三角函数公式

[编辑](javascript:;)

[三角函数](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0/1652457)是数学中属于[初等函数](https://baike.baidu.com/item/%E5%88%9D%E7%AD%89%E5%87%BD%E6%95%B0/846442)中的[超越函数](https://baike.baidu.com/item/%E8%B6%85%E8%B6%8A%E5%87%BD%E6%95%B0/3365811)的函数。它们的本质是任何角的集合与一个比值的集合的变量之间的映射。通常的三角函数是在[平面直角坐标系](https://baike.baidu.com/item/%E5%B9%B3%E9%9D%A2%E7%9B%B4%E8%A7%92%E5%9D%90%E6%A0%87%E7%B3%BB/1296463)中定义的。其[定义域](https://baike.baidu.com/item/%E5%AE%9A%E4%B9%89%E5%9F%9F/7879679)为整个[实数](https://baike.baidu.com/item/%E5%AE%9E%E6%95%B0)域。另一种定义是在[直角三角形](https://baike.baidu.com/item/%E7%9B%B4%E8%A7%92%E4%B8%89%E8%A7%92%E5%BD%A2/234388)中，但并不完全。现代数学把它们描述成无穷数列的[极限](https://baike.baidu.com/item/%E6%9E%81%E9%99%90/3564509)和[微分方程](https://baike.baidu.com/item/%E5%BE%AE%E5%88%86%E6%96%B9%E7%A8%8B/4763)的解，将其定义扩展到复数系。

三角函数公式看似很多、很复杂，但只要掌握了三角函数的本质及内部规律，就会发现三角函数各个公式之间有强大的联系。而掌握三角函数的内部规律及本质也是学好三角函数的关键所在。

**中文名**

三角函数公式

**外文名**

Formulas of trigonometric functions

**应用学科**

数学、物理、地理、天文地理等

**适用领域范围**

几何，代数变换，数学、物理、地理、天文等

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2. 2 [函数关系](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#2)
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10. ▪ [万能公式](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#4_6)
11. ▪ [辅助角公式](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#4_7)
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13. ▪ [正弦定理](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#5_1)
14. ▪ [余弦定理](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#5_2)
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18. ▪ [泰勒展开式](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#5_6)
19. ▪ [万能公式](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#5_7)
20. ▪ [傅里叶级数](https://baike.baidu.com/item/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=aladdin#5_8)

定义式

[编辑](javascript:;)

|  |  |  |
| --- | --- | --- |
|  | **锐角三角函数** | **任意角三角函数** |
| 图形 | [直角三角形](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/7af40ad162d9f2d34d021b03abec8a136227ccce?fr=lemma&ct=single)直角三角形 | [任意角三角函数](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/cdfe7281be28419cbc3e1ec3?fr=lemma&ct=single)任意角三角函数 |
| [正弦](https://baike.baidu.com/item/%E6%AD%A3%E5%BC%A6)（sin） | https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D64/sign=bbca315479cb0a468122883d6a63e789/35a85edf8db1cb13fe6defbddf54564e92584b1c.jpg | https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D64/sign=c535097d8518367aa9897cd92f73bdf8/b17eca8065380cd7e44766b9a244ad345982810d.jpg |
| [余弦](https://baike.baidu.com/item/%E4%BD%99%E5%BC%A6/73670)（cos） | https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D66/sign=a9128bed9d510fb37c197491d8334474/730e0cf3d7ca7bcb4599027bbd096b63f724a880.jpg | https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=400d237f0df41bd5de53ebf151da7529/908fa0ec08fa513d1b84a88a3e6d55fbb2fbd9bc.jpg |
| [正切](https://baike.baidu.com/item/%E6%AD%A3%E5%88%87)（tan或tg） | https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D67/sign=15cc11a8adc379317968852eeac4518c/d52a2834349b033b36892f7417ce36d3d539bd1f.jpg | https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D66/sign=582f966dca3d70cf48faa90bf8dc6b2a/060828381f30e9249f8858654f086e061d95f7b2.jpg |
| [余切](https://baike.baidu.com/item/%E4%BD%99%E5%88%87)（cot或ctg） | https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=4c588197cb3d70cf48faa908f9dc6bb4/060828381f30e9248bff4f9f4e086e061c95f7c1.jpg | https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=6d48a340bd0e7bec27da00e42e2e2bf8/50da81cb39dbb6fd062f52690c24ab18972b370c.jpg |
| [正割](https://baike.baidu.com/item/%E6%AD%A3%E5%89%B2)（sec） | https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=da579e4fec24b899da3c7a3d6f068ea1/4a36acaf2edda3cc8eb60c6f04e93901213f922b.jpg | https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=a21aa49fb1fb43161e1f797f21a4938c/ae51f3deb48f8c540b99017b39292df5e0fe7f18.jpg |
| [余割](https://baike.baidu.com/item/%E4%BD%99%E5%89%B2)（csc） | https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=d6ce10ba69600c33f479ddcd1b4ce17c/dc54564e9258d1090e737a1cd358ccbf6d814d88.jpg | https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D65/sign=b641fdb3922397ddd2799b015882df92/bba1cd11728b47106a6b3998c0cec3fdfc03231a.jpg |

*表格参考资料来源：[现代汉语词典](https://baike.baidu.com/item/%E7%8E%B0%E4%BB%A3%E6%B1%89%E8%AF%AD%E8%AF%8D%E5%85%B8" \t "_blank)*[1]  ．

函数关系

[编辑](javascript:;)

倒数关系：①

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D99/sign=a2f1d24f2d2eb938e86d76fbd46227ef/80cb39dbb6fd52662600edbaa918972bd4073671.jpg

；②

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D97/sign=8b9f8b760fd162d981ee6e1b10df854c/00e93901213fb80ef27ba46533d12f2eb83894de.jpg

；③

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商数关系：①

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；②

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D92/sign=d0163316d63f8794d7ff442cd31b7aa2/f3d3572c11dfa9ecc84086b960d0f703918fc136.jpg

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平方关系：①

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；②

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；③

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D127/sign=59cbcca61d30e924cba498337b096e66/d6ca7bcb0a46f21fe0ca8ffdf4246b600d33aea6.jpg

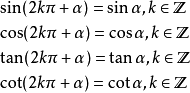
诱导公式

[编辑](javascript:;)

**公式一：**设

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=c18078950c24ab18e416e53734fa4b14/91529822720e0cf3442cd3480f46f21fbf09aaae.jpg

为任意角，终边相同的角的同一三角函数的值相等：



**公式二：**设

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=dde67ea96559252da7171904359bf9d3/32fa828ba61ea8d3f900e8b7920a304e251f5866.jpg

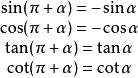
为任意角，

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D37/sign=ef462cb6e0dde711e3d245f1a6ef21df/9f510fb30f2442a7e3176116d443ad4bd1130263.jpg

与

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=7b0ea554942397ddd2799c045882dfe4/c9fcc3cec3fdfc038c737f0bd13f8794a4c2267d.jpg

的三角函数值之间的关系：



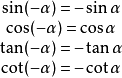
**公式三：**任意角

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D20/sign=4f44080882d6277fed1235382938b4f4/caef76094b36acafe6980a4f79d98d1001e99c0d.jpg

与

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=b4511d0c895494ee83220b192cf53643/4afbfbedab64034f3f7f5eb5aac379310b551dd6.jpg

的三角函数值之间的关系：



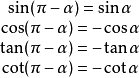
**公式四**：

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与

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=30929082243fb80e08d165d737d12caf/5366d0160924ab18a6507d8430fae6cd7b890b3a.jpg

的三角函数值之间的关系：



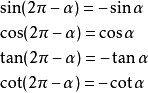
**公式五：**

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D45/sign=17c160be75cf3bc7ec00cce9d0006a7d/e824b899a9014c081eceb4160f7b02087af4f488.jpg

与

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=f24a589cbb096b6385195a500c33e43d/9358d109b3de9c822954663c6981800a19d84354.jpg

的三角函数值之间的关系：



**公式六**：

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D39/sign=a818a3749552982201333fcad6caecf5/bd315c6034a85edf638a61904c540923dd54757b.jpg

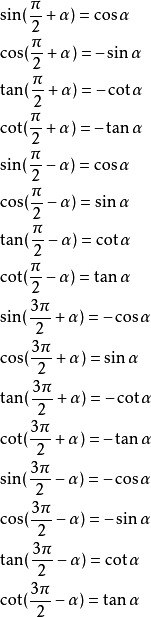
及

https://gss0.bdstatic.com/-4o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D48/sign=5ff600e7064f78f0840b9bfb783154d7/242dd42a2834349ba720f036ccea15ce36d3be5b.jpg

与

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D10/sign=29013c89354e251fe6f7e0f8a68613e5/86d6277f9e2f0708400db4b3ec24b899a901f27d.jpg

的三角函数值之间的关系：

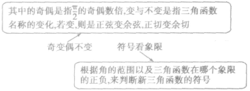


记背诀窍：奇变偶不变，符号看象限[2]  ．即形如（2k+1）90°±α，则函数名称变为余名函数，正弦变余弦，余弦变正弦，正切变余切，余切变正切。形如2k×90°±α，则函数名称不变。

**诱导公式口诀“奇变偶不变，符号看象限”意义：**

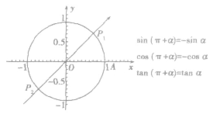
k×π/2±a(k∈z)的三角函数值．(1)当k为偶数时，等于α的同名三角函数值，前面加上一个把α看作锐角时原三角函数值的符号；  
　　(2)当k为奇数时，等于α的异名三角函数值，前面加上一个把α看作锐角时原三角函数值的符号。

**记忆方法一：奇变偶不变，符号看象限：**

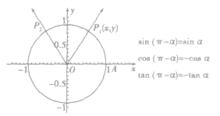
[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/f31fbe096b63f624e0f4b9658044ebf81a4ca32f?fr=lemma&ct=single)

**记忆方法二：无论α是多大的角，都将α看成锐角．**

以诱导公式二为例：

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/78310a55b319ebc4f799f6ff8526cffc1f1716f6?fr=lemma&ct=single)

若将α看成锐角（终边在第一象限），则π+α是第三象限的角（终边在第三象限），正弦函数的函数值在第三象限是负值，余弦函数的函数值在第三象限是负值，正切函数的函数值在第三象限是正值．这样，就得到了诱导公式二．  
　　以诱导公式四为例：

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/50da81cb39dbb6fd4db965400e24ab18962b37f1?fr=lemma&ct=single)

若将α看成锐角（终边在第一象限），则π-α是第二象限的角（终边在第二象限），正弦函数的三角函数值在第二象限是正值，余弦函数的三角函数值在第二象限是负值，正切函数的三角函数值在第二象限是负值．这样，就得到了诱导公式四．

**诱导公式的应用：**

运用诱导公式转化三角函数的一般步骤：

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/fc1f4134970a304edc1a239cd6c8a786c9175c98?fr=lemma&ct=single)

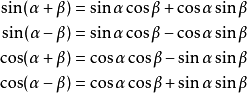
特别提醒：三角函数化简与求值时需要的知识储备：①熟记特殊角的三角函数值；②注意诱导公式的灵活运用；③三角函数化简的要求是项数要最少，次数要最低，函数名最少，分母能最简，易求值最好。

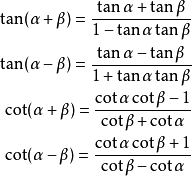
基本公式

[编辑](javascript:;)

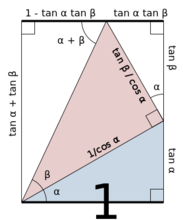
和差角公式

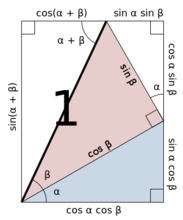
**二角和差公式**





证明如图：负号的情况只需要用-*β*代替*β*即可．cot(*α*+*β*)推导只需把角*α*对边设为1，过程与tan(*α*+*β*)相同．

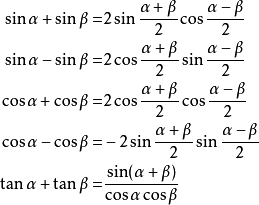
[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/55e736d12f2eb93868d5c2edd6628535e5dd6f71?fr=lemma&ct=single)证明正切的和差角公式

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/7acb0a46f21fbe09ded77b0b68600c338744ad20?fr=lemma&ct=single)证明正弦、余弦的和差角公式

**三角和公式**

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D597/sign=8edbf1a3376d55fbc1c6762f5a234f40/c8ea15ce36d3d5391e4c92be3087e950352ab05e.jpg

和差化积公式



口诀：正加正，正在前，余加余，余并肩，正减正，余在前，余减余，负正弦．

积化和差公式

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D273/sign=39774d12dfc451daf2f60bec85fc52a5/1f178a82b9014a90d4a193f1ab773912b21beedb.jpg

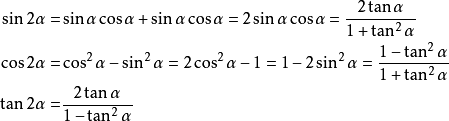
https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D273/sign=e7957f327dd98d1072d40b36123eb807/574e9258d109b3de55ca0da6cdbf6c81810a4c83.jpg

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D277/sign=fd709e578326cffc6d2ab8b58e004a7d/63d9f2d3572c11dfe3d957ed622762d0f603c2f0.jpg

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D274/sign=5f6e77bebc51f819f525044deeb54a76/58ee3d6d55fbb2fb256aa987434a20a44723dce6.jpg

倍角公式

**二倍角公式**



**三倍角公式**

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D176/sign=8e5dd9b198510fb37c197390ef32c893/377adab44aed2e73e34411248101a18b87d6fa0d.jpg

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D180/sign=daad732b75f08202299295377bfafb8a/b999a9014c086e066a73a84304087bf40bd1cbdb.jpg

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D282/sign=ca2973f27dcb0a4681228c315962f63e/ac345982b2b7d0a2ca2e9595cdef76094a369ad7.jpg

https://gss2.bdstatic.com/9fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D287/sign=b34b1023b651f819f5250442edb54a76/09fa513d269759eeb734f6c3b4fb43166d22df56.jpg

https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D279/sign=bbc8798c4ced2e73f8e9812bbe00a16d/58ee3d6d55fbb2fbcea4ce1a494a20a44623dc2f.jpg

*证明：*

**sin3a**

=sin(a+2a)

=sin2a·cosa+cos2a·sina

=2sina(1-sin2a)+(1-2sin2a)sina

=3sina-4sin3a

**cos3a**

=cos(2a+a)

=cos2acosa-sin2asina

=(2cos2a-1)cosa-2(1-cos2a)cosa

=4cos3a-3cosa

**sin3a**

=3sina-4sin3a

=4sina(3/4-sin2a)

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D260/sign=49f65b339c52982201333ec5e7c87b3b/902397dda144ad345657e073dca20cf430ad855d.jpg

=4sina(sin60°+sina)(sin60°-sina)

=4sina×2sin[(60+a)/2]cos[(60°-a)/2]×2sin[(60°-a)/2]cos[60°+a)/2]

=4sinasin(60°+a)sin(60°-a)

**cos3a**

=4cos3a-3cosa

=4cosa(cos2a-3/4)

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D172/sign=46101fcc62061d957946333f49f50a5d/eaf81a4c510fd9f9383b339d292dd42a2834a412.jpg

=4cosa(cosa-cos30°)(cosa+cos30°)

=4cosa×2cos[(a+30°)/2]cos[(a-30°)/2]×{-2sin[(a+30°)/2]sin[(a-30°)/2]}

=-4cosasin(a+30°)sin(a-30°)

=-4cosasin[90°-(60°-a)]sin[-90°+(60°+a)]

=-4cosacos(60°-a)[-cos(60°+a)]

=4cosacos(60°-a)cos(60°+a)

上述两式相比可得：

**tan3a**=tana·tan(60°-a)·tan(60°+a)

**四倍角公式**

sin4a=-4×[cosa·sina·(2×sin2a-1)]

cos4a=1+(-8×cos2a+8×cos4a)

tan4a=(4×tana-4×tan3a)/(1-6×tan2a+tan4a)

**五倍角公式**

https://gss0.bdstatic.com/94o3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D263/sign=f13d6174c5cec3fd8f3ea073e589d4b6/aa64034f78f0f736e7ecd0b90c55b319ebc41373.jpg

https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D269/sign=447e32d849c2d562f608d7ebde1090f3/cdbf6c81800a19d8417818bd35fa828ba61e460b.jpg

https://gss1.bdstatic.com/-vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D273/sign=57bdfda417dfa9ecf92e511051d1f754/d52a2834349b033bfedb636213ce36d3d539bd33.jpg

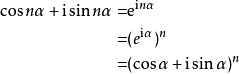
***n*倍角公式**

应用[欧拉公式](https://baike.baidu.com/item/%E6%AC%A7%E6%8B%89%E5%85%AC%E5%BC%8F)：

https://gss3.bdstatic.com/7Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D132/sign=7fcb2990307adab439d01f40b9d5b36b/4e4a20a4462309f7178037aa740e0cf3d6cad69f.jpg

.

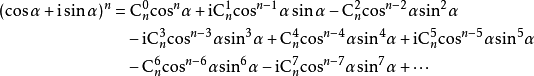
上式用于求n倍角的三角函数时，可变形为：



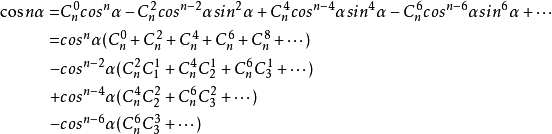
所以

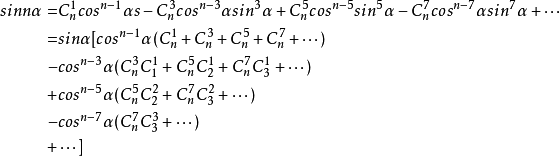
https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D208/sign=c9b2cfb90c55b31998f985757ba88286/6f061d950a7b02081d85bc5964d9f2d3572cc83e.jpg

其中，Re表示取实数部分，Im表示取虚数部分．而

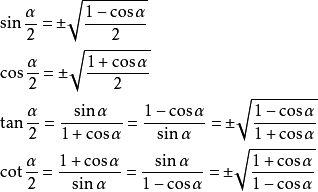


所以





半角公式

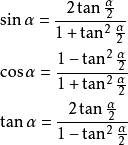


（正负由

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D31/sign=11184a251d30e924cba49a304c088f5f/2fdda3cc7cd98d1017f95b1c233fb80e7bec90e9.jpg

所在的象限决定）

万能公式



辅助角公式

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D339/sign=50c83f21ce1349547a1eee676f4f92dd/f9dcd100baa1cd117c9cf580bc12c8fcc3ce2d63.jpg

证明：

由于

https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D67/sign=da1fdeee7df0f736dcfe4f060b555352/6159252dd42a283445e0de9a5eb5c9ea14cebfd9.jpg

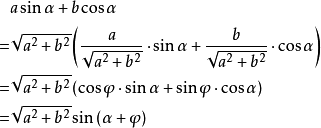
，显然

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D35/sign=0da7e8dbdfb44aed5d4eb8e1b21cbb17/96dda144ad34598239080e0309f431adcaef84a5.jpg

，且

https://gss3.bdstatic.com/-Po3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D247/sign=fbb8916427f5e0feea188e056b6134e5/6609c93d70cf3bc7949a5391db00baa1cd112a30.jpg

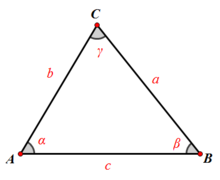
故有：



其他公式

[编辑](javascript:;)

正弦定理

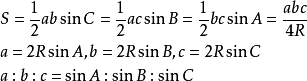
[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/8ad4b31c8701a18b715f3b239c2f07082838fe22?fr=lemma&ct=single)余弦定理

*详见词条：*[*正弦定理*](https://baike.baidu.com/item/%E6%AD%A3%E5%BC%A6%E5%AE%9A%E7%90%86)

在任意△*ABC*中，角*A*、*B*、*C*所对的边长分别为*a*、*b*、*c*，三角形[外接圆](https://baike.baidu.com/item/%E5%A4%96%E6%8E%A5%E5%9C%86" \t "_blank)的半径为*R*．则有[3]  ：

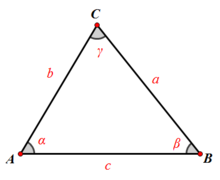
https://gss1.bdstatic.com/9vo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D190/sign=b214ddd4ad64034f0bcdc60f9fc27980/03087bf40ad162d90b3c711517dfa9ec8b13cded.jpg

正弦定理变形可得：

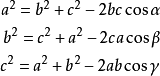


余弦定理

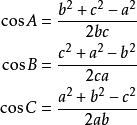
*详见词条：*[余弦定理](https://baike.baidu.com/item/%E4%BD%99%E5%BC%A6%E5%AE%9A%E7%90%86)

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/8ad4b31c8701a18b715f3b239c2f07082838fe22?fr=lemma&ct=single)余弦定理

对于如图所示的边长为*a*、*b*、*c*而相应角为*α*、*β*、*γ*的△*ABC*，有：



也可表示为：



降幂公式

sin²α=[1-cos(2α)]/2

cos²α=[1+cos(2α)]/2

tan²α=[1-cos(2α)]/[1+cos(2α)]

三角和

sin(α+β+γ)=sinα·cosβ·cosγ+cosα·sinβ·cosγ+cosα·cosβ·sinγ-sinα·sinβ·sinγ

cos(α+β+γ)=cosα·cosβ·cosγ-cosα·sinβ·sinγ-sinα·cosβ·sinγ-sinα·sinβ·cosγ

tan(α+β+γ)=(tanα+tanβ+tanγ-tanα·tanβ·tanγ)÷(1-tanα·tanβ-tanβ·tanγ-tanγ·tanα)

幂级数

c0+c1x+c2x2+...+cnxn+...=∑cnxn (n=0..∞)

c0+c1(x-a)+c2(x-a)2+...+cn(x-a)n+...=∑cn(x-a)n (n=0..∞)

它们的各项都是[正整数](https://baike.baidu.com/item/%E6%AD%A3%E6%95%B4%E6%95%B0" \t "_blank)幂的[幂函数](https://baike.baidu.com/item/%E5%B9%82%E5%87%BD%E6%95%B0), 其中c0,c1,c2,...cn...及a都是常数， 这种级数称为幂级数。

泰勒展开式

泰勒展开式又叫幂级数展开法

https://gss2.bdstatic.com/-fo3dSag_xI4khGkpoWK1HF6hhy/baike/s%3D467/sign=c91d3b06711ed21b7dc92fe39a6fddae/79f0f736afc3793144dc2cdde7c4b74542a91189.jpg

实用幂级数：

ex= 1+x+x2/2!+x3/3!+…+xn/n!+…,x∈**R**

ln(1+x)=x-x2/2+x3/3-…+(-1)k-1xk/k, x∈(-1,1)

sin x = x-x3/3!+x5/5!-…+(-1)k-1x2k-1/(2k-1)!+…, x∈**R**

cos x = 1-x2/2!+x4/4!-…+(-1)kx2k/(2k)!+…, x∈**R**

arcsin x = x + x3/(2×3) + (1×3)x5/(2×4×5) + (1×3×5)x7/(2×4×6×7)…+(2k+1)!!×x2k+1/(2k!!×(2k+1))+…, x∈(-1,1)（!!表示双阶乘）

arccos x = π/2 -[x + x3/(2×3) + (1×3)x5/(2×4×5) + (1×3×5)x7/(2×4×6×7)……], x∈(-1,1)

arctan x = x - x3/3 + x5/5 -…, x∈(-∞,1)

sinh x = x+x3/3!+x5/5!+…+x2k-1/(2k-1)!+…, x∈**R**

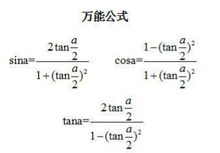
cosh x = 1+x2/2!+x4/4!+…+x2k/(2k)!+…, x∈**R**

arcsinh x =x - x3/(2×3) + (1×3)x5/(2×4×5) -(1×3×5)x7/(2×4×6×7)…, x∈(-1,1)

arctanh x = x + x3/3 + x5/5 + …, x∈(-1,1)

在解初等三角函数时，只需记住公式便可轻松作答，在竞赛中，往往会用到与图像结合的方法求三角函数值、三角函数[不等式](https://baike.baidu.com/item/%E4%B8%8D%E7%AD%89%E5%BC%8F" \t "_blank)、面积等等。

万能公式

[](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733/0/c9fcc3cec3fdfc038b017081d93f8794a4c22689?fr=lemma&ct=single)

傅里叶级数

[傅里叶级数](https://baike.baidu.com/item/%E5%82%85%E9%87%8C%E5%8F%B6%E7%BA%A7%E6%95%B0)又称三角级数

f(x)=a0/2+∑(n=0..∞) (ancosnx+bnsinnx)

a0=1/π∫(π..-π) (f(x))dx

an=1/π∫(π..-π) (f(x)cosnx)dx

bn=1/π∫(π..-π) (f(x)sinnx)dx

词条图册[更多图册](https://baike.baidu.com/pic/%E4%B8%89%E8%A7%92%E5%87%BD%E6%95%B0%E5%85%AC%E5%BC%8F/4374733?fr=lemma)