

Is $(\lambda \vec{v}) \times (\alpha \vec{w}) = \lambda \alpha (\vec{v} \times \vec{w})$? YES

$$\vec{v} = \langle v_1, v_2, v_3 \rangle, \quad \vec{w} = \langle w_1, w_2, w_3 \rangle.$$

$$\lambda \vec{v} = \langle \lambda v_1, \lambda v_2, \lambda v_3 \rangle, \quad \alpha \vec{w} = \langle \alpha w_1, \alpha w_2, \alpha w_3 \rangle$$

$$\lambda \vec{v} \times \alpha \vec{w} = \langle \lambda v_1, \lambda v_2, \lambda v_3 \rangle \times \langle \alpha w_1, \alpha w_2, \alpha w_3 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \lambda v_1 & \lambda v_2 & \lambda v_3 \\ \alpha w_1 & \alpha w_2 & \alpha w_3 \end{vmatrix} = (\lambda v_2 \alpha w_3 - \lambda v_3 \alpha w_2) \vec{i} - (\lambda v_1 \alpha w_3 - \lambda v_3 \alpha w_1) \vec{j} + (\lambda v_1 \alpha w_2 - \lambda v_2 \alpha w_1) \vec{k}$$

$$= \lambda \alpha \left[(v_2 w_3 - v_3 w_2) \vec{i} - (v_1 w_3 - v_3 w_1) \vec{j} + (v_1 w_2 - v_2 w_1) \vec{k} \right]$$

$$= \lambda \alpha \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

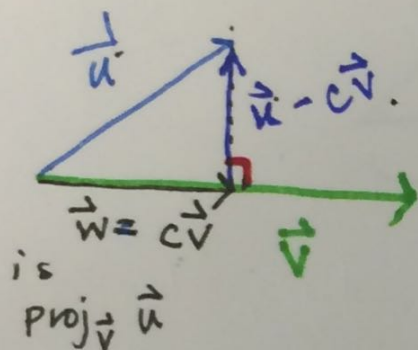
$$= \lambda \alpha (\vec{v} \times \vec{w}).$$

Theorem 3, P. 681, Projection $\vec{v} \neq \vec{0}$.

Projection of \vec{u} along \vec{v} is the vector

$$\text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$$

Proof: (sketch)



$$\vec{u} - c\vec{v} \perp \vec{v} \quad \text{so } \dots$$

$$(\vec{u} - c\vec{v}) \cdot \vec{v} = 0$$

$$\vec{u} \cdot \vec{v} - c\vec{v} \cdot \vec{v} = 0$$

$$\vec{u} \cdot \vec{v} = c\vec{v} \cdot \vec{v}$$

$$\vec{v} \cdot \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} = c(\vec{v} \cdot \vec{v})$$

$$\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v} = c\vec{v}$$

$$c\vec{v} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$$

$$\therefore \text{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \right) \vec{v}$$