

## Fall 2017 Exam 2 Problem 4 Solution:

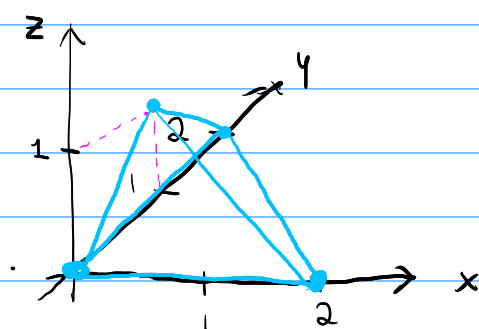
Each vertex of the tetrahedron is the intersection of three of the planes:

$$\begin{array}{lcl} V1 & \begin{array}{l} x=0 \\ z=0 \\ y=z \end{array} & \Rightarrow (0,0,0) \end{array}$$

$$\begin{array}{lcl} V2 & \begin{array}{l} x=0 \\ z=0 \\ x+y+z=2 \end{array} & \Rightarrow (0,2,0) \end{array}$$

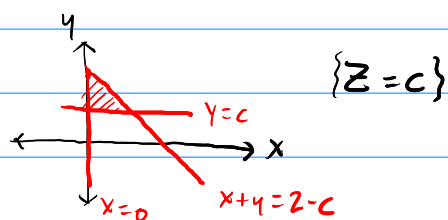
$$\begin{array}{lcl} V3 & \begin{array}{l} x=0 \\ y=z \\ x+y+z=2 \end{array} & \Rightarrow (0,1,1) \end{array}$$

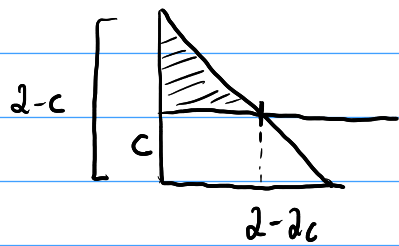
$$\begin{array}{lcl} V4 & \begin{array}{l} z=0 \\ y=z \\ x+y+z=2 \end{array} & \Rightarrow (2,0,0) \end{array}$$



Consider the  $z=c$  cross-section. ( $0 \leq c \leq 1$ )  
This is a triangle with edges given by

$$\begin{array}{l} x=0 \\ y=c \\ x+y=2-c \end{array}$$





Area of  $z=c$  cross section is  $\frac{(2-2c)^2}{2}$

Thus volume of tetrahedron is:

$$\begin{aligned}
 \int_0^1 \frac{(2-2c)^2}{2} dc &= 2 \int_0^1 (1-c)^2 dc \\
 &= 2 \left[ -\frac{(1-c)^3}{3} \right] \Big|_0^1 \\
 &= \frac{2}{3}
 \end{aligned}$$