No data? No problem! A Search-based Recommendation System with Cold Starts*

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October 2020

Recommendation systems are essential ingredients in producing matches between products and buyers. Despite their ubiquity, they face two important challenges. First, they are data-intensive, a feature that precludes sophisticated recommendations by some types of sellers, including those selling durable goods. Second, they often focus on estimating fixed evaluations of products by consumers while ignoring state-dependent behaviors identified in the Marketing literature.

We propose a recommendation system based on consumer browsing behaviors, which bypasses the "cold start" problem described above, and takes into account the fact that consumers act as "moving targets," behaving differently depending on the recommendations suggested to them along their search journey. First, we recover the consumers' search policy function via machine learning methods. Second, we include that policy into the recommendation system's dynamic problem via a Bellman equation framework.

When compared with the seller's own recommendations, our system produces a profit increase of 33%. Our counterfactual analyses indicate that browsing history along with past recommendations feature strong complementary effects in value creation. Moreover, managing customer churn effectively is a big part of value creation, whereas recommending alternatives in a forward-looking way produces moderate effects.

1 Introduction

Recommendation systems have been used in several industries to match products and buyers. Often relying on machine learning techniques, researchers have suggested recommendation systems that can affect customers' choices and predict their preferences (e.g. Amazon recommendations, Netflix ratings). Despite their popularity, modern recommendation technologies face two challenges. The first, the "cold start" problem, arises when the data initially required to inform the recommendation system is unavailable. The problem arises in multiple contexts: Small companies often lack the resources necessary to acquire and maintain customer- and product-related data, not to mention maintain the required technological infrastructure. Durable goods sellers may also find it challenging to acquire rich customer data: In our context, the case of a North American online used-car seller, customers usually buy at most one good, and thus generate little to no purchase history data. Growing privacy concerns also limit firms' access to rich consumer-level data. For example, the General Data Protection Regulation (GDPR) rules out "use-bundling," forcing sellers to disclose each intended use of the data to be collected when asking consumers' permissions, which is widely viewed as constraining for firms (Martin, Matt, Niebel, and Blind, 2019).

The second challenge – as stated in Adomavicus and Tuzhilin (2005) – is that "in its most common formulation, the recommendation problem is reduced to the problem of estimating ratings for the items that have not been seen by a user. (...) Once we can estimate ratings for the yet unrated items, we can recommend to the user the item(s) with the highest estimated rating(s)." Despite its prevalence, this approach ignores the findings from the consumer search literature surrounding the marked path-dependent behaviors by consumers.

In this paper, we develop a recommendation system that tackles these two challenges. The recommendation system is rooted on an estimated consumer "search/purchase" policy function. In line with the literature on consumer search in Marketing, our recommendation system incorporates the fact that consumers are a "moving target," changing their beliefs and behaviors as they learn more about the products offered by the seller. This is done

by first estimating the consumer's dynamic policy function and second, using the estimated policy function to inform the dynamic problem of the recommendation system. In addition to determining the ex-ante first-best recommendation policy, we conduct a series of counterfactual scenarios in order to determine the characteristics of the recommendation system that drive value.

Our approach relies on the Bellman framework to propose recommendations at each step of the consumer journey. This is important because we know, from the literature on consumer search, that consumer behavior is complex and path-dependent. For example, the seminal paper characterizing stylized sequential search (Weitzman, 1979) has found counterpoints by De los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2017), both of which find evidence towards fixed-sample search behaviors. Bronnenberg, Kim, and Mela (2016) document non-trivial consumer search behaviors, such as the focus on only a few characteristics in the attribute space, and state dependence while navigating alternatives. Ke, Shen, and Villas-Boas (2016) and Ursu, Wang, and Chintagunta (2020) incorporate the continuous-type aspect of consumer search, and Gardete and Antill (2020) account for piecemeal search for correlated characteristics of alternatives, as observed in browsing data.

This complex, path-dependent perspective is underrepresented in the Computer Science literature on recommendation systems, whose main object is to estimate consumers' expected utilities for different alternatives. In that literature, recommendation systems are often organized into two main types: Collaborative filtering and Content-based filtering (see Adomavicus and Tuzhilin (2005) for a review). Collaborative filtering methods rely on the existence of users who exhibit overlaps in terms of products bought (i.e., "Buyers of this product also bought..."). As for content-based filtering methods, these use the characteristics of the items that users liked, bought, etc., and match those behaviors to characteristics of other items available for sale. For example, a user that listens to a particular music genre may be recommended more songs of the same genre. Just like collaborative filtering, content-based filtering also relies on broad availability of user-data, such as past purchases.

In practice, these methods rely on a static view of consumer-product match values, and can perform only when there exist significant amounts of data.

A third approach, relying on multi-armed bandits, assigns values to multiple arms, i.e., the alternatives to be recommended. While these methods are useful to address the cold start problem, they require significant transformation in order to incorporate the dynamics of consumer search. Finally, even recent advances in context-aware recommender systems (e.g., Song, Tekin, and van der Schaar, 2016) tend to rely on static contexts.[1]

Taking advantage of a dataset containing clickstream data from a North American online used car seller, we first estimate the consumers' "search/purchase" policy functions flexibly via machine learning methods.[2] This procedure allows us to characterize the effect of product recommendations and other customer actions on the subsequent probabilities of search and conversion. To address the scalability issues associated with the high number of alternatives involved, we rely on alternative-level clustering (as in Song, Tekin, and van der Schaar (2016)). In this case, "the recommendations are made at a cluster level instead of an item level." The consumer policy is estimated via multiple machine learning methods and for several numbers of clusters. We analyze a number of out-of-sample fit metrics in order to decide the optimal pair of estimation method and number of clusters to be used.[3]

Once consumer search is characterized, we calculate the ex-ante performance of the seller's recommendation system (i.e., the status quo), and compare it with the performance of the ex-ante first-best system. Our dynamic framework is able to anticipate complex consumer search strategies. For example, our recommendation system may engage in active learning by suggesting a potentially low-valued but highly informative alternative, in order to transition the consumer to a state that makes her more amenable to buying. Our model can also weigh

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[1]These methods also tend to optimize against click-through rates rather than actual profitability. See Kulkarni and Rodd (2020) for a review of context-aware recommendation systems. See also Agrawal (2019) for a review of MABs in the context of sequential decision-making. In Marketing, multi-armed bandit models have been used for a variety of tasks, including advertising placement (Schwartz, Bradlow, and Fader, 2017).

[2]See Bajari, Nekipelov, Ryan, and Yang (2015) for a review and application of several machine learning methods for estimating demand models.

[3]An important question is the validity of the recovered policy function when conducting counterfactual analyses. We take advantage of our institutional setup, as discussed in detail in Section 3.

promoting consumer engagement against the likelihood of consumer churn.[4]

When compared with the status quo case, the first-best recommendation system increases expected seller profits by 33%. The system tends to concentrate vehicle recommendations as the consumer search journey progresses, being more likely to recommend vehicles from different clusters near the beginning, and honing into a few or a single recommended cluster down the line.

We conduct a series of counterfactual analyses in order to add interpretability to the model's performance. First, despite the sizable increment in profitability, we find that the status quo recommendation system does extremely well in terms of maximizing profit based only on the last alternative viewed by the consumer. In fact, our model is unable to attain a statistically significantly better performance when constrained to optimize solely on the last alternative browsed. This could make it seem that previous browsing history and past recommendations, excluding the alternative currently being viewed, are relatively much more important for profitability. Our findings point otherwise. When basing the recommendation system only on previous consumer and recommendation histories while ignoring the last alternative viewed, profits increase by only 2.7%. We find that there is a significant complementarity between past history and information on the current vehicle being viewed, which combined account for the 33% maximum increase in profitability. Additional counterfactual analyses reveal that value creation depends less on moving consumers towards high-margin alternatives, and more on 1) managing consumer churn and 2) being forward-looking in terms of anticipating the effect of product recommendations on the consumer journey as a whole.

The next section describes the dataset. Section 3 describes the estimation and identification procedure for the consumer policy function. Section 4 describes vehicle clustering and formalizes the recommendation system. Section 5 presents the estimation results, and

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[4]See also the related work by Chen and Yao (2017) and De los Santos and Koulayev (2017). The latter "propose a method of determining the ranking of search results that maximizes consumers' click-through rates." In contrast to this work, our model focuses on profitability rather than click-through rates, and takes the stage of the consumer search journey into account when recommending alternatives.

Section 6 presents the findings from the counterfactual analyses. Section 7 concludes.

2 Data

Our data was collected from a online platform that sells used cars, Shift.com. This seller operates in multiple North American cities, and allows buyers to book test-drives before they buy. The dataset covers the period between February and September 2016. It includes the full consumer clickstream data as well as the characteristics of the vehicles on sale during the time period. The dataset is explained in detail by Gardete and Antill (2020). In contrast with their work, we use all of the vehicle categories available for the analysis.

We observe a total of 71,143 users and 442,392 pageviews (Table 1). Each vehicle page was viewed an average of 130 times in our sample and we denote significant variance: One vehicle exhibited a single visit while another was visited more than 2,000 times.

Consumer visits are tracked via IP address and cookies. A consumer visit yields an average of 7 vehicle pageviews and 95% of visitors perform fewer than 22 searches (see Figure 1).

Table 1: Descriptive Statistics for Browsing Data

Variables	Count	Mean	Std. Dev.	Min	Median	Max
Number of pageviews per user	71,143	7.01	15.58	1	3	783
Conversion rate (%)	71,143	0.029	0.167	0	-	1
Number of visits per vehicle	3,795	131.5	121.9	1	101	2,268
Browsing time / pageview (min.)	442,391	1.42	3.20	0	0.51	30

Like Gardete and Antill (2020), the conversion rate measures the probability of booking a vehicle test-drive on the platform. We use test-drives as proxies for purchase because 1) more than 90% of consumers who book test drives end up buying the vehicle; 2) because of point 1, there is little data available after the first test-drive (e.g., consumers seldom book a second one). We do not expect the relationship between test-drive and purchases to be

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altered by changes to the recommendation system, and so our counterfactual analyses should speak both to conversions and to final purchases.

We also include vehicles' Carfax valuations at the time of the sample, in order to construct potential vehicle selling margins. The main goal with these data is to proxy for differential margins across vehicles during counterfactual analyses. These data allow us to understand the impact of having the recommendation system prioritize vehicles with higher margins, for example.[5]

Figure 1 presents the histogram of the number of vehicle pageviews per user. Across their search journey, most users (30%) visit 2 vehicles. Moreover, behavior follows a long tail: While most users (85%) perform fewer than 10 searches, others search beyond 30 (less than 3%).

Figure 1: Histogram of Pageviews per User
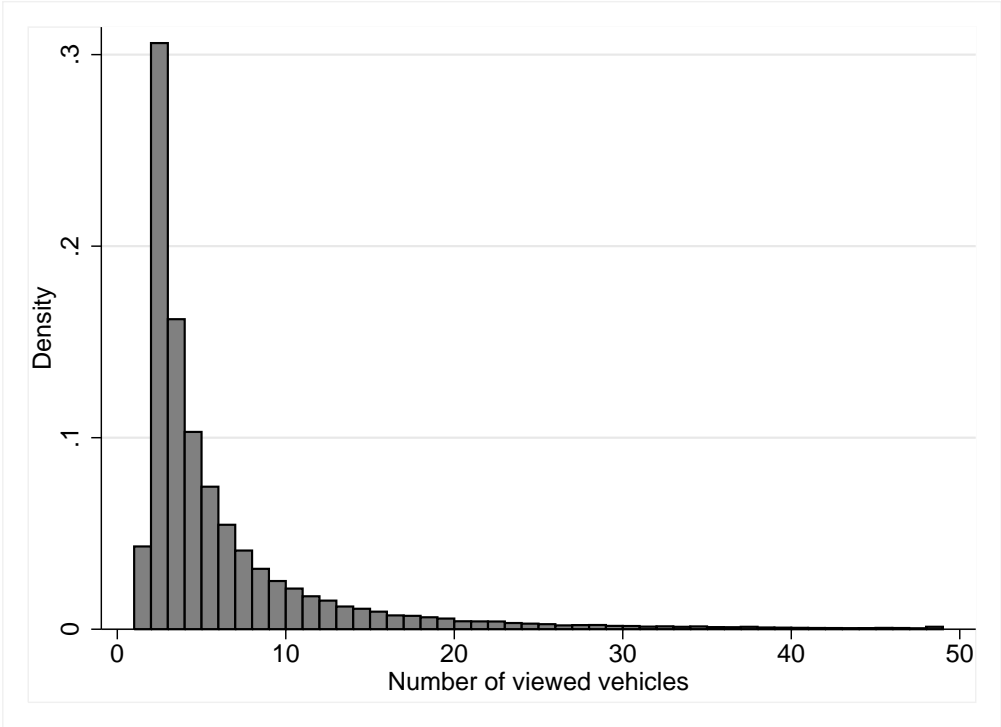
Note: Distribution of pageviews per user, truncated at the 99th percentile.

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[5]In those analyses we assume 30% margins, and exclude outliers below/above the 5/95% percentiles.

Status Quo Recommendation System. Every time a user browses a vehicle, she is recommended 3 additional alternatives. Figure 2 shows the vehicle recommendations to a user who viewed the vehicle profile page of a 2016 Toyota Corolla S Plus.

Figure 2: Recommendation example
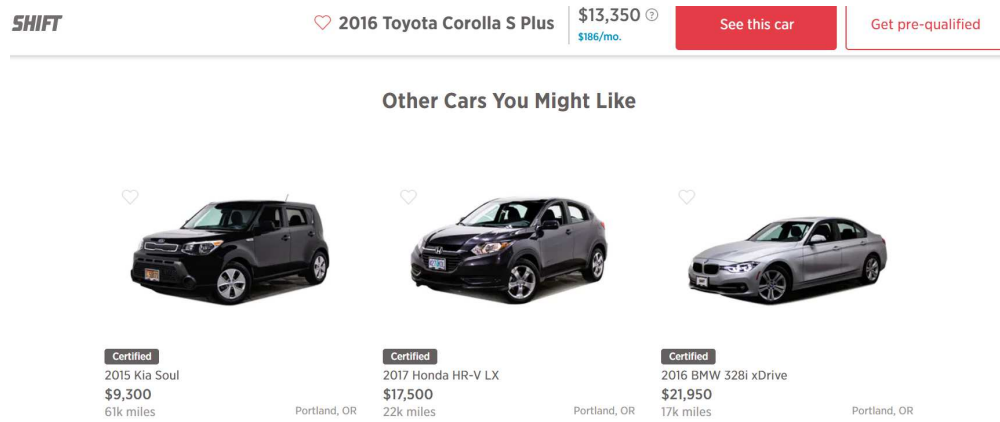
The website's recommendation system suggests vehicles based on the vehicle currently being viewed. These vehicles may be more or lesse expensive than the focal vehicle. During the data collection period, the seller's recommendation system was static: If a consumer visits a vehicle and returns to it later, she will observe the same vehicle recommendations. In other words, the status quo recommendation system does not track consumer behavior in order to customize recommendations.

Figure 2 presents a probabilistic recommendation matrix calculated from the status quo recommendation system. For this exercise, vehicles were grouped into four distinct segments according to cluster analysis (details are presented later, in Section 4.1). Each row depicts a visit to a vehicle of one of the four clusters and each column corresponds to a recommendation of a vehicle from those same clusters.

Table 2: Recommendation Probabilities - 4 Clusters

| | | Vehicle Recommendation | | | |
	Cluster	1	2	3	4
	1	0.513	0.144	0.146	0.197
Vehicle	2	0.145	0.433	0.193	0.229
Viewed	3	0.191	0.252	0.362	0.195
	4	0.221	0.232	0.156	0.392

Note: Above, probability of recommending a vehicle in a column, conditional on a consumer browsing a vehicle in a row.

The main diagonal reveals that the current recommendation system is not random: The algorithm is most likely to recommend vehicles that have similar characteristics to the ones just viewed by consumers.

Vehicle characteristics. Our data comprises characteristics of 4,140 vehicles, specifically, make, model, color, year, price, mileage, number of owners, market valuation, body style, transmission, drivetrain, and number of accidents.

Table 3: Descriptive Statistics for Vehicle Data (metric variables).

Variables	Mean	Std. Dev.	Min	Median	Max
Year	2011	2.89	2001	2011	2016
Number of Owners	1.38	0.94	0	1	7
Number of Accidents	0.08	0.30	0	0	3
Mileage	100	27.15	83.49	132.57	220.74
Price	100	10.1	86.07	97.37	223.37
Market Value	100	11.41	81.81	97.11	234.57
N:	4,140				

Note: Descriptive statistics for the vehicles available on the seller's platform. Above, variables mileage, price and market value are not comparable, as each is transformed via a different normalization.

The ordinal vehicle characteristics are presented in Table 3. The price and mileage variables have been scaled and mean-shifted for confidentiality purposes. The largest vehicle

class is of sedans with front wheel drive and automatic transmission.

3   Consumer Behavior

3.1   Policy Function

The Marketing literature often uses dynamic programs to characterize consumer search behaviors (Sun (2006), Kim, Albuquerque, and Bronnenberg (2010), Yao and Mela (2011), Seiler (2013), Ursu, Wang, and Chintagunta (2020), Gardete and Antill (2020)). The idea is that consumers procure information strategically to find product matches. In turn, the recommendation system incorporates the consumer's dynamic behavior into its own dynamic optimization problem: It decides what to recommend to consumers at each stage of their journey. We start out by defining the consumers' policy function that maximizes their value:

$$Pr(y_{it} = j | \Theta_{it}), \, j \in \{0..2J + 1\} \tag{1}$$

Above, y_{it} is consumer i's action at time t and Θ_{it} is the set of the consumer's current state variables. Set $\{0..2J+1\}$ includes all of the consumer's allowed actions at time t: Converting immediately to one of the available vehicles ($J + 1$ actions, including the outside option), or deciding to search a specific vehicle (J additional actions).

The consumer's policy function can be recovered flexibly from choice data. Indeed, the goal of this first stage is to characterize consumer behavior so it can be incorporated into the recommendation system's optimization problem. When solving its own optimization problem, the recommendation system will decide which vehicles to serve by incorporating consumers' behaviors as described in (1). Counterfactual analyses rely on the consumer policy being recovered properly. We discuss the identification of causal effects in the next section.

Once the consumer policy is estimated, it is used to calculate recommendations across

several counterfactual scenarios. Consumer policy (1) is likely to remain valid across these analyses. Specifically, we do not expect consumers to ascribe strategic behavior to the recommendation system as we switch recommendation regimes. For one, such changes are hard to identify from the consumer's point of view. Even if consumers could indeed detect changes in recommendations, policy (1) remains valid as long as they do not act strategically by attempting to influence or game the recommendation system through their actions. While such behaviors could occur in settings where consumers gain a lot of experience with the platform's recommendation system (e.g., Amazon, YouTube), it is unlikely to arise in the context of the website of a relatively small used car seller, where consumers often make at most one purchase.[6]

We include three groups of behavioral state variables while estimating the consumer's policy function: The order of the interaction (t), the vehicles browsed by the consumer up to t (H_t), and the history of vehicle recommendations (R_t). Moreover, we decompose the vehicles browsed, H_t, into $H_t = \{a_t, A_t\}$, where a_t is the vehicle clicked by the user at time t and A_t is the set of vehicles searched before time t.[7] The state space for consumer i can be written as $\Theta_{it} = \{t_i, a_{it}, A_{it}, R_{it}\}$. This space is used for policy estimation as well as for the purposes of the recommendation system. We discuss the state space and its transitions in more detail in Section 4.

3.2 Identification

It is worth considering how different recommendations interact with policy (1). Assigning a new set of recommendations has two effects on consumers. First, it moves consumers onto new information sets. For example, instead of learning about a "green Mini," the consumer may be recommended a "tempting convertible Beetle." As long as policy (1) is well recovered, we can use the behavior of consumers in comparable states (i.e., those

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[6]Note also that this assumption is used extensively in the empirical literature on consumer search in Marketing and Economics and in the Recommendation System literature in Computer Science.

[7]This separation allows us to easily consider the case when the recommendation system reacts only to the vehicle currently being viewed (a_t) rather than the full search history $H_t = a_t \cup A_t$.

who were recommended the convertible Beetle) to predict the counterfactual behavior of the focal consumer. This strategy builds on i) policy (1) being recovered flexibly and ii) the counterfactual analyses staying within the "support" of the original recommendation system. We address the first criterion by estimating policy (1) via multiple machine learning methods across several numbers of vehicle clusters, and selecting the winning pair via a number of out-of-sample criteria. As for the support condition, we take advantage of the fact that, as vehicles sell, the seller's recommendation system picks new ones to recommend to customers. This natural rotation scheme allows us to observe different vehicles being recommended to consumers in the same state.[8] We illustrate identification through the example depicted in Figure 3.

Figure 3: Identification of Causal Effects of Recommendations

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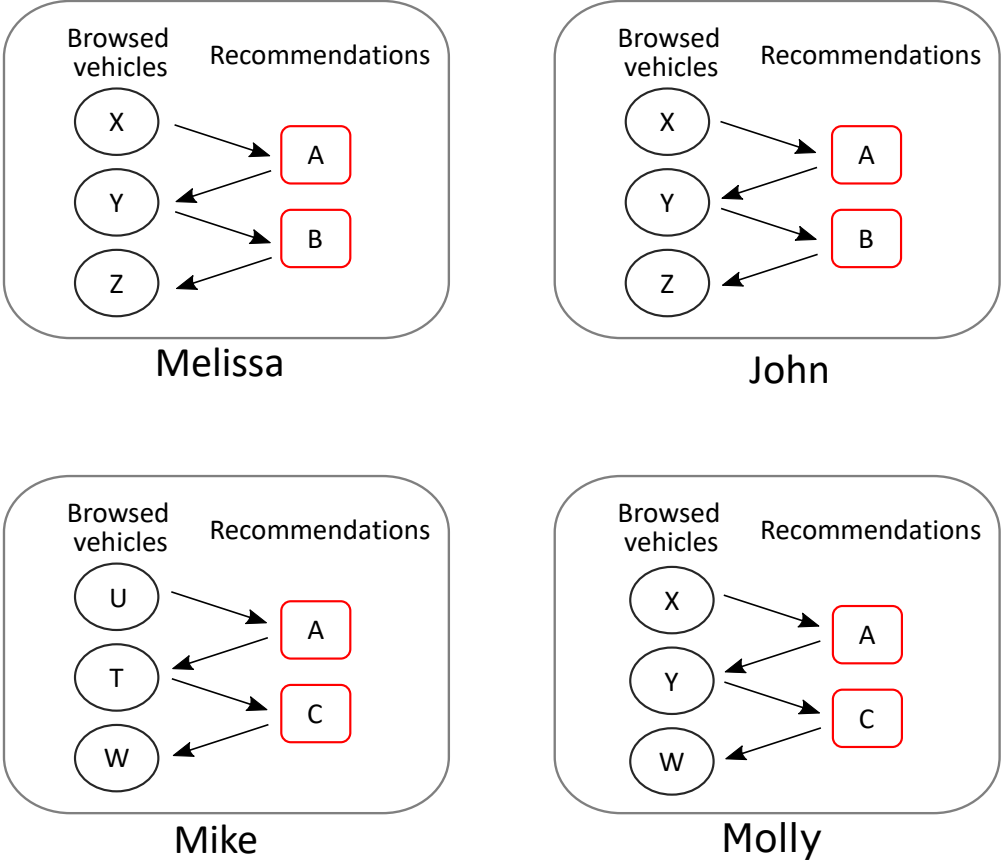
[8]See also Tables 2 and 6, which show strictly positive recommendation probabilities across all clusters, upon visiting each given vehicle class.

In the example, four consumers browsed three vehicles and were exposed to two product recommendations after each click. The leftmost consumers (Mike and Melissa) are shown different recommendations after inspecting their second vehicle. One could compare their subsequent behaviors (visiting vehicle Z vs. W) to infer the effect of recommending vehicle B rather than C. However, these consumers also feature different browsing histories, a likely reflection of their preferences. Interpreting differences in browsing behaviors by Mike and Melissa solely as the consequence of the recommendations they were exposed to would not produce valid causal effects of recommendations.

Melissa and John (topmost row of Figure 3) feature similar browsing histories as a result of their similar preferences for vehicles. However, in the example, both were exposed to the same vehicles by the recommendation system. In this case the estimator faces a crucial lack of variation. The examples discussed above illustrate that, for proper identification of the effects of recommendations (i.e., selection avoidance), we need consumers with *similar* preferences to be exposed to *different* recommendations.

We take advantage of the fact that vehicle recommendations change as vehicles are sold in the platform. This explains the different recommendation exposures by Mike and Molly in the example above, and produces exogenous variation in the recommendations. We also benefit from the fact that the status quo recommendation system only uses the current vehicle being viewed as the basis for producing recommendations, which allows us to control for it in the model directly, and ensures that other sources of endogeneity are absent.

Estimation. The estimation procedure proceeds as follows:

1. Vehicles are grouped according to cluster analyses performed on car characteristics. The analyses are performed once for each number of vehicle clusters, from 3 to 10 clusters.

2. Consumer and vehicle data are organized according to the clusters from step 1. For example, in the 3-cluster scenario, vehicle visits are organized according to vehicle-cluster membership: A visit to vehicles 1 and 2, both belonging to cluster 3, are re-coded as two

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visits to vehicles of cluster 3.

3. Flexible supervised learning methods are used to estimate the consumer policy (1), once for each number of clusters (3 through 10; a total of $3 \times 8{=}24$ estimations). We implement decision trees (random forests and boosting), and multinomial logit methods.[9]

4. The estimation method is selected together with the number of clusters, based on the cluster analysis silhouette values and out-of-sample model fit metrics.

4 Recommendation System

4.1 State Space: Vehicle Segments

As referred in the previous section, the recommendation system suggestions rely on a dynamic program. We employ standard dynamic-programming techniques to keep the size of the state space manageable. First, we aggregate the 4,000 vehicles in our dataset into to clusters. The goal is to produce near-homogeneous groups of vehicles to make the problem estimable and more interpretable. As explained before, we do not set the number of vehicle clusters a priori. Instead, we conduct cluster analyses for multiple numbers of clusters, and select the preferred number only after having estimated consumer behavior.

To overcome the local minimum problem of cluster analysis, we start with a simple hierarchical solution (Ward's method) and use the resulting allocation as the starting values for k-means clustering. This strategy is likely to produce better allocations than using random initial seeds. Vehicles are clustered by normalized values of their characteristics, namely, body style, transmission type, drivetrain type, number of accidents, number of owners, price, mileage, and markup. Categorical variables are weighted by the number of different categories.[10]

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[9]We also estimated a support vector machine model, but discarded it due to the lack of fit and overall heavy computational burden.

[10]We employed two normalization alternatives for the continuous variables: A min-max normalization and a quantile normalization. The advantage of the quantile normalization is that it is invariant to order-preserving transformations. Nonetheless, our results suggest that the min-max normalization of the log-

We conduct cluster analyses while varying the number of clusters from 3 to 10. Figure 4 reports the silhouette value for each of these analyses. The silhouette value is often used in machine learning applications to select the optimal number of clusters. In our case, the silhouette value is maximized at 8 clusters.

Figure 4: Silhouette Values by Number of Clusters
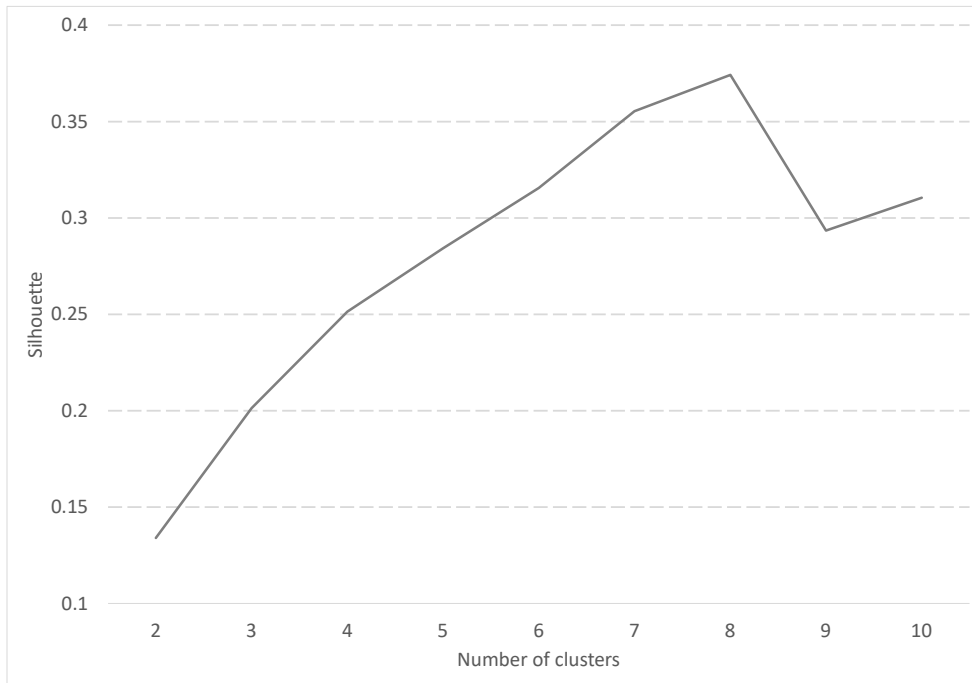
We present some characteristics of the vehicle clusters in Table 4, for the case of 8 clusters. Cluster 1 may be labeled as rear-wheel drive, since all its vehicles have rear-wheel drivetrains. Similarly, all vehicles in cluster 2 are front-wheel drive vehicles. Cluster 3 is solely composed of vehicles with manual transmission and cluster 4 captures vehicles for which we have not been able to determine the drivetrain with certainty from the website (and neither would consumers, since we observe the same information they have had access to). Clusters 5, 6, and 7 feature vehicles with CVT, AWD and 4WD transmissions, respectively. Finally, all vehicles in cluster 8 feature at least one accident. In addition to the features we highlight

transformed variables, which we have opted for, performs the best. Results are available from the authors.

here, the remaining vehicle characteristics also vary across clusters, as can be verified in
Table 4.

Table 4: Cluster Centroids - Relative Frequencies

Cluster/Segment	1	2	3	4	5	6	7	8	Total
Bodystyle									
Convertible	15%	3%	10%	8%	1%	2%	1%	6%	6%
Coupe	18%	4%	28%	12%	4%	7%	0%	10%	10%
Hatchback	1%	20%	26%	14%	34%	3%	1%	15%	15%
SUV	6%	17%	7%	13%	16%	59%	78%	23%	21%
Sedan	52%	51%	25%	51%	41%	24%	0%	40%	42%
Truck	7%	0%	2%	0%	0%	0%	19%	1%	2%
Van	0%	4%	0%	1%	0%	1%	0%	2%	1%
Wagon	0%	2%	2%	2%	4%	4%	1%	2%	2%
Transmission									
Automatic	100%	100%	0%	100%	0%	100%	100%	79%	78%
CVT	0%	0%	0%	0%	100%	0%	0%	11%	11%
Manual	0%	0%	100%	0%	0%	0%	0%	11%	12%
Drivetrain									
Rear WD	100%	0%	31%	0%	0%	0%	0%	18%	22%
AWD	0%	0%	11%	0%	19%	100%	0%	17%	14%
4WD	0%	0%	4%	0%	1%	0%	100%	8%	5%
Front WD	0%	100%	35%	0%	64%	0%	0%	41%	44%
NA	0%	0%	18%	100%	16%	0%	0%	17%	14%
Number of accidents									
No accidents	100%	100%	100%	100%	100%	100%	100%	0%	93%
At least one accident	0%	0%	0%	0%	0%	0%	0%	100%	7%
Number of owners									
New car	18%	17%	19%	18%	14%	16%	21%	0%	16%
1 owner	28%	48%	42%	37%	60%	38%	47%	45%	43%
2 owners	38%	28%	27%	34%	22%	35%	26%	42%	31%
3 or more owners	15%	8%	12%	11%	4%	12%	6%	13%	10%
Normalized									
Log Transformed Price	0.48	0.38	0.44	0.40	0.41	0.51	0.54	0.40	0.43
Log Transformed Mileage	0.86	0.85	0.83	0.85	0.83	0.86	0.85	0.88	0.85
Log Transformed Margin	0.42	0.34	0.39	0.38	0.36	0.45	0.47	0.36	0.38
Number of vehicles	725	1270	445	380	427	445	161	287	4,140

Note: Cluster centroids for the eight cluster solution. Centroids are the proportion (%) of vehicles with a given
characteristic, except for the continuous variables (price, mileage and margin) where the (logarithm) of the
variables are min-max normalized.

4.2 Dynamic Approach

The recommendation system suggests vehicles to consumers after each click. The dynamic problem faced by the recommendation system can be written through the following Bellman equation (subscript i omitted):

$$\underbrace{V(t, a_t, A_t, R_t)}_{\Theta_t} = \max_{r_{t+1}} \pi(t, a_t, A_t, R_t) + E\left[V(t+1, a_{t+1}, A_{t+1}, R_{t+1}) | a_t, A_t, R_t, r_{t+1}\right] \quad (2)$$

The timing is a bit subtle and deserves discussion. Above, a_t is the vehicle the consumer has just clicked in order to inspect it. After this click, the recommendation system calculates the vehicles to be recommended and the time period advances from t to $t+1$. Variables A_t and R_t contain the vehicles inspected and recommended previously, respectively. Finally, r_{t+1} is the set of vehicles to be featured by the recommendation system at time $t+1$ - based on current period information. Much like online advertisement impressions, recommendation r_{t+1} is calculated immediately after a consumer clicks on the link to a vehicle's webpage (a_t), but before the new vehicle's page is rendered on the user's screen.[11] The recommendation policy calculates three vehicle recommendations, in line with the seller's website during the data collection period. We denote the recommendation function as:

$$r_{t+1} = r^*(t, a_t, A_t, R_t) \quad (3)$$

The result of this calculation is incorporated into the state variable $R_{t+1} = f(R_t, t, r_{t+1})$, where $f(\cdot)$ is a transition function.

State Variable Transitions. The transition of variable t, the order of the consumer's action, is straightforward. The consumer's action a_t originates from her policy, as characterized by expression (1). As for variables A_t and R_t, these keep track of the vehicles viewed

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[11]Recommendations can also be precalculated, and then selected depending on the consumer's choice of vehicle to inspect.

by and recommended to the user in the past. Specifically, the variables keep track of the relative frequencies of vehicle clusters viewed and suggested in the past. For example, if in the past a consumer has visited vehicles from cluster 1 three times and has visited clusters 2 and 3 once each, the corresponding state variable is equal to $A_t = \{0.6, 0.2, 0.2\}$. This approach is based on the work by Santos (2020). The advantage of keeping track of relative frequencies is that it reduces the size of the state space by generating the same number of support points as of a multinomial distribution. Following standard dynamic techniques (state interpolation), we select grid values for the support of A_t and R_t and solve the value function through backward iteration.[12] We also take into account that the same values of A_t and R_t may mean different things to consumer behavior, depending on the stage of the customer's journey. For this reason, we introduce the time period t as a state variable, making the problem non-stationary. The introduction of state variable t allows us to characterize behavior differently at times, say, 3 and 5, even though $A_3 = A_5$. The added flexibility also applies, by construction, to all remaining state variables.

Example. Figure 5 illustrates the relationship between the consumer's search journey and the state variable transitions. The consumer starts out by inspecting a vehicle from group 3 and, based on this choice, the recommendation system serves up recommendations $\{3, 2, 2\}$, i.e., one vehicle from cluster 3 and two vehicles from cluster 2 are recommended.[13] The web server then serves up the vehicle profile page along with the vehicle recommendations.

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[12]We simulate back from 22 consumer searches, which in the dataset covers approximately 95% of the data.

[13]Variables A_1 and R_1 are initialized at zero; these are placeholder values for $t = 1$, since there is no history yet, given the cold start nature of the problem.

Figure 5: Illustration of State Variable Transitions
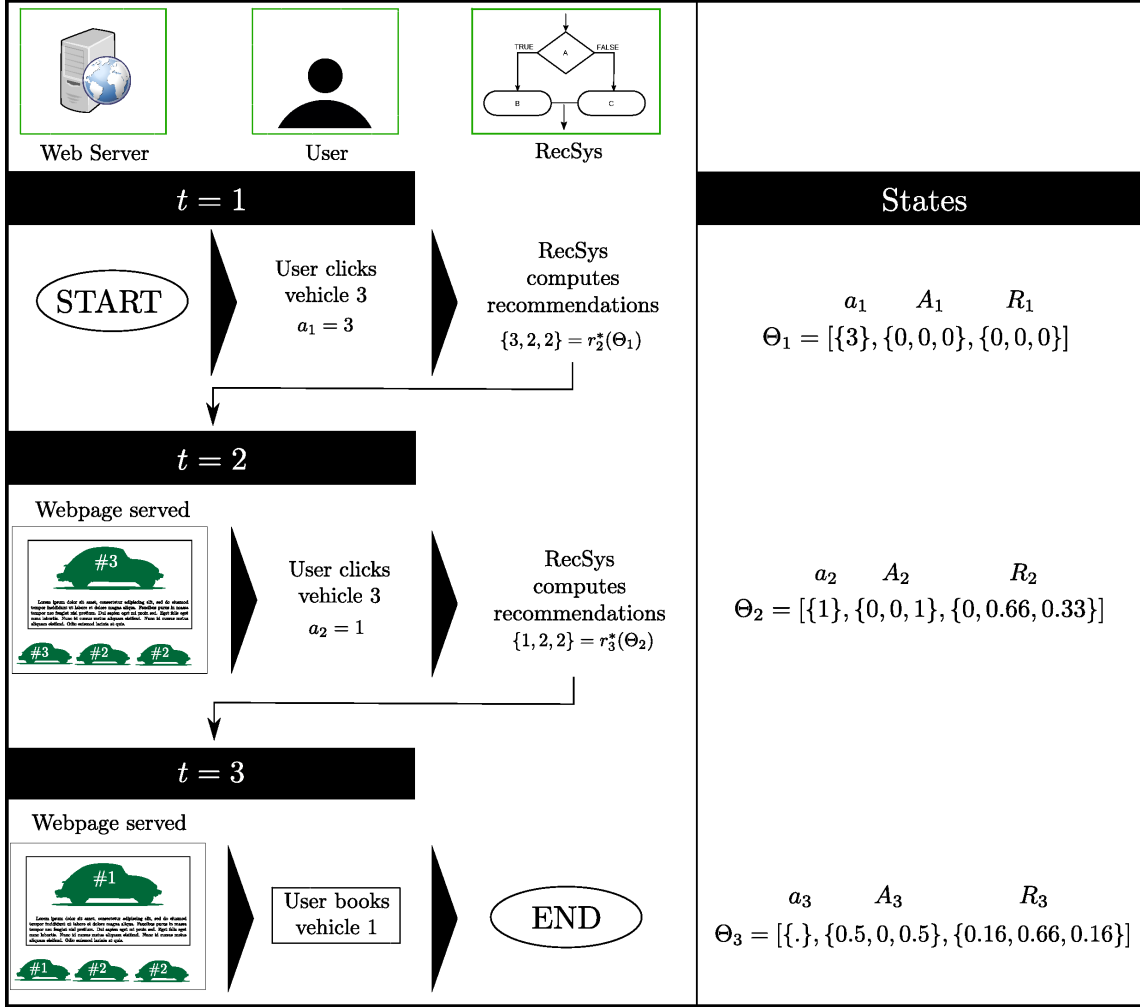
Note: In the example above there exist three groups of vehicles that can be browsed by the consumer and recommended by the recommendation system at each point in time. State variable t omitted from the state variables for compactness.

In period $t = 2$, the customer decides to visit a vehicle from group 1. At this moment the recommendation system is called upon again to decide which vehicles to feature. The state corresponding variables are given by

$$\Theta_2 = \{t, a_2, A_2, R_2\} = \{2, 1, \{0, 0, 1\}, \{0, 0.66, 0.33\}\} \tag{4}$$

Variables $t = 2$ and a_2 are straightforward. Variable A_2 comprises the relative frequencies

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of vehicle groups shown to the consumer in the past, in this case, 100% of past visits have been of vehicles in group 3. Similarly, R_2 stores the relative frequencies of previously recommended vehicle groups: two vehicles of group 2 and one vehicle of group 3 have been recommended, giving rise to state $R_2 = \{0, 0.66, 0.33\}$. In the example, the recommendation system calculates recommendations through function

$$r_3 = r^* (\Theta_2) = \{1, 2, 2\} \tag{5}$$

Once the web server serves the requested profile page and recommendations, the consumer converts to a vehicle of group 1, thus terminating the recommendation system's decision problem. [14]

5 Empirical Results

In order to estimate equation (1) in the most flexible way, we implement three machine learning techniques: multinomial logit, random forests, and boosting. Estimation is performed across methods and numbers of clusters. Finally, we compare out-of-sample fit metrics to assess which method/number-of-clusters combination to use for the counterfactual analysis.

5.1 Out-of-Sample Fit

We start by comparing fit across the different specifications and for different numbers of segments. We keep a random sample with 40% of individuals as a holdout, for which we construct our fit metrics.

The related literature contains a vast set of metrics to compare fit for a fixed number of cluster segments (i.e. holding fixed the number of categories that the dependent variable can take). The most common fit metric in the literature is accuracy, i.e., the number of correctly

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[14]The intertwining dynamic problems faced by the consumers and the recommendation system are presented in Appendix A.

predicted cases divided by the total number of cases. However, by using predicted classes instead of probabilities, accuracy serves our purpose poorly. For example, in a model with two classes, a trained prediction with 51%/49% probabilities produces the same accuracy metric than a prediction with 99%/1% probabilities. This is a well-known issue of the accuracy criterion. In order to incorporate predicted *probabilites* rather than predicted *classes* into our analysis, we include the log-loss criterion (equivalent to the log-likelihood value: $\sum_j y_j . \ln \hat{p}_j$, where j is a class) and the difference between predicted and observed probabilities. A popular choice for the latter is the use of the Helling distance metric given by $\sum_j \left(y_j^{\frac{1}{2}} - \hat{p}_j^{\frac{1}{2}} \right)^2$, which we use.[15]

Both of these metrics account for fit using predicted probabilities rather than predicted classes. An important note, which we will revisit, is that none of these metrics are expected to remain constant as the number of classes/clusters changes. Therefore, the emphasis of this first analysis is to determine which method appears to fit the out-of-sample data best, across different numbers of clusters.

Table 5 exhibits fit metrics across models and numbers of classes, from 3 to 10. Model performance is extremely similar across methods and numbers of clusters, and no dominant method emerges. For example, the multinomial logit model is more accurate than the random forest method when 3 clusters are considered, but comes out behind for 8 vehicle clusters. Also, when the number of clusters is fixed, no model dominates another across criteria. The last column of Table 5 presents a simple average of the different criteria across methods. There we find that the random forest method is always at least as good as the remaining ones across criteria, when equal weights are used to average across clustering scenarios.

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[15]Absolute differences in probabilities generate the same qualitative implications for our analysis.

Table 5: Fit metrics

	3	4	5	6	7	8	9	10	Simple Average
				Number of segments					
	Tree Ensemble (Random Forest)								
Accuracy	47.1%	34.2%	33.3%	28.6%	29.4%	29.7%	22.1%	22.5%	30.86%
Log-Loss	0.361	0.445	0.483	0.522	0.550	0.571	0.615	0.628	0.522
Prob. difference	0.122	0.113	0.097	0.086	0.077	0.069	0.065	0.059	0.086
	Tree Ensemble (Boosting)								
Accuracy	46.8%	33.8%	33%	30.2%	30.0%	29.1%	22.2%	22.1%	30.9%
Log-Loss	0.362	0.446	0.484	0.523	0.55	0.57	0.615	0.627	0.522
Prob. difference	0.122	0.113	0.097	0.086	0.077	0.069	0.065	0.059	0.086
	Multinomial Logit								
Accuracy	47.6%	34.5%	33.1%	30.4%	30.2%	29.5%	21.5%	21.4%	31%
Log-Loss	0.365	0.453	0.492	0.53	0.559	0.579	0.623	0.635	0.53
Prob. difference	0.123	0.115	0.098	0.087	0.078	0.07	0.066	0.06	0.087

Note: All metrics are calculated in the houldout sample (40% of the sample not used for estimation). Accuracy is defined as the number of correctly predicted cases divided by the number of cases. Log-loss is equivalent to the log likelihood value. Probability difference is the difference between the averaged probability predicted by the model and the outcomes.

Due to their very nature, the metrics in Table 5 are expected to be monotonic with the number of clusters. In order to compare fit across numbers of clusters, we examine two additional metrics: Lift and the Nagelkerke's pseudo-R^2. Figure 6 presents both metrics based on the random forest model, across numbers of clusters. It also includes the silhouette metric from the cluster analysis step, also reported in Figure 4.

We find that the lift criterion is maximized at 8 clusters and the pseudo-R^2 remains relatively stable between 7 and 10 clusters, being highest at 9 clusters. The silhouette statistic from the cluster analysis is maximized at 8 clusters. The last result is striking, since the silhouette criterion is estimated independently from the remaining metrics.

Figure 6: Fit Metrics for the Random Forest Model: Lift, Pseudo-R^2, and Silhouette
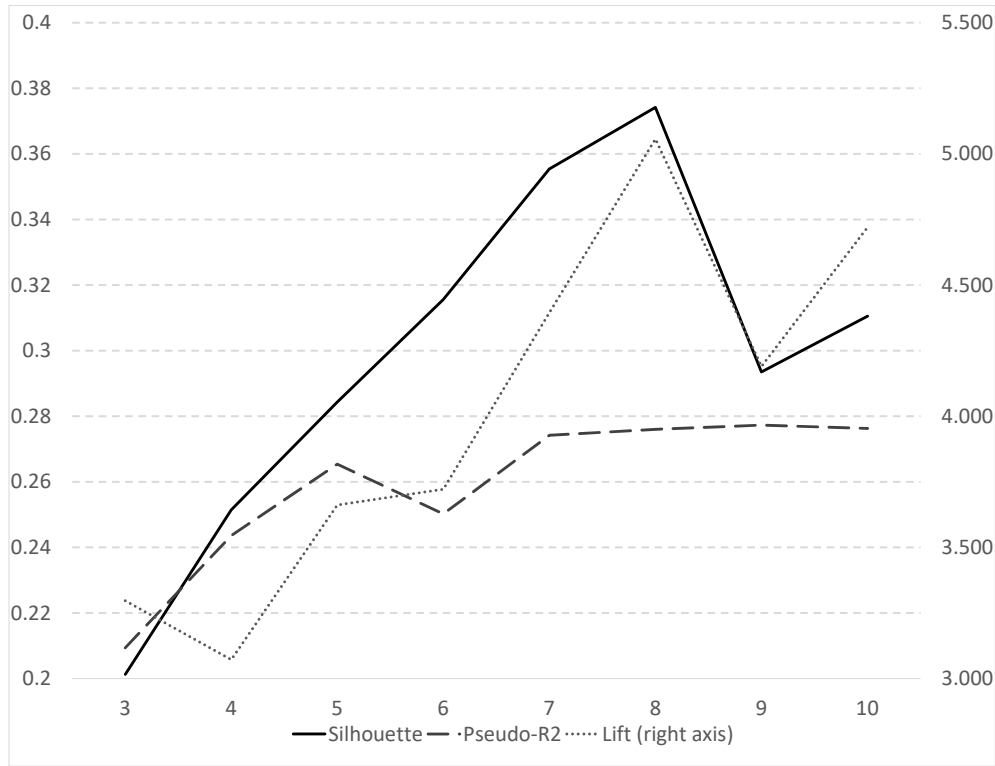
Note: Lift is defined as the number of correctly predicted cases divided by the number of cases that would be predicted by a random (uniform) model.

Given the sharp decline of the lift and silhouette criteria at 9 clusters, and the relative stability of the pseudo-R^2 metric around the same region, we opt for the random forest model with 8 clusters from here on. Given how close model performance is for any given number of clusters, we expect that conducting the subsequent analyses with other methods would yield very similar results.

5.2 Model Predictions

The status quo recommendation policy can be summarized by a matrix that assigns probabilistic recommendations to each type of vehicle viewed. Table 6 displays the recovered status quo recommendation policy at the selected number of clusters (eight).

Table 6: Recommendation Policy - Status Quo

		Vehicle Recommendation							
	Cluster	1	2	3	4	5	6	7	8
	1	**0.493**	**0.139**	**0.104**	0.064	0.037	0.097	0.018	0.046
	2	**0.141**	**0.398**	0.096	**0.1**	0.086	0.095	0.017	0.067
	3	**0.209**	**0.189**	**0.326**	0.065	0.048	0.081	0.026	0.058
Vehicle	4	**0.194**	**0.273**	**0.117**	**0.127**	0.066	**0.11**	0.033	0.079
Viewed	5	**0.133**	**0.317**	0.089	0.078	**0.201**	**0.105**	0.033	0.046
	6	**0.236**	**0.184**	0.071	**0.105**	0.04	**0.259**	0.063	0.042
	7	**0.172**	**0.194**	0.099	0.081	0.041	**0.212**	**0.165**	0.036
	8	**0.216**	**0.256**	**0.125**	**0.106**	0.079	**0.136**	0.03	0.053

Note: Above, probability of recommending a vehicle in a column, conditional on a consumer browsing a vehicle in a row. Probabilities > 10% are written in bold font.

The results are intuitive. First, the current algorithm privileges recommending vehicles of the type being browsed, as visible in the bold diagonal of vehicle clusters 1 through 7. In addition, vehicle clusters 1 and 2 are also recommended more frequently, regardless of the vehicle currently being viewed, which is unsurprising given that they represent almost half of the vehicles in the sample. Vehicle cluster 6 is also recommended across multiple cases.

Using the estimated policy function (equation 1), we now compare its search, conversion, and recommendation predictions with the analogue moments in the data. The results are presented in Table 7.

The model replicates the moments in the data extremely well, especially in terms of recommendations and search decisions. The test-drive decision (conversion) is not approximated as well for a number of segments.

Table 7: Probabilities (Search, Recommendation, and Conversion) - Data vs. Model Predictions

	Probabilities					
	Conversions		Recommendations		Search	
	Data	Simulated	Data	Simulated	Data	Simulated
Segment 1	0.53	0.30	25.93	24.71	20.59	16.00
Segment 2	0.92	0.64	24.65	24.94	20.63	19.23
Segment 3	0.21	0.30	12.87	12.78	10.91	10.24
Segment 4	0.31	0.31	8.79	8.86	7.01	6.75
Segment 5	0.30	0.34	6.67	6.92	5.74	6.78
Segment 6	0.33	0.43	12.30	12.69	10.38	11.60
Segment 7	0.06	0.06	3.28	3.53	2.76	3.87
Segment 8	0.20	0.18	5.51	5.57	5.35	5.43
Outside option	97.14	97.44	-	-	-	-

Note: Probabilities for conversion, searches and recommendations observed in the overall data (working and holdout samples) and predicted by the model (simulation). Data truncated at 22 search actions, covering approximately 95% of users.

This may be due to the relatively low conversion rate observed in the data. We prefer not to 'tweak' the model: Matching this moment better could come at the cost of overfitting the data. As is common in the Marketing literature, all subsequent counterfactual scenarios are calculated within the estimated model, so that fit issues do not affect the interpretation of the predictions.

6 Counterfactual Analyses

We now characterize the first-best recommendation system in depth, and compare it with the status quo case. We then analyze the value-drivers of the recommendation system in Section 6.2.

6.1 First-Best Recommendation System

The first-best recommendation system is obtained by solving the dynamic problem of offering up recommendations. It incorporates the consumers' decision probabilities and maximizes

expected profit in a forward-looking manner. Formally, for some consumer i (subscript omitted), the Bellman equation is given by:

$$V\left(t, a_t, A_t, R_t\right) = \max_{\varphi(\cdot)} \pi\left(t, a_t, A_t, R_t\right) + E\left[V\left(t+1, a_{t+1}, A_{t+1}, R_{t+1}\right) \middle| a_t, A_t, R_t, \varphi\left(\cdot\right)\right] \quad (6)$$

where the state variables $\{t, a_t, A_t, R_t\}$ characterize the probabilistic consumer search and conversion decisions, as discussed in Section 3. The payoff function $\pi\left(\cdot\right)$ represents the expected instantaneous profit for the seller: Here, the margins of the different vehicle clusters are weighted by the respective instantaneous conditional purchase probabilities. The expectation operator $E\left(\cdot\right)$ is a consequence of the uncertainty about the consumer's subsequent action, conditional on the current state variables. The uncertainty is characterized by the probability mass function

$$\Pr(a_{t+1}|t, a_t, A_t, R_t, \varphi\left(\cdot\right)) = \Pr(a_{t+1}|t, a_t, A_t, R_t \cup r_{t+1}), \quad (7)$$

as induced by the estimated consumer's policy function.

The recommendation function is given by $\varphi\left(t, a_t, A_t, R_t\right)$: It determines the set of three recommendations to be shown to user i immediately after she visits a vehicle profile. In between the user's "click" and the impression of the new page, the recommendation system analyzes the consumer's current state, given by $\{t, a_t, A_t, R_t\}$, and computes recommendation $\varphi\left(\cdot\right)$. Finally, once the vehicle profile page is displayed to the consumer, state variable R_t transitions to $R_{t+1} = \{R_t \cup \varphi\left(t+1, a_{t+1}, A_{t+1}, R_t\right)\}$, where the "union" operator denotes the relative frequency update of variable R_t via the recommendations in φ. Variable A_{t+1} is updated via a_t through an analogue procedure.

We calculate the first-best recommendation policy via backward induction. In order to interpret the ex-ante first-best policy, we first calculate the implied average transition matrix. In line with the status quo recommendation matrix presented in Table 6, we consider only the last action taken by each consumer, and note the probability of each vehicle recommendation

offered by the first-best policy. We present the results in Table 8.

Table 8: Recommendation Policy - First-Best

		Vehicle Recommendation							
	Cluster	1	2	3	4	5	6	7	8
	1	0.021	**0.506**	0.001	0.003	**0.143**	**0.122**	**0.159**	0.044
	2	0.001	**0.232**	0.035	0.003	**0.465**	**0.222**	0.038	0.004
	3	0.001	**0.788**	0.038	0.003	0.010	0.078	0.072	0.009
Vehicle	4	0.001	**0.592**	0.009	**0.1**	0.054	**0.126**	**0.117**	0.000
Viewed	5	0.046	**0.326**	**0.152**	**0.381**	0.021	0.062	0.008	0.004
	6	0.000	**0.716**	**0.123**	0.001	0.037	0.025	0.040	0.058
	7	0.025	**0.779**	0.021	0.030	0.046	0.040	0.019	0.039
	8	0.059	**0.627**	0.064	0.098	0.003	0.013	**0.110**	0.026

Note: Above, probability of recommending a vehicle in a column, conditional on a consumer browsing a vehicle in a row. Remaining states are averaged out. Probabilities > 10% are written in bold font.

Figure 7: Distribution of Vehicle Cluster Recommendations Over Time
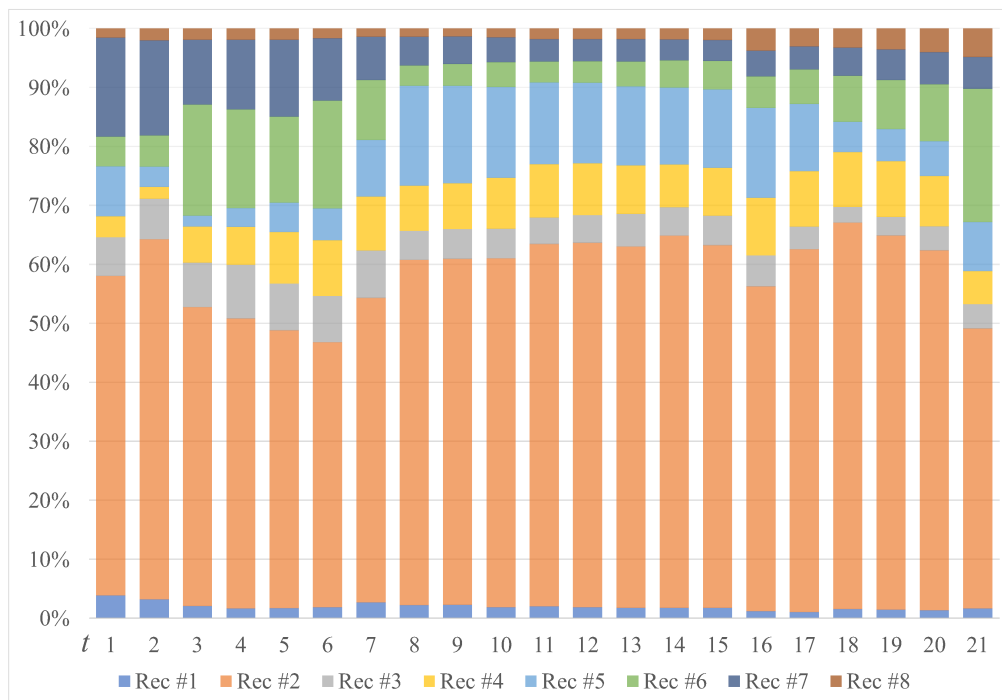
Note: Above, we depict 21 recommendations shown to consumers after each click. In the model we consider a total of 22 "clicks," the first occurring before recommendations are made. In our data, 95% of consumers search vehicles at most 22 times.

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In comparison with the status quo policy (Table 6), we find that the first-best policy 1) relies less on the last vehicle viewed by the consumer and 2) is more likely to recommend vehicles from cluster 2, independently of the previous action.

Table 8 does not tell the full story, since in reality it depends on a number of variables in addition to the type of the last vehicle viewed. Figure 7 shows the relative frequency of recommended vehicle clusters over time.

The results in the figure above are averaged across states uniformly, so that they can be interpreted independently of consumers' behaviors. Figure 7 reveals that the first-best recommendation system is far from stationary. For example, near the beginning of the consumer journey, vehicles from clusters 7 are relatively more likely to be recommended. By search number 8 however, cluster 7 is seldom recommended, giving space to vehicles from clusters 2 and 5. Near the end of the consumer journey, these clusters are substituted, on average, by recommendations of vehicles from cluster 6.

Figure 8 depicts the relative frequencies of three recommendation cases. The top line (blue) depicts the number of times the recommendation system opts to serve all recommendations from a single cluster (e.g., three vehicles from cluster 2). The middle line (orange) depicts the number of times vehicles from two clusters are recommended (e.g., one vehicle from cluster 1 and two vehicles from cluster 2). The bottom line (gray) depicts the number of times all three vehicles originated from different clusters (e.g., one vehicle from clusters 1, 2, and 7 each).

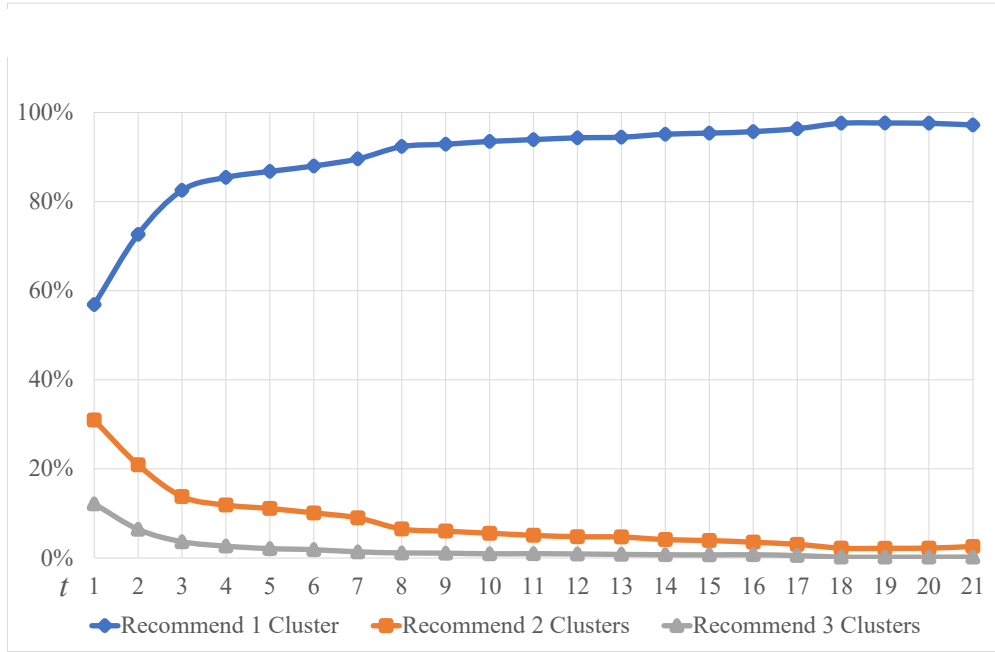Figure 8: Distribution of Different Cluster Recommendations Over Time

Figure 8 shows that recommendations become more concentrated as consumers progress in their search journeys. From around a 60% likelihood in the beginning, by period 20 the chances that all recommendations belong to the same cluster have practically increased to 100%. This striking pattern is consistent with the recommendation system learning over time: As consumers reveal more information through their search actions, the recommendation system hones down on their "types" and recommends vehicles from the most appropriate cluster, transitioning from an exploration to an exploitation regime.

6.2   Recommendation System Value Decomposition

In this section, we aim to understand how different recommendation system features and data drive value creation for the company. Our approach is based on "comparative statics": We turn different features on and off to assess their effects on the performance of the rec-

ommendation system, as measured by expected profit. This approach is in contrast with the majority of the literature on recommendation systems, which tends to focus on model performance rather than ask why certain performance gains arise.

Recommendation Matrices. We start out by comparing four scenarios. Scenario 1 corresponds to the performance of the status quo recommendation policy depicted in Table 6; Scenario 2 reoptimizes the stationary status quo probabilities (i.e., it optimizes the status quo recommendation system); Scenario 3 considers a recommendation system based on time-dependent recommendation matrices. In this case, the consumer is exposed to different recommendation policies, drawn from different recommendation matrices, as she proceeds through the website.

Figure 9: Performance Benchmarks and Rec. Matrix Optimization
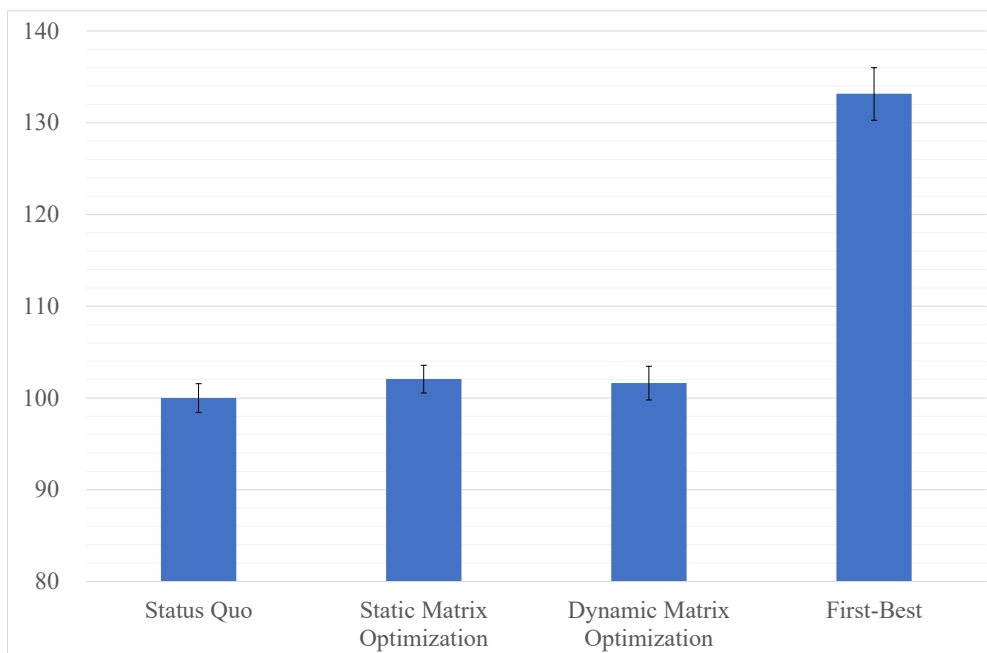
Note: Values of expected profit are presented in relation to scenario "Status Quo", which has been normalized to 100. Bands represent one standard deviation of estimated profit.

Scenario 4 corresponds to the first-best scenario. The results are presented in Figure 9.[16]

We start out by normalizing the expected profit of the Status Quo scenario (current recommendation system) to 100. As such, all other scenarios can be understood as relative changes to the status quo case (i.e., an expected profit of 110 means the expected profit is 10% higher than the status quo case). We do this for ease of comparison and for confidentiality purposes.

Analysis of Figure 9 reveals that the seller's recommendation system does extremely well when compared to the second scenario, in which we reoptimize the recommendation matrix (profit increases to 102.1, and difference is not statistically significant). The third scenario, in which we allow for different recommendation matrices for each time period (i.e., the order of vehicles viewed) also does not produce a meaningfully different profit. In fact, we find that expected profit is equal to 101.6, which is above the status quo scenario, but below the static recommendation matrix case.[17] Finally, by conducting bootstrap sampling of the consumer policy, we find that the expected profits of the three regimes are not statistically significantly different (p-values marginally above 0.05).

The last column of Figure 9 depicts expected profit when full maximization is employed (i.e., recommendation policy is optimized for each possible consumer state). In this case, we find that the highest expected profit the company should expect is of 133.2. Thus, we estimate that there is significant margin to optimize the recommendation system, although none of the matrix methods is able to capture a significant fraction of that potential.

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[16]All results from the counterfactual analyses are shown simultaneously in Appendix C, Table 10.

[17]Optimizing recommendation matrices can be a more demanding task than finding the "unconstrained" first-best recommendation policy. We describe the approximation methods employed in Appendix B.

Value of History. Figure 10 presents how the use of different data affect the performance of the recommendation system. In all scenarios below, we calculate the first-best recommendation system, while introducing data constraints. In the first case, we optimize the recommendation system only based on the consumer's previous browsing actions, ignoring the current vehicle being viewed. In the second scenario we introduce previous vehicle recommendations to the optimization program. Finally, the difference between the second scenario and the first-best case is the introduction of the vehicle presently being viewed by the consumer into the recommendation system optimization.

Figure 10: Performance of Different Data Usage Scenarios
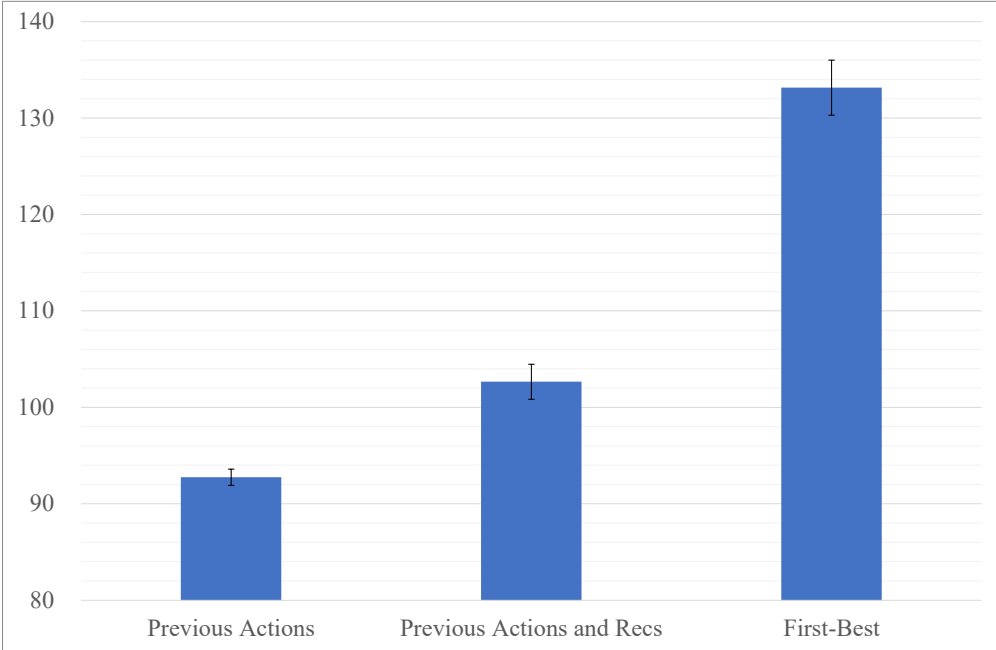
Note: Values of expected profit are presented in relation to scenario "Status Quo", which has been normalized to 100. Bands represent one standard deviation of estimated profit.

First, we find that optimizing the recommendation system based solely on consumers' previous actions does significantly worse than the status quo (expected profit=93). This result suggests that consumers' past actions are not necessarily very indicative of preferences,

since history does not lend itself to producing a large portion of future recommendation value. This result is in contrast with the view that past behavioral data is fundamental to predict future behavior. In our case, actions taken before the present one produce little value. It is possible that this insight applies mostly to search contexts, in which alternatives that have been passed on are likely to reveal more about what consumers *are not* interested in, rather than what consumers *are* interested in.

Including recommendations shown to the consumer in the past increases the recommendation system performance significantly above the status quo, to 102.7 (p<0.01). More noticeable is the difference between the second and third columns, which is explained by the introduction of the current vehicle being viewed in to the recommendation system optimization. In this case (the first-best), expected profit increases from 102.7 to 133. Ensuring that the current option being browsed feeds into the recommendation system seems to be the most important behavioral data piece, as it generates the highest increase in expected profit. Together with the previous results, this analysis proposes a strong complementary effect between the vehicle currently being viewed and the remaining data available to the recommendation system.

Recommendation Goals. Next, we describe how different features of the recommendation system drive expected profit (Figure 11). In this analysis, the recommendation system is optimized based on the full dataset. However, the objective function is altered in each scenario.

The first bar corresponds to the case where the recommendation system assigns a low probability to the event of consumers abandoning the website (outside option). In this case, the probability of selecting the outside option is reduced to zero during model optimization, while the probabilities of the remaining actions are increased in proportion. In other words, in this case the recommendation system acts as if the consumer will buy for sure. The analysis reveals that ignoring the possibility of consumer churn limits expected profit significantly,

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to 120, as compared with the first-best scenario of 133.

Figure 11: Performance of Different Rec. System Features
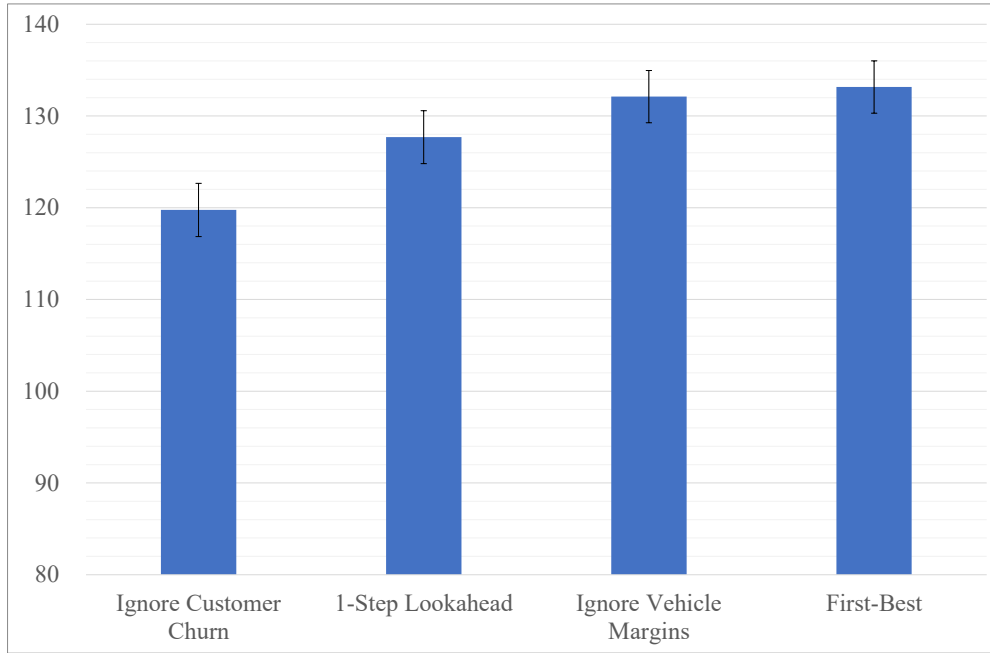
Note: Values of expected profit are presented in relation to scenario "Status Quo", which has been normalized to 100. Bands represent one standard deviation of estimated profit.

The second column turns off forward-looking behavior by the recommendation system. In this case, the recommendation system acts so as to optimize the next period's payoff, but ignores any subsequent ones. The underlying goal of this analysis is to understand the value of "strategic behavior" in recommendations. For example, when the recommendation system is perfectly forward looking, it can suggest vehicles that may not appear optimal immediately – and may not be even bought – but may lead consumers to explore other vehicles that will later increase the probability of conversion and margins realized. Our analysis estimates that ignoring forward-looking behavior decreases expected profits from 133 to 127.7. The difference is significant both in magnitude (5% of status quo profits) as well as statistically.

We conclude by conducting a counterfactual scenario in which the recommendation sys-

tem ignores the fact that different vehicles yield different profit margins for the firm. For example, in the first-best scenario the algorithm may recommend products with higher margins, on average, whereas in the "Ignore Business Goals" scenario it cares only about the probability of conversion. Perhaps surprisingly, expected profit changes very little (133 to 132.1) when product margins are ignored, and the difference is not statistically significant.

As a whole, these results indicate that one of the the most important tasks for a recommendation system is to keep the consumer engaged and away from churning. In contrast, forward-looking recommendations generate relatively modest benefits, and taking product margins into consideration affect profits only slightly.

7 Final Remarks

Recommendation systems are extremely popular among sellers. So far, the emphasis on improving such systems is through the acquisition of larger and better customer data, and the development of hybrid recommendation methods. This trend has left small businesses, sellers constrained by the very nature of their transactions, and others abiding by strict privacy regulations, out of the conversation. In this paper we develop a recommendation system that addresses the "cold start" problem, a blocking factor for such actors.

The model is inspired in a fundamental insight from the search literature: Consumers do not hold static views on products, but they update them as they acquire information. As a result, it is important to take the consumer's state into account when calculating recommendations. For example, our model takes into account that a given recommendation may affect subsequent consumer search, leading to a different set of future search and conversion actions. The model relies on standard Bellman equation formulation and dynamic programming techniques (state aggregation and value function interpolation). It takes advantage of the institutional features of our setting, which allow us to recover the causal effect of recommendations from revealed-preference data. Our analysis suggests that the model transitions

from exploration to exploitation, as it suggests more similar alternatives along the consumer search journey.

Finally, we investigate the resulting 'black box' by conducting a series of counterfactual analyses. We find that it is the joint effect of past behavioral and recommendation data with the current vehicle being browsed the allows the model to drive the most value to the seller. On their own, each of these data components account for small improvements in ex-ante profits. The final set of counterfactuals reveals that managing consumer churn is a primary concern for the recommendation model. In addition, the recommendation system benefits from thinking not only about the next consumer decision, but the whole path until conversion.

Currently, most recommendation systems rely on a hybridization of collaborative filtering and content-based filtering. Despite their popularity, these systems tend to rely on an implicit estimate of each product's worth to each consumer. It would be interesting to investigate the extent to which our approach can be combined with the traditional ones, so as to alleviate the cold start problem while incorporating the sequential nature of consumer search into product recommendations.

Appendix

A   The Consumer's Dynamic Search Problem

A.1   Dynamic Problem

Consumer i derives utility u_{ijt} from consuming alternative $j \in \{1, ..., J\}$. The utility from the outside good $j = 0$ is normalized to $u_0 = \epsilon_0$. Consumers do not know u_{ijt} a priori, but hold prior beliefs about the distribution of u_{ijt}, $F(u_{ijt}|\Theta'_{it})$. That prior distribution depends on the information state, Θ'_{it} at every period $t = 0, .., T$. We omit the individual-level subscripts from here on, for simplicity.

In every period, the consumer decides between (i) searching an alternative $j \in \{1, ..., J\}$ (extensive margin); (ii) terminating search and purchase an alternative ($j = 0, ..., J$), where $j = 0$ is the outside good.

Performing a search action has a cost of c.[18] The dynamic problem faced by the consumer can be described by the solution to the following Bellman equation

$$V\left(\Theta'_t\right) = \max \left\{ \underbrace{E(u_{ijt}|\Theta'_t)\Big|_{j=0,...,J}}_{Conversion\ Actions}, \underbrace{-c_i + E(V(\Theta_{i,t+1}|\Theta'_t)\Big|_{j=J+1,...2J+1}}_{Search\ Actions} \right\} \tag{8}$$

In each period, the consumer can make one of $2J+1$ decisions. As researchers, we observe the decisions related with search and conversion. The solution to the problem above, y_{it}, as a function of state space Θ'_{it}, is the relationship the researchers observe and estimate from the data.

A.2   A Tale of Two Dynamic Problems

We now clarify how the consumers' dynamic problem intertwines with the recommendation system's. Table 9 exemplifies the decisions and state transitions of both parties.

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[18]Notice that our approach also allows the search cost to be time dependent.

Table 9: Sequence of Consumer and Recommendation System Decisions

t	User State $\left(\Theta_t'\right)$	User Action		Recommendation System's State (Θ_t)	Rec. System Action
$t = 1$	$\Theta_1' : t = 1$ $a_0 = \emptyset$ $A_0 = \emptyset$ $R_1 = \emptyset$	$a_1\left(\Theta_1'\right) = 3$	$\rightarrow$	$\Theta_1 : t = 1$ $a_1 = 3$ $A_1 = \emptyset$ $R_1 = \emptyset$	$r_2\left(\Theta_1\right) = 1$
$t = 2$	$\Theta_2' : t = 2$ $a_1 = 3$ $A_1 = \emptyset$ $R_2 = \{1, 0, 0\}$	$a_2\left(\Theta_2'\right) = 1$	$\rightarrow$	$\Theta_2 : t = 2$ $a_2 = 1$ $A_2 = \{0, 0, 1\}$ $R_2 = \{1, 0, 0\}$	$r_3\left(\Theta_2\right) = 3$
$t = 3$	$\Theta_3' : t = 3$ $a_2 = 1$ $A_2 = \{0, 0, 1\}$ $R_3 = \{1/2, 0, 1/2\}$	$a_3\left(\Theta_3'\right) = 2$	$\rightarrow$	...	

At $t = 1$, the user arrives to the seller's homepage and selects a vehicle, $a_1\left(\Theta_1'\right) = 3$. At this point, her state is mostly empty, since no history exists. The recommendation system then proposes a vehicle from cluster 1, $r_2\left(\Theta_1\right) = 1$ (assume a single recommendation for clarity).[19] Notice that the states used by each party to inform decision-making are different. Specifically, the recommendation is calculated upon the consumer's last decision, and the consumer's decision is based on the recommendation system's last recommendation.

At $t = 2$, the consumer's state comprises the time period, the last consumer action $(a_1 = 3)$, the actions before that $(A_1 = \emptyset)$ and the previous recommendations, $R_2 = R_1 \cup r_2 = \{1, 0, 0\}$. Based on these data, the consumer opts for decision $a_2\left(\Theta_2'\right) = 1$. In response, the recommendation system updates its state to $\{t = 2, a_2 = 1, A_2 = \{0, 0, 1\}, R_2 = \{1, 0, 0\}\}$.

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[19] Also, since the first consumer action is made without recommendations (a click on the homepage), r_1 is not defined.

At this point, a_2 captures the vehicle that the consumer wants to browse now, whereas $A_2 = \{0, 0, 1\}$ holds the consumer's first browsing decision, i.e., $A_2 = A_1 \cup a_1$. Finally, the recommendation system opts to recommend a vehicle from cluster 3, and we move to period 3. The consumer's state updating proceeds in the same fashion, with state variable R_3 being calculated via relative frequencies of past recommendations, $R_3 = \frac{1}{2}(R_1 + R_2) = \{1/2, 0, 1/2\}$.

B Optimization of the Static and Dynamic Matrices

Consider the value function faced by the recommendation system once again:

$$V(t, a_t, A_t, R_t) = \max_{\varphi(\cdot)} \pi(t, a_t, A_t, R_t) + E\left[V(t+1, a_{t+1}, A_{t+1}, R_{t+1})\,|\, a_t, A_t, R_t, \varphi(\cdot)\right] \quad (9)$$

For counterfactuals "Static Matrix Optimization" and "Dynamic Matrix Optimization", problem (9) is solved under the constraint that $\varphi(\cdot)$ is an 8×8 recommendation matrix of probabilities, which is constant or can change over time, depending on the scenario of interest. This optimization has many control variables (in the dynamic case, this yields 1,232 variables) and is infeasible through direct optimization methods. Our optimization approach is to solve problem (9) once, based on the original dataset, and then, for each bootstrap sample, approximate problem (9) quadratically via Taylor series expansion, and calculate the resulting maximizer analytically.

Let the original policy function, estimated from the original dataset, induce the following dynamic problem for the recommendation system:

$$V(t, a_t, A_t, R_t) = \max_{\varphi(\cdot)} \pi(t, a_t, A_t, R_t) + E_0\left[V(t+1, a_{t+1}, A_{t+1}, R_{t+1})\,|\, a_t, A_t, R_t, \varphi(\cdot)\right]$$

where E_0 incorporates the consumer's policy function recovered via machined learning esti-

mation. We first define the following non-optimized function

$$W\left(\varphi, f\left(\cdot\right)\right) := \pi\left(t, a_t, A_t, R_t\right) + E\left[V\left(t+1, a_{t+1}, A_{t+1}, R_{t+1}\right) \middle| a_t, A_t, R_t, \varphi\right] \qquad (10)$$

Note that function W is a functional of the conditional probability mass function

$$f(\cdot) = f\left(a_{t+1} \middle| t, a_t, A_t, R_t, \varphi\right) \qquad (11)$$

which is the estimated object summarizing the consumer policy function. Denote the analogue object recovered from the original dataset by f_0. Through classical numerical solvers, we can calculate

$$\varphi_0^* = \operatorname{argmax}_\varphi W\left(\varphi, f_0\right) \qquad (12)$$

where φ_0^* is a large matrix. Taylor expansion yields the following quadratic approximation for function W evaluated at bootstrap policy sample b:

$$\widetilde{W}\left(\varphi, f_b\right) = W\left(\varphi_0^*, f_b\right) + D'.\left(\varphi - \varphi_0^*\right) + \frac{1}{2}\left(\varphi - \varphi_0^*\right)^T.H.\left(\varphi - \varphi_0^*\right) \qquad (13)$$

where $D = \frac{\partial}{\partial \varphi}W\left(\varphi, f_b\right)$ is a matrix of first partial derivatives of W, and $H = \frac{\partial^2}{\partial \varphi^2}W\left(\varphi, f_b\right)$ is the (symmetric) Hessian matrix of W, both evaluated at $\varphi = \varphi_0^*$. Taking the first-order condition of the right-hand side w.r.t. φ yields:

$$\frac{\partial}{\partial \varphi}\widetilde{W}\left(\varphi, f_b\right) = 0$$

$$\Leftrightarrow 0 + D' + \frac{1}{2}\left(2\varphi'H - 2\varphi_0^{*'}H\right) = 0$$

$$\varphi'H = -D' + \varphi_0^{*'}H$$

$$\varphi' = \varphi_0^{*'} - D'H^{-1}$$

$$\varphi = \varphi_0^* - H^{-1}D$$

where we have taken advantage of the fact that H is a symmetric matrix. For each sample f_b, this procedure yields a matrix recommendation φ_b, at the cost of evaluating the first and second matrices of derivatives of W at each estimated bootstrap policy function f_b, calculated through finite differences. The optimization of the static matrix scenario across bootstrap samples took the longest to calculate, and was parallelized within and across computers (takes about 22 days on a top-of-the-line 8-core pc).

C   Counterfactual Analyses

Table 10: Expected Profits and their Standard Deviations for each Counterfactual Scenario

Scenario	Expected Profit	Std. Deviation
Status Quo	100.0	1.57
Static Matrix Optimization	102.1	1.51
Dynamic Matrix Optimization	101.6	1.84
Previous Actions Only	92.8	0.84
Previous Actions and Recommendations	102.7	1.81
Ignore Vehicle Margins	132.1	2.85
Ignore Customer Churn	119.8	2.85
1-Step Look-ahead	127.7	2.90
First-Best Policy	133.2	2.89

Note: Standard deviations calculated based on 50 bootstrap replications of the consumer policy.

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