

Let (X, μ) be a measure space. enumerate

Prove that $f_n \rightarrow f$ in measure iff for all $\varepsilon > 0$ there is N such that $\mu(\{x : |f_n(x) - f(x)| \geq \varepsilon\}) < \varepsilon$ for all $n \geq N$.

Suppose $f_n \rightarrow f$ in measure and $g_n \rightarrow g$ in measure. Prove that $f_n + g_n \rightarrow f + g$ in measure, and if $\mu(X) < \infty$ then $f_n g_n \rightarrow fg$ in measure.