



## Modeling and forecasting return jumps using realized variation measures

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### ABSTRACT

This paper proposes a simple HAR-RV-based model to predict return jumps through a conditional density of jump size with time-varying moments. We model jump occurrences based on a version of the autoregressive conditional hazard model that relies on past continuous realized volatilities. Applying our methodology to seven equity indices on the U.S. and Chinese stock markets, we reach the following key findings: (i) jump occurrence and size are dependent on past realized volatility, (ii) the proposed model yields superior in- and out-of-sample jump size density forecasts compared to an ARMA(1,1)-GARCH(1,1) model, (iii) and the occurrence and sign of return jumps are predictable to some extent.

### 1. Introduction

The distributional properties of speculative prices are among the most studied phenomena in empirical finance. Numerous problems in finance, such as risk management, portfolio choice, and derivatives pricing require a full characterization of the distribution of returns. Seminal studies, dating back to the work of Mandelbrot (1963) and Fama (1965), document the unconditional distributions of daily and longer horizon asset returns exhibit excess kurtosis and fatter tails compared to the normal distribution. To explain this empirical evidence, early studies focus on modeling returns using alternative nonnormal distributions (e.g., Blattberg and Gonedes, 1974; Clark, 1973; Fama, 1965; Fielitz and Rozelle, 1983; Kon, 1984; Mandelbrot, 1963; Praetz, 1972; Westerfield, 1977). More recently, a large literature stream on time-series analyses and derivatives pricing incorporates the presence of jumps in equilibrium prices, along with GARCH and stochastic volatility models, to more effectively capture leptokurtosis in the return distribution (e.g., Andersen et al., 2002; Bakshi et al., 1997; Ball and Torous, 1983, 1985; Bates, 1996, 2000; Beckers, 1981; Bollerslev, 1987; Chernov et al., 2003; Duffie et al., 2000; Eraker, 2004; Eraker et al., 2003; Johannes, 2004; Jorion, 1988; Li et al., 2008; Maheu and McCurdy, 2004; Merton, 1976; Press, 1967; Xu et al., 2011, 2016). In these models based on low-frequency data jumps are treated as latent, which not only complicates estimation procedures but also leads to statistically insignificant or even inconsistent results.

The past decade witnessed an increase in the popularity of realized variation measures in volatility modeling and forecasting (e.g., Andersen et al., 2003, 2009, 2012; Barndorff-Nielsen and Shephard, 2004, 2006; Barndorff-Nielsen et al., 2008; Çelik and Ergin, 2014; Christensen et al., 2010; Corsi, 2009; Corsi et al., 2010; Liu et al., 2017; Ma et al., 2017; Podolskij and Vetter, 2009a, 2009b; Zhang et al., 2005), as well as jump detection methods entailing the use of high-frequency intraday data (e.g., Aït-Sahalia and Jacod, 2009; Aït-Sahalia et al., 2012; Andersen et al., 2010, 2011; Christensen et al., 2014; Lee and Mykland, 2008, 2012). The observability of jumps allows researchers greater flexibility in modeling their dynamics.

In parametric jump-diffusion models, jumps are assumed as unobservable and jump sizes are typically modeled as time-invariant and independent and identically distributed (i.i.d.) over time to ensure the feasibility of model evaluation and statistical inference, based on low-frequency data. However, several studies suggest jump intensities and sizes are both time-varying and autocorrelated (e.g., Andersen et al., 2007; Johannes, 2004). For instance, Andersen et al. (2011) use intraday data to model squared jump sizes through a log-linear heterogeneous autoregressive (HAR) structure (Corsi, 2009), coupled with an autoregressive conditional hazard (ACH) model (Hamilton and Jordà, 2002) for time-varying jump intensities. While the simple log-linear HAR model captures the long-memory or persistent effects of past return variations for squared jump sizes, the ACH model is consistent with the more recent empirical evidence on self-exciting jump clustering (e.g., Aït-Sahalia

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et al., 2015; Fulop et al., 2015).

First, this paper contributes to the above literature by proposing a new model for return jumps, based on the realized variance and bipower variation measures developed by Barndorff-Nielsen and Shephard (2004, 2006). As per Andersen et al. (2011), we model both jump sizes and occurrence. Although a considerable body of literature focuses on detecting jumps and studying their dynamics, few studies (e.g., Tauchen and Zhou, 2011) focus on modeling return jump dynamics directly. We first extend the specification of Andersen et al. (2011) for squared jump sizes by deriving a simple conditional density for jump sizes with time-varying moments. The asymmetry of jump size density is modeled as a function of lagged realized variances, and the conditional mean, variance, and skewness of jump sizes can evolve dynamically over time. Moreover, we adopt the ACH model to describe the occurrence of return jumps. However, unlike Andersen et al. (2011), we use lagged realized variances as exogenous variables to model the conditional hazard rate of jumps in returns because the literature indicates a state-dependence of jump frequency on time-varying volatility (e.g., Bates, 2000; Eraker, 2004).

This paper is closely related to those of Andersen et al. (2010, 2011), which provide nonparametric jump detection methods and models for the continuous and discontinuous parts of realized volatility. Among the studies decomposing total quadratic variation into its continuous and discontinuous sample paths and separately analyzing their dynamics (e.g., Andersen et al., 2011; Busch et al., 2011; Duong and Swanson, 2015; Patton and Sheppard, 2015), few have studied the benefits of incorporating realized volatility or its components into the return distribution. On the other hand, Maheu and McCurdy (2011) propose a bivariate model of returns and realized volatility, which improves forecasts of return distributions, but do not decompose realized volatility or separate jumps from the continuous component. We focus on the direct modeling of jumps in returns, which allows us to characterize the density and signs of jumps and link realized volatility and its components to the conditional moments of return jumps. This approach may further help in forecasting the density of returns as a whole. All models in this paper are estimated using maximum likelihood. Their empirical application to high-frequency prices for seven major equity indices on the U.S. and Chinese stock markets reveal, that for most indices, asymmetry exists in jump size density and lagged realized variances have a considerable impact on jump occurrence.

We subsequently use our model to forecast jumps in returns. Profitable trading and asset allocation strategies are all based on the successful forecasting of market direction or return signs. Recent studies reveal asset return signs can be successfully forecasted (e.g., Chevapatrakul, 2013; Christoffersen and Diebold, 2006). As such, forecasting return jumps is essential in financial trading activities because they represent sudden and substantial price movements, such as those observed during the stock market crash of 1987. Moreover, Fleming and Paye (2011) suggest the asymmetry and abnormality of return distributions stem mainly from jumps. Specifically, the return sign could be primarily determined by the sign of a jump. As a result, decomposing returns into jump and continuous parts and separately analyzing their dynamics and signs hold promise for achieving a more reliable characterization of returns distribution, which may further enhance the performance of forecasting market direction or return signs, and thereafter increase the profitability of asset allocation strategies.

Hence, we also contribute to the literature by applying our model to forecasting the signs and occurrence of jumps. Forecast accuracy is statistically tested using the probability forecast evaluation of Cox (1958). The empirical results show our jump model provides accurate in- and out-of-sample probability forecasts for both jump occurrence and signs.

The third contribution of our paper to the literature is evaluating our model's accuracy in forecasting the distribution density of return jumps. Risk management, portfolio choice, and derivative pricing require a full characterization of the distribution of returns. Several studies (e.g., Andersen et al., 2000, 2003; 2010; Hansen et al., 2012) examine the

distribution of returns in the context of realized volatility. On the other hand, Maheu and McCurdy (2011) and Hua and Manzan (2013) study the accuracy of return density forecasts produced by alternative realized volatility models. Fleming and Paye (2011) suggest jumps should be included in price models and the abnormality of returns is mainly caused by jumps. Nevertheless, few studies consider the importance of modeling jumps for return density forecasting.

Accordingly, we examine whether our jump model could improve forecasting the density of return jumps, which is critical to deriving return density as a whole. To this end, we adopt the predictive likelihood approach (e.g., Amisano and Giacomini, 2007; Diebold and Mariano, 1995; Maheu and McCurdy, 2011) to evaluate the accuracy of the model's density forecast. Compared with benchmark model ARMA(1, 1)-GARCH(1,1), our model produces superior density forecasts for jump sizes, both in- and out-of-sample.

The rest of this paper is organized as follows. Section 2 introduces realized variation measures and the jump detection method used in this study. Section 3 presents the data used in our empirical applications. Then, our model for return jump sizes and occurrence is discussed and estimated in Section 4. Model-based forecasts for the occurrence, signs, and density of jumps are reported and evaluated in Sections 5 and 6, respectively. Finally, Section 7 provides conclusions.

## 2. Realized variation measures and jump detection

Quadratic variation can be approximated by the following realized variance (Andersen et al., 2011):

$$RV_t = \sum_{j=1}^M r_{t,j}^2, \quad (1)$$

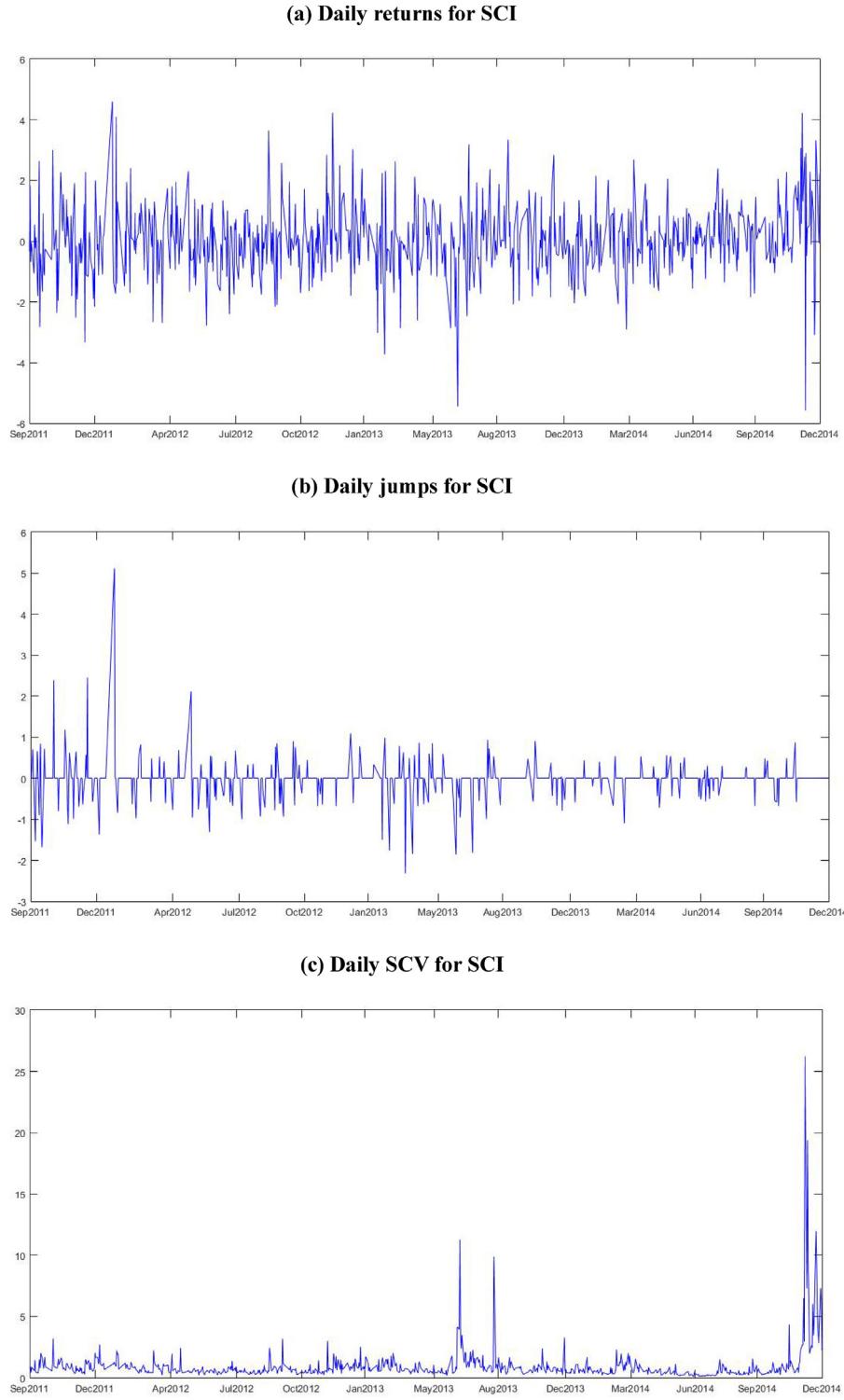
$$RV_t \xrightarrow{P} \int_{t-1}^t \sigma_s^2 ds + \sum_{i=1}^{N_t} \kappa_{t,i}^2 \text{ as } M \rightarrow \infty, \quad (2)$$

**Table 1**  
Sample period and trading days for analyzed data.

	In-sample		Out-of-sample	
	Sample period	Trading days	Sample period	Trading days
SCI	09/06/2011–01/ 30/2014	569	02/07/2014–02/ 05/2015	248
CIS	09/08/2011–01/ 30/2014	563	02/07/2014–02/ 06/2015	249
CSI300	09/08/2011–01/ 30/2014	563	02/07/2014–02/ 05/2015	248
HSI	07/12/2012–01/ 30/2014	375	02/04/2014–02/ 05/2015	240
SP500	06/29/2012–02/ 04/2014	393	02/05/2014–02/ 05/2015	252
DJIA	06/29/2012–02/ 04/2014	394	02/05/2014–02/ 05/2015	239
Nasdaq	07/02/2012–02/ 04/2014	385	02/05/2014–02/ 05/2015	252

**Table 2**  
Summary statistics for SCI and SP500.

	SCI			SP500		
	$r_t$	$J_t$	$SCV_t$	$r_t$	$J_t$	$SCV_t$
Mean	0.029	-0.026	1.014	0.068	0.062	0.252
Std.dev.	1.201	0.448	1.600	0.751	0.497	0.247
Skewness	-0.224	0.029	8.474	0.021	0.386	2.460
kurtosis	7.249	46.979	106.219	5.084	8.770	11.523
Min	-8.018	-4.950	0.101	-2.533	-2.381	0.006
Max	4.636	5.121	26.216	4.189	3.561	2.060
Obs.	817	817	817	645	645	645



**Fig. 1.** Daily returns, jumps, and continuous realized variances for SCI.

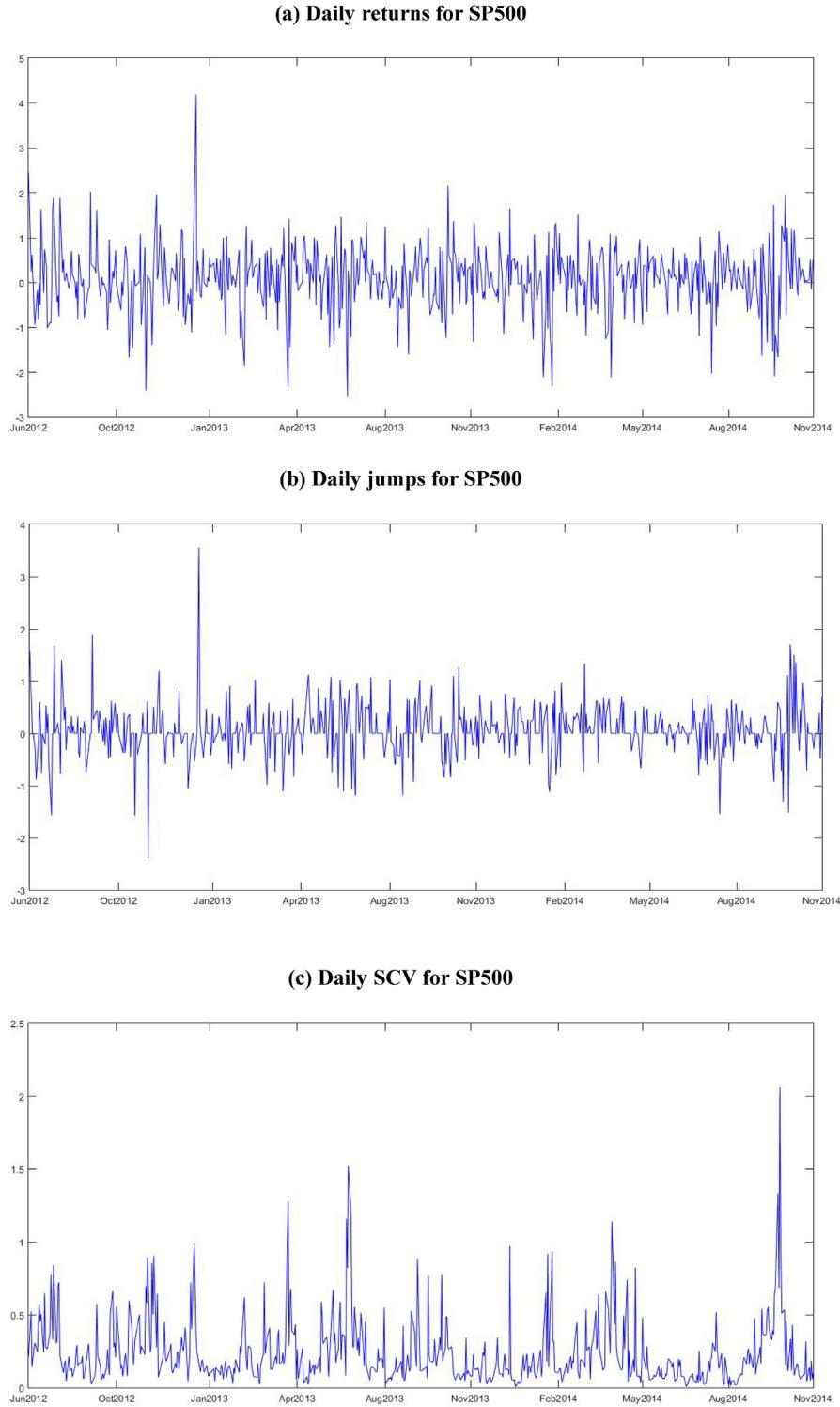
where  $r_{t,j}$  is the  $j$ th intraday log return on day  $t$ ,  $\kappa_{t,i}$  the  $i$ th jump occurring over day  $t$ ,  $M$  the number of intraday returns,  $N_t$  the number of intraday jumps, and  $\int_{t-1}^t \sigma_s^2 ds$  the integrated variance. Barndorff-Nielsen and Shephard (2004, 2006) reveal the realized bipower variation is a consistent estimator for integrated variance:

$$RBV_t = \mu_1^{-2} \left( \frac{M}{M-2} \right) \sum_{j=3}^M |r_{t,j-2}| |r_{t,j}|, \quad (3)$$

where  $\mu_1 = \sqrt{2/\pi}$ , and

$$RBV_t \xrightarrow{P} \int_{t-1}^t \sigma_s^2 ds, \quad RV_t - RBV_t \xrightarrow{P} \sum_{i=1}^{N_t} \kappa_{t,i}^2 \text{ as } M \rightarrow \infty. \quad (4)$$

The definition of  $RBV_t$  in Eq. (3) is adopted from Huang and Tauchen



**Fig. 2.** Daily returns, jumps, and continuous realized variances for SP500.

(2005) and Andersen et al. (2011), which is different from the original measure suggested by Barndorff-Nielsen and Shephard (2004), as an additional stagger has been employed to alleviate microstructure concerns. Andersen et al. (2011) propose the following test statistic for detecting significant jumps:

$$Z_t = \frac{\frac{RV_t - RBV_t}{RV_t}}{\sqrt{\left[ \left( \frac{\pi}{2} \right)^2 + \pi - 5 \right] \frac{1}{M} \max \left( 1, \frac{RTQ_t}{RBV_t^2} \right)}} \rightarrow N(0, 1), \quad (5)$$

where  $RTQ_t$  is staggered realized tripower quarticity, being defined as follows:

$$RTQ_t = \mu_{4/3}^{-3} \left( \frac{M^2}{M-4} \right) \sum_{j=5}^M |r_{t,j-4}|^{4/3} |r_{t,j-2}|^{4/3} |r_{t,j}|^{4/3} \quad (6)$$

and  $\mu_{4/3} = 2^{2/3} \Gamma(7/6)/\Gamma(1/2) \approx 0.8309$ .  $Z_t$  is asymptotically standard normally distributed under a null hypothesis of no within-day jump over

**Table 3**  
Jump size model estimates.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$\alpha_0$	-0.815 [0.000]	-0.864 [0.003]	-0.751 [0.000]	-0.759 [0.000]	-1.278 [0.041]	-0.629 [0.801]	-0.457
$\alpha_d$	-0.178 [0.815]	0.110 [0.916]	0.029 [0.931]	-0.025 [0.779]	0.267 [0.210]	0.443 [0.012]	0.406
$\alpha_w$	0.488 [0.490]	0.202 [0.585]	0.297 [0.258]	0.517 [0.004]	0.495 [0.027]	0.355 [0.226]	0.343 [0.300]
$\alpha_m$	0.761 [0.007]	0.885 [0.596]	0.779 [0.107]	0.178 [0.509]	-0.316 [0.336]	-0.232 [0.448]	-0.137 [0.925]
$\kappa_0$	1.261 [0.000]	0.836 [0.599]	1.192 [0.003]	0.766 [0.055]	0.811 [0.025]	1.014 [0.005]	0.641 [0.000]
$\kappa_d$	0.148 [0.700]	0.033 [0.852]	0.016 [0.955]	0.280 [0.099]	-0.027 [0.981]	-0.032 [0.902]	-0.192 [0.944]
$\kappa_w$	0.100 [0.702]	-0.137 [0.935]	0.037 [0.924]	-0.409 [0.098]	-0.054 [0.632]	0.246 [0.811]	-0.328 [0.626]
$\kappa_m$	-0.189 [0.330]	0.040 [0.967]	-0.175 [0.535]	0.182 [0.619]	-0.295 [0.522]	-1.626 [0.470]	0.550 [0.812]
$\omega_0$	0.346 [0.328]	0.399 [0.000]	0.064 [0.000]	0.443 [0.000]	0.396 [0.000]	1.432 [0.000]	0.374 [0.000]
$\omega_p$	0.014 [0.708]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]	0.000 [1.000]
$\omega_q$	0.686 [0.029]	0.635 [0.488]	0.932 [0.002]	0.693 [0.000]	0.851 [0.000]	0.086 [0.874]	0.829 [0.000]
LogL	-117.380	-158.222	-141.236	-297.968	-263.639	-197.525	-300.717
Obs.	156	156	166	344	359	359	351

Notes: This table reports the MLEs with the  $p$  values in square brackets.

day  $t$ . Consequently, the realized variance can be decomposed into two parts, namely discontinuous component  $JV_t$  and continuous component  $CV_t$ :

$$JV_t = I_{\{Z_t > \phi_\alpha\}} \cdot (RV_t - RBV_t), \quad (7)$$

$$CV_t = RV_t - JV_t = I_{\{Z_t \leq \phi_\alpha\}} \cdot RV_t + I_{\{Z_t > \phi_\alpha\}} \cdot RBV_t, \quad (8)$$

where  $I_Q$  is the indicator function and  $\Phi_\alpha$  the critical value of significance level  $\alpha$  from the standard normal distribution. We follow the procedure of Andersen et al. (2011) and set  $\alpha = 0.99$ .

Similarly, the method developed by Andersen et al. (2010) can be used to detect sequent jumps in returns. Specifically, when jumps occur on day  $t$ , statistic  $Z_t$  in Eq. (5) is significantly nonzero. Therefore, the highest figure for intraday returns  $\{r_{t,1}, \dots, r_{t,M}\}$  in terms of absolute value is treated as the first observation for jumps occurring over day  $t$ ,  $\kappa_{t,1} = \text{sgn} \left( \max_{j \in \{1, \dots, M\}} |r_{t,j_1}| \right) \max_{j \in \{1, \dots, M\}} |r_{t,j_1}|$ . To measure the possible second intraday jump, the realized variance of day  $t$  becomes  $RV_{t,2} = \frac{M}{M-1} \sum_{j \in \{1, \dots, M\} \setminus j_1} r_{t,j}^2$ , which replaces  $RV_t$  in Eq. (5) and consequently updates the value of  $Z_t$ . If  $Z_t$  is still significantly different from zero, the second observation for jumps is defined as  $\kappa_{t,2} = \text{sgn} \left( \max_{j_2 \in \{1, \dots, M\} \setminus j_1} |r_{t,j_2}| \right) \max_{j_2 \in \{1, \dots, M\} \setminus j_1} |r_{t,j_2}|$ ; otherwise, only one jump occurs in returns over day  $t$ . If a total of  $N_t$  jumps occur during day  $t$ , we define jump size  $J_t$  of the return on day  $t$  as follows:

$$J_t = \sum_{i=1}^{N_t} \kappa_{t,i}, \quad \kappa_{t,i} = \text{sgn} \left( \max_{j_i \in \{1, \dots, M\} \setminus \{j_1, \dots, j_{i-1}\}} |r_{t,j_i}| \right) \max_{j_i \in \{1, \dots, M\} \setminus \{j_1, \dots, j_{i-1}\}} |r_{t,j_i}|. \quad (9)$$

Andersen et al. (2010) define the realized variance caused by sequent jumps as

$$SJV_t = \sum_{i=1}^{N_t} SJV_{t,i}, \quad (10)$$

$$SJV_{t,i} = I_{\{Z_{t,i} > \phi_{1-\alpha}\}} \left\{ \max_{j_i \in \{1, \dots, M\} \setminus \{j_1, \dots, j_{i-1}\}} r_{t,j_i}^2 - \frac{1}{M-N_t} \sum_{k \in \{1, \dots, M\} \setminus \{j_1, \dots, j_{N_t}\}} r_{t,k}^2 \right\}, \quad (11)$$

where  $Z_{t,i}$  represents the  $i$ th updated value of the test statistic. Accordingly, the continuous part of the realized variance can be defined as

$$SCV_t = RV_t - SJV_t. \quad (12)$$

### 3. Data and summary statistics

Our data comprise five-minute prices for seven major equity indices on the U.S. and Chinese stock markets: Shanghai Composite Index (SCI), Compositional Index of Shenzhen (CIS), China Securities Index 300 (CSI300), Hang Seng Index (HSI), Standard & Poor's 500 Index (SP500), the Dow Jones Industrial Average (DJIA), and NASDAQ. We obtained the indices from Shanghai Great Wisdom Co., Ltd. and chose five-minute sampling frequencies for the best trade-off between market microstructure noise effects and measurement accuracy (e.g., Andersen and Bollerslev, 1998; Corsi et al., 2010; Liu et al., 2015). After removing errors from raw transaction prices, we derive the sample period and number of trading days for each index, as per Table 1. To demonstrate data properties, Table 2 presents selected reports for SCI and SP500, that is, summary statistics of daily log returns  $r_t$ , estimated  $SCV_t$ , and  $J_t$  computed from the five-minute log returns scaled by 100. Figs. 1 and 2 respectively show sample paths for the two indices, indicating the dynamic properties of the data. In the middle panels of Figs. 1 and 2 we plot daily jumps  $J_t$  (including the overnight jumps) detected over the entire sample period. For SCI (SP500), 20.3% (57.2%) of the total number of trading days in the sample are associated with jumps. The proportion decreases to 7.0% (11.9%) if only intraday jumps are considered, which indicates more jumps occur overnight. The intraday jump intensity for SP500 in our sample (i.e. 11.9%) is close to that (13.3%) estimated in Tauchen and Zhou (2011). The realized volatility literature deals with this non-trivial overnight variation in different ways (e.g., Andersen et al., 2011, and the references therein). In this paper, we assume overnight jumps come from the same process that generates intraday jumps, and thus treat an overnight jump as another intraday jump to obtain a measure (i.e.  $J_t$ ) for the entire day jump. This is because Andersen et al. (2011) suggest both

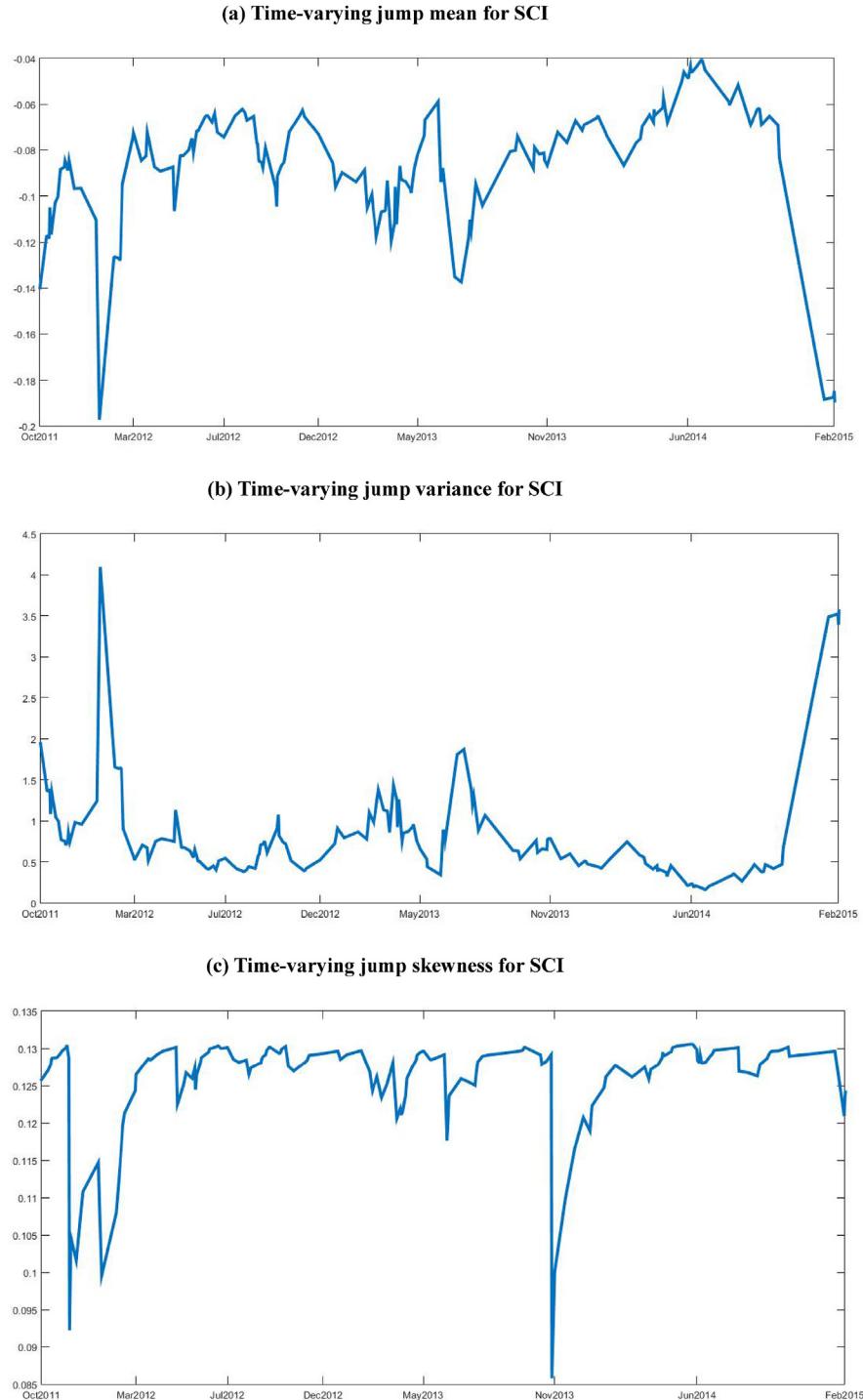


Fig. 3. Conditional moments of jump size for SCI.

intraday and overnight jumps are associated with past continuous variations and we are especially interested in exploiting the explanatory power of one-week or one-month lagged continuous volatilities (in HAR structures) in modelling daily jumps. Additionally, the sample correlation coefficient between squared daily jump size  $J_t^2$  (when at least one jump occurs within the entire day) and continuous realized volatilities  $SCV_t$  is 0.56 for SCI and 0.28 for SP500. The sample correlation coefficient between daily jump occurrence (i.e.,  $I_{t(J \neq 0)}$ ) and continuous realized volatilities  $SCV_t$  is -0.12 for SCI and -0.23 for SP500. These features indicate a distinct dynamic dependency between our estimates for daily jumps and continuous variations, which are accounted for in the

subsequent sections.

#### 4. Jump modeling

Our modeling for jumps in returns involves two components: (1) modeling jump sizes and (2) modeling jump occurrence. We begin with modeling of jump sizes.

##### 4.1. Modeling jump sizes

Most jump-diffusion models assume the sizes of jumps become i.i.d.

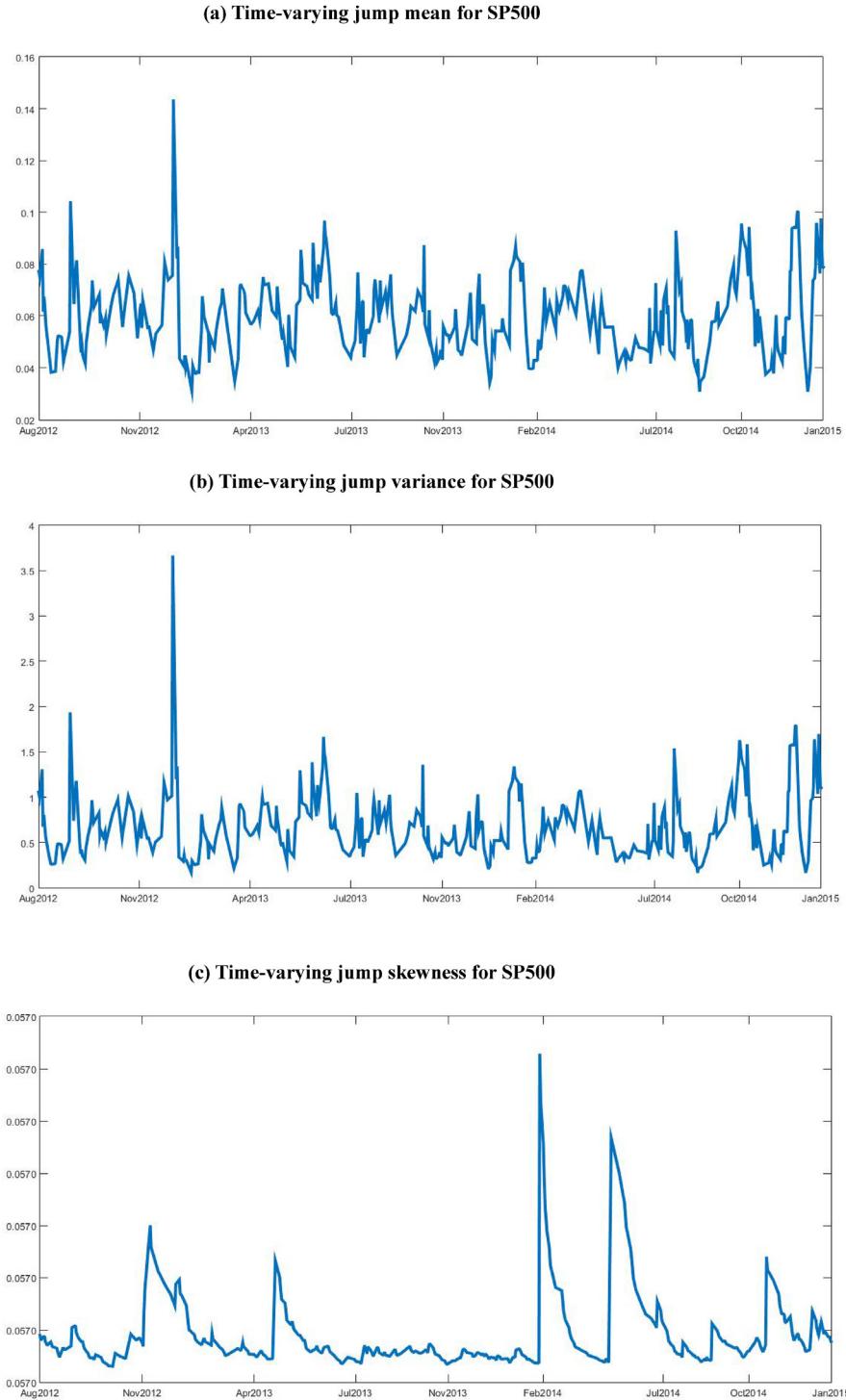


Fig. 4. Conditional moments of jump size for SP500.

over time. By directly observing squared jumps, Andersen et al. (2011) propose the HAR-J model for squared jump sizes. Specifically, they parameterize conditional squared jump sizes as function of past continuous realized variances with a GARCH(1,1)-t error structure

$$\ln JV_{t(i)} = \beta_0 + \beta_{CD} \ln CV_{t(i)-1} + \beta_{CW} \ln CV_{t(i)-5,t(i)-1} + \beta_{CM} \ln CV_{t(i)-22,t(i)-1} + \varepsilon_{t(i)},$$

$$\varepsilon_{t(i)} = \sigma_{t(i)} \sqrt{\frac{\nu-2}{\nu}} z_{t(i)}, \quad z_{t(i)} \sim t(\nu),$$

$$\sigma_{t(i)}^2 = \omega + \alpha e_{t(i-1)}^2 + \beta \sigma_{t(i-1)}^2, \quad (13)$$

where  $t_{(i)}$  maps jump counter  $i$  into the corresponding trading day  $t$  and  $CV_{t,k,t-1} = k^{-1}[CV_{t-k} + CV_{t-k+1} + \dots + CV_{t-1}]$ . We extend the specification in Eq. (13) by deriving the density of return jump sizes. Specifically, if we refer to the density of log squared jump size  $\ln J_t^2$  as  $f_{\ln J_t^2}(x)$ , the density of jump size  $f_{J_t}(x)$  satisfies the following equations:

$$f_{J_t}(x) + f_{J_t}(-x) = \frac{2}{x} f_{\ln J_t^2}(\ln x^2), \quad x > 0, \quad (14)$$

**Table 4**  
Augmented ACH model estimates.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$\beta_0$	2.191 [0.000]	4.649 [0.000]	0.672 [0.000]	0.220 [0.517]	1.137 [0.000]	0.292 [0.000]	1.366 [0.170]
$\beta_1$	0.085 [0.000]	0.133 [0.527]	0.050 [0.363]	0.099 [0.022]	0.143 [0.000]	0.000 [1.000]	0.112 [0.323]
$\beta_2$	0.474 [0.000]	0.000 [0.999]	0.795 [0.000]	0.750 [0.000]	0.179 [0.000]	0.802 [0.000]	0.000 [1.000]
$\beta_d$	-0.481 [0.000]	0.103 [0.059]	-0.111 [0.225]	0.022 [0.986]	0.040 [0.004]	0.109 [0.270]	0.056 [0.932]
$\beta_w$	2.725 [0.000]	-0.004 [0.988]	0.446 [0.000]	0.003 [0.999]	-0.019 [0.626]	-0.347 [0.000]	0.009 [0.979]
$\beta_m$	-1.980 [0.000]	-0.299 [0.272]	-0.229 [0.018]	0.020 [0.984]	0.053 [0.139]	0.467 [0.000]	0.574 [0.302]
LogL	-388.104	-390.493	-404.783	-398.473	-421.304	-411.183	-414.978
Obs.	817	812	811	615	645	633	637

Notes: This table reports MLEs with the  $p$  values in square brackets.

$$f_{J_t}(x) + f_{J_t}(-x) = -\frac{2}{x^2} \ln J_t^2 (\ln x^2), \quad x < 0. \quad (15)$$

If we assume

$$f_{J_t}(-x) = k_t f_{J_t}(x), \quad k_t > 0, \quad \text{for } \forall x > 0, \quad (16)$$

where  $k_t$  can be a constant or function, we can easily obtain the following:

$$\Pr(J_t > 0) = \frac{1}{1 + k_t}, \quad \Pr(J_t < 0) = \frac{k_t}{1 + k_t}. \quad (17)$$

If we further assume  $\ln J_t^2 \sim \text{Normal}(\mu_t, \sigma_t^2)$ , then<sup>1</sup>

$$E[J_t] = \frac{1 - k_t}{1 + k_t} e^{\frac{\mu_t}{2} + \frac{1}{2}\sigma_t^2},$$

$$\text{var}[J_t] = e^{\mu_t + \frac{1}{4}\sigma_t^2} \left[ e^{\frac{1}{2}\sigma_t^2} - \left( \frac{1 - k_t}{1 + k_t} \right)^2 \right],$$

$$\text{skew}[J_t] = \frac{1 - k_t}{1 + k_t} \frac{\exp\left(\frac{\sigma_t^2}{4}\right) \left[ \exp\left(\frac{\sigma_t^2}{2}\right) - 3 \right] - 2 \left( \frac{1 - k_t}{1 + k_t} \right)^2}{\left[ \exp\left(\frac{\sigma_t^2}{4}\right) - \left( \frac{1 - k_t}{1 + k_t} \right)^2 \right]^{\frac{3}{2}}}. \quad (18)$$

Hence,  $k_t$  characterizes the asymmetry of the jump size distribution: the distribution is skewed if  $k_t \neq 1$ . Substituting Eq. (16) into Eqs. (14) and (15), we formally assume the density of return jump size  $J_{t(i)}$  to be

$$f(J_{t(i)}) = \begin{cases} \frac{2}{J_{t(i)}(1 + k_{t(i)})} g(J_{t(i)}), & J_{t(i)} > 0 \\ -\frac{2k_{t(i)}}{J_{t(i)}(1 + k_{t(i)})} g(J_{t(i)}), & J_{t(i)} < 0 \end{cases}, \quad (19)$$

where

$$g(J_{t(i)}) = \frac{1}{\sqrt{2\pi}\sigma_{t(i)}} e^{-\frac{(\ln J_{t(i)} - \mu_{t(i)})^2}{2\sigma_{t(i)}^2}},$$

$$\mu_{t(i)} = \alpha_0 + \alpha_d \ln SCV_{t(i)-1} + \alpha_w \ln SCV_{t(i)-5,t(i)-1} + \alpha_m \ln SCV_{t(i)-22,t(i)-1},$$

$$k_{t(i)} = \kappa_0 + \kappa_d \ln SCV_{t(i)-1} + \kappa_w \ln SCV_{t(i)-5,t(i)-1} + \kappa_m \ln SCV_{t(i)-22,t(i)-1},$$

$$\sigma_{t(i)}^2 = \omega_0 + \omega_q \sigma_{t(i-1)}^2 + \omega_p (\ln J_{t(i-1)}^2 - \mu_{t(i-1)})^2.$$

Specifically, the log squared jump size is conditionally normally distributed and the conditional variance follows GARCH(1,1). Notably,  $\lim_{J_t \rightarrow 0} f(J_t) = 0$ .

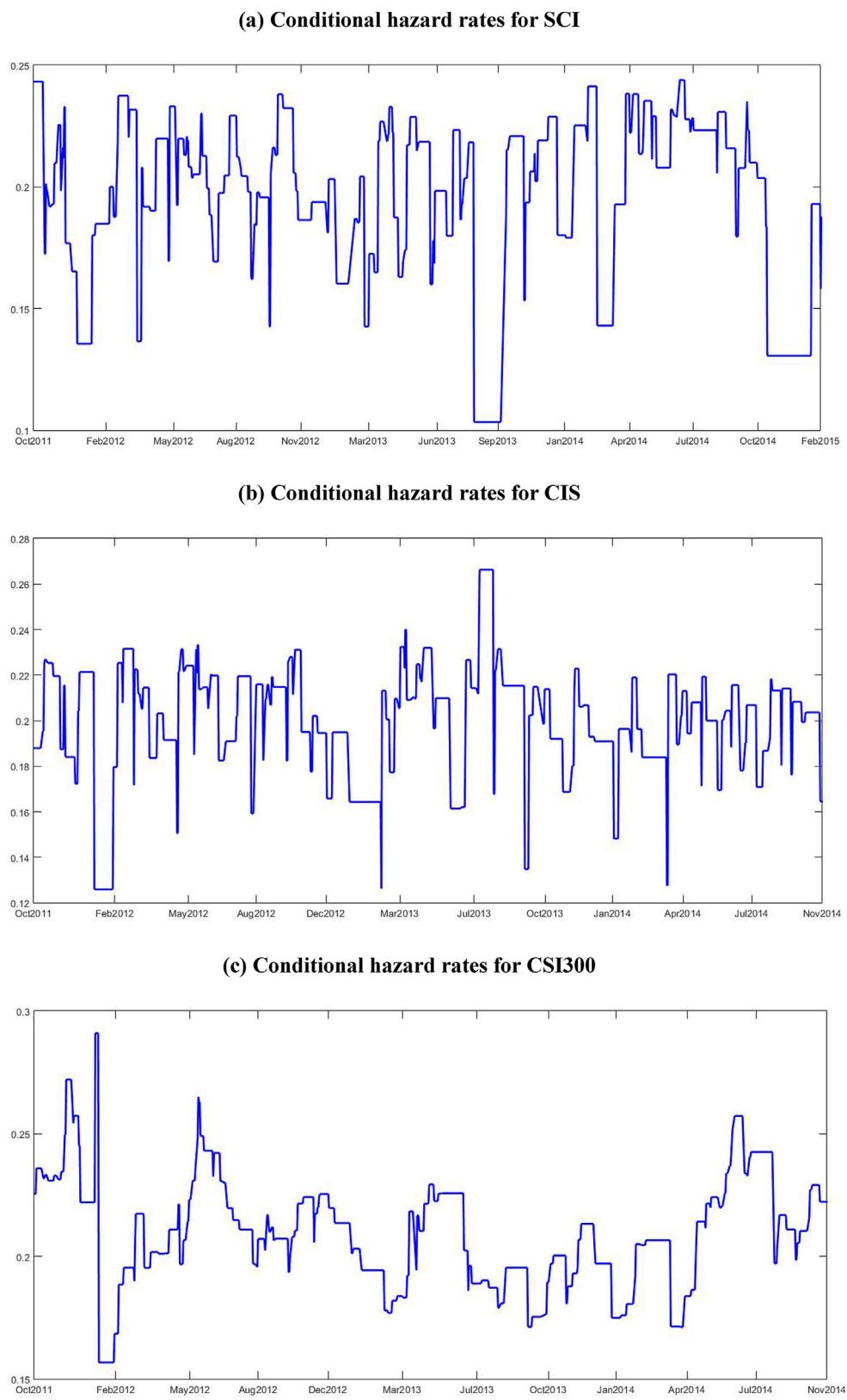
[Table 3](#) reports the resulting maximum likelihood estimates (MLEs) with the  $p$  values in square brackets. As we can see, the effects of lagged realized continuous variances on the conditional mean of the log squared jump size vary among indices. For the conditional variance of the log squared jump size, a significant positive autoregressive relationship exists in the indices, except for CIS and DJIA. Moreover, the coefficient estimates for the squared lagged innovation are all close to zero and insignificant. Although the lagged realized continuous variances have limited explanatory power for the dynamics of jump size asymmetry, the estimates for the constant  $\kappa_0$  are all positive and significant for most of the indices. [Tauchen and Zhou \(2011\)](#) preliminarily discuss the time-varying mean and volatility for daily return jumps through rolling estimation of a Poisson-mixing-normal jump model. By contrast, our model further characterizes the conditional skewness of jump size. To effectively illustrate the dynamics of time-varying moments of jump size, [Figs. 3 and 4](#) plot for SCI and SP500 respectively the time series of conditional mean, variance, and skewness of jump size (given in Eq. (18)) derived from the MLEs in [Table 3](#). The bottom panels of [Figs. 3 and 4](#) show daily return jumps are right skewed for both SCI and SP500, which is consistent with the unconditional skewness reported in [Table 2](#).

#### 4.2. Modeling jump occurrences

We adopt the ACH model developed by [Hamilton and Jordà \(2002\)](#), which is consistent with recent empirical evidence of self-exciting jump clustering (e.g., [Aït-Sahalia et al., 2015](#); [Fulop et al., 2015](#)), to model the occurrence of return jumps. The ACH model improves the original autoregressive conditional duration model proposed by [Engle and Russell \(1998\)](#) by incorporating new information as it becomes available when modeling dynamic dependences in jump duration. [Andersen et al. \(2011\)](#) propose the use of weekday dummies and number of days to macroeconomic news announcements as information variables for modeling the probability of jumps in realized variance. However, they report insignificant coefficients for such variables. By contrast, we use lagged realized variations as exogenous variables to model the conditional hazard rate for jumps in returns. This specification is used to test the hypothesis that lagged variations in returns have a significant effect on jump probability, because the literature indicates the existence of a state-dependence of jump frequency on time-varying volatility (e.g., [Bates, 2000](#); [Eraker, 2004](#)). The augmented ACH(1,1) is specified as follows.

Let  $x_t$  denote the jump indicator, and  $x_t = 1$  when jumps occur on day  $t$  and  $x_t = 0$  otherwise.  $N_t = \sum_{i=1}^t x_t$  is the cumulative number of jumps observed as of day  $t$ . Let  $d_{N_t}$  denote the duration between the  $n$ th and the

<sup>1</sup> Note  $\left(\frac{1-k_t}{1+k_t}\right)^2 < 1$  always holds for  $k_t > 0$ .

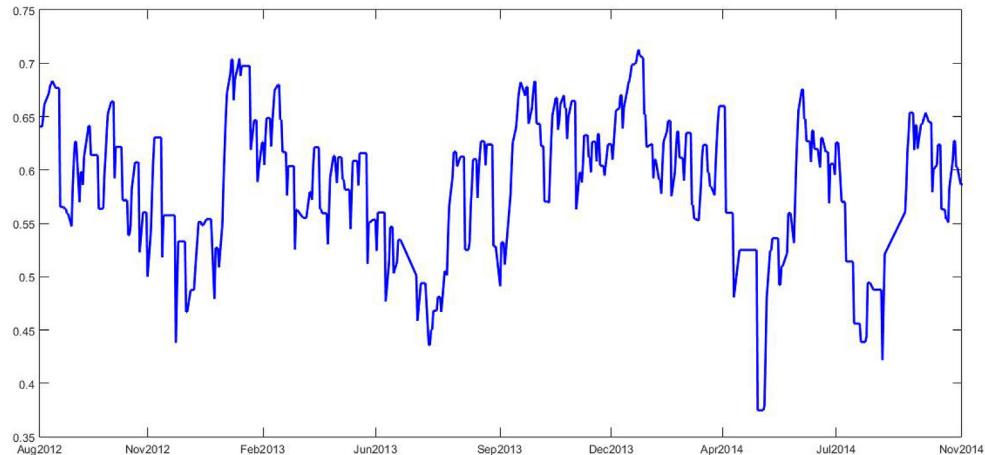


**Fig. 5.** Conditional hazard rates from the augmented ACH(1,1) model.

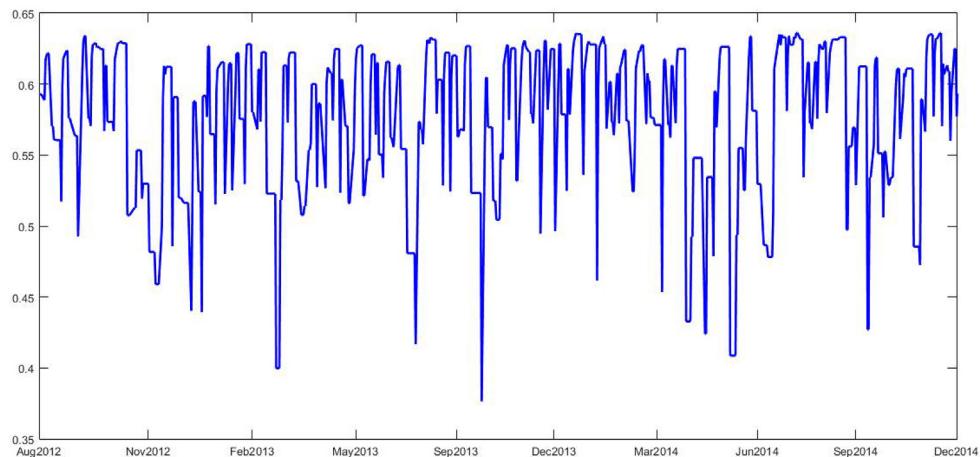
$(n - 1)$ th jump and  $\psi_{N_t} = E_t[d_{N_t+1}]$  the conditional expectation of  $d_{N_t+1}$  at day  $t$ . If no new information appears after the  $n$ th jump, we obtain

$$\psi_{N_t} = E_t[d_{N_t+1}] = \sum_{j=1}^{\infty} j h_{t+1} (1 - h_{t+1})^{j-1} = \frac{1}{h_{t+1}}, \quad (20)$$

(d) Conditional hazard rates for HSI



(e) Conditional hazard rates for SP500



(f) Conditional hazard rates for DJIA

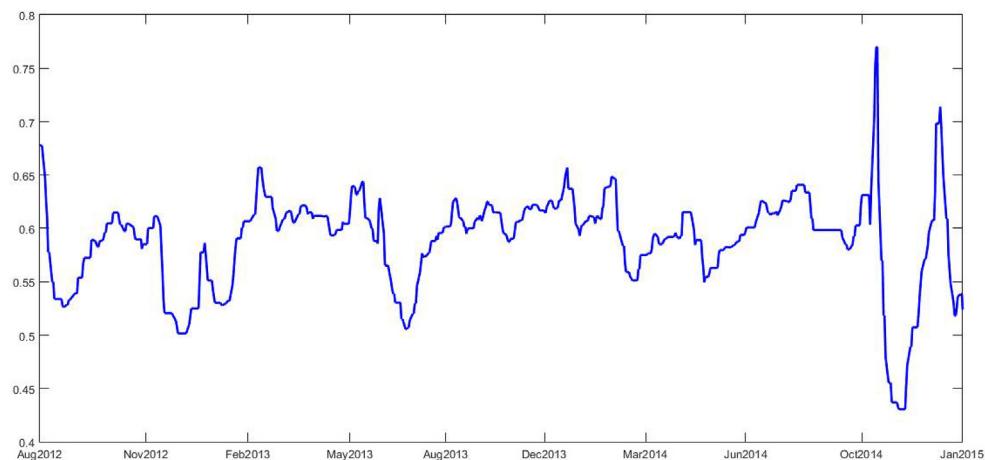


Fig. 5. (continued).

where  $h_{t+1} = \Pr_t[N_{t+1} \neq N_t]$  is the hazard rate or conditional probability of a jump on day  $t + 1$  given the information as of day  $t$ . Additionally, we set  $\psi_{N_0} = \frac{1}{h_1}$ . The basic ACH(1,1) model assumes the following specification for  $\psi_{N_t}$ :

$$\psi_{N_t} = \beta_0 + \beta_1 d_{N_t} + \beta_2 \psi_{N_{t-1}}. \quad (21)$$

To determine the effect of return variances on the occurrence of return jumps, we augment Eq. (21) through lagged continuous realized variances, which serve as new information between jumps and are

(g) Conditional hazard rates for Nasdaq

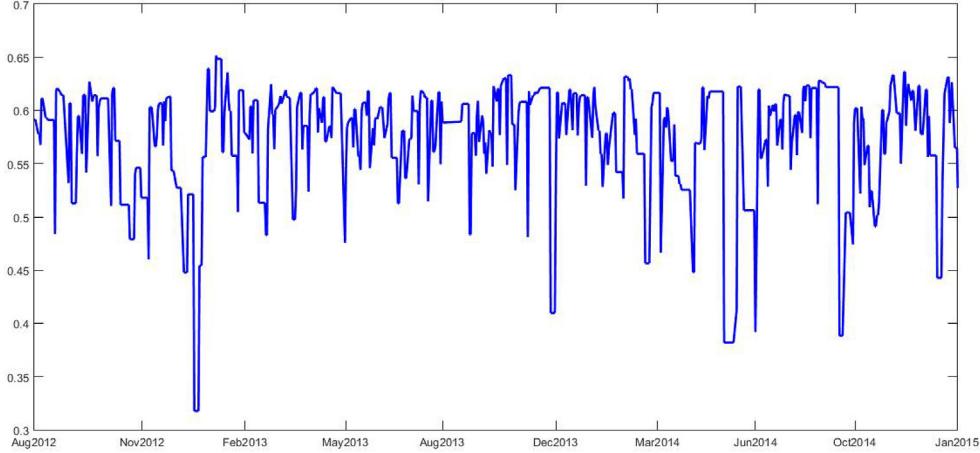


Fig. 5. (continued).

Table 5

Cox test statistics for one-day-ahead in-sample probability forecasts.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$h_t$	0.173 [0.917]	0.008 [0.996]	0.001 [1.000]	0.038 [0.981]	0.004 [0.998]	0.018 [0.991]	0.086 [0.958]
$Pr_{t+}$	0.600 [0.741]	0.144 [0.931]	0.346 [0.841]	1.239 [0.538]	0.829 [0.661]	0.280 [0.869]	0.151 [0.927]
Obs.	569	563	563	375	393	394	385

Notes: The Cox test statistics for the one-day-ahead in-sample forecasts of  $h_t$  and  $Pr_{t+}$  are reported in the first and second rows, respectively.  $p$  values are presented in square brackets.

Table 6

Cox test statistics for one-day-ahead out-of-sample probability forecasts.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$h_t$	2.179 [0.336]	1.686 [0.430]	5.620 [0.060]	4.559 [0.102]	1.367 [0.505]	8.900 [0.012]	3.178 [0.204]
$Pr_{t+}$	4.227 [0.121]	4.624 [0.099]	2.644 [0.267]	2.799 [0.247]	1.868 [0.393]	0.773 [0.680]	1.351 [0.509]
Obs.	248	249	248	240	252	239	252

Notes: The Cox test statistics for the one-day-ahead out-of-sample forecasts of  $h_t$  and  $Pr_{t+}$  are reported in the first and second rows, respectively.  $p$  values are presented in square brackets.

relevant for predicting the conditional hazard rate for these jumps. Specifically,

$$h_{t+1} = \frac{1}{\psi_{N_t} + \beta_d SCV_t + \beta_w SCV_{t-4,t} + \beta_m SCV_{t-21,t}}. \quad (22)$$

Since  $h_{t+1}$  is a probability, the maximum likelihood algorithm has to ensure it does not select a value for  $h_{t+1}$  outside (0,1). We adopt the numerical search procedure proposed by Hamilton and Jordà (2002), which sets  $h_{t+1}$  to a constant slightly below unity whenever the denominator of Eq. (22) becomes too small.<sup>2</sup> Given  $h_{t+1}$  and the jump indicator  $x_{t+1}$ , the probability of observing  $x_{t+1}$  is

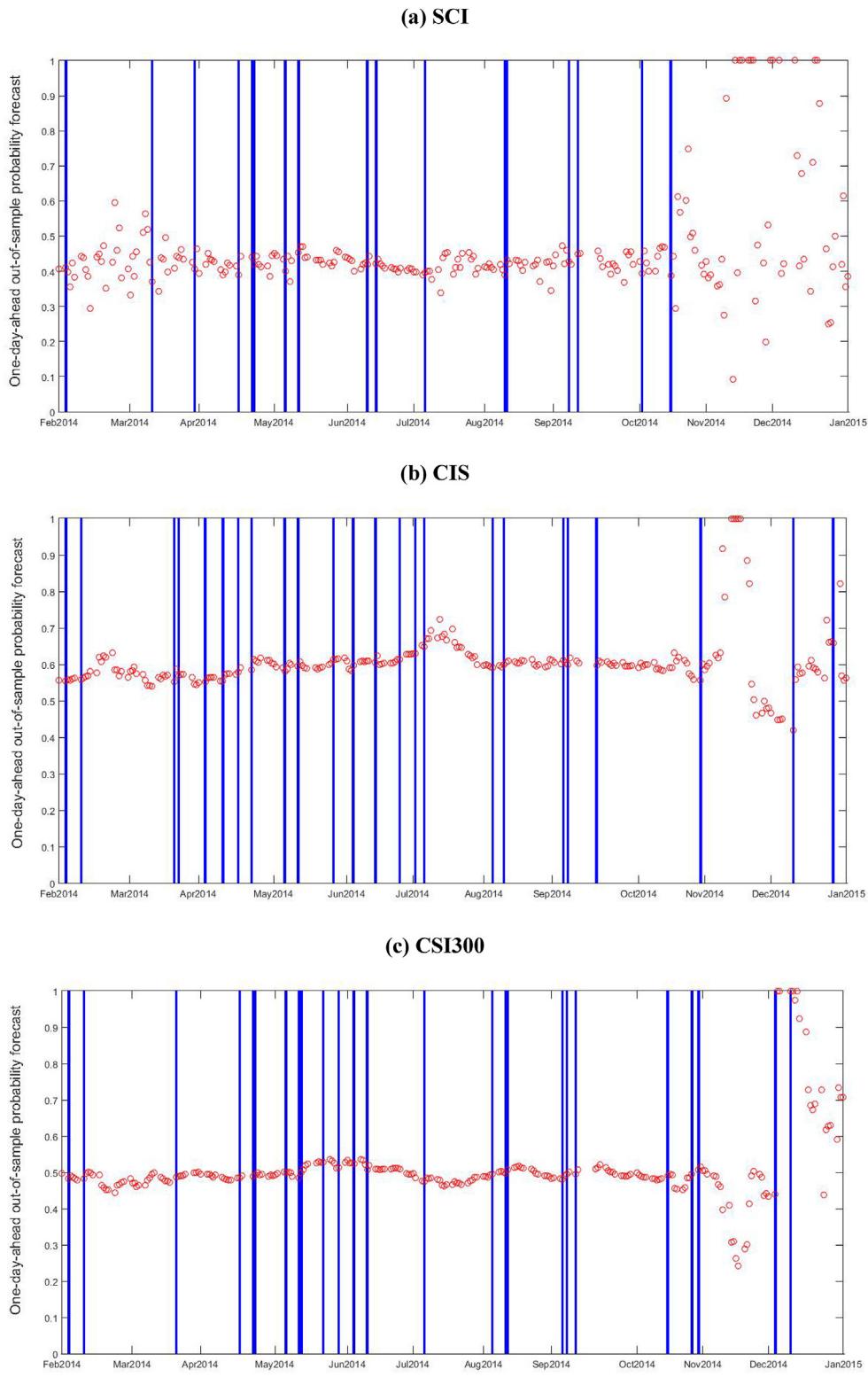
$$p(x_{t+1}|\theta) = (h_{t+1})^{x_{t+1}} (1 - h_{t+1})^{1-x_{t+1}}, \quad (23)$$

where  $\theta = \{\beta_0, \beta_1, \beta_2, \beta_d, \beta_w, \beta_m\}$ . Thus, the conditional log likelihood is

$$L(\theta) = \sum_{t=1}^T [x_t \ln h_t + (1 - x_t) \ln(1 - h_t)]. \quad (24)$$

Eq. (24) is then maximized numerically with respect to  $\theta$ . We follow Hamilton and Jordà (2002) by imposing on parameters restrictions  $\beta_1 \geq 0, 0 \leq \beta_2 \leq 1$  to ensure the robustness of the numerical maximization. The MLEs for the augmented ACH(1,1) are shown in Table 4. The lagged realized variances indicate a considerable impact on jump occurrence, and there is a positive autoregressive relationship in the jump durations for most indices. Our estimation results that jump durations, or hazard rates, exhibit distinct persistence indicate jump clustering and are in line with the recent empirical findings (e.g., Ait-Sahalia et al., 2015; Andersen et al., 2011; Fulop et al., 2015). To effectively illustrate the dynamics of conditional hazard rates for jumps  $h_t$ , Fig. 5 plots the time series of the estimated hazard rates for each equity index. The figure reveals a general pattern in that daily jumps seem more than three times as likely for the U.S. equity indices compared to their counterparts in Mainland China over most of the sample period. We believe this is partly due to the daily price limit, that is, a daily price up/down limit of 10% imposed on trading of stocks on the Shanghai and Shenzhen Stock Exchange.

<sup>2</sup> See Eq. (8) in Hamilton and Jordà (2002).



**Fig. 6.** One-day-ahead probability forecasts for positive return jumps during the out-of-sample period. The circles denote probability forecasts and the solid bars the days positive jumps occur.

## 5. Jump signs and occurrence forecasting

Profitable trading and asset allocation strategies are all based on the successful forecasting of market direction or return signs. This section focuses on the accuracy of forecasting the signs of jumps that can cause

considerable return variation. Since determining jump signs relies on the forecast of  $h_t$  (i.e., hazard rate for jumps), we also examine whether the augmented ACH(1,1) model can accurately forecast jump occurrence. Specifically, the conditional probabilities of jump signs are

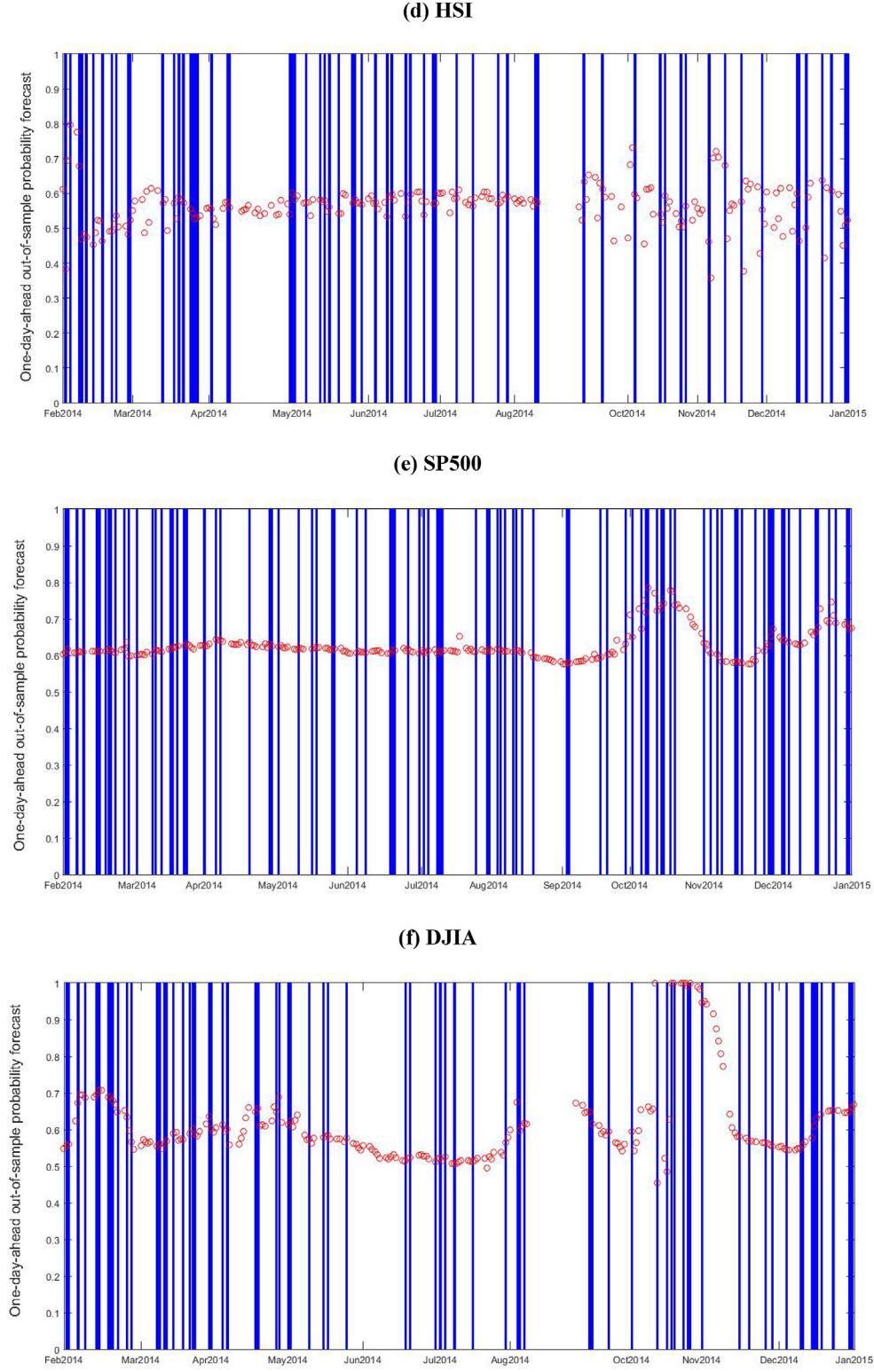


Fig. 6. (continued).

$$\Pr(J_t > 0, x_t = 1) = \frac{1}{1 + k_t} \cdot h_t, \quad \Pr(J_t < 0, x_t = 1) = \frac{k_t}{1 + k_t} \cdot h_t. \quad (25)$$

For the mutually exclusive events in Eq. (25), we forecast only the probability of a positive jump, which is hereafter referred to as  $\Pr_{t+}$ .

We statistically test the accuracy of our forecasts for  $h_t$  and  $\Pr_{t+}$  by using the probability forecast evaluation approach of Cox (1958). Specifically, let  $Y_1, Y_2, \dots, Y_n$  be random variables taking 0 or 1 and  $p_1, p_2, \dots,$

$p_n$  a set of numbers and  $0 \leq p_t \leq 1$  for  $t = 1, 2, \dots, n$ . Cox (1958) proposes a statistic for testing the null hypothesis:

$$\Pr(Y_t = 1) = p_t, \quad t = 1, 2, \dots, n. \quad (26)$$

Under the null hypothesis, the distribution of the following statistic is nearly  $\chi^2$  with two degrees of freedom:

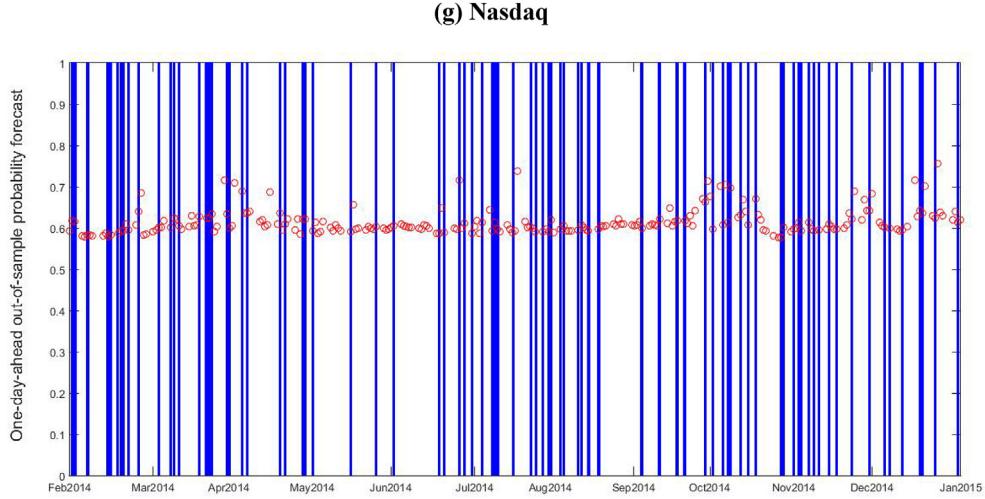


Fig. 6. (continued).

**Table 7**

Diebold-Mariano test statistics for one-day-ahead in-sample jump size density forecasts.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$D_{jump}$	-0.974	-1.243	-1.082	-1.095	-0.964	-0.779	-1.110
$D_{GARCH}$	-2.668	-2.895	-2.696	-1.312	-1.050	-1.025	-1.237
$t_{jump, GARCH}$	2.584	4.031	3.786	3.678	0.821	2.383	1.383
	[0.010]	[0.000]	[0.000]	[0.000]	[0.412]	[0.017]	[0.167]

Notes: The one-day-ahead in-sample average predictive likelihoods produced by our jump model and ARMA(1,1)-GARCH(1,1) are reported in the first and second rows, respectively. The third row reports the Diebold-Mariano test statistics, with the  $p$  values provided in square brackets.

**Table 8**

Diebold-Mariano test statistics for one-day-ahead out-of-sample jump size density forecasts.

	SCI	CIS	CSI300	HSI	SP500	DJIA	Nasdaq
$D_{jump}$	-0.537	-0.932	-0.790	-1.128	-1.214	-0.722	-1.101
$D_{GARCH}$	-2.351	-4.837	-4.625	-1.280	-0.985	-1.010	-1.302
$t_{jump, GARCH}$	4.979	2.119	2.002	1.458	-0.784	3.570	3.800
	[0.000]	[0.034]	[0.045]	[0.145]	[0.433]	[0.000]	[0.000]

Notes: The one-day-ahead out-of-sample average predictive likelihoods produced by our jump model and ARMA(1,1)-GARCH(1,1) are reported in the first and second rows, respectively. The third row reports the Diebold-Mariano test statistics, with the  $p$  values provided in square brackets.

$$(x - E(X), y - E(Y)) \begin{pmatrix} V(X) & C(X, Y) \\ C(X, Y) & V(Y) \end{pmatrix}^{-1} \begin{pmatrix} x - E(X) \\ y - E(Y) \end{pmatrix} \quad (27)$$

where

$$x = \sum X_t, \quad y = \sum Y_t,$$

$$X_t = \begin{cases} \log(2p_t) & \text{when } Y_t = 1; \\ \log[2(1-p_t)] & \text{when } Y_t = 0, \end{cases}$$

$$E(X) = n \log 2 + \sum p_t \log p_t + \sum (1-p_t) \log(1-p_t),$$

$$V(X) = \sum p_t (1-p_t) \{\log[p_t/(1-p_t)]\}^2,$$

$$E(Y) = \sum p_t,$$

$$V(Y) = \sum p_t (1-p_t),$$

$$C(X, Y) = \sum p_t (1-p_t) \log[p_t/(1-p_t)].$$

The first rows of Tables 5 and 6 report the Cox test statistics for the

one-day-ahead in- and out-of-sample forecasts of  $h_t$ , and  $p$  values are presented in square brackets. The obtained forecasts are based on an expanding window.  $Y_t = 1$  when jumps occur on day  $t$  and the null hypothesis is

$$\Pr(Y_t = 1) = h_t. \quad (28)$$

Similarly, the second rows of Tables 5 and 6 report the results for  $\Pr_{t+}$  and the null hypothesis is

$$\Pr(Y_t = 1) = \Pr_{t+}, \quad (29)$$

where  $Y_t = 1$  denotes positive jumps occur on day  $t$ . If the proposed model is appropriate for forecasting the occurrence and signs of jumps, that is, the null hypotheses in Eqs. (28) and (29) are true, any Cox test statistic in Tables 5 and 6 should be close to 0 from the right with the associated  $p$  value close to 1.

The statistics demonstrate the augmented ACH can produce accurate in-sample forecasts for jump occurrence because all  $p$  values are close to 1 in the first row of Table 5. For out-of-sample forecasts, only two of the seven equity indices report  $p$  values below 0.1 in the first row of Table 6. We obtain similar results for forecasting  $\Pr_{t+}$ . The  $p$  values for in-sample testing are all above 0.5, and the lowest  $p$  value is 0.099 in out-of-sample tests. Therefore, our jump size model can be used to determine the signs

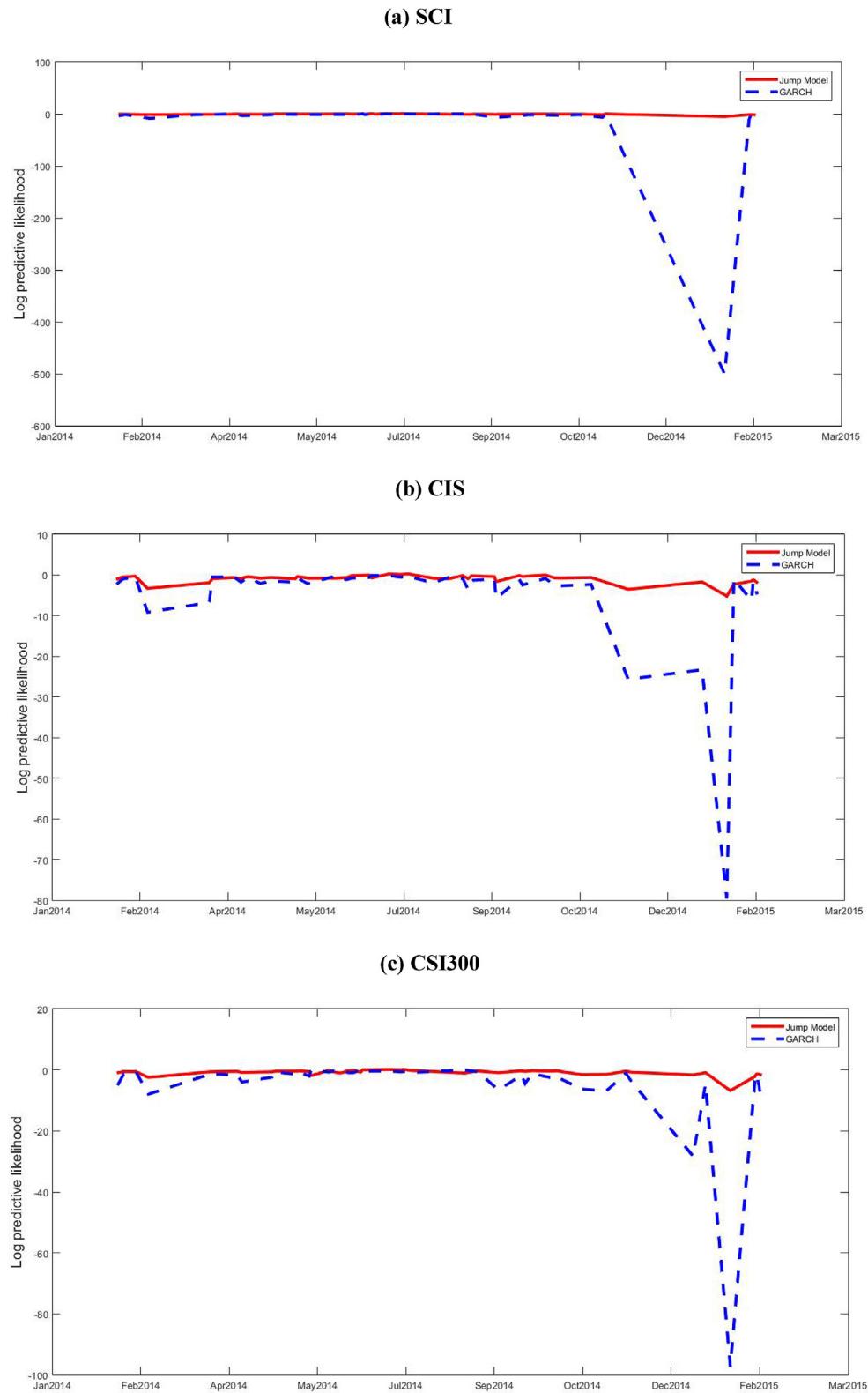


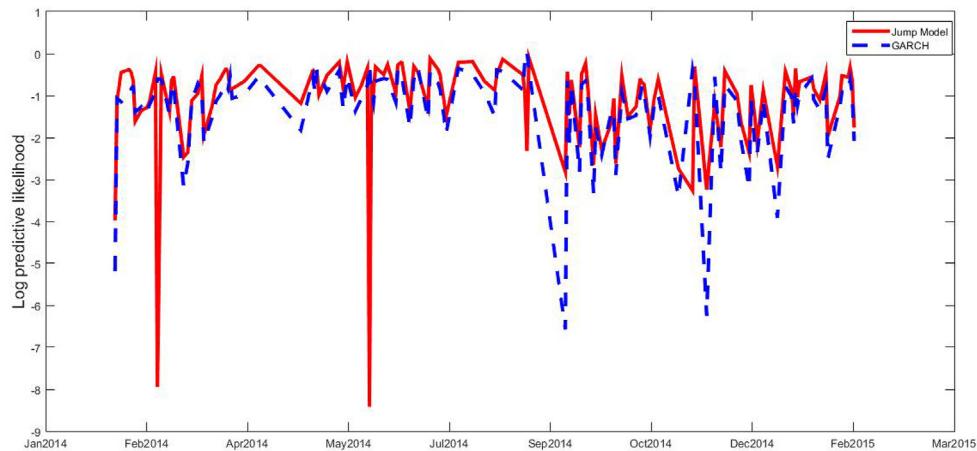
Fig. 7. Time series of one-day-ahead log predictive likelihoods over the out-of-sample period.

of return jumps. To effectively illustrate the model's forecasting performance, Fig. 6 also plots one-day-ahead probability forecasts for positive jumps during the out-of-sample period, showing that it is possible to detect jumps and their signs in advance to some extent.

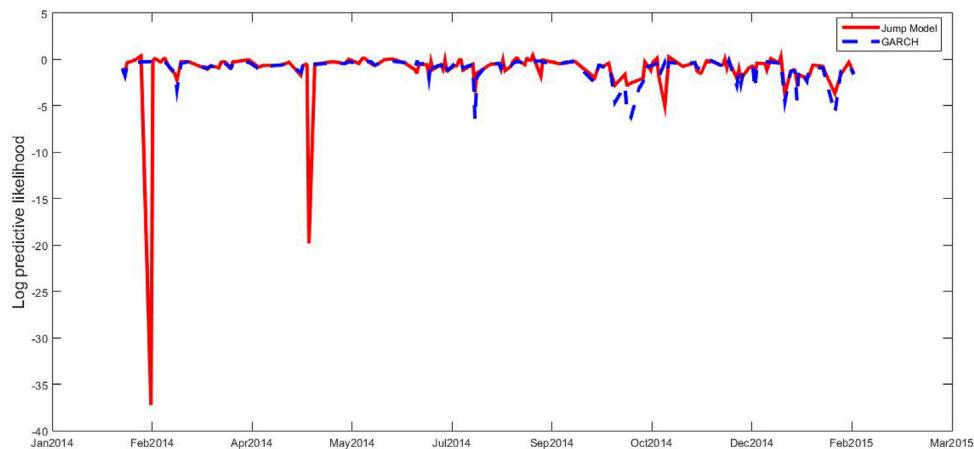
## 6. Jump size density forecast

Many problems in finance require a full characterization of the distribution of returns. As such, this section examines whether our jump size model  $f(J_t)$  in Eq. (19), based on realized variance, could enhance

(d) HSI



(e) SP500



(f) DJIA

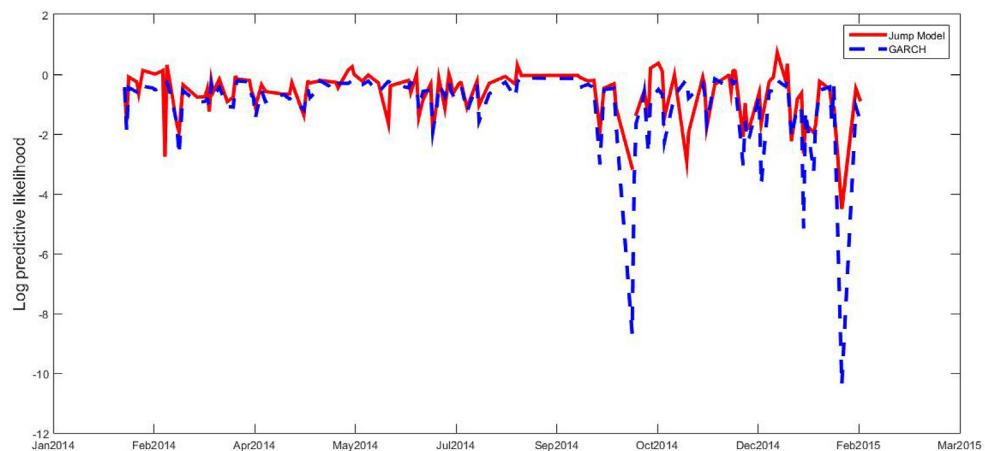


Fig. 7. (continued).

forecasting the conditional density of jump size, which is critical to return density as a whole.

One recently proposed approach for assessing the accuracy of a model's density forecast is predictive likelihood (e.g., Amisano and Giacomini, 2007; Diebold and Mariano, 1995; Maheu and McCurdy, 2011). The average predictive likelihood produced by model  $M$  over

observations  $t = 1, 2, \dots, T$  is

$$D_M = \frac{1}{T} \sum_{t=1}^T \ln f_{M,t}(J_{t+1}), \quad (30)$$

where  $f_{M,t}(\cdot)$  is the one-day-ahead predictive density provided by model

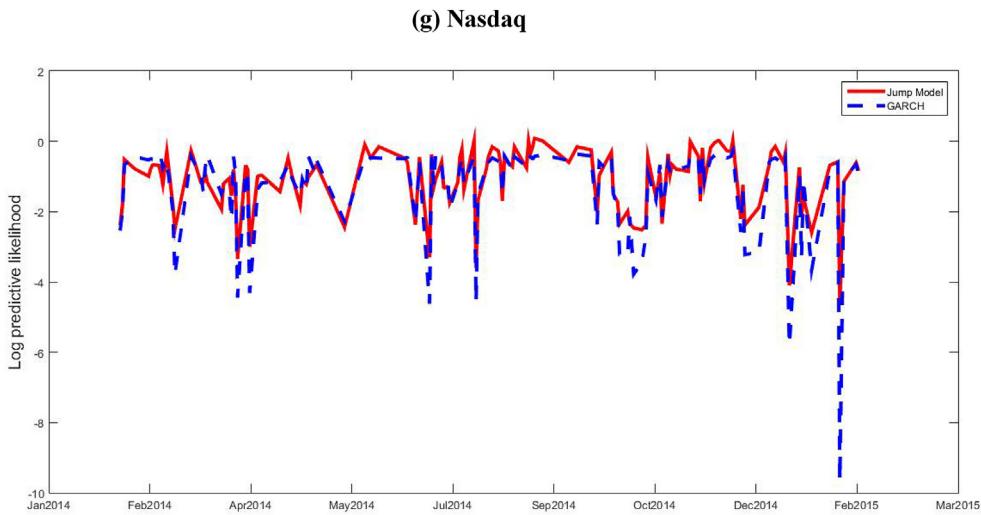


Fig. 7. (continued).

$M$  at time  $t$ , evaluated at realized jump  $J_{t+1}$ . The models that provide a more accurate density forecast produce a higher  $D_M$  value. To statistically compare the density forecast accuracies of alternative models, Maheu and McCurdy (2011) use the following Diebold and Mariano (1995) test statistic, based on the work of Amisano and Giacomini (2007):

$$t_{AB} = \frac{D_A - D_B}{\frac{1}{\sqrt{T}}\widehat{\sigma}_{AB}}, \quad (31)$$

where  $\widehat{\sigma}_{AB}$  is the Newey-West long-run sample variance estimate for  $d_t = \ln f_{A,t}(J_{t+1}) - \ln f_{B,t}(J_{t+1})$ . Under the null hypothesis of equal performance for models  $A$  and  $B$ ,  $t_{AB}$  is asymptotically standard normal, for which a higher positive value rejects the null hypothesis in favor of model  $A$ .

Tables 7 and 8 present the results of the Diebold-Mariano test for the one-day-ahead in-sample and out-of-sample jump size density forecasts. The benchmark model for comparison is ARMA(1,1)-GARCH(1,1), where  $D_{jump}$  represents the average predictive likelihood produced by our jump size model and  $D_{GARCH}$  that produced by the benchmark. The third rows of Tables 7 and 8 present the values of the Diebold-Mariano test statistic  $t_{jump, GARCH}$  with the  $p$  values provided in square brackets. For in-sample forecasts, the test statistics are all positive and in favor of our model. The  $p$  values are below the 5% significance level for five out of the seven equity indices. We obtain similar results for out-of-sample forecasts, where only two of the seven equity indices report  $p$  values higher than 0.05. To graphically represent how successful our out-of-sample forecasting is, Fig. 7 also plots the time series for one-day-ahead log predictive likelihoods  $\ln f_{jump,t}$  and  $\ln f_{GARCH,t}$  over the out-of-sample period. The graphical evidence is consistent with the test statistics in Table 8, that is, our jump model produces, on average, higher predictive likelihoods than ARMA(1,1)-GARCH(1,1), except for the Hang Seng Index and SP500. This indicates we outperform the benchmark model and provide superior density forecasts.

## 7. Conclusion

This paper proposes a simple realized-variation-based model for forecasting return jumps. The asymmetry of jump size density is modeled as a function of lagged realized variances, where the conditional mean, variance, and skewness of jump sizes can evolve dynamically over time. We also augment the basic ACH model using realized variances to predict the conditional hazard rate for return jumps. The empirical application of our model to equity indices demonstrates asymmetry exists in jump size density and realized variances considerably affect both jump size and occurrence. The performance of our model is statistically evaluated in

terms of forecasting jump signs and the distribution density. The results demonstrate the model can provide accurate in- and out-of-sample probability forecasts for both jump occurrence and signs. Compared with the benchmark ARMA(1,1)-GARCH(1,1) model, our model produces superior density forecasts for jump size, both in- and out-of-sample.

Modeling jumps is critical to characterize the distribution of returns. As such, our results suggest a simple procedure of modeling asset returns by decomposing them into jump and continuous parts nonparametrically by using high-frequency data. Combining the results with the recent findings of normality for returns (e.g., Fleming and Payne, 2011), our jump model can be extended to a distribution for returns easily applied for return sign forecasting, risk management, and asset allocation. For example, in the econometrics of portfolio choice problems, the precision of the plug-in estimator depends on the precision of the returns' mean and volatility estimates (Brandt, 2009). Specifically, jointly modeling the jump and continuous component of returns based on our model holds promise for achieving a more reliable characterization of returns distribution and improving estimation precision for portfolio management. Moreover, our results are suggestive of option modeling with realized jumps and volatility (e.g., Christoffersen et al., 2014, 2015; Corsi et al., 2013), to elucidate the empirical inconsistencies regarding the role of jumps in option pricing.

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