① Base $(n=1)$: $1 = \frac{(1+1).1}{2} = \frac{2}{2} = 1$ OK!

Hipótese Indutiva: é (V) para n, isto é, $1+2+3+\dots+n = \frac{(n+1)n}{2}$

Tese: é (V) para $n+1$ $\therefore$ $\underbrace{\cancel{1+2+3+\dots+n}}_{HI} + n+1 = \frac{(n+1+1)(n+1)}{2}$

$= \frac{(n+2)(n+1)}{2}$ (distributiva)

$= \frac{n(n+1)+2(n+1)}{2}$

$= \underbrace{\cancel{\frac{n(n+1)}{2}}}_{HI} + \frac{\cancel{2}(n+1)}{\cancel{2}}$

$\therefore$ $n+1 = n+1$ OK! ■

② Base $(n=1)$: $2.1-1 = 1^2$

$1 = 1$ OK!

HI: é (V) para n, então $1+3+\dots+2n-1 = n^2$

Tese: é , para $n+1$, então $\underbrace{\cancel{1+3+\dots+2n-1}}_{HI} + 2(n+1)-1 = (n+1)^2$

$= \underbrace{\cancel{n^2}}_{HI} + 2n+1$

$\therefore$ $2n+2-1 = 2n+1$

$\therefore$ $2n+1 = 2n+1$ OK! ■

③ Base $(n=0)$: $F(0+1).F(0) = (F(0))^2$

$F(1).F(0) = 0^2$

$F(0) = F(0)$ OK!

HI: é (V) para n, então $F(n+1).F(n) = (F(0))^2 + (F(1))^2 + \dots + (F(n))^2$

Tese: é (V) para $n+1$, então $F((n+1)+1).F(n+1) = \underbrace{(F(0))^2+(F(1))^2+\dots+(F(n))^2}_{HI} + (F(n+1))^2$

$\therefore$ $F(n+2).F(n+1) = F(n+1).F(n) + (F(n+1))^2$

$= F(n+1).F(n) + F(n+1).F(n+1)$

$= F(n+1)(\underbrace{F(n)+F(n+1)}))$

$= F(n+1) \,.\, F(n+2)$ (tese)

OK! ■

④ Base $(n=1)$: $1^3 = \left(\frac{(1+1).1}{2}\right)^2 \Rightarrow 1 = \left(\frac{\cancel{2}}{\cancel{2}}\right)^2 \Rightarrow 1 = 1$ OK

HI: é (V) para n

$\therefore$ $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{(n+1).n}{2}\right)^2$

Tese: é (V) para $n+1$

$$\therefore \underbrace{1^3 + 2^3 + 3^3 + \ldots + n^3}_{HI} + (n+1)^3 = \left(\frac{((n+1)+1)\cdot(n+1)}{2}\right)^2$$

$$\therefore \left(\frac{(n+1)\cdot n}{2}\right)^2 + (n+1)^3 = \left(\frac{(n+2)\cdot(n+1)}{2}\right)^2$$

$$= \left(\frac{n(n+1) + 2(n+1)}{2}\right)^2$$

$$= \left(\frac{n(n+1)}{2} + \frac{\cancel{2}(n+1)}{\cancel{2}}\right)^2$$

$$= \left(\frac{n(n+1)}{2}\right)^2 + \frac{\cancel{2}n(n+1)(n+1)}{\cancel{2}} + (n+1)^2$$

↳ (colocando $(n+1)^2$ em evidência)

$$= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^2(n+1)$$

$$= \frac{n(n+1)}{2} + (n+1)^3 \quad \text{OK!}$$

■

5 Base $(n=1)$: $S(1) = \dfrac{-1}{1+1}$?

$$\therefore F(1) = \frac{-1}{2}$$

$$\frac{-1}{1(1+1)} = \frac{-1}{2} \quad \text{OK}$$

HI: é (V) para n tq $F(n) = \dfrac{-1}{n(n+1)}$

$$\therefore S(n) = \frac{-n}{n+1}$$

Tese: é (V) para $n+1$ tq $F(n+1) = \dfrac{-1}{(n+1)(n+1+1)} = \dfrac{-1}{(n+1)(n+2)}$

$$\therefore S(n+1) = \frac{-(n+1)}{n+1+1}$$

$$\therefore \underbrace{S(n)}_{HI} + F(n+1)$$

$$= \frac{-n}{n+1} + \frac{(-1)}{(n+1)(n+2)}$$

$$= \frac{-n(n+2)}{(n+1)(n+2)} - \frac{1}{(n+1)(n+2)}$$

$$= \frac{-n^2 - 2n - 1}{(n+1)(n+2)} = \frac{-(n^2+2n+1)}{(n+1)(n+2)}$$

$$= \frac{-(n+1)^{\cancel{2}}}{(\cancel{n+1})(n+2)} = \frac{-(n+1)}{n+2} = \frac{-n+1}{n+1+1} \rightarrow \text{Tese}$$

OK

■

⑥ Base $(n=1)$: $F(2.1-1).F(2.1) = F(2.1)$?

$$\therefore F(1).F(2) = F(2)$$

$$F(2) = F(2)$$ OK!

HI: é (V) para n

$$\therefore F_0F_1 + F_1F_2 + F_2F_3 + \dots + F_{(2n-1)-1} \cdot F_{(2n)-1} + F_{2n-1} \cdot F_{2n} = (F_{2n})^2$$

Tese: é (V) para $n+1$

$$\therefore F_0F_1 + F_1F_2 + F_2F_3 + \dots + F_{(2(n+1)-1)-1} \cdot F_{(2(n+1))-1} + F_{2(n+1)-1} \cdot F_{2(n+1)} = (F_{2(n+1)})^2$$

$$\Rightarrow F_0F_1 + F_1F_2 + F_2F_3 + \dots + F_{2n+1-1} \cdot F_{2n+1} + F_{2(n+1)-1} \cdot F_{2(n+1)} = (F_{2(n+1)})^2$$

$$\Rightarrow \underbrace{F_0F_1 + F_1F_2 + F_2F_3 + \dots + F_{2n-1} \cdot F_{2n}}_{HI} + F_{2n-\cancel{1}+\cancel{1}} \cdot F_{2n+1} + F_{2n+1} \cdot F_{2n+2} = (F_{2n+2})^2$$

$$\cancel{(F_{2n})^2} + F_{2n+1}(F_{2n} + F_{2n+2}) = (F_{2n+1} + F_{2n})^2$$

$$= (F_{2n+1})^2 + 2F_{2n+1} \cdot F_{2n} + \cancel{(F_{2n})^2}$$

$$\therefore \cancel{F_{2n+1}}(F_{2n} + F_{2n+2}) = \cancel{F_{2n+1}}(F_{2n+1} + 2F_{2n})$$

$$\therefore F_{2n} + F_{2n} + \cancel{F_{2n+1}} = \cancel{F_{2n+1}} + 2F_{2n}$$

$$\Rightarrow 2F_{2n} = 2F_{2n}$$ OK!

$\blacksquare$

⑦ Base $(n=1)$: $f_{1+1}.f_{1-1} - f_1^2 = (-1)^1$?

$$f_2.\underbrace{\cancel{f_0}}_{0} - f_1 = -1$$

$$-1 = -1$$ ok

HI: é (V) para n

$$\therefore f_{n+1} \cdot f_{n-1} - f_n^2 = (-1)^n$$

Tese: é (V) para $n+1$

$$\therefore f_{n+1+1} \cdot f_{n+1-1} - f_{n+1}^2 = (-1)^{n+1}$$

$$\therefore f_{n+2} \cdot f_n - f_{n+1}^2 = (-1)^n \cdot (-1)^1$$

$$\therefore (f_{n+1} + f_n)f_n - (f_n + f_{n-1})^2 = -\underbrace{(-1)^n}_{HI}$$

$$f_{n+1}f_n + f_n^2 - (f_n^2 + 2f_nf_{n-1} + f_{n-1}^2) = -(f_{n+1} \cdot f_{n-1} - f_n^2)$$

$$(f_n + f_{n-1})f_n + \cancel{f_n^2} - f_n^2 \quad 2f_nf_{n-1} \quad f_{n-1}^2 = -(f_{n+1} \cdot f_{n-1}) + \cancel{f_n^2}$$

$$\cancel{f_n^2} + f_{n-1}f_n - \cancel{f_n^2} - 2f_nf_{n-1} - f_{n-1}^2$$

$$-\cancel{f_{n-1}}(-f_n + 2f_n + f_{n-1}) = -f_{n+1} \cdot \cancel{f_{n-1}}$$

$$-(f_n + f_{n-1})$$

$$-(f_{n+1}) = -f_{n+1}$$ OK!

$\blacksquare$

⑧ Base $(n=1)$: $1(1+1) = \frac{1(1+1)(1+2)}{3}$

$2 = \frac{2.\cancel{3}}{\cancel{3}}$ OK!

HI: é (V) para n

$\therefore\ 1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Tese: é (V) para $n+1$

$\therefore\ \underbrace{1.2 + 2.3 + 3.4 + \dots + n(n+1)}_{HI} + (n+1).(n+1+1) = \frac{(n+1)(n+1+1)(n+1+2)}{3}$

$\therefore\ \frac{n(n+1)(n+2)}{3} + \frac{(n+1)(n+2)3}{3} = \frac{(n+1)(n+2)(n+3)}{3}$

$\therefore\ \frac{(n+1)(n+2)(n+3)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$ ∎

⑨ Base $(n=1)$: $\sum_{k=1}^{1} k.2^k = (1-\cancel{1}).2^{1+1} + 2$?

$\therefore\ 1.2^1 = 0 + 2$

$2 = 2$ OK!

HI: é (V) para n

$\therefore\ \sum_{k=1}^{n} k.2^k = (n-1).2^{n+1} + 2$

Tese: é (V) para $n+1$

$\therefore\ \sum_{k=1}^{n+1} k.2^k = (n+\cancel{1}-\cancel{1}).2^{n+1+1} + 2$

$\therefore\ \underbrace{\sum_{k=1}^{n} k.2^k}_{HI} + (n+1).2^{n+1}$

$\therefore\ (n-1).2^{n+1} + 2 + (n+1)2^{n+1}$

$\therefore\ 2^{n+1}(n-\cancel{1}+n+\cancel{1}) + 2 = n.2^{n+1+1} + 2$

$\therefore\ 2^{n+1}.2^1 n + 2$

$\therefore\ 2^{n+1+1}n + 2 = n.2^{n+1+1} + 2$ OK! ∎

10) Base $(n=1)$: $\frac{1}{\sqrt{1}} > 2(\sqrt{1+1}-1)$?

$\frac{1}{1} > 2\sqrt{2} - 2$

$1 > \sqrt{8} - 2$ $\quad (\sqrt{8} < \sqrt{9} \Rightarrow \sqrt{8} < 3 \Rightarrow \sqrt{8} - 2 < 3 - 2$
$\Rightarrow \sqrt{8} - 2 < 1)$

$\therefore\ 1 > \sqrt{8} - 2$ OK!

HI: é (V) para n

$\therefore\ 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1)$

Tese: é (V) para $n+1$

$\therefore\ \underbrace{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots \frac{1}{\sqrt{n}}}_{HI} + \frac{1}{\sqrt{n+1}} > 2(\sqrt{n+1+1} - 1)$

$2(\sqrt{n+1} - 1) + \frac{1}{\sqrt{n+1}} > 2(\sqrt{n+2} - 1)$

$> 2\sqrt{n+2} - 2$

$\frac{2(\cancel{\sqrt{n+1}})^{\cancel{2}} - 2\sqrt{n+1} + 1}{\sqrt{n+1}} > 2\sqrt{n+2} - 2$

$2(n+1) - 2\cancel{\sqrt{n+1}} + 1 > 2\sqrt{(n+2)(n+1)} - 2\cancel{\sqrt{n+1}}$

$4(n+1)^2 + 4(n+1) + 1 > 4(n+2)(n+1)$

$4(n+1)(n+1+1) + 1$

$\cancel{4(n+1)(n+2)} + 1 > \cancel{4(n+2)(n+1)}$

$1 > 0$ OK!