— Simulado — Integrais:

1º) Resolva a integral $\int_0^{\pi} x^2 \operatorname{sen} x \, dx$

2º) Calcule $\int_0^{2\pi} \operatorname{sen}^3 x \cos^4 x \, dx$

3º) Resolva a integral indefinida: $\int \frac{x^3}{(1+x^2)^2} \, dx$

4º) Calcule a área do semicírculo limitado pela equação $y = \sqrt{4 - x^2}$ e pelo eixo x.

1º) $\int_0^{\pi} \underbrace{x^2}_{u} \underbrace{\text{sen}\, x\, dx}_{dv}$ $\Rightarrow du = 2x\, dx \,,\; v = -\cos x$

Obs.: $\int_a^b u\, dv = (u.v)\Big|_a^b - \int_a^b v\, du$

$$\Rightarrow \int_0^{\pi} x^2 \text{sen}\, x\, dx = \left(x^2.(-\cos x)\right)\Big|_0^{\pi} - \int_0^{\pi} (-\cos x).2x\, dx$$

$$= \underbrace{\left(x^2(-\cos x)\right)\Big|_0^{\pi}} + 2\int_0^{\pi} \underbrace{\cos x.}_{dv}\, \underbrace{x}_{u}\, dx \Rightarrow \begin{array}{l} du = 1\, dx \\ v = \text{sen}\, x \end{array}$$

Integrando por partes de novo:

$$\int_0^{\pi} x^2 \text{sen}\, x\, dx = \left(\pi^2.(-\cos(\pi))\right) - (0) + 2\left[(x.\text{sen}\, x)\Big|_0^{\pi} - \int_0^{\pi} \text{sen}\, x\, dx\right]$$

$$= \pi^2 + 2\left[(\pi.\underbrace{\text{sen}\,\pi}_{0} - 0) - (-\cos(x))\Big|_0^{\pi}\right]$$

$$= \pi^2 + 2\left[0 + \underbrace{(\cos x)\Big|_0^{\pi}}_{-2}\right] = \boxed{\pi^2 - 4}$$

2º) $\int_0^{2\pi} \text{sen}^3 x \cos^4 x\, dx$

$$= \int_0^{2\pi} \text{sen}^2 x.\cos^4 x.\text{sen}\, x\, dx$$

fazemos $u = \cos x$, $du = -\text{sen}\, x\, dx$ e $\text{sen}^2 x = 1 - \cos^2 x = 1 - u^2$

Note que para $x = 0$, tenmos $u = \cos(0) = 1$

e, para $x = 2\pi$, temos $u = \cos(2\pi) = 1$

logo:

$$\int_0^{2\pi} \text{sen}^2 x.\cos^4 x.\text{sen}\, x\, dx = \int_1^1 (1-u^2).u^4.(-du) = 0 \quad !!!$$

0

3:) $\displaystyle\int \frac{x^3}{(1+x^2)^2}\,dx$

Temos: $\displaystyle\int (1+x^2)^{-2} \cdot x^3\,dx = \int (1+x^2)^{-2} \cdot x^2 \cdot x\,dx$

seja $u = 1+x^2$, $du = 2x\,dx$ e note que $x^2 = u-1$

então:

$$\int (1+x^2)^{-2} \cdot x^2 \cdot x\,dx = \int u^{-2} \cdot (u-1) \cdot \frac{du}{2}$$

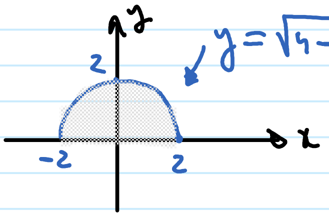
$$= \frac{1}{2}\int (u^{-2} \cdot u - u^{-2} \cdot 1)\,du$$

$$= \frac{1}{2}\int (u^{-1} - u^{-2})\,du$$

$$= \frac{1}{2}\ln|u| - \left(\frac{u^{-1}}{-1}\right) + C$$

$$= \frac{1}{2}\ln|u| + \frac{1}{u} + C$$

$$= \frac{1}{2}\ln(1+x^2) + \frac{1}{1+x^2} + C.$$

4:) 

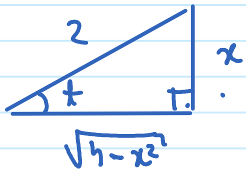
$y = \sqrt{4-x^2}$

(obs.: Temos $y = \sqrt{4-x^2} \Leftrightarrow \underbrace{x^2+y^2=4}_{\text{Eq de uma circunferência de } R=2 \text{ e } C(0,0).},\ y>0$)

A área é dada por:

$$A = \int_a^b (f(x) - g(x))\,dx = \int_{-2}^{2} \sqrt{4-x^2}\,dx$$

fazemos $x = 2\sin t$, $dx = 2\cos t\,dt$



Note: $x = 2 \Rightarrow 2 = 2\sin t \Rightarrow \sin t = 1 \Rightarrow t = \pi/2$

$x = -2 \Rightarrow -2 = 2\sin t \Rightarrow \sin t = -1 \Rightarrow t = -\pi/2$

logo: $$\int_{-2}^{2} \sqrt{4-x^2}\,dx = \int_{-\pi/2}^{\pi/2} \sqrt{4-4\,\mathrm{sen}^2 t}\,.\,2\cos t\,dt$$

$$= \int_{-\pi/2}^{\pi/2} 2\cos t\,.\,2\cos t\,dt$$

$$= 4 \int_{-\pi/2}^{\pi/2} \cos^2 t\,dt$$

$$= \cancel{4}\,2 \int_{-\pi/2}^{\pi/2} \frac{1+\cos(2t)}{\cancel{2}}\,dt$$

$$= 2\left[t + \frac{\mathrm{sen}(2t)}{2} \right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= 2\left[\frac{\pi}{2} + \underbrace{\cancel{\frac{\mathrm{sen}(\pi)}{2}}}_{0} - \left(-\frac{\pi}{2} + \underbrace{\cancel{\frac{\mathrm{sen}(-\pi)}{2}}}_{0} \right) \right]$$

$$= 2\left[\frac{\pi}{2} + \frac{\pi}{2} \right] = \boxed{2\pi}$$

de Fato: $$A = \frac{\pi R^2}{2} = \frac{\pi\,.\,2^2}{2} = \boxed{2\pi}$$