

# Continuous-Time Ultra-Wideband-Inertial Fusion

Kailai Li <sup>1</sup>, Ziyu Cao <sup>2</sup>, and Uwe D. Hanebeck <sup>3</sup>

<sup>1</sup>Department of Electrical Engineering, Linköping University, Sweden

<sup>2</sup>Department of Informatics, Karlsruhe Institute of Technology, Germany

**This work was published in IEEE Robotics and Automation Letters, vol. 8, no. 7, pp. 4338-4345, July 2023**



## Abstract

We introduce a novel framework of continuous-time ultra-wideband-inertial sensor fusion for online motion estimation. Quaternion-based cubic cumulative B-splines are exploited for parameterizing motion states continuously over time. Systematic derivations of analytic kinematic interpolations and spatial differentiations are further provided. Based thereon, a new sliding-window spline fitting scheme is established for asynchronous multi-sensor fusion and online calibration. We conduct a dedicated validation of the quaternion spline fitting method, and evaluate the proposed system, SFUISE (spline fusion-based ultra-wideband-inertial state estimation), in real-world scenarios using public data set and experiments. The proposed sensor fusion system is real-time capable and delivers superior performance over state-of-the-art discrete-time schemes. We release the source code and own experimental data at <https://github.com/ASIG-X/SFUISE>.

## Background

Recent advancements in UWB technology have offered promising alternative solutions to localization in GPS-denied environments, ideal for indoor scenarios due to their low cost, scalability, and lightweight nature. However, technical issues such as non-line-of-sight, multipath conditions, and non-Gaussian noise patterns persist, making high-performance and reliable UWB-based tracking still very challenging in practice. These issues can be mitigated by deploying inertial measurement units (IMUs) to provide instantaneous and higher-order motion information. In this work, we focus on continuous-time state estimation based on UWB-inertial fusion.

## Contributions

- SFUISE - Spline Fusion-based Ultra-wideband-Inertial State Estimation
- Quaternion-based cubic cumulative B-splines for continuous-time 6-DoF pose representation + Analytic kinematic interpolations and Jacobians w.r.t. control points
- Efficient sliding-window spline fitting scheme for UWB-inertial fusion at raw time instants in real time + option for online calibration
- Extensive real-world evaluations for UWB-inertial tracking (ToA & TDoA)
- Open-source implementation together with own experimental datasets

## State Estimation on B-Splines

We deploy cubic cumulative B-splines for continuous-time state representation

$$\underline{x}(t) = [\underline{r}(t)^\top, \underline{s}(t)^\top, \underline{b}(t)^\top]^\top, \quad \text{with control points} \quad \underline{x} = [\underline{q}^\top, \underline{p}^\top, \underline{\ell}^\top]^\top \in \mathbb{R}^{13}.$$

States at arbitrary time instants can be interpolated w.r.t. a local set of knots as follows

$$\begin{array}{ll} \text{quaternion state} & \underline{r}(t) = \underline{q}_{i-2} \otimes \prod_{j=1}^3 \text{Exp}_{\mathbb{H}}(\lambda_j(u) \underline{\delta}_j), \\ \text{linear state} & \underline{s}(t) = \underline{p}_{i-2} + \sum_{j=1}^3 \lambda_j(u) \underline{\delta}_j. \end{array}$$

## Sliding-Window Spline Fitting

For online performance, we formulate the sensor fusion problem in the form of optimization over a window of recent  $\tau_w$  knots  $\mathcal{X}_w = [\mathbf{x}_1, \dots, \mathbf{x}_{\tau_w}] \in \mathbb{R}^{13 \times \tau_w}$ , namely,

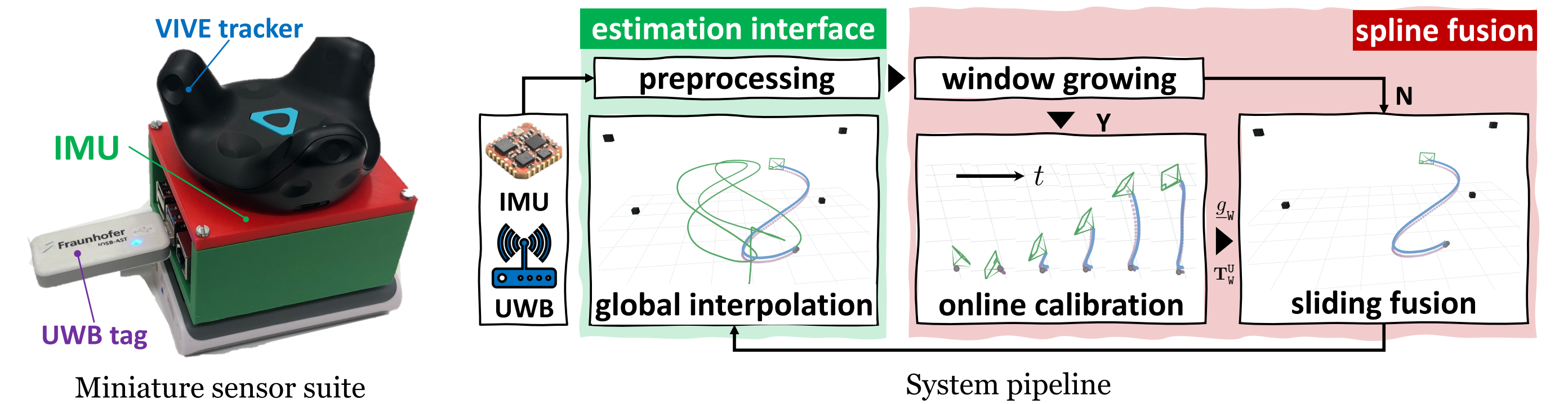
$$\mathcal{X}_w^* = \arg \min_{\mathcal{X}_w} \mathcal{F}(\mathcal{X}_w).$$

The objective function incorporating UWB and IMU residuals is formulated as

$$\mathcal{F}(\mathcal{X}_w) = v_u \sum_{i=1}^m \left\| \mathcal{E}_u(\mathcal{X}_w, \hat{z}_{u,i}) \right\|_{\mathbf{C}_u}^2 + v_i \sum_{k=1}^n \left\| \mathcal{E}_i(\mathcal{X}_w, \hat{z}_{i,k}) \right\|_{\mathbf{C}_i}^2.$$

## System Design

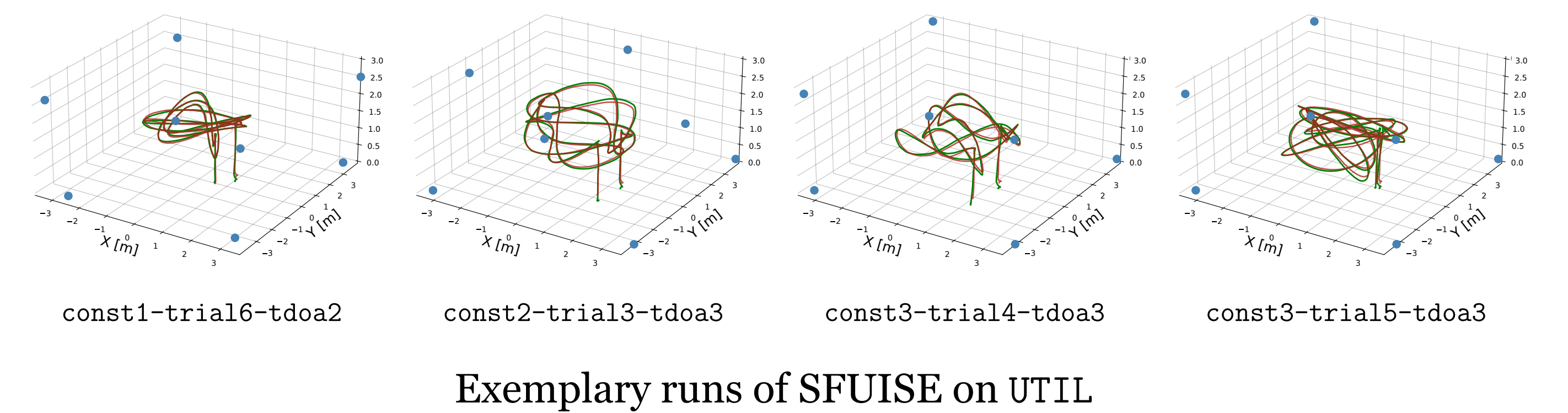
The system SFUISE composes an estimation interface and a functional core of sliding-window spline fitting. As UWB and IMU measurements arrive, a cumulative B-spline composing the continuous-time states is fitted first over a window-growing stage together with the online calibrations and afterward over sliding windows. Resulting estimates are sent back to estimation interface for visualization.



To evaluate the proposed scheme on UWB-inertial fusion, a miniature sensor suite based on time-of-arrival (ToA) ranging has been instrumented. It is composed of a UWB tag and an IMU, both mounted to a Raspberry Pi for sensor coordination and data recording. An additional VIVE tracker is added to provide the ground truth.

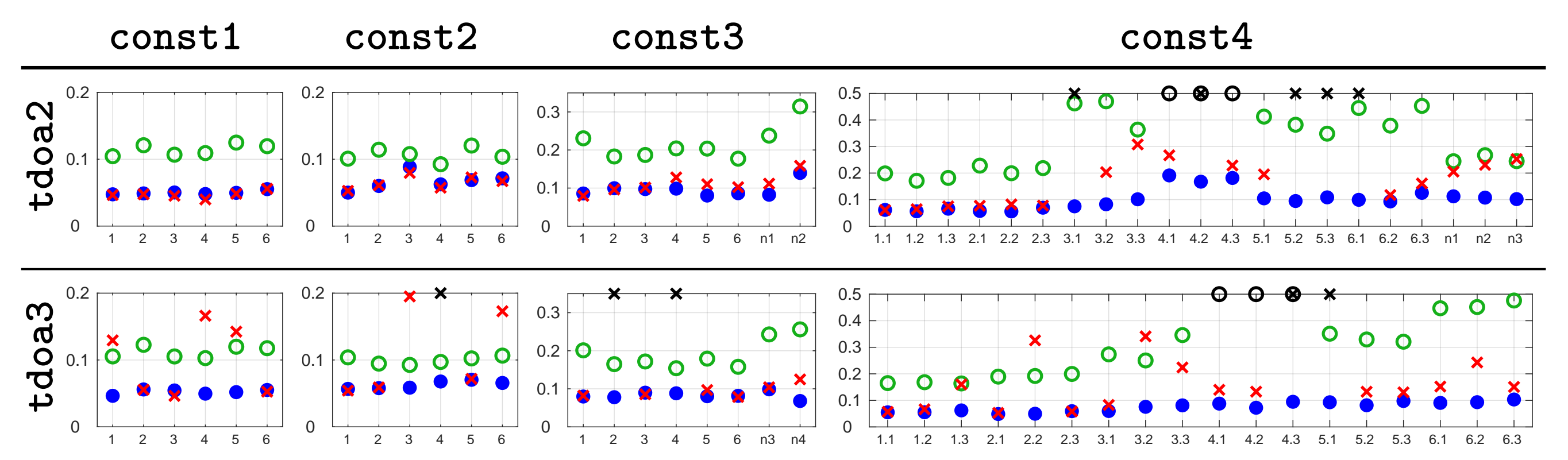
## Evaluation & Conclusion

SFUISE has been evaluated in real-world scenarios based on the public dataset UTIL ([utiasdsl.github.io/util-uwb-dataset](https://utiasdsl.github.io/util-uwb-dataset)) and own experiments with our miniature suite, incorporating both ToA and TDoA UWB data.



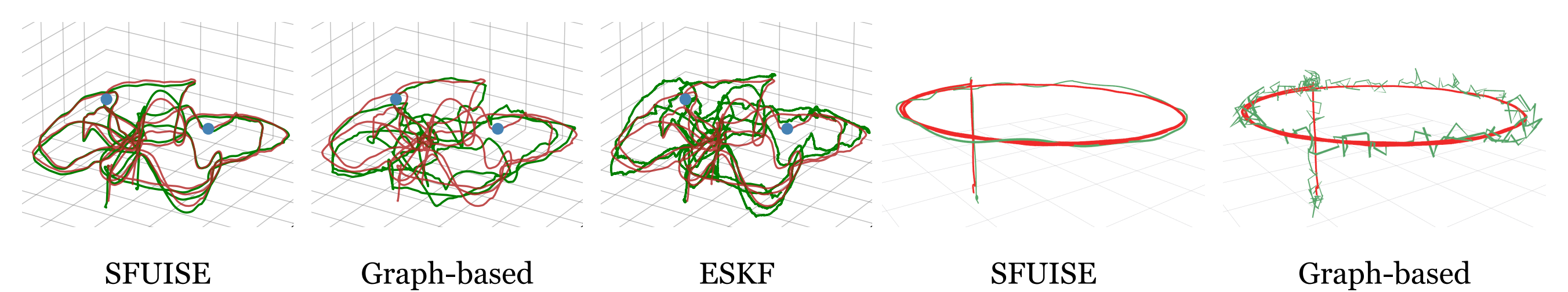
### Exemplary runs of SFUISE on UTIL

Two major discrete-time sensor fusion schemes of state of the art are considered for comparison: (1) an own composition of graph-based UWB-inertial fusion system (similar to VINS-Mono) and (2) an error-state Kalman filter (ESKF) provided by UTIL.



APEs (RMSE) obtained from benchmark on all sequences of UTIL dataset. Results from SFUISE are plotted with  $\bullet$ . Results from ESKF and the graph-based methods are given by  $\circ$  and  $\times$ , with  $\circ$  and  $\times$  indicating tracking fails, respectively.

The strength of spline fusion can be further highlighted in UWB-only tracking. The proposed spline-based scheme delivers motion estimates of inherent kinematic consistency in an efficient representation. We look forward to further establishing the continuous-time paradigm in areas of planning, control, odometry and mapping, etc.



## Comparisons with discrete-time methods

UWB-only tracking

Find our open-source implementation & Visit our group's homepage:



**Acknowledgement:** German Federal Ministry of Education and Research (BMBF) under grant POMAS FKZ 01IS17042, German Research Foundation (DFG) under grant HA 3789/25-1, Swedish Research Council under grant Scalable Kalman Filters, Linköping University under the ZENITH grant Computational Agile Sensing and Inference for Intelligent Systems.