LISTA 1 - AVLC

Kailane Eduarda Felix da Silva

CPF: 125. 769. 454 - 57

① $\|\bar{U}\| = 4$ $\quad \|\bar{U}+\bar{V}\|^2 = (\bar{U}+\bar{V})\cdot(\bar{U}+\bar{V})$

$\|\bar{V}\| = 3$ $\quad \|\bar{U}+\bar{V}\|^2 = \|U\|^2 + \bar{U}\cdot\bar{V} + \bar{V}\cdot\bar{U} + \|V\|^2$

$\theta = 60°$ $\quad \|\bar{U}+\bar{V}\|^2 = 4^2 + 2\cdot(\bar{U}\cdot\bar{V}) + 3^2$

$\|\bar{U}+\bar{V}\|^2 = 16 + 2\cdot(\|U\|\cdot\|V\|\cdot\cos 60) + 9 \quad (\bar{V}\cdot\bar{U} = \|V\|\cdot\|U\|\cdot\cos\theta)$

$\|\bar{U}+\bar{V}\|^2 = 16 + 2\cdot(4\cdot 3\cdot 1/2) + 9$

$\|\bar{U}+\bar{V}\|^2 = 16 + 2\cdot 6 + 9$

$\|\bar{U}+\bar{V}\|^2 = 37, \quad \|U+V\| = \sqrt{37}$

Para $\bar{U}-\bar{V}$:

$\|\bar{U}-\bar{V}\|^2 = (\bar{U}-\bar{V})\cdot(\bar{U}-\bar{V})$

$\|\bar{U}-\bar{V}\|^2 = \|U\|^2 - \bar{U}\cdot\bar{V} + \bar{U}\cdot\bar{V} + \|V\|^2$

$\|\bar{U}-\bar{V}\|^2 = 4^2 - 2(4\cdot 3\cdot 1/2) + 3^2 \quad (\bar{V}\cdot\bar{U} = \|V\|\cdot\|U\|\cdot\cos\theta)$

$\|\bar{U}-\bar{V}\|^2 = 16 - 12 + 9$

$\|\bar{U}-\bar{V}\|^2 = 13, \quad \|U-V\| = \sqrt{13}$

②

a) $A(1,2,-1)$ $\quad$ Para $\hat{A} = 90°$, $\vec{AB}\cdot\vec{AC} = 0$

$B(-1,0,-1)$ $\quad (-2\cdot 1) + (-2\cdot -1) + (0\cdot 3) = 0$

$C(1,-1,3)$ $\quad (-2) + (2) + (0) = 0$

$-2 + 2 = 0$

$0 = 0$*

* logo, $\hat{A} = 90°$.
O triângulo é retângulo em A.

b) $\bar{BA} = (2,2,0)$, $\bar{BC} = (3,1,3)$ $\quad P_{\frac{\bar{BA}}{\bar{BC}}} = \left[\dfrac{\bar{BA}\cdot\bar{BC}}{\|BC\|^2}\right]\cdot\bar{BC}$ $\quad \therefore P_{\frac{\bar{BA}}{\bar{BC}}} = \left[\dfrac{8}{19}\right]\cdot(3,1,3)$ → vetor projeção

* produto escalar $(\bar{BA}\cdot\bar{BC})$

$(2\cdot 3) + (2\cdot 1) + (0\cdot 3) =$

$6 + 2 = 8$

* módulo $\|BC\|^2$

$\|BC\|^2 = 3^2 + 1^2 + 3^2 =$

$\|BC\|^2 = 19$

* medida (módulo) de $P_{\frac{\bar{BA}}{\bar{BC}}}$

$\|P_{\frac{\bar{BA}}{\bar{BC}}}\| = \dfrac{8}{19}\sqrt{3^2+1^2+3^2}$

$\|P_{\frac{\bar{BA}}{\bar{BC}}}\| = \dfrac{8}{19}\cdot\sqrt{19}$

c) Seja "H" o pé da altura do triângulo relativa ao vértice A, temos que:

$\bar{BH} = P_{\frac{\bar{BA}}{\bar{BC}}}$

$8/19\cdot(3,1,3)$

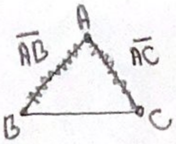
$\bar{BH} = (x_H+1,\ y_H-0,\ z_H+1) = \left[\dfrac{24}{19}, \dfrac{8}{19}, \dfrac{24}{19}\right]$

$x_H = \dfrac{24}{19} - 1$

$y_H = \dfrac{8}{19}$ $\quad H = \left(\dfrac{43}{19}, \dfrac{8}{19}, \dfrac{43}{19}\right)$

$z_H = \dfrac{24}{19} - 1$

③ $A(x,1,1)$
$B(1,-1,0)$
$C(2,1,-1)$



* $\overline{AB} \times \overline{AC}$ $\left(\overline{AB}(1-x,-2,-1), \overline{AC}(2-x,0,-2)\right)$

$$D = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-x & -2 & -1 \\ 2-x & 0 & -2 \end{bmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1-x & -2 \\ 2-x & 0 \end{matrix}$$

$D = 4\hat{i} + \hat{j}(-2+x) \cancel{+0} - \hat{j}(-2+2x) \cancel{-0} - \hat{k}(-4 \ldots$

$D = 4\hat{i} \cancel{-2\hat{j}} + \hat{j}x \cancel{+2\hat{j}} - 2\hat{j}x + 4\hat{k} - 2\hat{k}x$

$D = 4\hat{i} + \hat{j}x - 2\hat{j}x + 4\hat{k} - 2\hat{k}x$

$D = 4\hat{i} - \hat{j}x + 4\hat{k} - 2\hat{k}x$

$D = 4\hat{i} - \hat{j}x + \hat{k}(4-2x)$

$\vec{v} = (4, -x, 4-2x)$

$|\vec{v}| = \sqrt{4^2+(-x)^2+(4-2x)^2}$

$$\text{ÁREA} = \frac{\|\overline{AB} \times \overline{AC}\|}{2} = \frac{\sqrt{29}}{2}$$

$$\left(\frac{\sqrt{29}}{2^2}\right)^{\cancel{2}} = \left(\frac{\sqrt{4^2+(-x)^2+(4-2x)^2}}{2^2}\right)^{\cancel{2}}$$

$$\frac{29}{4} = \frac{4^2+(-x)^2+(4-2x)^2}{4}$$

$$29 = \frac{16 + x^2 + 16 - 2\cdot 4 \cdot -2x + (-2x)^2}{\cancel{4}} \cdot \cancel{4}$$

$29 = 16 + \cancel{x^2} + 16 + 16x + \cancel{4x^2}$

$29 = 5x^2 - 16x + 32$

$5x^2 - 16x + 32 - 29 = 0$

$5x^2 - 16x + 3 = 0$

$a = 5$
$b = -16$
$c = 3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow x = \frac{+16 \pm \sqrt{16^2-4\cdot5\cdot3}}{2\cdot5}$$

$$x = \frac{+16 \pm \sqrt{256-60}}{10} \Rightarrow x = \frac{16 \pm 14}{10}$$

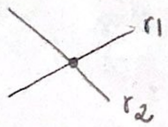
$$x_1 = \frac{30}{10} = 3$$

$$x_2 = \frac{2}{10} = \frac{1}{5}$$

Portanto, o valor de x para um triângulo de área $\frac{\sqrt{29}}{2}$ pode ser 3 ou $1/5$.

④

$$r_1 \begin{cases} y_1 = -3x + 2 \\ z_1 = 3x - 1 \end{cases} \qquad r_2 \begin{cases} x_2 = -T \\ y_2 = 1 + 2T \\ z_2 = -2T \end{cases}$$



* Definindo "T"

$y_1 = y_2$

$-3x + 2 = 1 + 2T$

$2 = 1 + 2T + 3x$

$2T + 3x = 1$

$\boxed{x = -T}$

$2T + 3(-T) = 1$

$2T - 3T = 1$

$-T = 1$

$\boxed{T = -1}$

* Substituindo "T" nas equações.

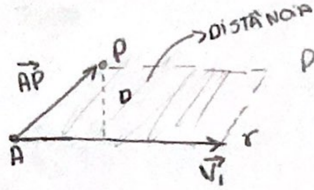
$x = -(-1)$ $\quad y = 1 + 2(-1)$ $\quad z = -2(-1)$

$\boxed{x = 1}$ $\quad \boxed{y = -1}$ $\quad \boxed{z = 2}$

Logo, o ponto de intersecção é $P_I(1,-1,2)$

⑤ $P(1,2,3)$

$$r = \begin{cases} x = 1 - 2t \\ y = 2t \\ z = 2 - t \end{cases}$$



Paralelogramo formado por $\vec{AP}$ e $\vec{v}_1$.

Tal que a área desse paralelogramo se dá por $\|\vec{AP} \times \vec{v}\|$.

* VETOR DIRETOR DA RETA r:

$\vec{v}_1 = (-2, 2, -1)$

* PONTO A:

$A = (1, 0, 2)$

* PRODUTO VETORIAL

$\vec{AP} = (1,2,3) - (1,0,2) = (0,2,1)$

$\vec{v} = (-2, 2, -1)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 2 \\ -2 & 2 \end{matrix}$$

$-2\hat{i} + (-2\hat{j}) + 0 - 0 - 2\hat{i} - (-4\hat{k})$

$-2\hat{i} - 2\hat{j} - 2\hat{i} + 4\hat{k}$

$-4\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{AP} \times \vec{v} = (-4, -2, 4)$

$\|\vec{AP} \times \vec{v}\| = \sqrt{(-4)^2 + (-2)^2 + 4^2}$

$\|AP \times \vec{v}\| = \sqrt{16 + 4 + 16}$

$\|AP \times \vec{v}\| = \sqrt{36}$

$\|AP \times \vec{v}\| = 6$

$$\|\vec{AP} \times \vec{v}\| = \overset{\|\vec{v}\|}{b} \cdot \underset{\text{distância (D)}}{h}$$

$$D = \frac{\|\vec{AP} \times \vec{v}\|}{\|\vec{v}\|} = \frac{6}{3} = 2$$

Logo, a distância entre o ponto (P) e a reta r é $\underline{\underline{2}}$ (unidades de comprimento).

* MÓDULO DE $\vec{v}$

$\|\vec{v}\| = \sqrt{(-2)^2 + 2^2 + (-1)^2}$

$\|\vec{v}\| = \sqrt{4 + 4 + 1}$

$\|\vec{v}\| = \sqrt{9}$

$\|\vec{v}\| = 3$

6

$\vec{U} = (1,1,0)$
$\vec{V} = (2,0,1)$

$w_1 = \overline{3U} - \overline{2V}$
$w_2 = \overline{U} + \overline{3V}$
$w_3 = \hat{i} + \hat{j} - 2\hat{k}$

* calculando w_1 =

$3(1,1,0) - 2(2,0,1)$

$(3,3,0) - (4,0,2)$

$w_1 = (-1,3,-2)$

* calculando w_2

$(1,1,0) + 3(2,0,1)$

$(1,1,0) + (6,0,3)$

$w_2 = (7,1,3)$

* calculando w_3

$(1,0,0) + (0,1,0) - 2(0,0,1)$

$(1,0,0) + (0,1,0) - (0,0,2)$

$w_3 = (1,1,-2)$

* PRODUTO MISTO

$$[w_1, w_2, w_3] = \left[\begin{array}{ccc} -1 & 3 & -2 \\ 7 & 1 & 3 \\ 1 & 1 & -2 \end{array}\right]\begin{array}{cc} -1 & 3 \\ 7 & 1 \\ 1 & 1 \end{array}$$

* $2 + 9 + (-14) - 42 - (-3) - (-2)$

$11 + (-14) - 42 + 3 + 2$

$11 - 14 - 42 + 5$

$11 - 14 + 42 + 5$

$-3 + 47 = 44.$

$\|[w_1, w_2, w_3\| = |44| = 44$ UNIDADES DE VOLUME.