Homework 0: Probability Review

CS 585, UMass Amherst, Fall 2017

1 Domain of a joint distribution

1.1

A and B are discrete random variables. A could take on one of 4 possible values. B could take on one of 3 possible values. (In other words, the size of domain(A) is 4, and the size of domain(B) is 3.) How many possible outcomes does the joint distribution P(A, B) define probabilities for?

ANS: 12

1.2

Say we have a sequence of n binary random variables A1, A2, . . . An. How many possible outcomes does the joint distribution P(A1, A2, . . . An) define probabilities for?

ANS: 2^n since domain of binary random variable is 2.

2 **Independence versus Basic Definitions**

Say we have three random variables A and B and C. Note that we're using standard probability theory notation where P(A, B) = P(B, A), which simply means the joint probability of both A and B occurring.

2.1 Which of the following statements is always true?

1. P(A|B) = P(B|A)	FALSE
2. P(A, B) = P(A|B)P(B)	TRUE IF P(B)!=0
3. P(A, B) = P(A)P(B)	FALSE
4. P(A|B) = P(A)	FALSE
5. P(A, B, C) = P(A)P(C)	FALSE
6. P(A, B, C) = P(A)P(B)P(C)	FALSE
7. P(A, B, C) = P(A)P(B|A)P(C|A, B)	TRUE
8. P(A) = P b∈domain(B) P(A, B = b)	TRUE
9. P(A) = P b∈domain(B) P(A|B = b)P(B = b)	TRUE

2.2

Now assume that A, B, and C are all independent of each other. Which of these statements is true?

1. P(A|B) = P(B|A)	FALSE
2. P(A, B) = P(A|B)P(B)	TRUE IF P(B)!=0
3. P(A, B) = P(A)P(B)	TRUE
4. P(A|B) = P(A)	TRUE
5. P(A, B, C) = P(A)P(C)	FALSE
6. P(A, B, C) = P(A)P(B)P(C)	TRUE
7. P(A, B, C) = P(A)P(B|A)P(C|A, B)	TRUE
8. P(A) = P b∈domain(B) P(A, B = b)	TRUE
9. P(A) = P b∈domain(B) P(A|B = b)P(B = b)	TRUE

3 Logarithms

3.1

Let p be a probability, so it is bounded to [0, 1] (between 0 and 1, inclusive). What is the range of possible values for log(p)? Please be specific about open versus closed intervals.

[- ∞,0] Closed Interval

3.2

Let p and q both be probabilities. What is the range of possible values for p/q?

[0, ∞] **Closed Interval**

3.3

What is the range of possible values for log(p/q)?

[-∞,∞]Closed Interval

4 Deriving Bayes Rule

If A and B are events with P(B)>0, then conditional probability of A given B is

$$P(A/B) = P(A, B)/P(B)$$

HENCE

i. $P(A, B) = P(A/B)\, P(B)$

ii. $P(B, A) = P(B/A)\, P(A)$ (Using conditional probability)

iii. $P(A, B) = P(B, A)$(both mean the same)

Substituting i and ii into iii we get

$$P(A/B)\, P(B) = P(B/A)\, P(A)$$

$$\mathbf{P(A/B) = \frac{(P(B/A)\, P(A))}{P(B)}}$$