LISTA II - AVLC
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① $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$, calcule:

a) adj A = a matriz adjunta de A, dada por $\bar{A}$, é a matriz formada pela trasposta da matriz dos cofatores de A.

* COFATORES $\left((-1)^{i+j} \cdot \det A_{ij} \right) \Rightarrow \begin{bmatrix} C(a_{11}) & C(a_{12}) & C(a_{13}) \\ C(a_{21}) & C(a_{22}) & C(a_{23}) \\ C(a_{31}) & C(a_{32}) & C(a_{33}) \end{bmatrix}$ Representação de C.

$\mathrm{COF}(a_{11}) = (-1)^{1+1} \cdot \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5$

$\mathrm{COF}(a_{12}) = (-1)^{1+2} \begin{vmatrix} 0 & 1 \\ 5 & 3 \end{vmatrix} = 6$

$\mathrm{COF}(a_{13}) = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} = -10$

$\mathrm{COF}(a_{21}) = (-1)^{2+1} \begin{vmatrix} 1 & -3 \\ 1 & 3 \end{vmatrix} = -6$

$\mathrm{COF}(a_{22}) = (-1)^{2+2} \cdot \begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} = 21$

$\mathrm{COF}(a_{23}) = (-1)^{2+3} \cdot \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} = 3$

$\mathrm{COF}(a_{31}) = (-1)^{3+1} \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 7$

$\mathrm{COF}(a_{32}) = (-1)^{3+2} \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = -2$

$\mathrm{COF}(a_{33}) = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 4$

$$\Downarrow$$

$$\begin{bmatrix} 5 & 6 & -10 \\ -6 & 21 & 3 \\ 7 & -2 & 4 \end{bmatrix} \text{ Matriz C.}$$

$$\Downarrow$$

A ADJUNTA = C^T

-> transposta de C, matriz na qual linhas se tornam colunas.

$$\begin{bmatrix} 5 & -6 & 7 \\ 6 & 21 & -2 \\ -10 & 3 & 4 \end{bmatrix}$$

ADJUNTA $(\bar{A})$ = C^T

b) $\det A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 5 & 1 \end{matrix} =$

$(12 + 5 + 0) - (-30 + 2 + 0) =$

$11 - (-28) = 45$ //

c) $A^{-1} = \dfrac{1}{\det} \cdot \mathrm{Adj}\, A$

$A^{-1} = \dfrac{1}{45} \cdot \begin{bmatrix} 5 & -6 & 7 \\ 6 & 21 & -2 \\ -10 & 3 & 4 \end{bmatrix} =$

$A^{-1} = \begin{bmatrix} 1/9 & -2/15 & 7/45 \\ 6/45 & -6/45 & -2/45 \\ -2/9 & 1/15 & 4/45 \end{bmatrix}$

② $\begin{vmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{vmatrix}$

a) pela definição:

$\det[a_{ij}] = \sum_{p} (-1)^J a_{j_1 1} a_{j_2 2} \dots a_{j_n n}$, que utiliza as permutações de J.

Assim, calculando o determinante pela regra de Sarrus, temos:

$\begin{vmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{vmatrix} \begin{matrix} 2 & 0 \\ 3 & 0 \\ 4 & -3 \end{matrix}$ $\quad DET = 9 - (-12) = 21$ //

$(0+0+9) - (0+(-12)+0)$

b) pelo desenvolvimento de La Place:

$\begin{vmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{vmatrix}$

* DET = soma entre os elementos multiplicados pelo seu cofator.

$DET = 0 \cdot Cof(a_{12}) + 0 \cdot Cof(a_{22}) + (-3) \cdot Cof(a_{32})$

$DET = 0 + 0 + (-3) \cdot (-1)^{3+2} \cdot \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$

$DET = 3 \cdot (4 - (-3)) = 21$ //

③ a) $\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} = -2 + 3 = 1$ //

$0 - 2 = -2 \qquad 3 - 0 = 3$

b) $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} = 4 - 1 = 3$ //

④ $\begin{cases} 3x + 5y + 12z + w = -3 \\ x + y + 4z - w = -6 \\ 2y + 2z + w = 5 \end{cases}$

* matriz associada:

$$\begin{bmatrix} 3 & 5 & 12 & 1 & -3 \\ 1 & 1 & 4 & -1 & -6 \\ 0 & 2 & 2 & 1 & 5 \end{bmatrix}$$

a)

* escalonamento =>

① $\frac{1}{3} \cdot R_1 \to R_1$ $\begin{bmatrix} 1 & 5/3 & 4 & 1/3 & -1 \\ 1 & 1 & 4 & -1 & -6 \\ 0 & 2 & 2 & 1 & 5 \end{bmatrix}$

② $R_2 - R_1 \to R_2$ $\begin{bmatrix} 1 & 5/3 & 4 & 1/3 & -1 \\ 0 & -2/3 & 0 & -2/3 & -5 \\ 0 & 2 & 2 & 1 & 5 \end{bmatrix}$

③ $\frac{-3}{2} \cdot R_2 \to R_2$ $\begin{bmatrix} 1 & 5/3 & 4 & -1/3 & -1 \\ 0 & 1 & 0 & 1 & 15/2 \\ 0 & 2 & 2 & 1 & 5 \end{bmatrix}$

④ $\frac{1}{2} R_3 \to R_3$ $\begin{bmatrix} 1 & 5/3 & 4 & -1/3 & -1 \\ 0 & 1 & 0 & 1 & 15/2 \\ 0 & 1 & 1 & 1/2 & 5/2 \end{bmatrix}$

⑤ $R_3 - 1 \cdot R_2 \to R_3$ $\begin{bmatrix} 1 & 5/3 & 4 & -1 & -1 \\ 0 & 1 & 0 & 1 & 15/2 \\ 1 & 0 & 1 & -3/2 & -5 \end{bmatrix}$

⑥ $R_1 - 4 \cdot R_3 \to R_1$ $\begin{bmatrix} 1 & 5/3 & 0 & 19/3 & 19 \\ 0 & 1 & 0 & 2 & 15/2 \\ 0 & 0 & 0 & -3/2 & -5 \end{bmatrix}$

... →

⑦ Forma Final =>

$R_1 - 5/3 \cdot R_2 \to R_1$ $\begin{bmatrix} 1 & 0 & 0 & 0 & 13/2 \\ 0 & 1 & 0 & 1 & 15/2 \\ 0 & 0 & 1 & -1/2 & -5 \end{bmatrix}$ $\Rightarrow \begin{cases} x = 13/2 \\ y + w = 15/2 \\ z + \frac{w}{2} = -5 \end{cases}$

$x = 13/2$

$y = 15/2 - w$

$z = -5 + \frac{w}{2}$

para qualquer "w"!

R = o sistema é possível e indeterminado. pois x é a única variável fixa.

b)

$z = -5 + \frac{w}{2}$

$2z + kw = 9$

$z = \frac{9 - kw}{2} = -5 + \frac{w}{2}$

$\frac{-10 + w}{\cancel{2}} = \frac{9 - kw}{\cancel{2}}$

$\boxed{k = \frac{19}{w} - 1}$ → $k = -1$, pois $\frac{19}{w} = 0$ é impossível.