Define $(a_i,b_i)' = (a,\infty)\cap(a_i,b_i)$ and $(a_i,b_i)'' = (-\infty,a]\cap(a_i,b_i)$. Notice $((a,\infty)\cap(a_i,b_i))\cup((-\infty,a]\cap(a_i,b_i)) = (a_i,b_i)$. Then, $m^*(a_i,b_i) = b_i - a_i = m^*((a_i,b_i)') + m^*((a_i,b_i)'')$.

Since $E \subset (a_i,b_i)$ for any i and $E\cap(a,\infty) \subset (a_i,b_i)\cap(a,\infty) \subset \bigcup_{i=1}^{\infty}(a_i,b_i)'$. So, by monotonicity and subadditivity of m^*,

$$m^*(E\cap(a,\infty)) \leq m^*\left(\bigcup_{i=1}^{\infty}(a_i,b_i)'\right) \leq \sum_{i=1}^{\infty} m^*((a_i,b_i)').$$

Similarly, since $E \subset (a_i,b_i)$ for any i and $E\cap(-\infty,a] \subset (a_i,b_i)\cap(-\infty,a] \subset \bigcup_{i=1}^{\infty}(a_i,b_i)''$.

So, by monotonicity and subadditivity of m^*,

$$m^*(E\cap(-\infty,a]) \leq m^*\left(\bigcup_{i=1}^{\infty}(a_i,b_i)''\right) \leq \sum_{i=1}^{\infty} m^*((a_i,b_i)''). \text{ Thus,}$$

$$m^*(E\cap(a,\infty)) + m^*(E\cap(-\infty,a]) \leq \sum_{i=1}^{\infty} m^*((a_i,b_i)') + \sum_{i=1}^{\infty} m^*((a_i,b_i)'') = \sum_{i=1}^{\infty}\left(m^*((a_i,b_i)') + m^*((a_i,b_i)'')\right)$$

$$= \sum_{i=1}^{\infty}(b_i - a_i) \leq m^*(E) + \varepsilon. \text{ Hence, } m^*(E) \geq m^*(E\cap(a,\infty)) + m^*(E\cap(-\infty,a]).$$

Thus, (a,∞) is m^*-measurable.

```
\item If $\{f_n \}_{n\in \N}$ is a sequence of Lebesgue measurable real-valued functions, prove that \\ $f=\liminf f_
\begin{pf}

\end{pf}

\item Let $f \colon \R \rightarrow [0, \infty)$ be Lebesgue measurable.
\begin{enumerate}[(a)] % (a), (b), (c), ...
\item Let $E_m=\{ x \in \R \colon f(x)> 1/m \}$. Use the monotone convergence theorem to show
\begin{equation*}
 \lim_{m \rightarrow \infty} \int_{E_m} f dm = \int_{\R} f dm.
\end{equation*}
\begin{pf}

\end{pf}
\item Prove that if $\int_\R fdm<\infty$, then for all $\varepsilon < \infty$, there exists $A \in \B_\R$ with $m(A)<
\begin{equation*}
  \int_{\R} f dm < \int_{A} f dm + \varepsilon.
\end{equation*}
\begin{pf}
        Assume $\int_\R fdm<\infty$. Then, $f \in L^+$ and \[
        \int_\R f \ d m = \sup \left\{ \int_\R  \phi\ d m : 0 \leq \phi \leq f, \ \phi \text{ simple} \right\}.
        \]
        Thus, for any $\epsilon>0$, we can find a simple function $\phi$ such that \[
         \int_\R f \ dm < \int_\R \phi \ dm + \epsilon
        \]
         Let \[
```

Dear LaTex,

I love your syntax. Thanks for the nice math formatting. Stay beautiful.

Kailee

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{ Then, } \int \phi \ dm = \sum_1^n a_j m(E_j).
 of all such integrals and $\int f \ dm < \infty$, $\int \phi \ dm < \inf
1^k E_j
. Also, $m(A_j) < \infty$ for all $j$ implies $m(\cup_1^k A_j) = m(A)
) = \sum_1^k a_j m(E_j) = \sum_1^k a_j m(A_j)= \int_A \phi \ dm.
nt_A \phi \ dm : 0 \leq \phi \leq f, \phi $ simple $ \}$, we have $\in
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\begin{homeworkProblem}[Exercise 3.2.21: Let $G$ be a group. The set $Z(G
\begin{proof}
        Note $e \in Z(G)$ since $eg=ge$ for all $g \in G$. Also, all $x \
\end{proof}
\noindent \textbf{(b)Show that $Z(G)=\bigcap_{a \in G}C(a)$. }
\begin{proof}
        \textbf{(show $Z(G)\subseteq \bigcap_{a \in G}C(a)$)} Consider an
        \textbf{(show $Z(G)\supseteq \bigcap_{a \in G}C(a)$)} Consider any $x \in \bigcap_{a \in G}C(a)$.
\end{proof}

\noindent \textbf{(c) Compute the center of $S_3$. }
Consider the multiplication table of $S_3$:
 \[
 \begin{array}{c||c|c|c|c|c|c}
  \circ & (1) & (12) & (13)& (23) & (123)& (132)\\
 \hline
  (1) & (1) & (12) & (13)& (23) & (123)& (132)\\
 \hline
  (12) & (12) & (1) & (132)& (123) & (23)& (13)\\
 \hline
   (13) & (13) & (123) & (1)& (132) & (12)& (23)\\
 \hline
   (23) & (23) & (132) & (123)& (1) & (13)& (12)\\
 \hline
   (123) & (123) & (13) & (23)& (12) & (132)& (1)\\
 \hline
   (132) & (132) & (23) & (12)& (123) & (1)& (123)\\
 \end{array}
 \]
 By inspection, $Z(G)=\{(1)\}$. Also note that $C(123) \cap C(12)=\{(1) \}$ so $Z(G)=\{(1)\}$ also follows
```

).

Proof. **(show $Z(G) \subseteq \bigcap_{a\in G} C(a)$)** Consider any $x \in Z(G)$. Then, for all $g \in G$, $xg = gx$. Equivalently for all $a \in G$, $xa = ax$. Thus, $x \in C(a)$ for all $a \in G$ so $x \in \bigcap_{a\in G} C(a)$.

(show $Z(G) \supseteq \bigcap_{a\in G} C(a)$) Consider any $x \in \bigcap_{a\in G} C(a)$. Then, for all $a \in G$, $x \in C(a)$. So, for all $a \in G$, $xa = ax$. Equivalently, for all $g \in G$, $xg = gx$ so $x \in Z(G)$. □

(c) Compute the center of S_3. Consider the multiplication table of S_3:

$\circ$	(1)	(12)	(13)	(23)	(123)	(132)
(1)	(1)	(12)	(13)	(23)	(123)	(132)
(12)	(12)	(1)	(132)	(123)	(23)	(13)
(13)	(13)	(123)	(1)	(132)	(12)	(23)
(23)	(23)	(132)	(123)	(1)	(13)	(12)
(123)	(123)	(13)	(23)	(12)	(132)	(1)
(132)	(132)	(23)	(12)	(123)	(1)	(123)