Getting Close Enough (2 + 2 = 5): The Art of Approximate Computing

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Introduction

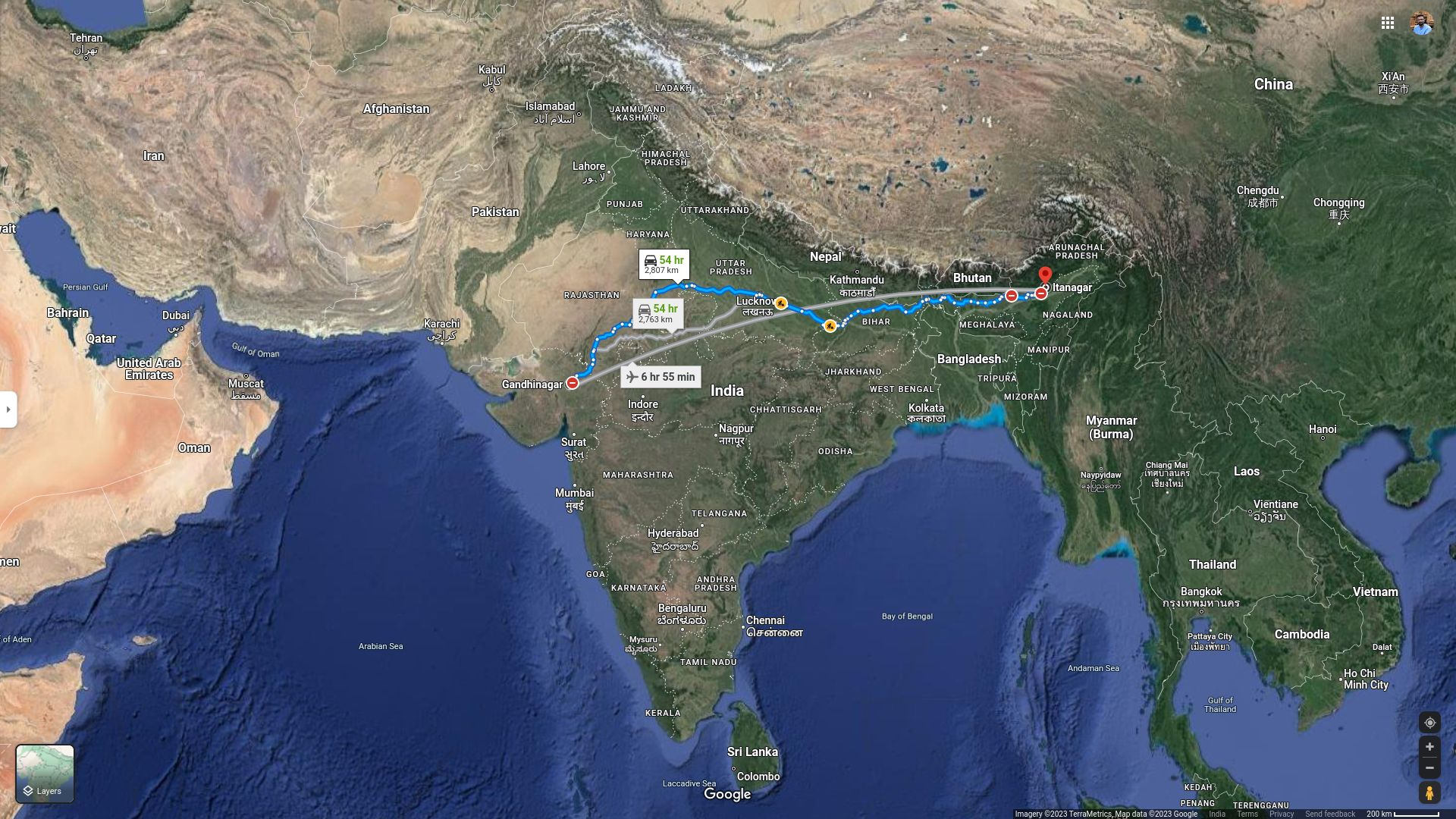
- Name: Kailash Prasad
- Education:
  - B.Tech. in Electronics and Communication Engineering from the National Institute of Technology Arunachal Pradesh (2018)
- Current Pursuit: Ph.D. in Electrical Engineering at the nanoDC Lab, IIT Gandhinagar
- Research Interests:
  - Hardware for machine learning
  - In-memory computing
  - Approximate computing
  - CAD tool design
  - SRAM memory subsystem design
- Recognitions and Scholarships:
  - Prime Minister Research Fellow
  - Intel India Research Fellowship recipient in 2020
  - SRC Research Scholar

Tenga Valley, Arunachal Pradesh

Govt. Higher Secondary School, Singchung

Kendriya Vidyalaya Tenga Valley

National Institute of Technology Arunachal Pradesh

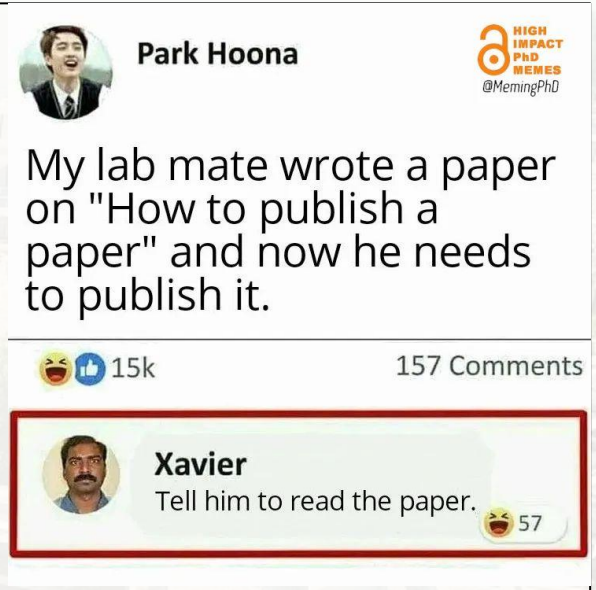

54 hr
2,807 km
54 hr
2,763 km
6 hr 55 min
Gandhinagar
Itanagar
Lucknow
Kathmandu
Tehran
Kabul
Islamabad
Lahore
Afghanistan
Iran
Pakistan
China
Nepal
Bhutan
India
Bangladesh
Myanmar (Burma)
Thailand
Laos
Vietnam
Cambodia
Oman
United Arab Emirates
Qatar
Bahrain
Dubai
Muscat
Karachi
Surat
Indore
Nagpur
Mumbai
Hyderabad
Bengaluru
Chennai
Kolkata
Sri Lanka
Colombo
Arabian Sea
Bay of Gulf
Andaman Sea
Laccadive Sea
Gulf of Thailand
Persian Gulf
Gulf of Oman
Chengdu
Chongqing
Xi'An
Hanoi
Vientiane
Bangkok
Ho Chi Minh City
Dalat
Pattaya City
Naypyidaw
Chiang Mai
Mysuru
Google
Layers
Imagery ©2023 TerraMetrics, Map data ©2023 Google
India
Terms
Privacy
Send feedback
200 km

Indian Institute of Technology Gandhinagar

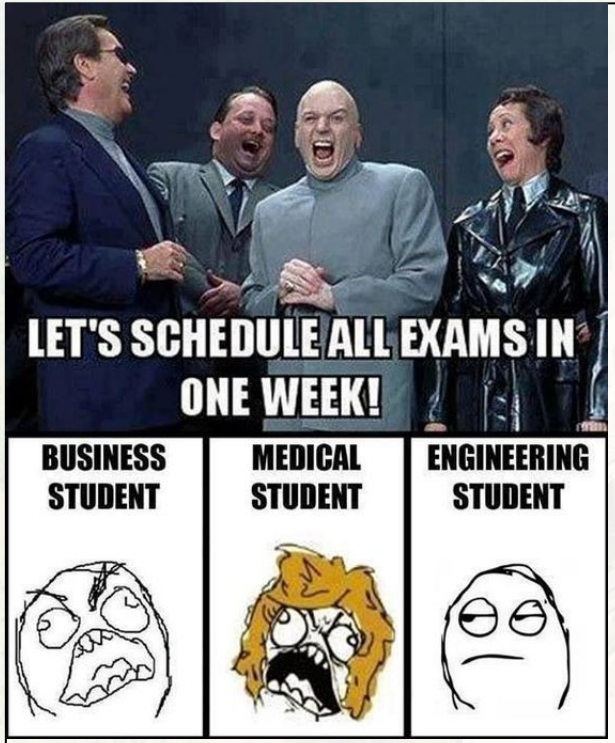
PhD Memes


HIGH IMPACT PhD MEMES
@MemingPhD
First year PhD
Last year PhD


Me when I realize that bananas without the 'b' is pineapple:


Park Hoona
HIGH IMPACT PhD MEMES
@MemingPhD
My lab mate wrote a paper on "How to publish a paper" and now he needs to publish it.
15k
157 Comments
Xavier
Tell him to read the paper.
57

Electrical Engineering Memes

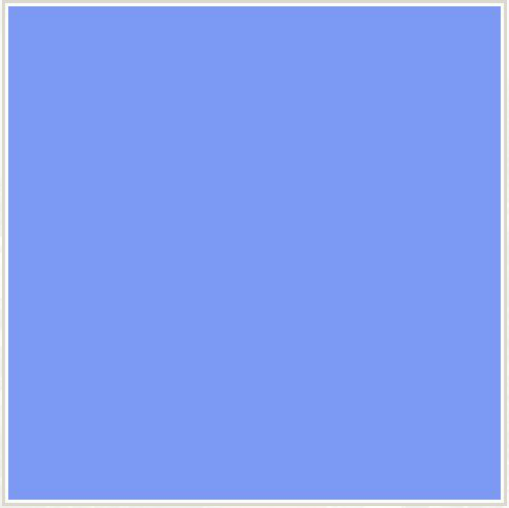


Why did the AC current break up with the DC current?

They just couldn't find a common ground.

Guess the Colour?

Blood Red

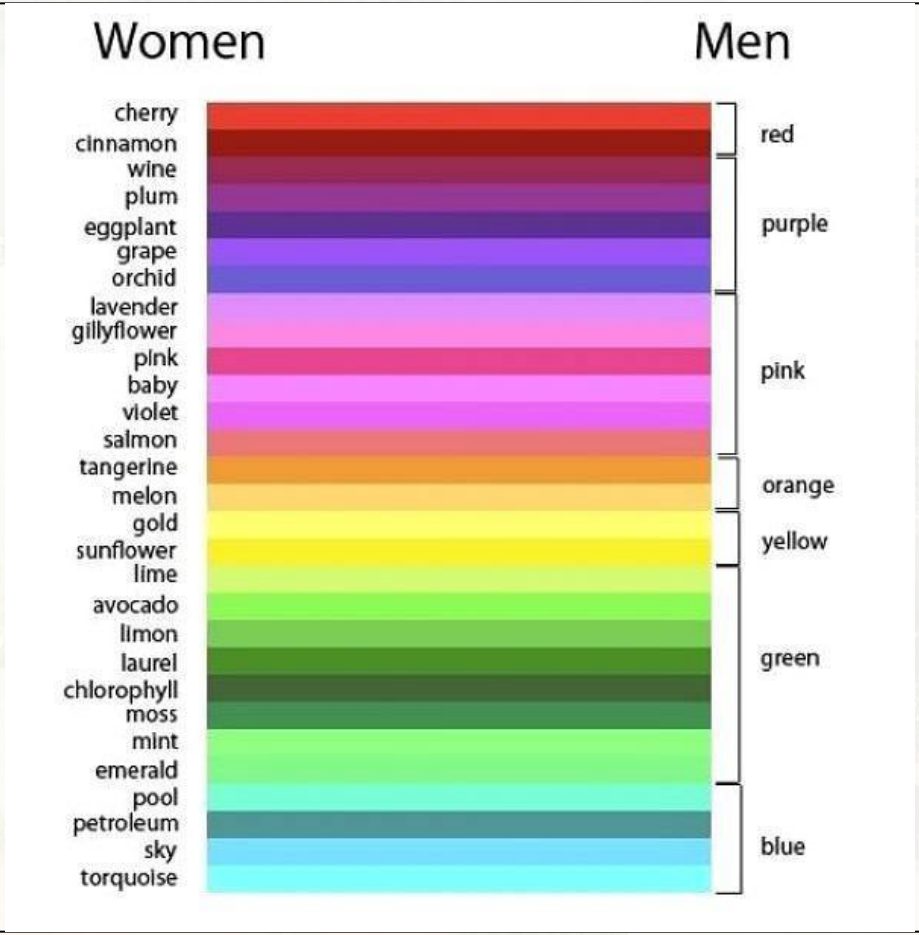


Cornflower

Evergreen

Many Shades of Blue

Name the Colors



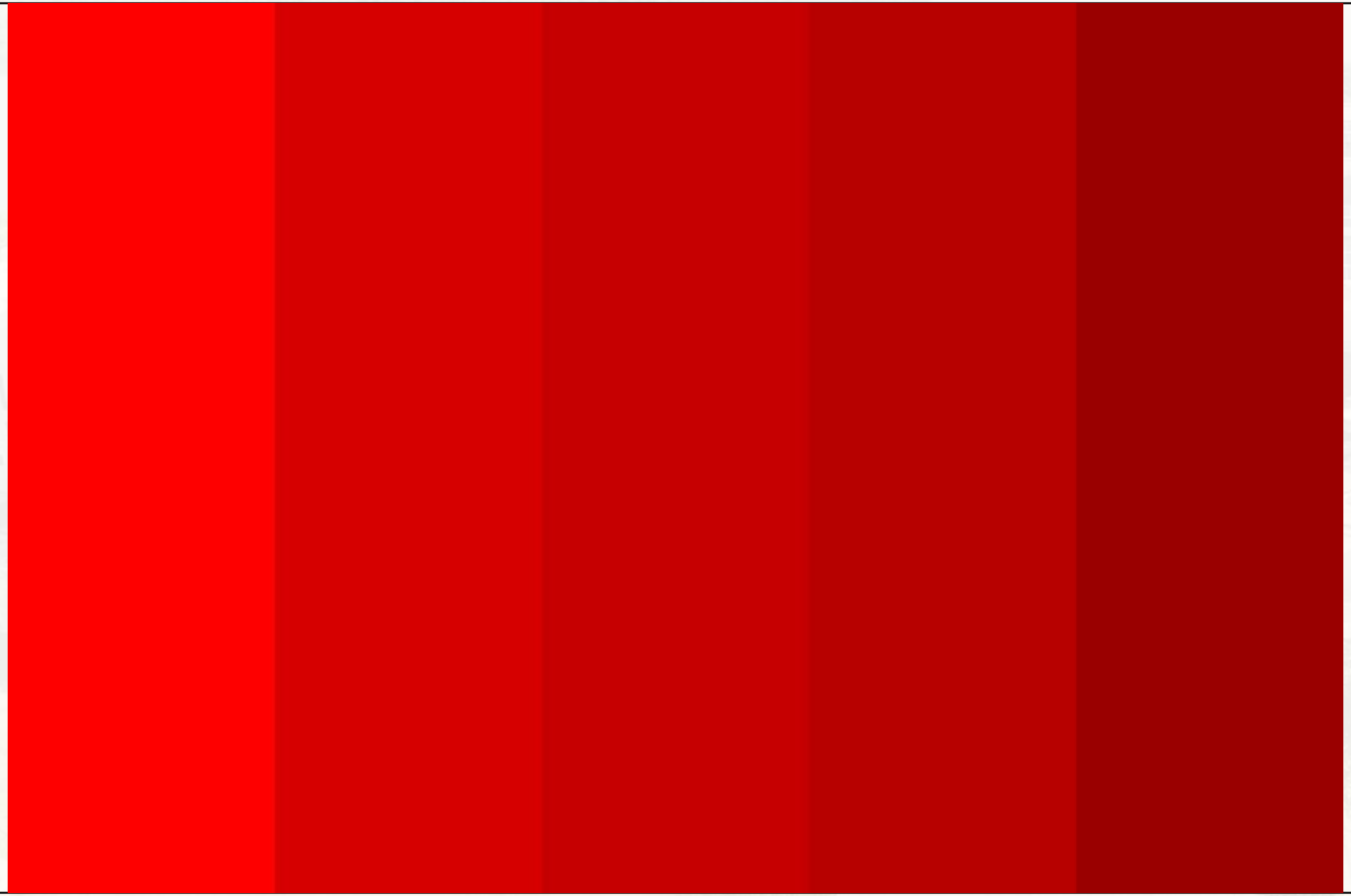
Many Shades of Blue

Shades of Red

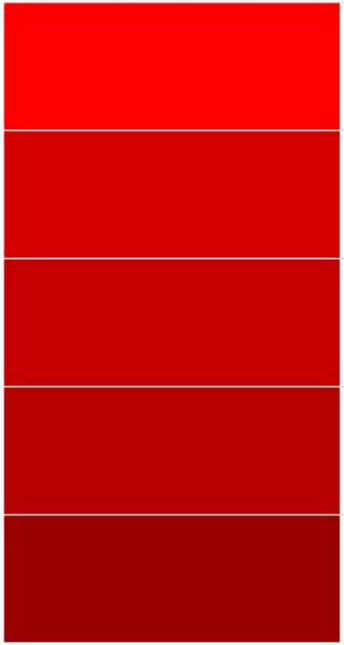
Shades of Red

Shades of Red

Shades of Red



Shades of Red



#ff0000	(255,0,0)
#d70000	(215,0,0)
#c60000	(198,0,0)
#b70000	(183,0,0)
#9b0000	(155,0,0)

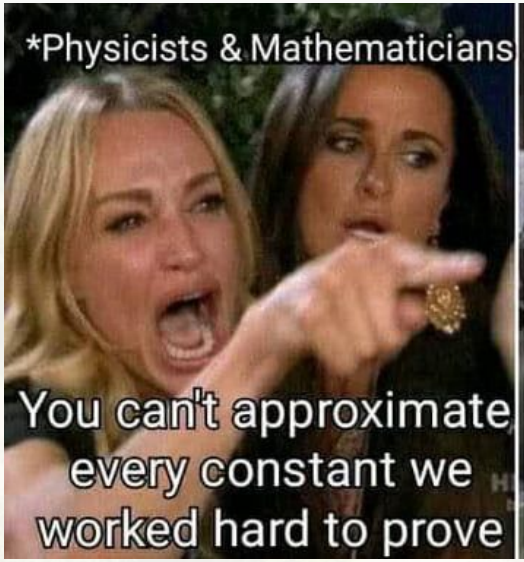
RGB(255, 0, 0)

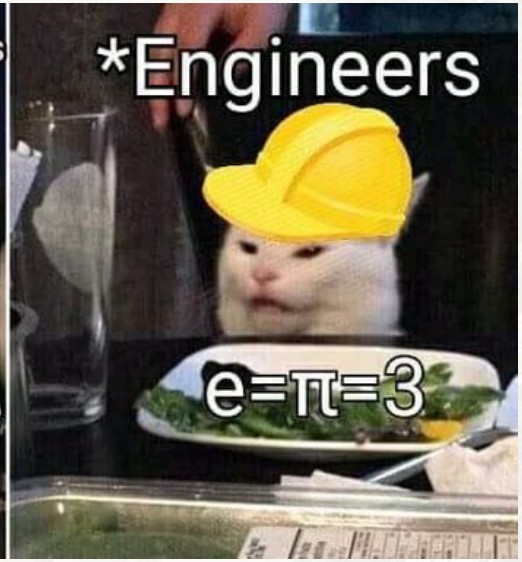
RGB(254, 0, 0)

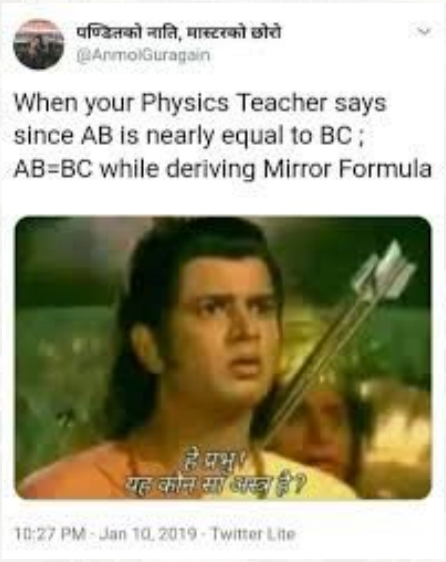
RGB(253, 0, 0)

Is it that you're content with any of the red shades, or are you scrutinizing the smallest details looking for flaws?

Approximation







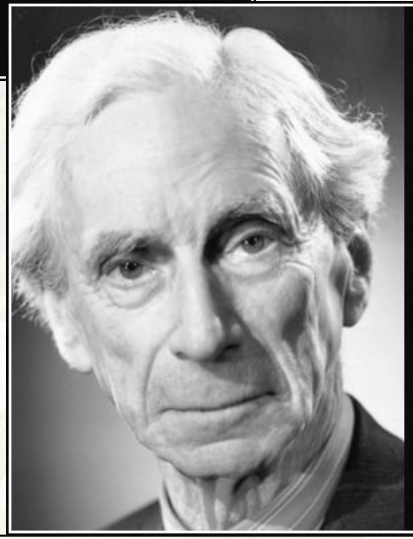
Approximation

Quotes

Truth is much too complicated to allow anything but approximations.

— John von Neumann —

AZ QUOTES



Although this may seem a paradox, all exact science is based on the idea of approximation. If a man tells you he knows a thing exactly, then you can be safe in inferring that you are speaking to an inexact man.

— Bertrand Russell —

AZ QUOTES

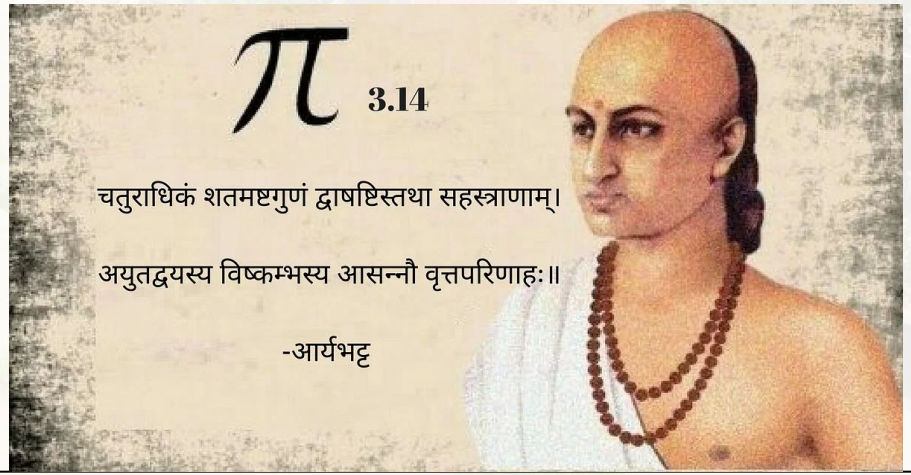
Approximation History

Approximation !!!

- One of Aryabhata's contribution in trigonometry was that, he gave methods of calculating their **approximate** numerical values.
- Bhaskara I (c. 600–680) expanded the work of Aryabhata in his books titled *Mahabhaskariya*, *Aryabhatiya-bhashya* and *Laghu-bhaskariya* and gave a rational **approximation** of the sine function.

Some facts

Aryabhata also demonstrated solutions to simultaneous quadratic equations, and produced an **approximation** for the value of π equivalent to 3.1416, correct to four decimal places. He used this to estimate the circumference of the Earth, arriving at a figure of **24,835 miles, only 70 miles off its true value.** But, perhaps even more astonishing, he seems to have been aware that π is an irrational number, and that any calculation with pi can only be an **approximation**, something not proved in Europe until 1761.



Approximation algorithm

- Iterative techniques for solving equations have been common in Indian astronomical calculations at least since the use in the **Paitāmahasiddhānta (fifth century)** of such a technique for correcting planetary longitudes.

- Parameśvara gives a more efficient **approximation** algorithm equivalent to what is now called the "secant method," in which the root of some function f(x) is approximated by

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

Value of Pi

- Π = 3 Baudhyana Shulba Sutra - (The Oldest **approximate** Value of Π)
- Π = 3 Mahabharata (Bhishmaparva, XII: 44)
- Π to be 18 * (3 – 2 $\sqrt{2}$) = 3.088 (other Shulba Sutras)
- Π to be 28/5= 3.125 (Manava Shulba Sutra)
- Π = $\sqrt{10}$ (Ancient Jaina School)
- Π = 62832/20000 = 3.1416. This was astonishingly correct to 4 decimal places Aryabhatta (476 AD)
- Madhava series (also Leibniz series) of Π/4

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}.$$

Square root of 2

- Rational **approximation** of root 2 occurs in Baudhayana, Apastamba and Katyayana Sulva Sutras

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34} = 1.4142156\ldots$$

- A commentator by name Rama who lived in the middle of the 15th century A.D., in a place called Naimis .a near modern Lucknow, improved upon this **approximation** and obtained

$$\sqrt{2} \approx 1 + \frac{1}{3} + \frac{1}{(3)(4)} - \frac{1}{(3)(4)(34)} - \frac{1}{((3)(4)(34)(33)} + \frac{1}{(3)(4)(34)(34)}$$

which gives a better **approximation**, correct up to seven decimal places

Approximation formula

The formula for calculating sin of an angle is given in verses 17 – 19, Chapter VII, Mahabhaskariya of Bhaskara I.

$$\sin x^\circ \approx \frac{4x(180 - x)}{40500 - x(180 - x)}$$

Bhaskara I's sine **approximation** formula can be expressed using the radian measure of angles as follows

$$\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}$$

Indian style of Mathematics

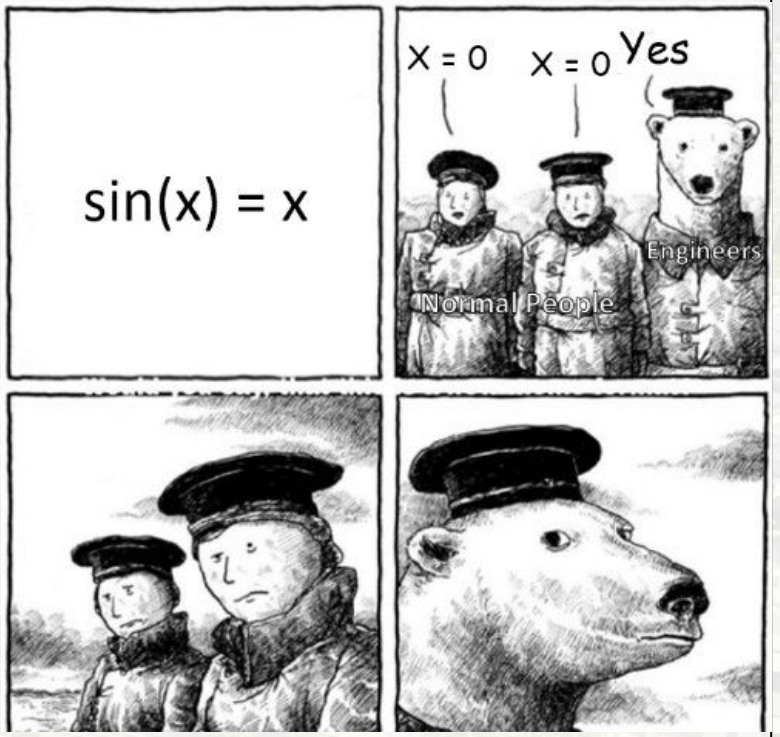
- The reason for this spectacular success of the Indian mathematicians lies in the explicitly algorithmic and computational nature of Indian mathematics.

- Indian mathematicians were not trying to discover the ultimate axiomatic truths in mathematics; they were interested in finding methods of solving specific problems that arose in the astronomical and other contexts.

- Therefore, Indian mathematicians were prepared to work with simple algorithms that may give only **approximate** solutions to the problem at hand; and they evolved sophisticated theories of error and recursive procedures to keep the approximations in check.

Do you use Approximation?

- Semiconductor device modelling
- Signal Processing
- Numerical Methods
  - Taylor Series
- Electromagnetic field analysis
  - Maxwell Equation
- Circuit Analysis
  - KCL and KVL



You are using approximation everytime you say

Ignore

Neglect

Assume

Error: Essential Part of the Design Process

John von Neumann's View on Error (1952):

"Our present treatment of error is unsatisfactory and ad hoc. … Error is viewed (in this work), therefore, not as an extraneous and misdirected or misdirecting accident, but as an essential part of the process under consideration …"

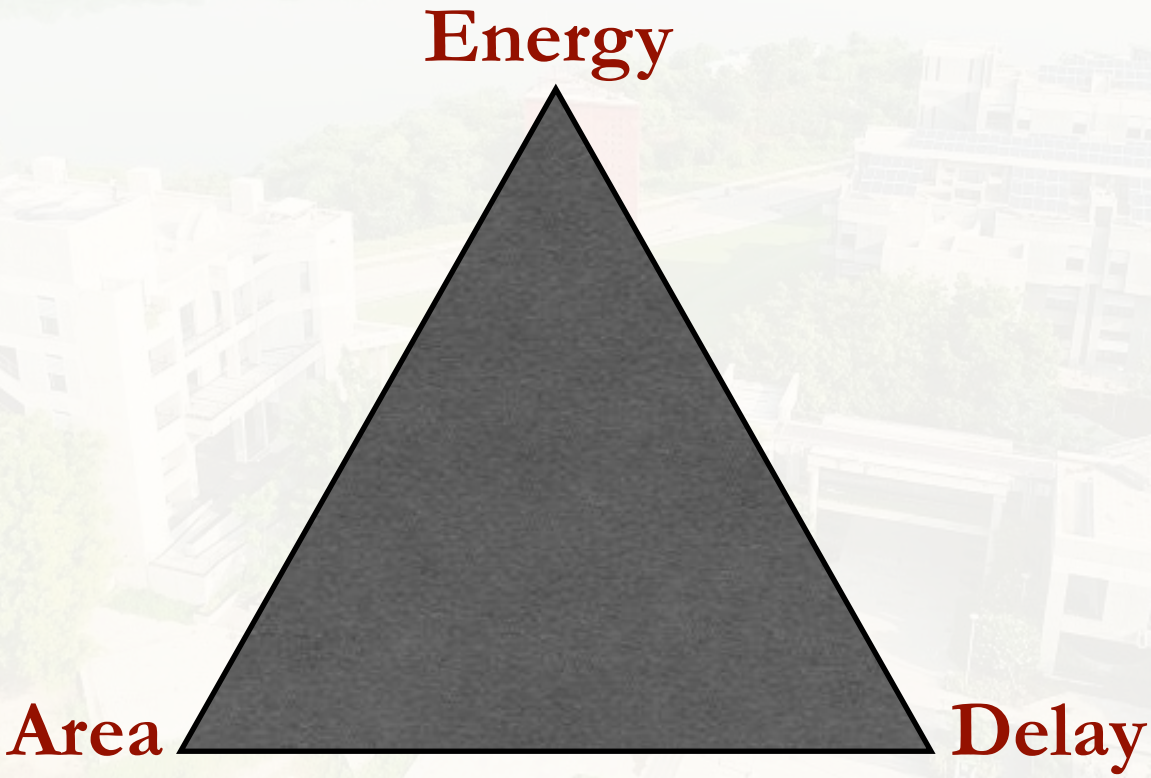
Source : https://ece.uwaterloo.ca/~ssundara/courses/prob_logics.pdf

What is Approximate Computing?

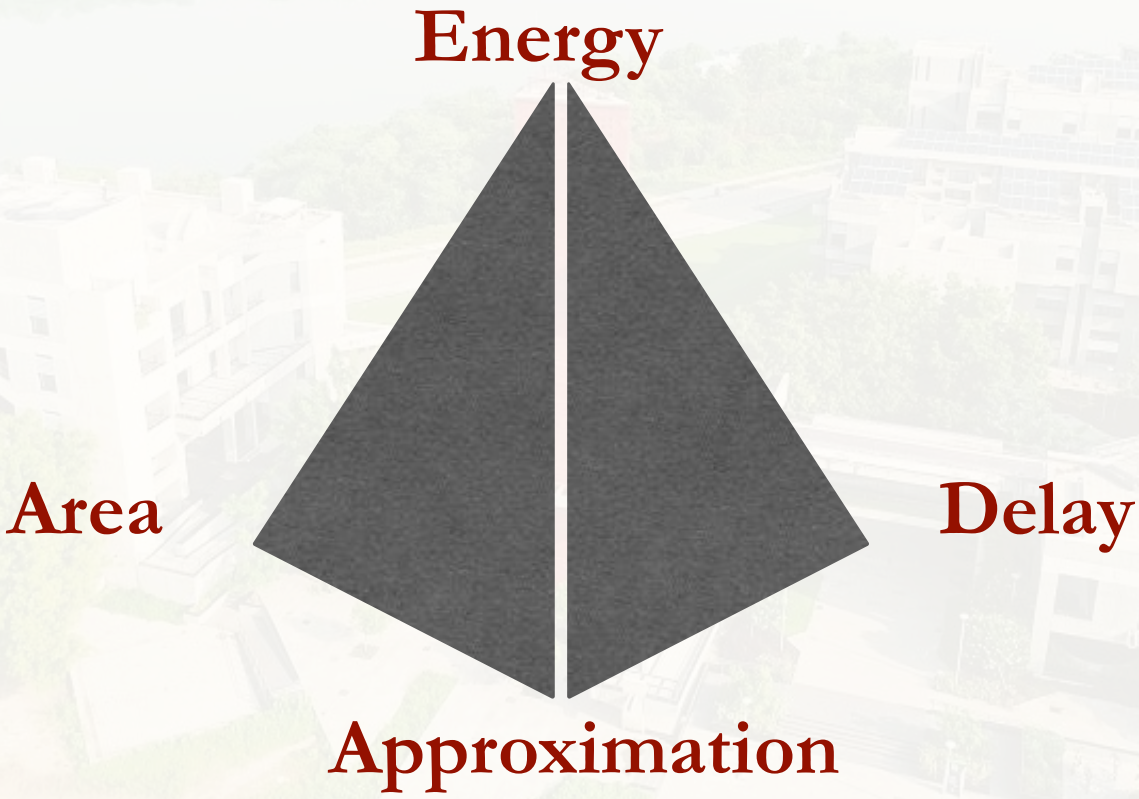
- Approximate computing is like trying to make a cake without measuring the ingredients. You just eyeball it and hope for the best. Sometimes it turns out great, other times it's a disaster. But hey, at least you didn't waste too much time trying to be precise, right? It's like the saying goes: close enough for computing!

- Relax the accuracy of computation
  - Improve Performance
  - Reduce Energy
  - Reduce Power
  - Reduce Delay

Why Approximate ?

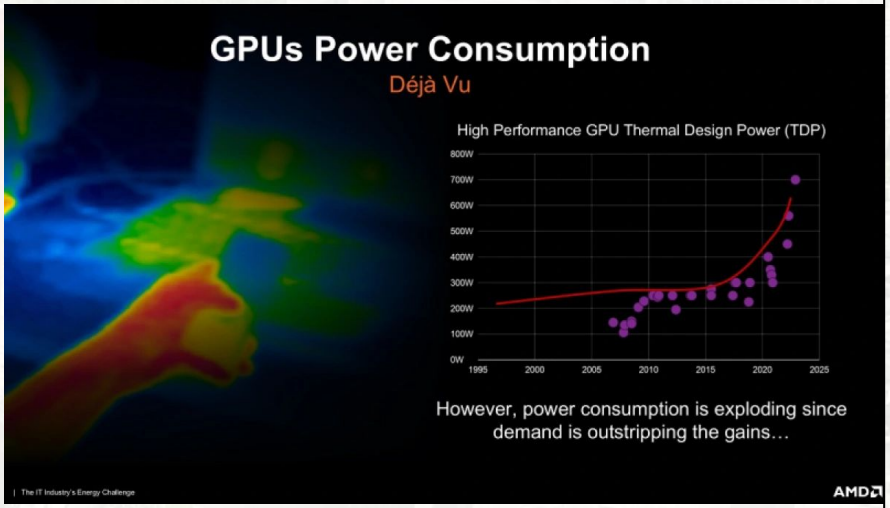


Why Approximate ?



AlphaGO

AlphaGO	Lee Se-dol
1202 CPUs, 176 GPUs,	1 Human Brain,
100+ Scientists.	1 Coffee.



Which One is Original Image?

Which One is Original Image?

Exact

```
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 ...,
 [ 43,  43,  50, ..., 104, 100,  98],
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 [ 44,  40,  49, ..., 104, 101,  99],
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 [ 42,  41,  56, ..., 105, 104, 109]]
```
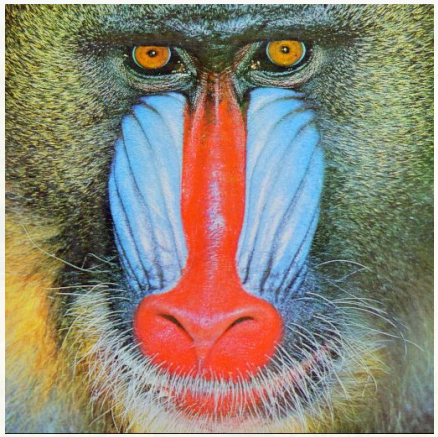
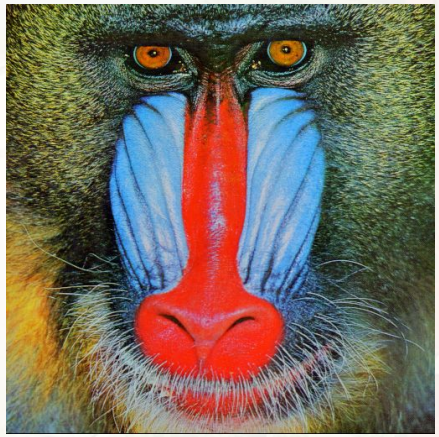
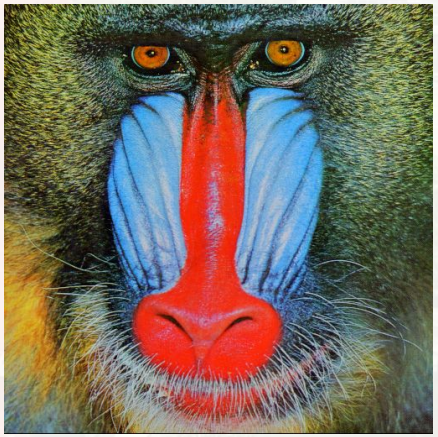

Approximate

Power Law Transformation Contrast Enhancement

$$EnhancedImage = Image^{\sqrt{2}}$$


$\sqrt{2} = 1.414$ $\sqrt{2} = 1.41$ $\sqrt{2} = 1.4$

Approximation is everywhere

IEEE 754 Single precision Arithmetic

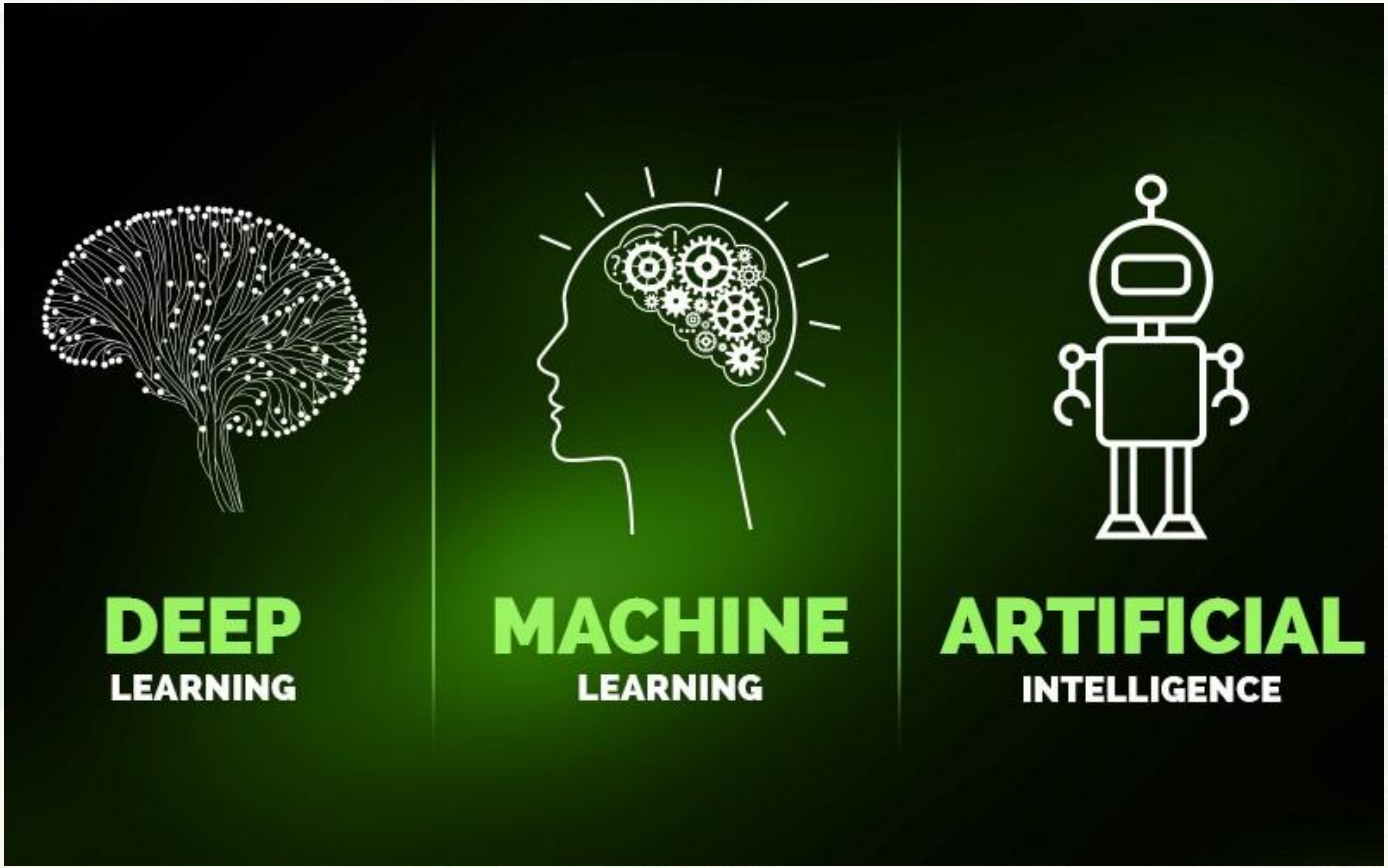
$$\left(\frac{27/10 - e}{\pi - \left(\sqrt{2} + \sqrt{3} \right)} \right)^{67/16}$$

Correct Answer: **302.8827196…**

IEEE 754 32 Bit Answer: **302.912...**

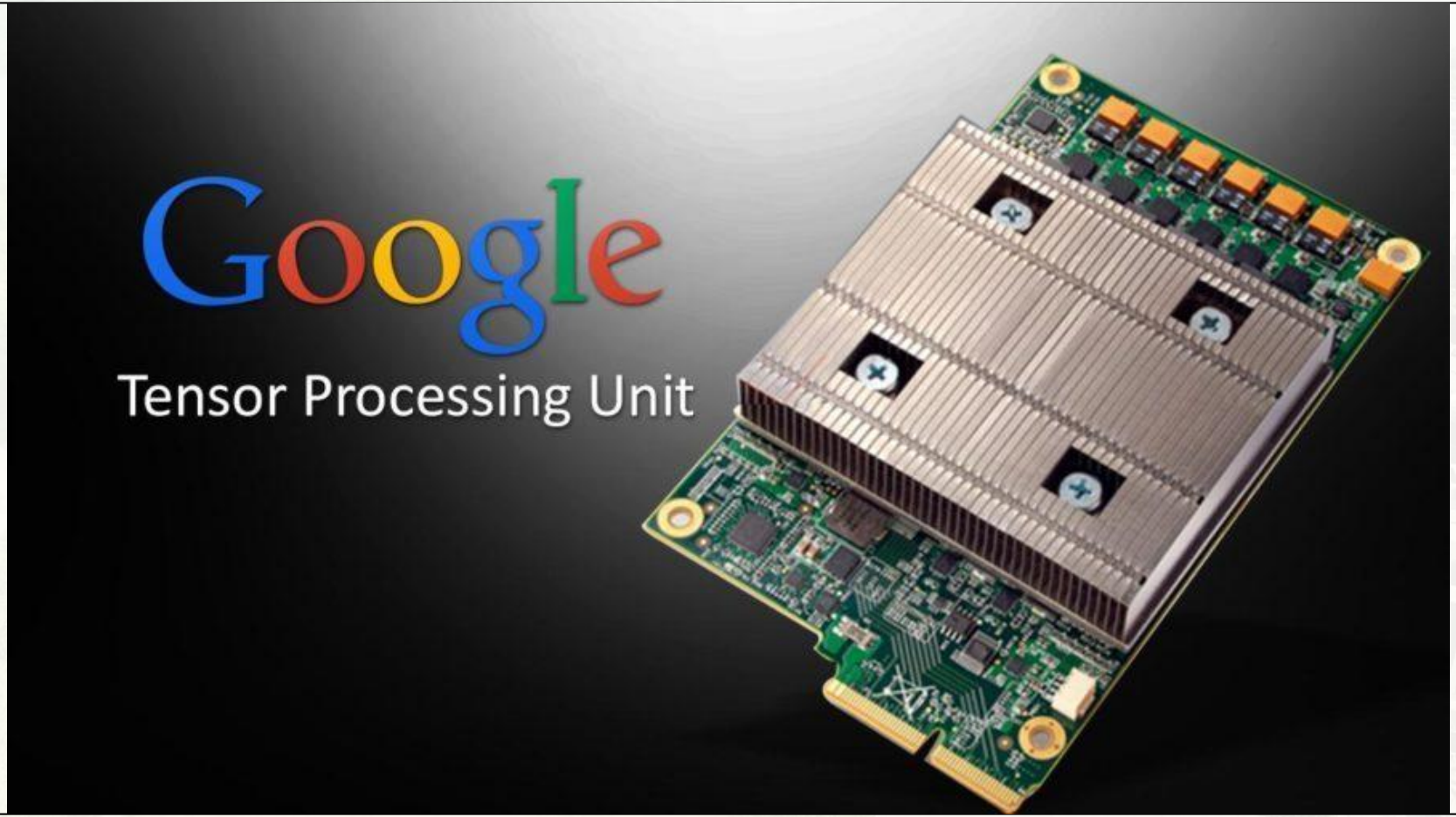
How much approximation you should do?

Biggest Application



Source: https://becominghuman.ai/artificial-intilligence-machine-learning-deep-learning-df6dd0af500e

Hardware

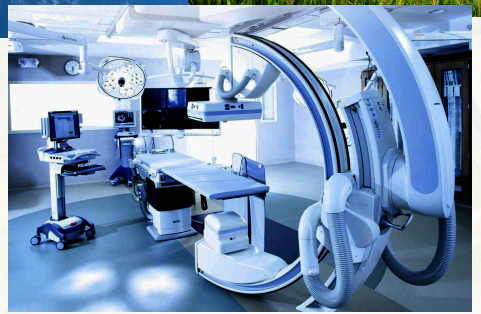


Should We approximate Everything?

What should and What should not be approximated?

High Precision Application

- Aeronautics - Chandrayaan 3
- Health care
- Nuclear Missions



Error Tolerant Application

- Image Processing
- Video Processing
- Machine Learning

Biggest Approximation of All time

Working in the digital domain, you're using approximations of things; the actual sound wave never enters the equation. You deal with sections of it, and you're able to do so much more by just reducing the information to a finite amount.

QUOTEHD.COM

Sean Booth
English Musician

nanoDC Family

nanoDC Family

Thank You

https://kailashprasad.com