Addressing asymmetrical ties in ranking systems of contestants and judges using primes and irrational numbers

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Proof School Math Burst 2020
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Ranking system premise & initial setup

- Given *n* judges and *k* contestants, assign each contestant a rank from 1 through *k*.
- We identified a couple immediate issues with commonplace ranking systems:
  - If there is an even number of judges, after averaging their votes, it is possible for contestants to tie for a position in the ranking.
  - One of the judges is crazy and does not vote with reason or logic.
  - Do judges rank each contestant, provide a rating, or pick their favorite *p* contestants?

Even number of judges?

If there were an even number of judges ranking their contestants, we can arbitrarily ignore a judge's ranking, making an odd number of opinions.

Crazy judge?

For logic and simplicity, we chose to assume that all judges voted under reason.

If a judge really is crazy, we disregard their ranking.

How do the judges cast their votes?

- We started with judges assigning each contestant a ranking from 1-10.
- Ranking each contestant from 1 - *k* produced better results.

Proposed ranking system and point assignment

- Each judge produces a ranking of contestants from integers 1 through *k*, with 1 being their most favored contestant and *k* being their least favorite. Each contestant must receive a unique ranking, and each ranking 1, ..., *k* must be assigned to a contestant.
  - An example of computing the total scores for a contestant would be 1 point for *k*'th place, 2 points for *k - 1*'th, and so on.
  - The optimal way to do this is to assign each position a prime or an irrational number, making it impossible for them to sum to each other.
- **Symmetrical ties** occur when contestants get the same set of ranks from different judges.
  - We found that this is not preventable.
- **Asymmetrical ties** occur when the total scores of two contestants are equal, but not symmetrical.
  - This occurs if the point values may sum to each other, but using primes and irrationals to assign ranking points, this is not possible.

Examples of symmetrical and asymmetrical ties

- Given judges A, B, and C, give X the rankings 1, 2, 3 and Y the rankings 2, 3, 1, they would be in a symmetrical tie.
- Given judges A, B, and C, give X the rankings 5, 3, 2 and Y the rankings 1, 5, 4, they would be in an asymmetrical tie.

Non prime or irrational ranking system example

- Each ranking has a point value of its distance from *(k+1)/2*.
- For each additional time that someone is ranked at a certain rank, they receive their rank points, plus additional rank points