MA component is present. We discuss estimation of MF models with an MA component in a separate section.

2.1 UMIDAS regressions and dynamic linear models

Let us assume that the DGP for the variable y and the N variables x is an ARDL(p, q) process (where ARDL is autoregressive distributed lag), as in Foroni et al. (2015):

$$a(L)y_{t_m} = b_1(L)x_{1t_m} + \dots + b_N(L)x_{Nt_m} + e_{yt_m}, \tag{1}$$

where $a(L) = 1 - a_1L - \dots - a_pL$, $b_j(L) = b_{j1}L + \dots + b_{jq}L^q$, $j = 1, \dots, N$, and the error e_{yt_m} is white noise. We assume, for simplicity, that $p = q$ and the starting values $y_{-p}, \dots, y_0$ and $x_{-p}, \dots, x_0$ are all fixed and equal to zero.

We then assume that x can be observed for each period t_m, while y can only be observed every m periods. We define $t = 1, \dots, T$ as the LF time unit and $t_m = 1, \dots, T_m$ as the HF time unit. The HF time unit is observed m times in the LF time unit. As an example, if we are working with quarterly (LF) and monthly (HF) data, it is $m = 3$ (i.e., 3 months in a quarter). Moreover, L indicates the lag operator at t_m frequency, while L^m is the lag operator at t frequency.

We also introduce the aggregation operator

$$\omega(L) = \omega_0 + \omega_1 L + \dots + \omega_{m-1}L^{m-1}, \tag{2}$$

which characterizes the temporal aggregation scheme. For example, $\omega(L) = 1 + L + \dots + L^{m-1}$ indicates the sum of the HF observations over the LF period, typically used in the case of flow variables, while $\omega(L) = 1$ corresponds to point-in-time sampling and is typically used for stock variables. As we will see, different aggregation schemes will play a role in generating MA components.

To derive the generating mechanism for y at MF, we introduce a polynomial in the lag operator, $\beta(L)$, whose degree in L is at most equal to $pm - p$ and which is such that the product $h(L) = \beta(L)a(L)$ only contains powers of L^m. This means that $h(L)$ is a polynomial of the form $h_0L^0 + h_1L^m + h_2L^{2m} + \dots + h_{pm-p}L^{pm-p}$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $a(L)$; see Marcellino (1999) for details.

In order to determine the AR component of the MF process, we then multiply both sides of Equation 1 by $\omega(L)$ and $\beta(L)$ to get

$$h(L)\omega(L)y_{t_m} = \beta(L)b_1(L)\omega(L)x_{1t_m} + \dots + \beta(L)b_N(L)\omega(L)x_{Nt_m} + \beta(L)\omega(L)e_{yt_m}. \tag{3}$$

Hence the autoregressive component only depends on LF values of y. Let us consider now the x variables, which are observable at HF t_m. Each HF x_{it_m} influences the LF variable y via a polynomial $\beta(L)b_j(L)\omega(L) = b_j(L)\beta(L)\omega(L)$, $j = 1, \dots, N$. We see that it is a particular combination of HF values of x_j, equal to $\beta(L)\omega(L)x_{jt_m}$, that affects the LF values of y.

Only under certain, rather strict conditions, it is possible to recover the polynomials $a(L)$ and $b_j(L)$ that appear in the HF model for y from the MF model, and in these cases also $\beta(L)$ can be identified. Therefore, when $\beta(L)$ cannot be identified, we can estimate a model as

$$\begin{gathered} c(L^m)\omega(L)y_{t_m} = \delta_1(L)x_{1t_m-1} + \dots + \delta_N(L)x_{Nt_m-1} + \epsilon_{t_m}, \\ t_m = m, 2m, 3m, \dots, \end{gathered} \tag{4}$$

where $c(L^m) = (1 - c_1L^m - \dots - c_cL^{mc})$, $\delta_j(L) = (\delta_{j,0} + \delta_{j,1}L + \dots + \delta_{j,v}L^v)$, $j = 1, \dots, N$.

We can focus now on the error term of Equation 3. In general, there is an MA component in the MF model, $q(L^m)u_{yt_m}$, with $q(L^m) = (1 + q_1L^m + \dots + q_qL^{mq})$. The order of $q(L^m)$, q, coincides with the highest multiple of m nonzero lag in the autocovariance function of $\beta(L)\omega(L)e_{yt_m}$. The coefficients of the MA component have to be such that the implied autocovariances of $q(L^m)u_{yt_m}$ coincide with those of $\beta(L)\omega(L)e_{yt_m}$ evaluated at all multiples of m. Consequently, also the error term ϵ_{t_m} in the approximate MF model (Equation 4), which is the UMIDAS model, in general has an MA structure.

It can be shown that the maximum order of the MA structure is p for average sampling and $p - 1$ for point-in-time sampling, where p is the order of the AR component in the HF model for y_{t_m}; see, for example, Marcellino (1999) for a derivation of this result.[4]

2.2 | Two analytical examples

In this section, we consider two simple DGPs and show that, even in these basic cases, an MA component appears in the MF model. In the first example, we consider an ARDL(1, 1) with average sampling, in the second one an ARDL(2, 2) with point-in-time sampling. In both cases, we work with monthly and quarterly variables; therefore $m = 3$, as in the empirical applications that will be presented later on. The examples could be easily generalized to consider higher order models and different frequency mismatches m.

2.2.1 | ARDL(1, 1) with average sampling

Let us assume an ARDL(1, 1) as HF DGP:

$$y_{t_m} = a y_{t_m-1} + b x_{t_m-1} + e_{yt_m}, \tag{5}$$

where y_{t_m} is a variable unobservable at HF, x_{t_m} is the HF variable, e_{yt_m} is white noise, and t_m is the HF time index. Although we do not observe y_{t_m}, we observe the quarterly aggregated values of the series.

In order to obtain the model for the quarterly aggregated series, let us write (5) as

$$(1 - aL)\, y_{t_m} = bLx_{t_m} + e_{yt_m}. \tag{6}$$

We consider average sampling, and therefore we define the aggregation operator $\omega(L) = 1 + L + L^2$. Then, we first introduce a polynomial in the lag operator, $\beta(L)$, which is such that the product $h(L) = \beta(L)(1 - aL)$ only contains powers of L^3. This polynomial exists and it is equal to $\left(1 + aL + a^2L^2\right)$. We then multiply both sides of Equation 6 by $\omega(L)$ and $\beta(L)$ and we obtain

$$\begin{aligned}\left(1 + aL + a^2L^2\right)(1 - aL)\left(1 + L + L^2\right) y_{t_m} = & \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} \\ & + \left(1 + aL + a^2L^2\right)\left(1 + L + L^2\right) e_{yt_m},\end{aligned} \tag{7}$$

or equivalently

$$\begin{aligned}\left(1 - a^3L^3\right) \tilde{y}_{t_m} = & \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} \\ & + \left[1 + (a + 1)L + \left(a^2 + a + 1\right)L^2 + \left(a^2 + a\right)L^3 + a^2L^4\right] e_{yt_m},\end{aligned} \tag{8}$$

where $\tilde{y}_{t_m} = \left(1 + L + L^2\right) y_{t_m}$ and $t_m = 3, 6, 9, \ldots$.

As we saw in Section 2.1, the order of the MA component coincides with the highest multiple of 3 nonzero lag in the autocovariance function of the error term in Equation 8, and it is bounded above by the AR order of the model for y_{t_m}.

Equation 8 is then estimated at quarterly frequency, but making use of all the information available in the HF variable x_{t_m}, and including the MA component, which is of order 1 in this case (being the relevant lag for the quarterly model L^3). The model in Equation 8 is therefore a UMIDAS-AR with an MA(1) component.

2.2.2 | ARDL(2, 2) with point-in-time sampling

Let us now assume an ARDL(2, 2) as HF DGP:

$$y_{t_m} = a_1 y_{t_m-1} + a_2 y_{t_m-2} + b_1 x_{t_m-1} + b_2 x_{t_m-2} + e_{yt_m}, \tag{9}$$

or equivalently

$$\left(1 - a_1L - a_2L^2\right) y_{t_m} = \left(b_1L + b_2L^2\right) x_{t_m} + e_{yt_m}, \tag{10}$$

where y_{t_m}, x_{t_m}, e_{yt_m} and t_m are defined as in the previous example.

[4]The fact that an aggregated model is misspecified and leads to biased and inconsistent estimators has been highlighted by Andreou, Ghysels, and Kourtellos (2010). However, contrary to us, they do not focus on the presence of an MA component in the aggregation.

We consider point-in-time sampling, and therefore $\omega(L) = 1$. Next, we need to multiply both sides of Equation 9 by $\omega(L)$ and find a polynomial $\beta(L)$ such that the product $h(L) = \beta(L)\left(1 - a_1L - a_2L^2\right)$ only contains powers of L^3. In can be easily shown that $\beta(L)$ exists and it is equal to

$$\left[1 + a_1L + \left(a_1^2 + a_2\right)L^2 - a_1a_2L^3 + a_2^2L^4\right].$$

The resulting MF model for the LF variable is

$$\begin{aligned}\left[1 - \left(a_1^3 + 3a_2a_1\right)L^3 - a_2^3L^6\right]y_{t_m} = &\left[1 + a_1L + \left(a_1^2 + a_2\right)L^2 - a_1a_2L^3 + a_2^2L^4\right]\left(b_1L + b_2L^2\right)x_{t_m}\\ &+ \left[1 + a_1L + \left(a_1^2 + a_2\right)L^2 - a_1a_2L^3 + a_2^2L^4\right]e_{yt_m},\end{aligned} \tag{11}$$

with $t_m = 3, 6, 9, \ldots$. Hence also in this case there is an MA component in the MF model for y. Its order coincides with the highest multiple of 3 nonzero lag in the autocovariance function of $\left[1 + a_1L + \left(a_1^2 + a_2\right)L^2 - a_1a_2L^3 + a_2^2L^4\right]e_{yt_m}$, and it is bounded above by the AR order of the model for y_{t_m} minus one, which is 1 in this example. Following the same line of reasoning as in the previous example, the MA component is of order 1.

3 | UMIDAS-ARMA AND MIDAS-ARMA: FORECASTING SPECIFICATIONS AND ESTIMATION

We describe now in more detail the model specifications we consider for forecasting, and the estimation details. We first recall the main features of the standard MIDAS regression, introduced by Ghysels et al. (2006), and its unrestricted version, as in Foroni et al. (2015). Then, we discuss their extensions to allow for an MA component and we discuss the estimation of the models.

The starting point for our MF models is Equation 4. In order to simplify the notation, we assume $\omega(L) = 1$ and one explanatory variable x_{t_m}.[5] Further, we allow for incorporating leads of the HF variable in the projections, which captures asynchronous releases.

The equation we are going to estimate to generate an h_m-step-ahead forecast is the following:

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \delta(L)x_{t_m-h_m+w} + \epsilon_{t_m}, \tag{12}$$

where $\tilde{c}(L^m)$ is a modified lag structure of Equation 4 to obtain a direct forecast and w is the number of months with which x is leading y.

If ϵ_{t_m} is serially uncorrelated, Equation 12 represents the UMIDAS-AR model. Given that the model is linear, the UMIDAS-AR regression can be estimated by simple OLS. Empirically, the lag length of the HF variable x is often selected by means of an information criterion, such as the Bayesian information criterion (BIC).

Adding an MA component to the UMIDAS-AR yields the UMIDAS-ARMA model:

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \delta(L)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m}, \tag{13}$$

where u_{t_m} is a (weak) white noise with $\mathrm{E}(u_{t_m}) = 0$ and $\mathrm{E}(u_{t_m}u'_{t_m}) = \sigma_u^2 < \infty$, and all the remaining terms stay the same, as in Equation 12. Given that MIDAS models are direct forecasting tools, we decided to follow a direct approach also when modeling the MA component. Note that if Equation 4 coincides with the DGP, then the errors in Equation 12 will be serially correlated. This provides an additional justification for the use of MA errors.

OLS estimation of the UMIDAS-ARMA model is no longer possible, because of the MA component in the residuals. We then estimate the model as in the standard ARMA literature, by maximum likelihood or, as we will actually do to be coherent with the MIDAS literature, by NLS.

The MIDAS-AR specification is a restricted version of the UMIDAS-AR. The MIDAS-AR model as in Ghysels et al. (2006), specified for forecasting h_m periods ahead, can be written as follows:

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \beta B(L, \theta)x_{t_m-h_m+w} + \epsilon_{t_m}, \tag{14}$$

[5]This is an innocuous simplification, as with a generic aggregation scheme $\omega(L) \neq 1$ we could just work with the redefined variable $\tilde{y}_{t_m} = \omega(L)y_{t_m}$.

where

$$B(L,\theta) = \sum_{j=0}^{K} b(j,\theta)L^j,$$

$$b(j,\theta) = \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K} exp(\theta_1 j + \theta_2 j^2)},$$

and K is the maximum number of lags included of the explanatory variable.

As is clear by comparing Equations (12) and 14, the MIDAS model is nested into the UMIDAS model.

The MIDAS-AR model in Equation 14 is estimated by NLS. Given that it is h_m-dependent, as in the UMIDAS case it has to be reestimated for each forecast horizon.

Exactly as for the UMIDAS, we extend the MIDAS-AR in Equation 14 to incorporate an MA component:

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \beta B(L,\theta)x_{t_m-h_m+w} + u_{t_m} + q(L^m)u_{t_m-h_m}, \tag{15}$$

where the error term is defined as in Equation 13. Given the nonlinearity of the model, we estimate its parameters by NLS. Supporting Information Appendix A provides additional details on the NLS estimation procedures.

To conclude, it is worth briefly comparing the use for forecasting of UMIDAS-ARMA versus the Kalman filter. The latter is clearly optimal in the presence of MF data and linear models. However, UMIDAS-ARMA is equivalent if it is theoretically derived from a known HF linear dynamic model, as UMIDAS-ARMA coincides with the MF DGP. The ranking of the two approaches is unclear if the HF model is misspecified. Moreover, the Kalman filter can incur computational problems when the frequency mismatch is large. Something similar happens to UMIDAS-ARMA due to parameter proliferation, and in this case the parsimony of MIDAS-ARMA can be particularly helpful. Bai et al. (2013) propose a more detailed comparison of the Kalman filter and the MIDAS approach.

4 | MONTE CARLO EVALUATION

We now assess the forecasting relevance of including an MA component in MIDAS and UMIDAS models by means of simulation experiments. We use two designs, closely related to the two analytical examples described in Section 2.2. We present first the Monte Carlo designs and then the results. Finally, we summarize the results obtained from other variations of the Monte Carlo design, aimed at making the evidence more robust. The detailed robustness checks are presented in Supporting Information Appendix B.

4.1 | Monte Carlo design

In the first design, the DGP is the HF ARDL(1, 1):

$$y_{t_m} = \rho y_{t_m-1} + \delta_l x_{t_m-1} + e_{y,t_m}, \tag{16}$$

where y_{t_m} is unobservable at HF, but available at LF, while x_{t_m} is the HF variable, t_m is the HF time index, the aggregation frequency is $m = 3$ (as in the case of quarterly and monthly frequencies), and t is the LF time index, with $t = 3t_m$. We assume that $\omega(L) = 1 + L + L^2$, corresponding to average sampling.

The shocks e_{y,t_m} are independent and sampled from a normal distribution. The variance of x and that of the error e_{y,t_m} are set in such a way that the R^2 in the model for y is equal to 0.9 in each simulation. We consider different combinations of ρ and δ_l, representing different degrees of persistence and correlation between the HF and the LF variables. In detail, we evaluate the following parameter sets:

$$(\rho, \delta_l) = \{(0.1, 0.1), (0.5, 0.1), (0.9, 1), (0.94, 1)\}, \tag{17}$$

and we would expect theoretically the relative importance of the MA component to increase with the value of ρ.

Finally, x_{t_m} is generated as an AR(1) with coefficient ρ. We highlight that there is no need to play on the persistence of x_{t_m} to obtain high or low R^2, but we can obtain the same results by changing the variance of the $e_{x_{t_m}}$, and consequently of x_{t_m}.

In the second design, the DGP is the HF ARDL(2, 2):

$$y_{t_m} = \rho_1 y_{t_m-1} + \rho_2 y_{t_m-2} + \delta_{l1} x_{t_m-1} + \delta_{l2} x_{t_m-2} + e_{y,t_m}. \tag{18}$$

We still assume $m = 3$ but now $\omega(L) = 1$, so that the LF variable is skip-sampled every $m = 3$ observations.
In this second DGP, we consider the following parameter combinations:

$$(\rho_1, \rho_2, \delta_{l1}, \delta_{l2}) = \{(0.05, 0.1, 0.5, 1), (0.125, 0.5, 0.125, 0.5), (0.25, 0.5, 0.5, 1)\}. \tag{19}$$

All the other design features are as in the first DGP.

We focus on typical sample sizes for the estimation sample, with $T = 50, 100$. The size of the evaluation sample is set to 50, and the estimation sample is recursively expanded as we progress in the recursive forecasting exercise. The number of replications is 500.

The competing forecasting models are the following:

1. a MIDAS-AR model, with 12 lags in the exogenous HF variable and 1 lag in the AR component;
2. a MIDAS-ARMA model, as in the previous point but with the addition of an MA component;
3. a MIDAS-ARMA model, with only 3 lags in the exogenous HF variable and 1 AR lag;
4. a UMIDAS-AR model, with lag length selected according to the BIC criterion, where the maximum lag length is set equal to 12;
5. a UMIDAS-ARMA model, as in the previous point, with the addition of an MA component;
6. a UMIDAS-ARMA, fixing at 3 the number of lags of the HF exogenous variable.

TABLE 1 Monte Carlo simulations results: MSE (model) relative to MSE (MIDAS)–DGP: ARDL(1, 1) with average sampling, $T = 100$

	Mean	10 pctl	25 pctl	Median	75 pctl	90 pctl
Panel A: $\rho = 0.94, \delta_l = 1$						
MIDAS-ARMA-12 (2)	0.974	0.986	0.981	0.968	0.970	0.967
MIDAS-ARMA-3 (3)	0.966	0.981	0.969	0.962	0.959	0.966
UMIDAS-AR (4)	0.997	0.997	0.986	1.005	0.994	0.990
UMIDAS-ARMA (5)	0.969	0.983	0.974	0.970	0.961	0.973
UMIDAS-ARMA-3 (6)	0.971	0.977	0.974	0.971	0.964	0.975
Panel B: $\rho = 0.9, \delta_l = 1$						
MIDAS-ARMA-12 (2)	0.976	0.989	0.984	0.979	0.973	0.976
MIDAS-ARMA-3 (3)	0.975	0.983	0.988	0.969	0.978	0.971
UMIDAS-AR (4)	1.030	1.024	1.023	1.029	1.038	1.040
UMIDAS-ARMA (5)	1.019	1.018	1.023	1.012	1.024	1.028
UMIDAS-ARMA-3 (6)	0.976	0.979	0.984	0.977	0.977	0.981
Panel C: $\rho = 0.5, \delta_l = 0.1$						
MIDAS-ARMA-12 (2)	0.986	0.990	0.994	0.984	0.983	0.978
MIDAS-ARMA-3 (3)	1.184	1.197	1.178	1.174	1.202	1.176
UMIDAS-AR (4)	1.005	1.000	1.003	1.006	1.013	0.995
UMIDAS-ARMA (5)	1.000	1.005	0.992	0.998	0.993	0.992
UMIDAS-ARMA-3 (6)	1.182	1.212	1.185	1.175	1.198	1.179
Panel D: $\rho = 0.1, \delta_l = 0.1$						
MIDAS-ARMA-12 (2)	0.981	0.985	0.989	0.983	0.980	0.972
MIDAS-ARMA-3 (3)	0.833	0.848	0.846	0.834	0.827	0.828
UMIDAS-AR (4)	0.825	0.837	0.834	0.823	0.824	0.819
UMIDAS-ARMA (5)	0.832	0.841	0.844	0.833	0.831	0.836
UMIDAS-ARMA-3 (6)	0.833	0.846	0.846	0.834	0.829	0.829

Note. The four panels report the results for four different DGPs for 1-quarter-ahead horizon (with the information of the first 2 months of the quarter available). The numbers (2)–(6) refer to the corresponding models described in Section 4. The results reported are the average, median, and the 10th, 25th, 75th, and 90th percentiles of the MSE of the indicated model relative to the average, median, and the 10th, 25th, 75th, and 90th percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

TABLE 2 Monte Carlo simulations results: MSE (model) relative to MSE (MIDAS)–DGP: ARDL(1, 1) with average sampling, $T = 50$

	Mean	10 pctl	25 pctl	Median	75 pctl	90 pctl
Panel A: $\rho = 0.94, \delta_l = 1$						
MIDAS-ARMA-12 (2)	0.985	0.975	0.996	0.983	0.990	0.968
MIDAS-ARMA-3 (3)	0.957	0.949	0.982	0.947	0.957	0.944
UMIDAS-AR (4)	0.982	0.984	0.998	0.968	0.986	0.979
UMIDAS-ARMA (5)	0.957	0.950	0.984	0.954	0.965	0.939
UMIDAS-ARMA-3 (6)	0.968	0.950	1.003	0.962	0.975	0.955
Panel B: $\rho = 0.9, \delta_l = 1$						
MIDAS-ARMA-12 (2)	0.997	1.001	1.012	0.994	0.986	0.982
MIDAS-ARMA-3 (3)	0.973	0.978	1.006	0.964	0.961	0.977
UMIDAS-AR (4)	1.033	1.074	1.041	1.025	1.034	1.013
UMIDAS-ARMA (5)	1.020	1.041	1.040	1.018	1.019	1.016
UMIDAS-ARMA-3 (6)	0.981	0.982	1.023	0.968	0.971	0.983
Panel C: $\rho = 0.5, \delta_l = 0.1$						
MIDAS-ARMA-12 (2)	1.013	1.007	1.010	1.022	0.999	1.014
MIDAS-ARMA-3 (3)	1.188	1.182	1.172	1.179	1.168	1.249
UMIDAS-AR (4)	1.038	1.056	1.054	1.026	1.046	1.064
UMIDAS-ARMA (5)	1.061	1.089	1.059	1.049	1.049	1.062
UMIDAS-ARMA-3 (6)	1.197	1.186	1.181	1.181	1.173	1.241
Panel D: $\rho = 0.1, \delta_l = 0.1$						
MIDAS-ARMA-12 (2)	0.984	0.987	0.989	0.983	0.989	0.973
MIDAS-ARMA-3 (3)	0.824	0.809	0.807	0.825	0.830	0.846
UMIDAS-AR (4)	0.810	0.791	0.814	0.819	0.814	0.820
UMIDAS-ARMA (5)	0.834	0.824	0.826	0.831	0.834	0.853
UMIDAS-ARMA-3 (6)	0.830	0.826	0.816	0.827	0.832	0.859

Note. See Table 1.

In all ARMA models there is an MA(1) component, in line with the theoretical results, but a higher order can be allowed. Further, a higher number of lags in the autoregressive component can also be included.[6]

We evaluate the competing one-step-ahead forecasts on the basis of their associated MSE, assuming that information on the first 2 months of the quarter is available (as is common in nowcasting exercises).

4.2 | Results

In Tables 1 –4 we report the mean relative MSE across simulations, and numbers smaller than one indicate that the model is better than the benchmark (model 1, MIDAS-AR). We also report the 10th, 25th, 50th, 75th, and 90th percentiles, to provide a measure of the dispersion in the results.

Tables 1 and 2 present the results for the first DGP (the ARDL(1, 1) with average sampling), using $T = 100$ in Table 1 and $T = 50$ in Table 2. The corresponding Tables 3 and 4 are based on the second DGP (the ARDL(2, 2) with point-in-time sampling).

A few key findings emerge. First, adding an MA component to the MIDAS model generally helps. The gains are not very large but they are visible at all percentiles, with a few exceptions for the second DGP. The gains are larger either with substantial persistence ($\rho = 0.9$ or $\rho = 0.94$ in the first DGP and $\rho_1 = 0.25$, $\rho_2 = 0.5$ in the second DGP) or with low persistence in the first DGP ($\rho = 0.1$), but in the latter case the result is mainly due to a deterioration in the absolute performance of the standard MIDAS model. The more parsimonious specification with 3 lags only of the HF variable is generally better, except when $\rho = 0.5$.

[6]We computed results with 3 lags in the MIDAS-AR and UMIDAS-AR, to check whether inserting more lags in the AR part serves as a proxy for the MA component. However, the relevance of the MA component is confirmed. Results are available upon request.

TABLE 3 Monte Carlo simulation results: MSE (model) relative to MSE (MIDAS)–DGP: ARDL(2, 2) with point-in-time sampling, $T = 100$

	Mean	10 pctl	25 pctl	Median	75 pctl	90 pctl
Panel A: $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1$						
MIDAS-ARMA-12 (2)	1.007	1.010	1.003	1.007	1.010	1.025
MIDAS-ARMA-3 (3)	1.006	1.006	0.997	1.003	1.014	1.018
UMIDAS-AR (4)	1.014	1.007	1.016	1.005	1.006	1.030
UMIDAS-ARMA (5)	1.015	1.000	1.014	1.007	1.014	1.026
UMIDAS-ARMA-3 (6)	1.007	0.998	1.004	1.006	1.016	1.023
Panel B: $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5$						
MIDAS-ARMA-12 (2)	0.956	0.955	0.960	0.963	0.949	0.959
MIDAS-ARMA-3 (3)	0.940	0.932	0.950	0.950	0.931	0.943
UMIDAS-AR (4)	0.938	0.921	0.938	0.945	0.929	0.946
UMIDAS-ARMA (5)	0.921	0.927	0.922	0.926	0.908	0.939
UMIDAS-ARMA-3 (6)	0.943	0.921	0.950	0.947	0.932	0.948
Panel C: $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1$						
MIDAS-ARMA-12 (2)	0.984	0.968	0.981	0.985	0.991	0.998
MIDAS-ARMA-3 (3)	0.980	0.981	0.982	0.968	0.981	0.999
UMIDAS-AR (4)	1.021	1.032	1.020	1.006	1.032	1.036
UMIDAS-ARMA (5)	0.992	0.987	0.986	0.988	1.001	1.004
UMIDAS-ARMA-3 (6)	0.983	0.978	0.979	0.980	0.985	0.998

Note. The four panels report the results for three different DGPs. The numbers (2)–(6) refer to the corresponding models described in Section 4. The results reported are the average, median, and the 10th, 25th, 75th, and 90th percentiles of the MSE of the indicated model relative to the average, median and the 10th, 25th, 75th, and 90th percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

TABLE 4 Monte Carlo simulation results: MSE (model) relative to MSE (MIDAS)–DGP: ARDL(2, 2) with point-in-time sampling, $T = 50$

	Mean	10 pctl	25 pctl	Median	75 pctl	90 pctl
Panel (A): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1$						
MIDAS-ARMA-12 (2)	1.020	1.003	1.024	1.021	1.009	1.017
MIDAS-ARMA-3 (3)	1.003	0.990	1.015	1.014	0.986	0.994
UMIDAS-AR (4)	1.006	0.982	1.036	1.019	1.011	0.988
UMIDAS-ARMA (5)	1.018	0.955	1.033	1.037	1.023	1.030
UMIDAS-ARMA-3 (6)	1.018	1.000	1.024	1.028	1.021	1.010
Panel (B): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1$						
MIDAS-ARMA-12 (2)	1.017	0.980	1.006	1.004	1.019	1.042
MIDAS-ARMA-3 (3)	0.967	0.934	0.970	0.995	0.953	0.991
UMIDAS-AR (4)	0.971	0.973	0.979	0.979	0.961	0.997
UMIDAS-ARMA (5)	0.983	0.983	0.977	1.000	0.958	0.980
UMIDAS-ARMA-3 (6)	1.009	0.970	1.002	1.016	1.000	1.023
PANEL (C): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1$						
MIDAS-ARMA-12 (2)	1.016	1.003	0.991	1.005	1.012	1.010
MIDAS-ARMA-3 (3)	0.990	0.993	0.965	0.984	0.988	0.979
UMIDAS-AR (4)	1.046	1.024	1.016	1.047	1.059	1.039
UMIDAS-ARMA (5)	1.041	1.051	1.035	1.018	1.045	1.038
UMIDAS-ARMA-3 (6)	1.016	1.008	1.014	0.991	1.024	1.018

Note. See Table 3.

TABLE 5 Data description

Series	Source	Source code	Transformation	Frequency
US data				
GDP deflator	FRED	GDPDEF	Log-difference	Quarterly
Real GDP	FRED	GDP	Log-difference	Quarterly
Private nonresidential fixed investment	FRED	PNFI	Level	Quarterly
Nonresidential (implicit price deflator)	FRED	A008RD3Q086SBEA	Level	Quarterly
Real private nonresidential fixed investment		PNFI/A008RD3Q086SBEA	Log-difference	Quarterly
Consumer price index (CPI)	FRED	CPIAUCSL	Log-difference	Monthly
Personal consumption expenditures: price index (PCE)	FRED	PCEPI	Log-difference	Monthly
Employment	FRED	PAYEMS	Log-difference	Monthly
Industrial production	FRED	INDPRO	Log-difference	Monthly

Second, adding an MA component to the UMIDAS model is also generally helpful, though the gains remain small.

Third, in general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor. This pattern is in contrast to the findings of Foroni et al. (2015), and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.

Finally, results are consistent across sample sizes, and the models do not seem sensitive to short sample sizes.

4.3 | Robustness checks

We run a battery of robustness checks on our Monte Carlo experiments, with the scope to strengthen the evidence emerging from our main results. For the sake of conciseness, here we report a summary of the checks and the main conclusions. The detailed results are presented in Supporting Information Appendix B.

As a first robustness exercise, we modify the setup by reducing the explanatory power of the x variable in such a way that the total R^2 in our DGP for y is equal to 0.3, 0.5, or 0.7. The MSE are obviously greater in absolute value, but the MA component is still generally improving the forecasting performance of the (U)MIDAS models. Second, we compute the results for a longer estimation sample, with $T_q = 200$, which corresponds to 50 years of quarterly observations, in line, for example, with empirical applications using long macroeconomic time series for the USA. The results show broadly the same features as when $T = 100$.

Finally, we consider not only nowcasting, but also 2- and 4-quarter-ahead forecasts. Results are overall robust at the 2-quarter-ahead horizon, while at the 4-quarter-ahead horizon results are much weaker. Theoretically, the relevance of the MA component should decline at longer horizons. Indeed, the Monte Carlo results show smaller gains from the use of an MA component for 2-quarter-ahead forecasting, and no gains at the 4-quarter horizon.

5 | EMPIRICAL APPLICATIONS

Applications of MIDAS regressions to forecasting macroeconomic variables is fairly standard by now, starting from the paper by Clements and Galvo (2008).[7] Our focus is on the inclusion of an MA component. Therefore, in this section, we look at the performance of our MA augmented MF models in forecasting exercises with actual data. The analysis focuses on forecasting quarterly US variables.

In particular, we consider three relevant quarterly US macroeconomic variables: real GDP growth, real PNFI growth and GDP deflator inflation. As monthly explanatory variables, we consider industrial production and employment for the real GDP growth and the PNFI growth, whereas we consider CPI inflation and personal consumption inflation for the GDP deflator. A complete description of data sources and transformations is available in Table 5.

The total sample spans over 50 years of data, from the first quarter of 1960 to the end of 2015. The forecasts are computed in pseudo real time, with progressively expanding samples. The evaluation period goes from 1980:Q1 to the end of the sample, covering roughly 35 years. At each point in time, we compute forecasts from 1 up to 4 quarters ahead. The

[7]Other studies which consider MIDAS applications are, among many others, Schumacher (2016) and Kim and Swanson (2017). For a comprehensive review, see Foroni and Marcellino (2013).

TABLE 6 Forecasting US GDP growth

	Explanatory variable: Industrial production growth						**Explanatory variable: Employment growth**					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**	**Value**	**Ratio**	**DM**
	$h = 1$						$h = 1$					
MIDAS-AR-12	4.06	1.00	NaN	1.58	1.00	NaN	4.84	1.00	NaN	1.73	1.00	NaN
MIDAS-ARMA-12lags	4.05	1.00	0.41	1.59	1.01	0.13	4.73	0.98	0.33	1.72	0.99	0.37
MIDAS-ARMA-3	4.27	1.05	0.07	1.60	1.01	0.22	4.76	0.98	0.39	1.72	0.99	0.42
UMIDAS-biclags	4.21	1.04	0.15	1.58	1.00	0.41	4.70	0.97	0.31	1.73	1.00	0.49
UMIDAS-ARMA-biclags	4.18	1.03	0.19	1.59	1.01	0.26	4.37	0.90	0.05	1.67	0.96	0.17
UMIDAS-ARMA-3	4.27	1.05	0.07	1.60	1.01	0.22	4.76	0.98	0.39	1.72	0.99	0.42
AR	7.60	1.87	0.01	1.97	1.25	0.01	7.60	1.57	0.02	1.97	1.14	0.05
	$h = 2$						$h = 2$					
MIDAS-AR-12	6.68	1.00	NaN	1.86	1.00	NaN	6.07	1.00	NaN	1.79	1.00	NaN
MIDAS-ARMA-12lags	6.35	0.95	0.03	1.84	0.99	0.29	5.58	0.92	0.03	1.76	0.98	0.22
MIDAS-ARMA-3	6.59	0.99	0.31	1.90	1.02	0.16	5.78	0.95	0.12	1.78	1.00	0.41
UMIDAS-biclags	7.05	1.05	0.00	1.94	1.04	0.00	6.07	1.00	0.06	1.79	1.00	0.05
UMIDAS-ARMA-biclags	6.89	1.03	0.12	1.92	1.03	0.02	5.81	0.96	0.15	1.78	1.00	0.45
UMIDAS-ARMA-3	7.04	1.05	0.09	1.89	1.01	0.19	6.10	1.00	0.46	1.82	1.01	0.20
AR	7.77	1.16	0.10	1.98	1.06	0.06	7.77	1.28	0.02	1.98	1.10	0.01
	h=3						h=3					
MIDAS-AR-12	8.14	1.00	NaN	2.01	1.00	NaN	8.85	1.00	NaN	2.10	1.00	NaN
MIDAS-ARMA-12lags	8.22	1.01	0.34	2.04	1.01	0.28	9.39	1.06	0.21	2.31	1.10	0.03
MIDAS-ARMA-3	7.44	0.91	0.00	1.89	0.94	0.00	7.06	0.80	0.00	1.90	0.90	0.00
UMIDAS-biclags	8.12	1.00	0.45	1.99	0.99	0.19	7.62	0.86	0.00	1.94	0.92	0.00
UMIDAS-ARMA-biclags	12.00	1.47	0.01	2.46	1.22	0.00	15.23	1.72	0.00	3.11	1.48	0.00
UMIDAS-ARMA-3	8.24	1.01	0.42	1.97	0.98	0.16	7.93	0.90	0.07	1.99	0.95	0.07
AR	8.77	1.08	0.11	2.05	1.02	0.19	8.77	0.99	0.43	2.05	0.98	0.19
	h=4						h=4					
MIDAS-AR-12	9.14	1.00	NaN	2.11	1.00	NaN	11.11	1.00	NaN	2.31	1.00	NaN
MIDAS-ARMA-12lags	8.56	0.94	0.09	2.07	0.98	0.24	11.20	1.01	0.46	2.43	1.05	0.13
MIDAS-ARMA-3	8.27	0.90	0.03	2.02	0.96	0.05	10.14	0.91	0.02	2.16	0.94	0.01
UMIDAS-biclags	8.77	0.96	0.19	2.05	0.97	0.10	10.37	0.93	0.05	2.17	0.94	0.01
UMIDAS-ARMA-biclags	8.91	0.98	0.40	2.10	0.99	0.45	11.13	1.00	0.49	2.38	1.03	0.30
UMIDAS-ARMA-3	10.01	1.09	0.30	2.08	0.99	0.40	9.52	0.86	0.00	2.10	0.91	0.00
AR	8.69	0.95	0.11	2.05	0.97	0.10	8.69	0.78	0.00	2.05	0.89	0.00

Note. The table reports the results on the forecasting performance of the different models. In the columns "Value" we report the MSE and the MAE respectively. In the columns "Ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the p-value of the Diebold–Mariano test. The forecasts are evaluated over the sample 1980:Q1–2015:Q4. The lowest values for each variable are underlined.

forecasting target is the annualized growth rate. Although the information contained in the monthly variables updates every month, we focus on the case in which the first 2 months of the quarter are already available.

We consider models (1)–(7) as described in Section 4.1 , plus a simple LF AR(1) model as a further benchmark for the usefulness of the MF data.[8] In particular, we consider the direct forecast resulting from the model:

$$y_t = c + \rho y_{t-h} + e_t. \tag{20}$$

[8]Despite, for a variable like inflation more accurate benchmarks could be chosen, we consider an AR process, which is nested in all the models under comparison, and we keep it the same for all the variables under analysis to test the usefulness of MF data.

TABLE 7 Forecasting US real private nonresidential fixed investment growth

	Explanatory variable: Industrial production growth						Explanatory variable: Employment growth					
	MSE			MAE			MSE			MAE		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
	$h = 1$						$h = 1$					
MIDAS-AR-12	30.70	1.00	NaN	4.40	1.00	NaN	33.01	1.00	NaN	4.56	1.00	NaN
MIDAS-ARMA-12lags	29.32	0.96	0.06	4.25	0.97	0.02	33.38	1.01	0.18	4.57	1.00	0.33
MIDAS-ARMA-3	<u>28.13</u>	0.92	0.01	<u>4.19</u>	0.95	0.01	33.15	1.00	0.40	4.57	1.00	0.37
UMIDAS-biclags	31.91	1.04	0.22	4.45	1.01	0.24	33.31	1.01	0.18	4.58	1.01	0.12
UMIDAS-ARMA-biclags	29.27	0.95	0.13	4.27	0.97	0.06	34.11	1.03	0.20	4.64	1.02	0.11
UMIDAS-ARMA-3	28.15	0.92	0.01	4.19	0.95	0.01	32.98	1.00	0.48	4.57	1.00	0.37
AR	44.91	1.46	0.01	5.11	1.16	0.01	44.91	1.36	0.02	5.11	1.12	0.05
	$h = 2$						$h = 2$					
MIDAS-AR-12	38.90	1.00	NaN	4.72	1.00	NaN	36.33	1.00	NaN	<u>4.71</u>	1.00	NaN
MIDAS-ARMA-12	38.59	0.99	0.34	4.74	1.00	0.37	<u>36.13</u>	0.99	0.40	4.74	1.01	0.32
MIDAS-ARMA-3	41.46	1.07	0.07	4.86	1.03	0.11	36.70	1.01	0.29	4.76	1.01	0.21
UMIDAS-biclags	41.92	1.08	0.00	4.87	1.03	0.00	37.84	1.04	0.03	4.78	1.02	0.08
UMIDAS-ARMA-biclags	44.76	1.15	0.00	5.11	1.08	0.00	38.40	1.06	0.03	4.85	1.03	0.07
UMIDAS-ARMA-3	43.93	1.13	0.03	4.92	1.04	0.06	36.69	1.01	0.36	4.80	1.02	0.11
AR	47.14	1.21	0.08	5.18	1.10	0.05	47.14	1.30	0.04	5.18	1.10	0.05
	h=3						$h = 3$					
MIDAS-AR-12	43.65	1.00	NaN	5.15	1.00	NaN	43.77	1.00	NaN	5.20	1.00	NaN
MIDAS-ARMA-12	41.97	0.96	0.07	4.99	0.97	0.05	<u>40.70</u>	0.93	0.02	<u>5.00</u>	0.96	0.01
MIDAS-ARMA-3	45.90	1.05	0.13	5.21	1.01	0.25	41.31	0.94	0.05	5.06	0.97	0.06
UMIDAS-biclags	52.95	1.21	0.02	5.51	1.07	0.00	45.51	1.04	0.05	5.25	1.01	0.17
UMIDAS-ARMA-biclags	47.52	1.09	0.07	5.26	1.02	0.17	42.62	0.97	0.16	5.06	0.97	0.06
UMIDAS-ARMA-3	45.86	1.05	0.14	5.21	1.01	0.28	41.36	0.95	0.07	5.08	0.98	0.11
AR	55.91	1.28	0.03	5.60	1.09	0.04	55.91	1.28	0.04	5.60	1.08	0.08
	$h = 4$						$h = 4$					
MIDAS-AR-12	52.70	1.00	NaN	5.72	1.00	NaN	69.36	1.00	NaN	6.18	1.00	NaN
MIDAS-ARMA-12	50.68	0.96	0.21	5.48	0.96	0.04	59.35	0.86	0.02	5.78	0.94	0.01
MIDAS-ARMA-3	50.04	0.95	0.17	5.44	0.95	0.02	<u>48.38</u>	0.70	0.00	<u>5.29</u>	0.86	0.00
UMIDAS-biclags	53.47	1.01	0.41	5.59	0.98	0.16	57.78	0.83	0.01	5.73	0.93	0.01
UMIDAS-ARMA-biclags	54.95	1.04	0.26	5.60	0.98	0.18	56.52	0.81	0.03	5.70	0.92	0.05
UMIDAS-ARMA-3	52.92	1.00	0.48	5.48	0.96	0.12	53.55	0.77	0.01	5.55	0.90	0.01
AR	63.44	1.20	0.04	5.91	1.03	0.17	63.44	0.91	0.01	5.91	0.96	0.03

Note. See Table 6.

We evaluate the forecasts both in terms of MSE and in terms of mean absolute errors (MAE). We then compare the forecasting performance relative to a standard MIDAS model with an autoregressive component and 12 lags of the explanatory variable (as the model (1) in Section 4).

In Tables 6– 8 we report the results for, respectively, the real GDP growth, the real PNFI growth and the GDP deflator inflation rate. Each table is organized in the same way: It reports the value of MSE and MAE for each model, the ratio of those criteria for each model relative to the MIDAS-AR, our benchmark model, and the *p*-value of the Diebold–Mariano test, to check the statistical significance of the differences in forecast measures with respect to the benchmark (see ; Diebold & Mariano, 1995).

The tables are broadly supportive of the inclusion of the MA component in the MF models, as the MSE and MAE ratios are often smaller than one for the MIDAS-ARMA and UMIDAS-ARMA models when compared with their versions without MA.[9] In more detail: For forecasting GDP growth, adding the MA component does not provide substantial

[9]The models which include an MA component are indicated in bold in the tables, while the lowest MSE and MAE values are underlined.

TABLE 8 Forecasting US GDP deflator

	Explanatory variable: CPI inflation						Explanatory variable: PCE inflation					
	MSE			MAE			MSE			MAE		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
	$h = 1$						$h = 1$					
MIDAS-AR-12	0.65	1.00	NaN	0.61	1.00	NaN	0.51	1.00	NaN	0.52	1.00	NaN
MIDAS-ARMA-12	0.65	0.99	0.42	0.61	1.00	0.45	0.50	0.98	0.37	0.51	0.99	0.28
MIDAS-ARMA-3	0.61	0.94	0.09	0.59	0.97	0.17	0.43	0.85	0.07	0.49	0.95	0.20
UMIDAS-biclags	0.65	0.99	0.42	0.60	0.98	0.22	0.53	1.04	0.24	0.52	1.00	0.47
UMIDAS-ARMA-biclags	0.61	0.94	0.09	0.59	0.97	0.17	0.51	1.01	0.44	0.52	1.00	0.47
UMIDAS-ARMA-3	0.61	0.94	0.09	0.59	0.97	0.17	0.49	0.96	0.11	0.51	0.98	0.14
AR	0.79	1.20	0.11	0.68	1.11	0.07	0.79	1.55	0.00	0.68	1.32	0.00
	$h = 2$						$h = 2$					
MIDAS-AR-12	0.79	1.00	NaN	0.67	1.00	NaN	0.55	1.00	NaN	0.55	1.00	NaN
MIDAS-ARMA-12	0.68	0.86	0.08	0.64	0.95	0.09	0.56	1.01	0.38	0.55	1.01	0.36
MIDAS-ARMA-3	0.68	0.86	0.13	0.64	0.96	0.16	0.50	0.90	0.14	0.53	0.96	0.24
UMIDAS-biclags	0.74	0.94	0.31	0.66	0.99	0.38	0.55	1.00	0.49	0.56	1.01	0.44
UMIDAS-ARMA-biclags	0.70	0.89	0.15	0.67	1.00	0.46	0.52	0.95	0.30	0.55	1.00	0.47
UMIDAS-ARMA-3	0.68	0.87	0.13	0.64	0.96	0.16	0.50	0.90	0.14	0.53	0.96	0.24
AR	0.82	1.04	0.43	0.71	1.05	0.27	0.82	1.48	0.00	0.71	1.28	0.00
	$h = 3$						$h = 3$					
MIDAS-AR-12	0.80	1.00	NaN	0.69	1.00	NaN	0.57	1.00	NaN	0.57	1.00	NaN
MIDAS-ARMA-12	0.87	1.08	0.29	0.73	1.07	0.15	0.64	1.13	0.04	0.61	1.06	0.07
MIDAS-ARMA-3	0.73	0.91	0.23	0.68	0.99	0.42	0.60	1.05	0.28	0.58	1.01	0.46
UMIDAS-biclags	0.84	1.05	0.31	0.72	1.05	0.17	0.71	1.24	0.05	0.63	1.10	0.03
UMIDAS-ARMA-biclags	0.84	1.05	0.36	0.74	1.07	0.15	0.76	1.33	0.00	0.66	1.15	0.01
UMIDAS-ARMA-3	0.75	0.93	0.25	0.68	0.99	0.43	0.60	1.05	0.27	0.58	1.01	0.45
AR	0.85	1.05	0.39	0.73	1.05	0.26	0.85	1.49	0.00	0.73	1.27	0.00
	$h = 4$						$h = 4$					
MIDAS-AR-12	0.86	1.00	NaN	0.74	1.00	NaN	0.63	1.00	NaN	0.61	1.00	NaN
MIDAS-ARMA-12	1.08	1.25	0.00	0.84	1.13	0.00	0.81	1.27	0.00	0.70	1.16	0.00
MIDAS-ARMA-3	0.85	0.98	0.42	0.73	0.99	0.36	0.69	1.09	0.09	0.63	1.04	0.14
UMIDAS-biclags	1.00	1.16	0.00	0.80	1.07	0.01	0.74	1.17	0.01	0.65	1.08	0.02
UMIDAS-ARMA-biclags	1.06	1.23	0.01	0.81	1.09	0.02	0.83	1.31	0.00	0.70	1.15	0.00
UMIDAS-ARMA-3	0.98	1.13	0.06	0.77	1.03	0.20	0.73	1.15	0.02	0.65	1.07	0.05
AR	0.89	1.03	0.41	0.76	1.01	0.41	0.89	1.41	0.00	0.76	1.25	0.00

Note. See Table 6.

improvements with respect to standard MF models for $h = 1$, with industrial production being the best indicator. When $h = 2$, employment becomes better than industrial production, and adding an MA term matters, with gains of 8% for the MIDAS-ARMA model. A similar results holds for $h = 3$, with gains increasing to 20%. Four quarters ahead, industrial production returns best, and MIDAS-ARMA leads to a decrease of 10% in the MSE. For PNFI growth, MIDAS-ARMA is best at all horizons, with employment preferred to industrial production except for $h = 1$. The gains are small for $h = 1$, 2, 3, in the range 1–8%, but increase to 30% for $h = 4$. For the GDP deflator, PCE inflation is systematically better than CPI, and MIDAS-ARMA yields gains for $h = 1$ and 2 of, respectively, 15% and 10%. It is also worth mentioning that MSE and MAE lead to the same rankings, and that the gains from adding the MA parts are generally statistically significant. Finally, the models perform well with respect to the AR benchmark. Confirming the widespread evidence in the literature, the MF models perform the best at short horizons. However, we get satisfactory results also up to $h = 4$.

TABLE 9 Bias/variance decomposition of MSE

		Bias				Variance			
		$h=1$	$h=2$	$h=3$	$h=4$	$h=1$	$h=2$	$h=3$	$h=4$
GDP with Industrial Production	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.72	0.68	0.59	0.89	1.00	0.97	1.04	0.94
	MIDAS-ARMA-3lags	0.85	0.97	1.01	0.90	1.06	0.99	0.91	0.90
	UMIDAS-biclags	1.09	1.04	1.08	0.87	1.04	1.06	0.99	0.97
	UMIDAS-ARMA-biclags	0.96	1.03	1.13	1.06	1.03	1.03	1.50	0.97
	UMIDAS-ARMA-3lags	0.85	1.12	1.23	0.98	1.06	1.05	1.00	1.10
	AR	3.06	1.02	1.23	1.01	1.84	1.17	1.07	0.95
GDP with Employment	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.26	0.70	2.22	1.69	0.98	0.92	0.95	0.93
	MIDAS-ARMA-3lags	0.18	1.44	0.51	0.65	0.98	0.94	0.82	0.94
	UMIDAS-biclags	5.60	1.00	0.51	0.74	0.97	1.00	0.89	0.95
	UMIDAS-ARMA-biclags	19.59	1.52	1.88	1.30	0.90	0.95	1.71	0.97
	UMIDAS-ARMA-3lags	0.18	1.40	0.56	0.61	0.98	1.00	0.93	0.88
	AR	348.15	4.11	0.87	0.56	1.51	1.24	1.00	0.81
PNFI with Industrial Production	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	5.39	0.94	0.86	0.86	0.95	0.99	0.96	0.96
	MIDAS-ARMA-3lags	10.55	1.13	1.54	1.12	0.91	1.07	1.05	0.95
	UMIDAS-biclags	0.09	1.38	1.31	1.01	1.04	1.08	1.21	1.01
	UMIDAS-ARMA-biclags	1.52	1.50	1.65	1.20	0.95	1.15	1.08	1.04
	UMIDAS-ARMA-3lags	10.59	1.54	1.54	0.92	0.91	1.13	1.05	1.01
	AR	33.09	2.46	2.77	2.10	1.45	1.20	1.27	1.19
PNFI with Employment	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.99	1.15	0.00	0.73	1.01	0.99	0.93	0.86
	MIDAS-ARMA-3lags	0.78	0.42	4.78	0.17	1.01	1.01	0.94	0.72
	UMIDAS-biclags	0.91	0.85	0.35	0.24	1.01	1.04	1.04	0.86
	UMIDAS-ARMA-biclags	0.81	0.32	7.20	0.23	1.04	1.06	0.97	0.84
	UMIDAS-ARMA-3lags	0.78	0.44	4.22	0.23	1.01	1.01	0.94	0.80
	AR	0.25	2.53	230.03	0.58	1.40	1.29	1.25	0.93
GDP Deflator with CPI inflation	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.08	0.79	0.83	0.95	0.98	0.88	1.15	1.35
	MIDAS-ARMA-3lags	0.87	0.61	0.51	0.72	0.94	0.92	1.03	1.07
	UMIDAS-biclags	0.63	0.61	0.63	0.98	1.02	1.01	1.18	1.21
	UMIDAS-ARMA-biclags	0.87	0.64	0.59	0.76	0.94	0.95	1.19	1.37
	UMIDAS-ARMA-3lags	0.87	0.61	0.53	0.69	0.94	0.92	1.06	1.27
	AR	0.79	0.62	0.67	0.89	1.24	1.13	1.17	1.07
GDP Deflator with PCE inflation	MIDAS1-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.23	1.00	0.88	1.04	0.97	1.01	1.19	1.33
	MIDAS-ARMA-3lags	0.45	0.58	0.62	0.77	0.87	0.95	1.15	1.17
	UMIDAS-biclags	0.70	0.58	0.60	0.87	1.06	1.07	1.39	1.24
	UMIDAS-ARMA-biclags	0.96	0.70	0.49	0.81	1.01	0.99	1.51	1.43
	UMIDAS-ARMA-3lags	0.90	0.58	0.62	0.73	0.96	0.95	1.15	1.26
	AR	2.09	1.16	1.24	1.45	1.52	1.53	1.54	1.40

Note. The table the decomposition of the MSE of the different models as presented in Section 4 into bias and variance, for different forecasting horizons. The forecasts are evaluated over the sample 1980:Q1–2015:Q4. The numbers reported are the ratio of the bias and of the variance of each model relative to the bias and variance of the MIDAS-AR model.

The empirical gains resulting from the use of the MA term in the MF models are somewhat larger than those in the Monte Carlo experiments. A possible reason is model misspecification, and in particular some form of parameter instability. In that case, the use of an MA component can be helpful to put the forecasts "back on track"; see, for example, Clements and Hendry (1998) for details.

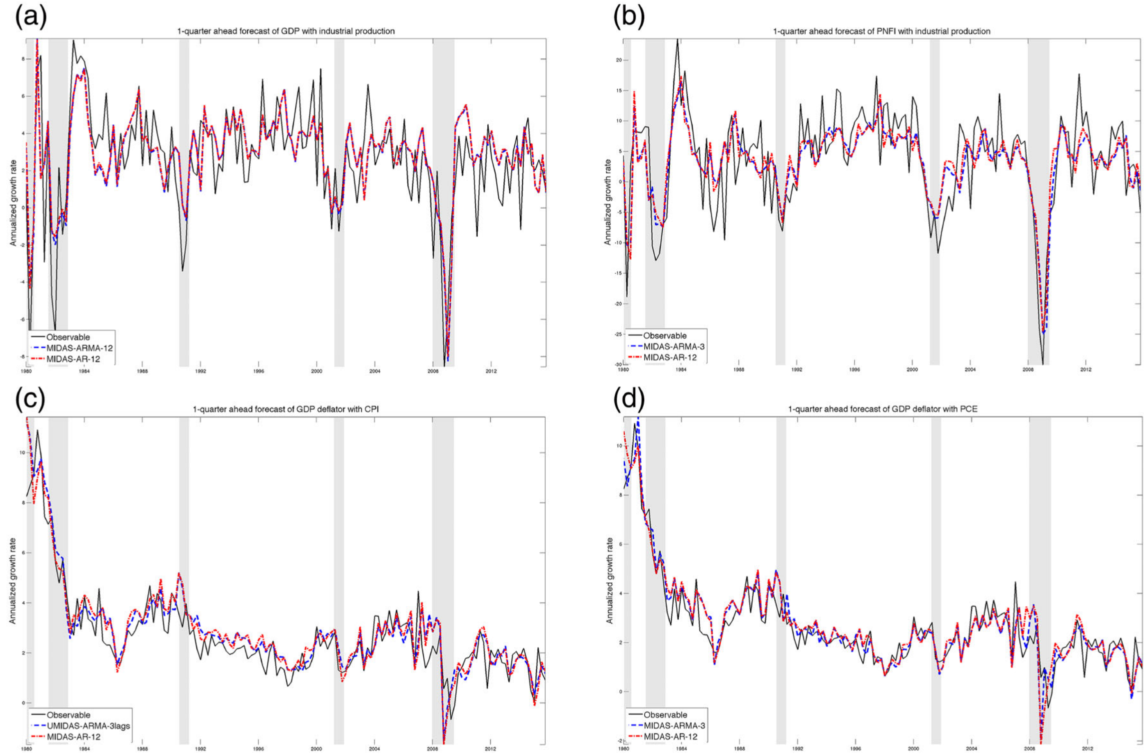

FIGURE 1 Out-of-sample performance: one quarter ahead. (a) GDP with monthly industrial production. (b) PNFI with monthly industrial production. (c) GDP deflator with monthly CPI. (d) GDP deflator with monthly PCE [Colour figure can be viewed at wileyonlinelibrary.com]

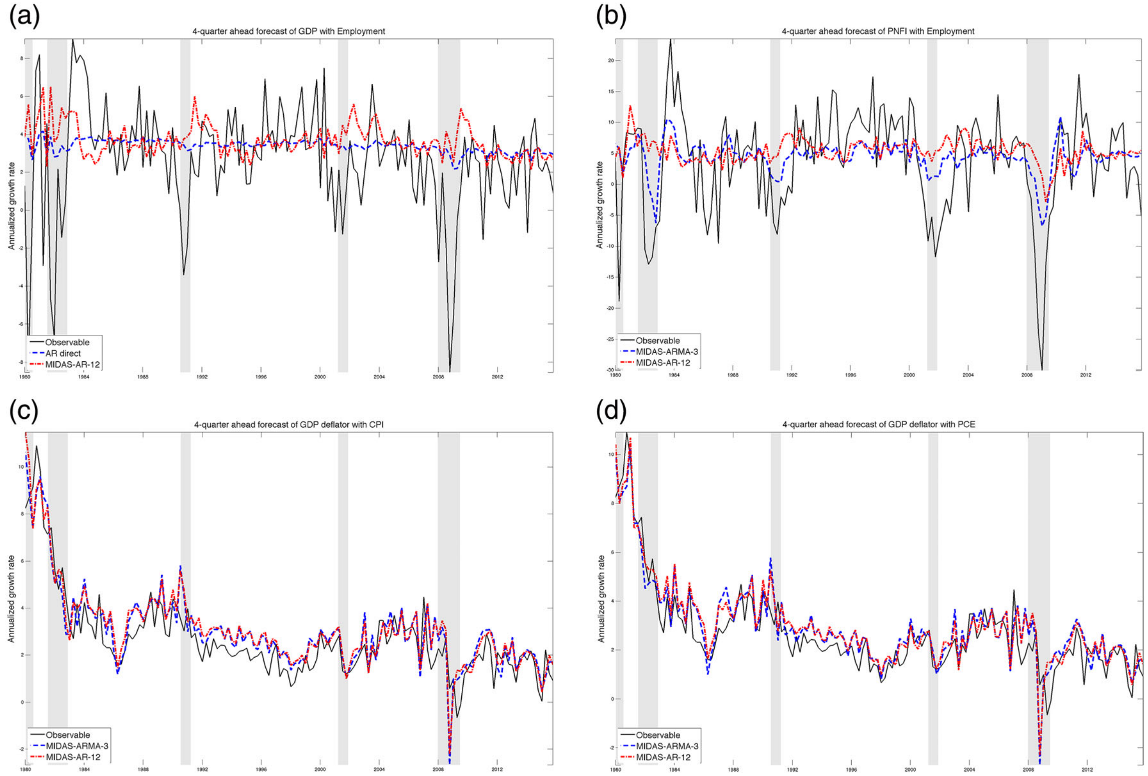

FIGURE 2 Out-of-sample performance: four quarters ahead. (a) GDP with monthly employment. (b) PNFI with monthly employment. (c) GDP deflator with monthly CPI. (d) GDP deflator with monthly PCE [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 10 Full-sample estimated MA coefficients

	Explanatory variable: Industrial production					Explanatory variable: Employment			
	$h=1$	$h=2$		$h=3$	$h=4$	$h=1$	$h=2$	$h=3$	$h=4$
	MA(1)	MA(1)	MA(2)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)	MA(1)
Forecasting US GDP growth									
MIDAS-ARMA-12lags	0.06	0.03	0.11	0.29	0.26	0.39	0.35	0.24	0.12
	(0.45)	(0.46)	(0.72)	(1.33)	(0.78)	(3.59)	(2.26)	(1.18)	(0.42)
MIDAS-ARMA-3lags	−0.10	0.05	0.17	0.31	0.32	0.41	0.44	0.37	0.26
	(−0.89)	(0.73)	(1.13)	(1.71)	(1.20)	(4.00)	(2.85)	(1.95)	(1.01)
UMIDAS-ARMA-biclags	0.01	0.06	0.19	0.11	0.25	0.05	0.42	0.25	0.28
	(0.08)	(0.84)	(1.14)	(0.43)	(1.22)	(0.18)	(2.98)	(1.47)	(1.15)
UMIDAS-ARMA-3lags	−0.10	0.05	0.17	0.31	0.16	0.41	0.45	0.33	0.27
	(−0.89)	(0.73)	(1.13)	(1.80)	(0.60)	(4.00)	(3.03)	(1.70)	(1.07)
Forecasting PNFI growth									
MIDAS-ARMA-12lags	−0.33	−0.14		0.13	−0.03	0.13	0.29	0.35	0.46
	(−3.82)	(−1.15)		(0.65)	(−0.13)	(1.27)	(2.22)	(2.37)	(2.67)
MIDAS-ARMA-3lags	−0.39	−0.14		0.15	−0.07	0.11	0.27	0.37	0.48
	(−5.83)	(−1.54)		(1.14)	(−0.43)	(1.23)	(2.86)	(2.65)	(2.94)
UMIDAS-ARMA-biclags	−0.31	0.05		0.17	−0.09	−0.04	0.26	0.35	0.48
	(−3.70)	(0.50)		(1.33)	(−0.51)	(−0.42)	(2.78)	(2.41)	(2.46)
UMIDAS-ARMA-3lags	−0.39	−0.12		0.15	−0.07	0.08	0.29	0.37	0.48
	(−5.83)	(−1.24)		(1.14)	(−0.43)	(0.83)	(3.00)	(2.65)	(2.98)

Note. The table reports the estimated values of MA coefficients using the full sample 1960–2015. The values in parentheses are t-statistics calculated with Newey–West standard errors to take into account the serial autocorrelation of order $h-1$, possibly induced by direct forecasting.

We now decompose the MSE in bias and variance, as

$$\text{MSE} = \underbrace{(E(e))^2}_{\text{bias}} + \underbrace{\text{var}(e)}_{\text{variance}} \tag{21}$$

with $e = y - \hat{y}$. We find that the MA part helps especially in reducing the bias, suggesting that the MA part is important to well approximate the conditional mean of y (the optimal forecast under the quadratic loss). When the models with the MA component are not performing well, this is due especially to the variance term, instead. Detailed results on the bias/variance decomposition are presented in Table 9. In particular, in the table we report the ratio of the bias and of the variance of each model relative to the bias and variance of the MIDAS-AR model, which is taken as a benchmark.

The MSE and MAE are computed over the entire evaluation sample. To check whether the performance of our models remains good across the entire sample, in Figure 1 we report the 1-quarter-ahead forecasts of the benchmark MIDAS-AR model and of one of the MA augmented models, together with the realized series. In Figure 2, instead, we report the 4-quarter-ahead forecasts.[10] It turns out that, on average, MIDAS models perform well throughout the sample, both with and without an MA component.

Tables 10 and 11 report the full-sample estimates of MA coefficients in MIDAS-ARMA and UMIDAS-ARMA models that have been used in the forecasting exercise. The corresponding t-statistics are shown in parentheses. We observe that many MA coefficients are significant. For instance, the MA(1) coefficient in MIDAS-ARMA-3lags model on GDP growth equation with employment growth is precisely estimated at horizons $h = 1, 2, 3$. This MIDAS-ARMA model was also the best in out-of-sample forecasting exercise (see Table 6). In case of PNFI, when forecasting 1 quarter ahead with industrial production as HF predictor, the MA coefficient is significant in all models. The same result holds at longer horizons with employment growth. When it comes to GDP deflator prediction, an interesting finding is that the MA(2) component is very strong and highly significant for most of the horizons and models.

[10]Figures 1 and 2 focus only on a small proportion of results that we have available. The same figures for other models, other forecast horizons, and other explanatory variables are available upon request.

TABLE 11 Full-sample estimated MA coefficients

	h = 1		h = 2		h = 3		h = 4	
	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)	MA(1)	MA(2)
Forecasting US GDP deflator								
Explanatory variable: Consumer price index								
MIDAS-ARMA-12lags	−0.09	−0.31	0.18	−0.30	0.29	−0.08	0.19	0.14
	(−1.03)	(−2.43)	(1.89)	(−2.78)	(2.78)	(−0.67)	(1.70)	(1.46)
MIDAS-ARMA-3lags	−0.09	−0.36	0.20	−0.34	0.28	−0.10	0.18	0.11
	(−1.13)	(−4.65)	(2.09)	(−4.48)	(2.99)	(−0.93)	(1.76)	(1.08)
UMIDAS-ARMA-biclags	−0.09	−0.36	0.18	−0.35	0.25	−0.12	0.16	0.09
	(−1.13)	(−4.65)	(1.78)	(−4.58)	(2.62)	(−1.09)	(1.58)	(0.91)
UMIDAS-ARMA-3lags	−0.09	−0.36	0.20	−0.34	0.28	−0.10	0.18	0.11
	(−1.13)	(−4.65)	(2.06)	(−4.44)	(2.99)	(−0.93)	(1.76)	(1.08)
Explanatory variable: PCE price index								
MIDAS-ARMA-12lags	−0.06	0.09	0.13	−0.22	0.25	−0.03	0.19	0.19
	(−0.68)	(0.55)	(1.50)	(−1.62)	(2.48)	(−0.30)	(1.84)	(2.07)
MIDAS-ARMA-3lags	0.15	0.32	0.14	−0.30	0.24	−0.05	0.18	0.16
	(1.97)	(3.25)	(1.56)	(−3.86)	(2.95)	(−0.53)	(1.91)	(1.53)
UMIDAS-ARMA-biclags	−0.13	−0.29	0.14	−0.30	0.22	−0.06	0.17	0.16
	(−1.84)	(−3.60)	(1.56)	(−3.86)	(2.63)	(−0.63)	(1.82)	(1.51)
UMIDAS-ARMA-3lags	−0.13	−0.29	0.14	−0.30	0.24	−0.05	0.18	0.16
	(−1.84)	(−3.60)	(1.56)	(−3.86)	(2.95)	(−0.53)	(1.91)	(1.53)

Note. The table reports the estimated values of MA coefficients using the full sample 1960–2015. The values in parentheses are t-statistics calculated with Newey–West standard errors to take into account the serial autocorrelation of order $h - 1$, possibly induced by direct forecasting.

In Supporting Information Appendix C we expand the empirical exercise along several dimensions. First, we analyze a shorter sample ending in 2007:Q3, to assess the effects of the recent crisis. Second, we report results for cases in which only one of the months of the quarters are available and when instead all 3 months are already available. Third, we use a real-time dataset with the different vintages.[11]

All in all, the robustness exercises confirm the evidence we find in this section. Excluding the crisis, results do not change substantially and remain broadly supportive of the inclusion of the MA component in the MF models. In most cases, the best-performing model up to 2007 remains the best in the full sample. The magnitude of improvements is also very comparable. For the other nowcasting horizons (i.e., including only 1 month of monthly information or, instead, all 3 months), results confirm that in most cases the best performance is obtained when the MA component is added. Finally, when using real-time data for the macroeconomic series considered, we see the same patterns as with pseudo real-time data.

6 | CONCLUSIONS

In this paper, we start from the observation that temporal aggregation in general introduces an MA component in the aggregated model. We show that a similar feature also emerges when not all but only a few variables are aggregated, which generates an MF model. Hence an MA component should be added to MF models, whereas this is generally neglected in the literature.

We illustrate in a set of Monte Carlo simulations that indeed adding an MA component to MIDAS and UMIDAS models further improves their nowcasting and forecasting abilities, though in general the gains are limited and particularly evident in the presence of persistence. Interestingly, the relative performance of MIDAS versus UMIDAS further improves when adding an MA component, with the latter attenuating the effects of imposing a particular polynomial structure in the dynamic response of the LF to the HF variable.

[11]As in the Monte Carlo, we could extend the number of lags in the AR component. However, we keep our benchmark specification of 1 AR lag, consistent with most of the empirical studies involving MIDAS. Results with 4 lags are available upon request.

A similar pattern emerges in an empirical exercise based on actual data. Specifically, we find that the inclusion of an MA component can substantially improve the forecasting performance of quarterly macroeconomic US variables, such as GDP growth, PNFI growth and GDP deflator inflation. MIDAS-ARMA models perform particularly well, suggesting that the addition of an MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure. Finally, full-sample estimates of MA coefficients are significant and important in most MIDAS-ARMA and UMIDAS-ARMA specifications.

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REFERENCES

Anderson, B. D., Deistler, M., Felsenstein, E., & Koelbl, L. (2016). The structure of multivariate AR and ARMA systems: Regular and singular systems; the single and the mixed frequency case. *Journal of Econometrics*, *192*(2), 366–373.

Andreou, E., Ghysels, E., & Kourtellos, A. (2010). Regression models with mixed sampling frequencies. Journal of Econometrics, *158*(2), 246–261.

Bai, J., Ghysels, E., & Wright, J. (2013). State space models and MIDAS regressions. *Econometric Reviews*, *32*, 779–813.

Clements, M. P., & Galvo, A. B. (2008). Macroeconomic forecasting with mixed-frequency data. *Journal of Business and Economic Statistics*, *26*, 546–554.

Clements, M., & Hendry, D. (1998). *Forecasting economic time series*. Cambridge, UK: Cambridge University Press.

Deistler, M., Koelbl, L., & Anderson, B. D. (2017). Non-identifiability of VMA and VARMA systems in the mixed frequency case. *Econometrics and Statistics*, *4*, 31–38.

Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, *13*(3), 253–263.

Dufour, J.-M., & Stevanovic, D. (2013). Factor-augmented VARMA models with macroeconomic applications. *Journal of Business and Economic Statistics*, *31*(4), 491–506.

Eraker, B., Chiu, C. W. J., Foerster, A. T., Kim, T. B., & Seoane, H. D. (2015). Bayesian mixed frequency VARs. *Journal of Financial Econometrics*, *13*(3), 698–721.

Faust, J., & Wright, J. H. (2013). Forecasting inflation. *Handbook of Economic Forecasting*, *2*(A), 1063–1119.

Foroni, C., & Marcellino, M. (2013). A Survey of Econometric Methods for Mixed-Frequency Data. (*Working Paper 2013/06*). Oslo, Norway: Norges Bank.

Foroni, C., Marcellino, M., & Schumacher, C. (2015). U-MIDAS: MIDAS regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society, Series A*, *178*(1), 57–82.

Ghysels, E. (2016). Macroeconomics and the reality of mixed-frequency data. *Journal of Econometrics*, *193*, 294–314.

Ghysels, E., & Qian, H. (2019). Estimating MIDAS regressions via OLS with polynomial parameter profiling. *Econometrics and Statistics*, *9*, 1–16.

Ghysels, E., Santa-Clara, P., & Valkanov, R. (2006). Predicting volatility: Getting the most out of return data sampled at different frequencies. *Journal of Econometrics*, *131*(1–2), 59–95.

Giannone, D., Reichlin, L., & Small, D. (2008). Nowcasting: The real-time informational content of macroeconomic data. *Journal of Monetary Economics*, *55*(4), 665–676.

Kim, H. H., & Swanson, N. R. (2017). Methods for backcasting, nowcasting and forecasting using factor-MIDAS: with an application to Korean GDP. *Journal of Forecasting*, *37*(3), 281–302.

Leroux, M., Kotchoni, R., & Stevanovic, D. (2017). Forecasting economic activity in data-rich environment. (*EconomiX Working Papers 2017-5*). Paris, France: University of Paris Nanterre.

Lutkepohl, H. (2006). Forecasting with VARMA Models, *Handbook of economic forecasting* (Vol. 1. pp. 287–325). Amsterdam, Netherlands: Elsevier.

Marcellino, M. (1999). Some consequences of temporal aggregation in empirical analysis. *Journal of Business and Economic Statistics*, *17*(1), 129–136.

Marcellino, M., Stock, J. H., & Watson, M. W. (2006). A comparison of direct and iterated multistep AR methods for forecasting macroeconomic time series. *Journal of Econometrics*, *135*(1–2), 499–526.

Mariano, R. S., & Murasawa, Y. (2010). A coincident index, common factors, and monthly real GDP. *Oxford Bulletin of Economics and Statistics*, *72*(1), 27–46.

Ng, S., & Perron, P. (2001). LAG Length selection and the construction of unit root tests with good size and power. *Econometrica*, *69*(6), 1519–1554.

Perron, P., & Ng, S. (1996). Useful modifications to some unit root tests with dependent errors and their local asymptotic properties. *Review of Economic Studies*, *63*(3), 435–463.

Ravenna, F. (2007). Vector autoregressions and reduced form representations of DSGE models. *Journal of Monetary Economics*, *54*(7), 2048–2064.

Schorfheide, F., & Song, D. (2015). Real-time forecasting with a mixed-frequency VAR. *Journal of Business and Economic Statistics*, *33*(3), 366–380.

Schumacher, C. (2016). A comparison of MIDAS and bridge equations. *International Journal of Forecasting*, *32*(2), 257–270.

Stock, J. H., & Watson, M. W. (2007). Why Has U.S. inflation become harder to forecast? *Journal of Money, Credit and Banking*, *39*(s1), 3–33.

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