vision math for rapid react, 2022

table of variables

variable name	meaning of variable	value
y_{img}	distance from the bottom of the screen in px	$0 < y_{img} < 240$
x_{img}	distance from the center horizontally in px, right = positive and left = negative	$160 < x_{img} < 320$
f	focal length (horizontal)	284.772
h	height from camera to tape in feet	4.8125
z	straight line distance to the target in feet	z
θ	horizontal angle between robot and target	θ
ρ	angle between robot and target, vertically	$arctan(\frac{h}{z}$
z_θ	distance to target if it's angled	z_θ
φ	vertical tilt up of limelight camera in RADIANS	φ
y_v	virtual y_{img}, where it would be if the FOV is centered	y_v

no tilt in any direction

Assumptions: robot is aligned, limelight FOV is 60 degrees and it's exactly 30 degrees on each side (FOV is centered)

because of how a pinhole camera works, and since no tilt in any axis,

$$x_{img} = 0$$

$$\frac{y_{img}}{f} = \frac{h}{z}$$

$$z = \frac{hf}{y_{img}}$$

alternatively,

$$\rho = arctan(\frac{y_{img}}{f}) = arctan(\frac{h}{z})$$

tilt in the horizontal direction

Assumptions: FOV is still centered.

Since there is horizontal tilt, $x_{img} \neq 0$. So we have a right triangle with legs x_{img}, in non-camera verison, and z, and hypotenuse z_θ, where z_θ is actual distance to target and z is the distance to the target if it were aligned. We already know that

$$z = \frac{hf}{y_{img}}$$

so we have that, by the pythagorean theorem,

$$z^2 + (\frac{x_{img}h}{y_{img}})^2 = {z_\theta}^2$$

solving for z_θ, we get

$$z_\theta = \sqrt{(\frac{hf}{y_{img}})^2 + (\frac{x_{img}h}{y_{img}})^2}$$

tilt in the vertical direction

Now the FOV is no longer centered. There is a tilt of φ radians up. For now, we will disregard horizontal tilt and assume we are aligned to the target.

Since $\theta = arctan(\frac{y_{img}}{f})$, we can get the vertical angle of the perceived target. Then since we are adding φ, the target appears further away than expected if φ is positive, so we take $tan(arctan(\frac{y_{img}}{f}) + \varphi)$ as the target is actually further up. Then finally, we take h divided by this to get the distance:

$$z_\varphi = \frac{h}{tan(arctan(\frac{y_{img}}{f}) + \varphi)}$$

We also get that $y_v = tan(arctan(\frac{y_{img}}{f} + \varphi)) \cdot f$.

all 3 axis together

So we have that

$$z_\theta = \sqrt{(\frac{hf}{y_{img}})^2 + (\frac{x_{img}h}{y_{img}})^2}$$

$$z_\varphi = \frac{h}{tan(arctan(\frac{y_{img}}{f}) + \varphi)}$$

When we derived z_θ, we had an intermediate equation $z^2 + (\frac{x_{img}h}{y_{img}})^2 = {z_\theta}^2$

Now we have that $z = z_\varphi$ and that y_v is actually $tan(arctan(\frac{y_{img}}{f}) + \varphi) \cdot f$

Plugging this in, we get

$$(\frac{h}{tan(arctan(\frac{y_{img}}{f}) + \varphi)})^2 + (\frac{x_{img}h}{tan(arctan(\frac{y_{img}}{f}) + \varphi) \cdot f})^2 = {z_\theta}^2$$

Or, solving for z_θ:

$$z_\theta = \sqrt{(\frac{h}{tan(arctan(\frac{y_{img}}{f}) + \varphi)})^2 + (\frac{x_{img}h}{tan(arctan(\frac{y_{img}}{f}) + \varphi) \cdot f})^2}$$